Representation in Collective Policymaking*

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Abstract

We study the impact and appeal of individual representatives in collective policymaking. A policy-motivated principal chooses a representative to participate in legislative bargaining over one-dimensional policy. The principal may prefer a biased representative who favorably constrains extremist proposers. This can entail (i) shifting a (de facto) veto player away from unfavorable extremists, or (ii) merely biasing towards veto players, which improves their policymaking expectations, narrows what would pass, and further constrains extremists. The principal may want to bias inward, but never outward. For a wide interval of principals, optimal representatives are unique, strictly increasing in the principal's ideal point, and biased inward toward a unique central location that varies with extremist proposal rights. Only at that location does the principal strictly prefer an unbiased representative. Additionally, a set of intermediate representatives are not optimal for any principal. In extensions, we study mass representation, the value of representation, and competitive representation.

Representation in collective policymaking is common but complex. Many policies are made in collective bodies by representatives, such as legislators representing their constituents, committee members representing their party leaders, and so on. Since those policies can depend on the overall composition of representatives, as well as their institutional rights and roles (Romer and Rosenthal 1978; Baron and Ferejohn 1989; Krehbiel 1998), the impact of individual representatives can be subtle (Miller and Stokes 1963; Eulau and Karps 1977).[1]

We study the impact and appeal of individual representatives in collective bodies. How do individual representatives impact collective policymaking? Which kinds of representatives are optimal? And, furthermore, how do they depend on the characteristics of other politicians or their institutional rights?

We aim to sharpen theoretical understanding by accounting for two key complications of collective policymaking. First, it is interdependent, as individual representatives may impact each other in a variety of ways (Harstad 2010; Gailmard and Hammond 2011). Second, it is uncertain, as forecasts about things like the duration, proposals, or outcomes are typically noisy (Fowler 2006). Moreover, not only can these two complications have common sources—e.g., voting rules, procedural powers, agenda congestion, as well as ideological heterogeneity or polarization—but they can also impact each other. Although these features and their connection have been incorporated into models of collective policymaking (e.g., Baron and Ferejohn 1987, 1989; Baron 1996; Banks and Duggan 2000, 2006), their consequences for representation are undeveloped.

We analyze a policy-motivated principal choosing the ideal point of their representative, who will bargain with other politicians over one-dimensional policy. We highlight how different representatives not only behave differently but also induce some of the other politicians to behave differently. Essentially, the representative's expected behavior impacts which policies

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[1] Miller and Stokes (1963) highlights that "[t]he legislator acts in a complex institutional setting in which he is subject to a wide variety of influences" (pg. 51) and Eulau and Karps (1977) echoes that "[. . .] representatives are influenced in their conduct by many forces or pressures or linkages [. . .]" (pg. 235). More broadly, Pitkin (1967) states that "representation is not any single action by any one participant, but the overall structure and functioning of the system, the patterns emerging from the multiple activities of many people" (pg. 221).

can pass and, in turn, affects proposals by extreme politicians. We show that a broad range of principals want to bias their representative inwards, to improve expectations of (de facto) veto players and further constrain extremists. Thus, we find a widespread aversion against more extreme representatives. Moreover, we show that under standard conditions all principals bias inward toward a unique central location, which is characterized by the balance of extremist proposal rights. Additionally, there is always a set of intermediate representatives who are not optimal for any principal.

Specifically, our collective policymaking setting consists of sequential bargaining over an infinite horizon à la Banks and Duggan (2000). In each period until agreement, a politician is recognized to propose and then a majoritarian vote determines whether their proposal passes or bargaining continues. The key heterogeneity between politicians is in their ideal points, but we also allow them to have different proposal rights (i.e., recognition probabilities). We fix those proposal rights, however, so that the principal can only choose the representative's ideal point. Accordingly, we isolate the impacts of individual ideological differences between representatives.

Once a representative is in place, equilibrium policymaking induces a unique lottery over policy (Cho and Duggan 2003; Cardona and Ponsati 2011). Whoever proposes first will pass their favorite policy in the set that a majority would pass (Banks and Duggan 2000). Furthermore, that set always coincides with the median politician's *acceptance set* (Duggan 2014), which is an interval of policies around her ideal point. Crucially, it is determined by her expectations about further policymaking—since policymaking can continue after rejected proposals—and thus depends on the profile of politician ideal points and their proposal rights.

The representative's ideology can indirectly affect what some of the other politicians would propose in equilibrium. It does so by changing the acceptance set, through shifting either the median's (i) location *or* (ii) policy expectations. The location channel is familiar (Gailmard and Hammond 2011; Klumpp 2010), but the other channel is less understood. In our setting, it is especially pervasive. Regardless of whether the representative would be

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the median, his mere presence affects the median's willingness to reject proposals due to the prospect that he might subsequently propose. Essentially, the representative's ideology can have *anticipation effects* (Friedrich 1937; Simon 1953). We parse these two channels and characterize how the acceptance set varies with the representative's ideal point. Over representatives who would be the median, it shifts monotonically with the representative's ideal point but its radius can change in different ways, depending on the distribution of proposal rights. And for the rest, it shrinks on both sides as the representative approaches the median since his continuation value from rejecting would improve.

Due to the representative's indirect impact on other politicians, the principal faces a classic tradeoff: a biased representative may induce extremists to propose policies more favorable to the principal (Schelling 1956). We highlight how this general tradeoff may be more widespread than previously appreciated, as it can arise from pervasive anticipation effects. Furthermore, we show that this distinct mechanism generates new implications for preferences over representatives. Additionally, we study how those preferences depend on the interaction between anticipation effects and the effects of shifting (de facto) veto players. Finally, throughout our analysis, we highlight how the balance of this tradeoff—and the resulting preferences over representatives—are shaped by the distribution of proposal rights.

We characterize general properties of optimal representatives and show that the principal is always more inclined towards moderation than extremism. Broadly, the principal never strictly prefers someone more extreme and instead always wants someone who is the median or biased in that direction. More precisely, if the principal is in a centrally located interval, then their optimal representative(s) will be the median but can be biased in either direction. Next, if the principal is in either of two intermediate intervals flanking that centrist interval, then their optimal representative(s) are biased strictly inwards, potentially enough to be the median. For any such principal, the downside from biasing their representative's proposal is outweighed by the upside from inducing extremists to further moderate their proposals. Finally, for the remaining (sufficiently extreme) principals, optimal representative(s) may be

more centrist but are never the median.

We fully characterize preferences over representatives if players have quadratic policy utility and proposal rights are (mostly) polarized. Three main insights are that (i) principals like moderation unless they are very extreme, (ii) polarization strengthens the desire for moderation, especially on the weaker side, and (iii) some intermediate representatives are not optimal for any principal. Unless the principal is very extreme, she has a unique optimal representative who is biased inwards toward a unique centrist location. That location, the *locus of attraction*, characterizes the unique principal who strictly prefers to have an unbiased representative.[2] It varies with the balance of extremist proposal rights, shifting away from the stronger side, but is always a median. Additionally, optimal representatives are ordered. Thus, the principals who want a median representative are an interval containing the medians along with some non-medians on at least one side. And, surrounding that interval are two intervals of principals who each want a representative who is more centrist but not a median. Furthermore, these preferences vary with the balance of extremist proposal rights: principals bias further away from the gaining side in order to further constrain those extremists. Finally, we show there is always a *dead zone* of median and non-median representatives who are not optimal for any principal.

To enrich the application and interpretation of our results, we explore three extensions. First, we study collective choice over the representative's ideal point — i.e., *mass representation.* We show that preferences over representatives satisfy a single-crossing condition as long as extremist proposal rights are not too high. Second, we study the principal's welfare gain from optimal representation — i.e., the *value of representation.* We show that if extremist proposal rights are balanced, then this value is greatest for principals around each moderate-extremist boundary. Third, we study two opposing principals each choosing a representative in a setting with two open positions — i.e., *competitive representation.* In equilibrium, both principals moderate their representative towards the median but, depending

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[2]An unbiased representative is weakly optimal for very extreme principals.

on the balance of extremist proposal rights, they may moderate more or less than in the baseline setting.

Contributions to the Literature

Our results provide insight into representation across various collective policymaking contexts. Our model of collective policymaking is a *minimal legislative process* (Baron 1994) with several interpretations.[3] For instance, it provides a lens for studying representation in separation-of-powers systems[4] (Epstein and O'Halloran 2001; Volden 2002) or, more narrowly, congressional committees.[5] Broadly, we emphasize the role of *ideological* factors for representation, complementing related work emphasizing *distributive* or *informational* factors.[6]

We shed new light on how biased representatives can provide a useful form of commitment (Schelling 1956; Sobel 1981) to improve *other* politicians' behavior enough to outweigh their *own* less-favorable behavior (e.g., Harstad 2010; Christiansen 2013; Loeper 2017).[7] One prominent mechanism is that a status-quo-biased representative with veto power will further constrain extreme proposals (Gailmard and Hammond 2011; Klumpp 2010). Our setting includes that mechanism but also features a different mechanism — the representative's effect on expectations about policymaking — that is lurking in well-known models of collective policymaking (e.g., Banks and Duggan 2000). Since both mechanisms may be present in

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[3]For discussion of interpretations and applications of our bargaining environment, see, e.g., Baron and Ferejohn (1989); Baron (1991); McCarty (2000); Kalandrakis (2006); Eraslan and Evdokimov (2019).

[4]In this vein, we add to Gailmard and Hammond (2011) and Epstein and O'Halloran (2001), who suggest "that theories of legislative organization should be brought out of the legislature and seen as part of our larger constitutional system of policy making" (pg. 391).

[5]For an overview of scholarship in committee composition, see Evans (2011). Theoretical work on committees has studied, e.g., their *representativeness* (Krehbiel 1990; Hall and Grofman 1990; Cox and McCubbins 2007), who serves on them (Rohde and Shepsle 1973), and the role of intercameral considerations (Diermeier and Myerson 1999; Gailmard and Hammond 2011).

[6]Echoing Fenno (1974), Epstein and O'Halloran (2001) claim that "each of the distributive, informational, and partisan theories predicts outcomes accurately in its own relevant domain; [...] so alternative explanations should be seen as complements rather than substitutes" (pg. 391).

[7]Also see, e.g., Persson and Tabellini (1992); Besley and Coate (2003). For a general overview of strategic pre-commitment in bargaining, see Miettinen (2022). For related strategic delegation incentives outside the political context, see Dixit (1980); Vickers (1985); Bulow et al. (1985) and Fershtman et al. (1991).

various settings (e.g., Banks and Duggan 2006), our results complement earlier work by showing how this strategic tension does not require the representative to be a veto player nor the status quo to be strategically relevant.

We highlight a new logic for how moderate representatives can be appealing by reducing extremism. This appeal can arise in in various aspects of collective policymaking. First, when allocating proposal rights, risk-averse politicians share an aversion to egalitarianism and would rather shift proposal rights towards moderate members — to make extreme proposals *less likely* (Diermeier et al. 2020).[8] In contrast, we fix (possibly unequal) proposal rights and show a widespread preference for relatively centrist representatives — to make extreme proposals *less extreme.*[9] Second, during bargaining that can continue with accepted policy as the new status quo, proposers may opt for a relatively centrist policy that directly increases the median's reservation value in *future* periods and thus constrains their opposition *in the future* (Baron 1996; Buisseret and Bernhardt 2017; Zápal 2020). In contrast, in our setting a more centrist representative increases the median's reservation value *today*, thereby constraining what extremists can pass *today.* Furthermore, in our analysis, moderate principals want to constrain extremists on both sides, not just their opponents. Third, interest groups seeking access may prefer to target more extreme representatives in order to increase the chances of moderating their proposals, thereby also improving centrist expectations and constraining what extremists can pass (Judd 2023). Our results highlight that beforehand, when the representatives are chosen, similar incentives encourage those groups to support the selection of more moderate candidates.

The moderation incentives we uncover also complement extremism incentives driven by collective policymaking in other settings. In a supermajoritarian take-it-or-leave-it setting, voters never want someone more moderate but may prefer a strictly more extreme represen-

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[8]For other related work on *endogenous procedures*, see Diermeier and Vlaicu (2011); Diermeier et al. (2015, 2016). In a dynamic setting with endogenous status quo, Duggan and Kalandrakis (2012) endogenize proposal rights but only study equilibrium existence. In a setting with distributive policy, Eguia and Shepsle (2015) endogenize the set of politicians and their proposal rights.

[9]Two other differences, motivated by our representative/delegate application, are that in our analysis (i) the location of the median policymaker can shift and (ii) we vary the principal's ideal point.

tative who would be a veto pivot (Kang 2017). In other contexts where policy is a weighted average of politician ideal points, extreme representatives can counterbalance extreme opponents (Alesina and Rosenthal 1996; Kedar 2005, 2009). Additionally, if principals care about how their representative will vote on an *exogenous* legislative agenda, then preferences can be asymmetric and favor extremism (Patty and Penn 2019). Understanding these different directions can help inform empirical interpretation and anticipation of future choices.[10]

Model

Players. There is a principal, P; a continuum of potential representatives; and a set of four auxiliary politicians, $K = \{L, \ell, r, R\}$.

Timing. The model has two stages. First, in the *appointment* stage, P selects a representative, denoted d, to bargain on her behalf.[11] Second, in the *bargaining* stage, the representative d interacts with the other politicians in K to collectively set a one-dimensional policy. Each bargaining period $t \in \{1, 2, ...\}$, a politician $i \in N = K \cup \{d\}$ is drawn from the recognition distribution ρ, where $\rho_i \in (0, 1)$ for all i and $\sum_{i \in N} \rho_i = 1$, and then i proposes a policy $x^t \in X = [0, 1]$. Next, all politicians vote to accept or reject x^t. The proposal is approved if and only if a simple majority of individuals approve. If x^t is approved by a simple majority of politicians, then it is implemented and the game ends. Otherwise, the proposal is rejected and the game moves to $t + 1$. Bargaining continues indefinitely until a proposal is accepted.

Preferences. All players are purely policy-motivated and each player has a unique ideal point $y_i \in X$. We denote the principal's ideal point as y_p and the ideal point of her chosen representative as y_d. Additionally, to streamline the main analysis, we assume the other politicians satisfy $y_L = 0 < y_\ell < y_r < 1 = y_R$ and $y_r - y_\ell < \min\{y_\ell, 1 - y_r\}$.

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[10]Extremism can also emerge if the principal does not know their appointee's ideology but does know they will serve on a collective body that sets policy at the median ideal point (Bailey and Spitzer 2018).

[11]We follow convention in the principal-agent literature by referring to a legislator (agent) as *he* and the principal as *her*.

Once a policy x is enacted, player i will receive policy utility $u(x, y_i) = 1 - (x - y_i)^2 \geq 0$ each period thereafter. Before then, every player receives zero utility in each period until agreement. Thus, our main analysis focuses on a *bad status quo* setting—i.e., all players receive higher utility from any agreement than they do from no agreement. This setting highlights the key forces, emphasizes the role of policymaking expectations, and sharpens our results. Regardless, however, our main insights are robust to including a status quo policy.

Cumulative payoffs are sums of per-period utilities, discounted by the common factor $\delta \in (0, 1)$. For convenience, we normalize per-period utility by the factor $1 - \delta$. Thus, if x is accepted in period t, then legislator i's payoff is $\delta^{t-1} u(x, y_i)$.

Information. All features of the game are common knowledge.

Strategies & Equilibrium concept. In the appointment stage, a pure strategy for the principal prescribes a choice of d's ideal point, y_d. In the bargaining stage, a pure *stationary strategy* for each individual $i \in N$ prescribes (i) a proposal, x_i, that he makes at any t he is selected to propose; and (ii) an acceptance set, A_i, that specifies a time-independent set of proposals that he accepts or rejects.[12] A *stationary subgame perfect equilibrium* in the bargaining subgame is a profile of stationary strategies that are mutual best responses in each subgame of the bargaining subgame. An *equilibrium* is a strategy profile in which (i) players in the bargaining subgame play a stationary subgame perfect equilibrium and (ii) P chooses y_d to maximize her expected payoff anticipating the distribution of policy outcomes that y_d will induce.

Analysis

To begin, we characterize equilibrium behavior during the bargaining stage, after y_d is chosen. Then, we trace how y_d affects d's behavior, as well as other politicians. Next, we study the

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[12]We focus on a standard class of bargaining strategies (Banks and Duggan 2000; Cardona and Ponsati 2011) that are relatively simple and focal (Baron and Kalai 1993; Baron 1994), with politicians always voting as if pivotal (Duggan and Fey 2006).

principal's preference over y_d and how her set of optimal representatives varies with her ideology. Finally, we study several extensions.

Equilibrium policymaking

Equilibrium policymaking is immediate from Banks and Duggan (2000) and Cardona and Ponsati (2011): bargaining ends immediately, with the first proposer proposing their favorite policy among those that will pass. We provide key details to explicitly highlight the role of the representative through y_d. By doing so, we pave the way for characterizing how y_d affects policymaking and, in turn, our main analysis: studying P's preferences over y_d.

In equilibrium, each politician always prefers to propose the closest majority-approved policy rather than delay. Thus, equilibrium policymaking is characterized by the *acceptance set* of policies that would pass if proposed.[13] That set is uniquely determined by the voting calculus of the politician with the median ideal point, denoted y_m. That politician will pass any proposal that she prefers over her continuation value from rejecting to continue bargaining, denoted V_m. Consequently, the acceptance set is $A(y_d) = [\underline{x}_m(y_d), \overline{x}_m(y_d)] \subset X$, where the lower bound is

$$\underline{x}_m(y_d) = \min \left\{ x \in X \mid x < y_m \text{ and } u(x, y_m) \geq \delta V_m(\underline{x}_m(y_d), \overline{x}_m(y_d)) \right\},$$

and the upper bound $\overline{x}_m(y_d)$ is analogous for $x > y_m$. In turn, $A(y_d)$ must be a compact interval that is centered at y_m and has a radius that depends on V_m. Furthermore, in equilibrium, $A(y_d)$ is consistent with each politician i proposing the unique acceptable policy closest to y_i. Specifically, given an arbitrary acceptance interval $[\underline{x}, \overline{x}]$,

$$V_m(\underline{x}, \overline{x}) \equiv P(\underline{x})u(\underline{x}, y_m) + (1 - P(\overline{x}))u(\overline{x}, y_m) + \sum_{i \in N: y_i \in (\underline{x}, \overline{x}]} \rho_i u(y_i, y_m), \tag{1}$$

where $P(x) \equiv \sum_{i \in N: y_i \leq x} \rho_i$ denotes the cumulative proposal rights of politicians left of x.[14]

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[13]These properties follow immediately from Cardona and Ponsati (2011).

[14]The first term in (1) is m's utility from the lower bound $\underline{x}$ weighted by the proposal rights of politicians with

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Lemma 1 summarizes properties of equilibrium policymaking given y_d. Figure 1 illustrates.

Lemma 1 (Cardona and Ponsati (2011)). *For each $y_d \in X$, equilibrium bargaining is characterized by a unique acceptance set $A(y_d) = [\underline{x}_m(y_d), \overline{x}_m(y_d)]$ and each politician i proposes the policy $x \in A(y_d)$ that minimizes $|x - y_i|$.*

Figure 1: Illustration of equilibrium policymaking (given y_d)

equilibrium acceptance set, $A^*(y_d)$
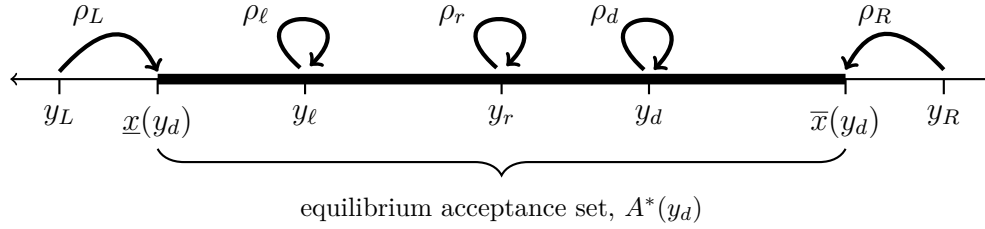
Note: Figure 1 illustrates Lemma 1 for a hypothetical legislature with $y_d > y_r$. The acceptance set is the bold interval, which is centered around $y_m = y_r$. Arrows point from legislators to their proposals (if recognized). Each legislator proposes the closest acceptable policy.

By Lemma 1, every y_d induces a unique equilibrium policy lottery. Essentially, the boundaries of $A(y_d)$ and the ideal points in its interior are each weighted by the recognition probability of the politicians who propose them.

Remark 1. *Given y_d, the unique equilibrium policy lottery puts probability $P(\underline{x}_m(y_d))$ on $\underline{x}_m(y_d)$; $1 - P(\overline{x}_m(y_d))$ on $\overline{x}_m(y_d)$; ρ_i on each y_i in $(\underline{x}_m(y_d), \overline{x}_m(y_d)]$; and zero otherwise.*

Remark 1 implies that each potential representative has a unique equilibrium value. Furthermore, it highlights that those values can depend on the representative through y_d and $A(y_d)$.

The representative's effects on policymaking

We now study how the representative's ideal point, y_d, affects policymaking. First, we characterize when it affects the representative's proposals. Then, we show how it affects the

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$y_i \leq \underline{x}$, the second term is analogous for the upper bound $\overline{x}$, and the last term is the proposal-rights-weighted sum of m's utility from $y_i \in (\underline{x}, \overline{x}]$.

acceptance set and, in turn, proposals by other politicians.

The acceptance set can vary with the representative's ideal point, y_d, through two channels. First, through m's ideal point, y_m, since:

$$y_m = \begin{cases} y_\ell & \text{if} \quad y_d < y_\ell \\ y_d & \text{if} \quad y_d \in [y_\ell, y_r] \\ y_r & \text{if} \quad y_d > y_r. \end{cases} \tag{2}$$

Second, through m's continuation value, V_m, by shifting y_m *or* d's proposal.

To streamline presentation, we focus on an intermediate range of δ for which ℓ and r are always able to pass their ideal point, but L and R are never able to pass their ideal point.[15]

Remark 2. *There are cutpoints satisfying $0 < \underline{\delta} < \overline{\delta} < 1$ such that $\delta \in (\underline{\delta}, \overline{\delta})$ implies $[y_\ell, y_r] \subset intA(y_d) \subset (0,1)$ for all $y_d \in X$.*

Assumption 1. *Suppose $\delta \in (\underline{\delta}, \overline{\delta})$.*

Intuitively, Assumption 1 reflects a situation in which every potential median would not settle for accepting the most extreme policies, but would pass both y_ℓ and y_r.

The representative's proposal varies with y_d if and only if he will not be constrained by the acceptance set — i.e., $y_d \in$ int $A(y_d)$. Lemma 2 shows that this property is fully characterized by an open interval containing y_ℓ and y_r.

Lemma 2. *There are unique $\underline{x}_\ell \in (0, y_\ell)$ and $\overline{x}_r \in (y_r, 1)$ such that $y_d \in intA^*(y_d)$ if and only if $y_d \in (\underline{x}_\ell, \overline{x}_r)$.*

Since $[y_\ell, y_r] \subset (\underline{x}_\ell, \overline{x}_r)$, we can partition representatives based on whether they would be the median politician or propose a boundary of the acceptance set. First, $y_d \in [y_\ell, y_r]$ would be the median and propose their (interior) ideal point. Second, $y_d \notin (\underline{x}_\ell, \overline{x}_r)$ would not be the median and would propose the nearest boundary of the acceptance set. Finally,

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[15]These observations follow from Banks and Duggan (2000) and Lemma 5 of Cardona and Ponsati (2011).

$y_d \in (\underline{x}_\ell, y_\ell) \cup (y_r, \overline{x}_r)$ would not be the median but would propose their (interior) ideal point. We label these three cases in Definition 1.

Definition 1. *A player i is **centrist** if $y_i \in [y_\ell, y_r]$; **extremist** if $y_i \notin (\underline{x}_\ell, \overline{x}_r)$; and **moderate** otherwise.*

In equilibrium, the acceptance set varies with y_d in several distinct ways, depending on whether the representative is centrist, moderate, or extremist. Broadly, $A(y_d)$ is (i) constant over aligned extremists, (ii) shrinks inward over aligned moderates, and (iii) shifts in the same direction as y_d over centrists. Moreover, the distance between y_d and y_ℓ or y_r will affect the rate of these changes.

These effects depend on how y_d impacts the center of the acceptance set, y_m, or radius, via V_m. First, centrist $y_d \in [y_\ell, y_r]$ impact both, as $y_m = y_d$ and V_m can also vary, since y_m shifts relative to the other potential proposers. These two effects can oppose each other but the first always dominates, so the acceptance set strictly increases over centrists. Second, moderate y_d on each side only impact V_m, as y_m is constant but changes in d's proposal will change V_m. Furthermore, V_m increases as y_d approaches y_m, so the acceptance set shrinks as y_d shifts inward over each interval $(\underline{x}_\ell, y_\ell)$ and $(y_r, \overline{x}_r)$. Yet, those changes vanish as y_d approaches the centrists—since u is strictly concave and differentiable, the effect through V_m goes to zero. More broadly, quadratic policy utility yields that the upper bound is piecewise convex on each interval $(\underline{x}_\ell, y_\ell)$, $(y_r, \overline{x}_r)$, and (y_ℓ, y_r); the lower bound is piecewise concave; and both are piecewise continuously differentiable.[16] Finally, extremist y_d on each side do not have any marginal impact, as y_m is constant and V_m is too since all of these representatives will propose the same boundary. Thus, (i) $y_d \leq \underline{x}_\ell$ implies $A(y_d) = [\underline{x}_\ell, \overline{x}_\ell]$ and (ii) $y_d \geq \overline{x}_r$ implies $A(y_d) = [\underline{x}_r, \overline{x}_r]$.

Lemma 3 summarizes these observations.[17] Figure 2 illustrates.

Lemma 3. *The correspondence A^* is continuous and $A^*(y_d) \subset [\underline{x}_\ell, \overline{x}_r]$ for all y_d, where:*

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[16]At y_ℓ and y_r there are kinks in both bounds of the acceptance set because it shifts more sharply with y_d over centrists.

[17]See Lemma A3.

1. $A^*(y_d) = [\underline{x}_\ell, \overline{x}_\ell]$ *for all* $y_d \le \underline{x}_\ell$;

2. $A^*(y_d) = [\underline{x}_\ell(y_d), \overline{x}_\ell(y_d)] \subset [\underline{x}_\ell, \overline{x}_\ell]$ *for each* $y_d \in (\underline{x}_\ell, y_\ell)$, *with* $\underline{x}_\ell(y_d)$ *strictly concave and increasing at a rate converging to zero as* $y_d \to y_\ell$, *and* $\overline{x}_\ell(y_d)$ *symmetric about* y_ℓ;

3. $A^*(y_d) = [\underline{x}_d(y_d), \overline{x}_d(y_d)] \subset [\underline{x}_\ell(y_\ell), \overline{x}_r(y_r)]$ *for each* $y_d \in [y_\ell, y_r]$, *with* $\underline{x}_d(y_d)$ *strictly concave and increasing, and* $\overline{x}_d(y_d)$ *strictly convex and increasing;*

4. $A^*(y_d) = [\underline{x}_r(y_d), \overline{x}_r(y_d)] \subset [\underline{x}_r, \overline{x}_r]$ *for each* $y_d \in (y_r, \overline{x}_r)$, *with* $\underline{x}_r(y_d)$ *strictly concave and decreasing at a rate converging to zero as* $y_d \to y_r$, *and* $\overline{x}_r(y_d)$ *symmetric about* y_r;

5. *and* $A^*(y_d) = [\underline{x}_r, \overline{x}_r]$ *for all* $y_d \ge \overline{x}_r$.

Lemma 3 has several additional implications for how the acceptance set varies with y_d. First, the representative can affect the acceptance set even without shifting the median. Second, extremist representatives induce a larger acceptance set than their aligned moderates and, moreover, it is more extreme on both ends — e.g., $y_d \in (\underline{x}_\ell, y_\ell]$ implies $[\underline{x}(y_d), \overline{x}(y_d)] \subset [\underline{x}_\ell, \overline{x}_\ell]$. Finally, we can order the boundaries of the two extremist acceptance sets: $\underline{x}_\ell < \underline{x}_r < \overline{x}_\ell < \overline{x}_r$.[18] Thus, extremist principals are always outside the acceptance set but are closest if d is an aligned extremist.

Figure 2: How the acceptance set varies with y_d
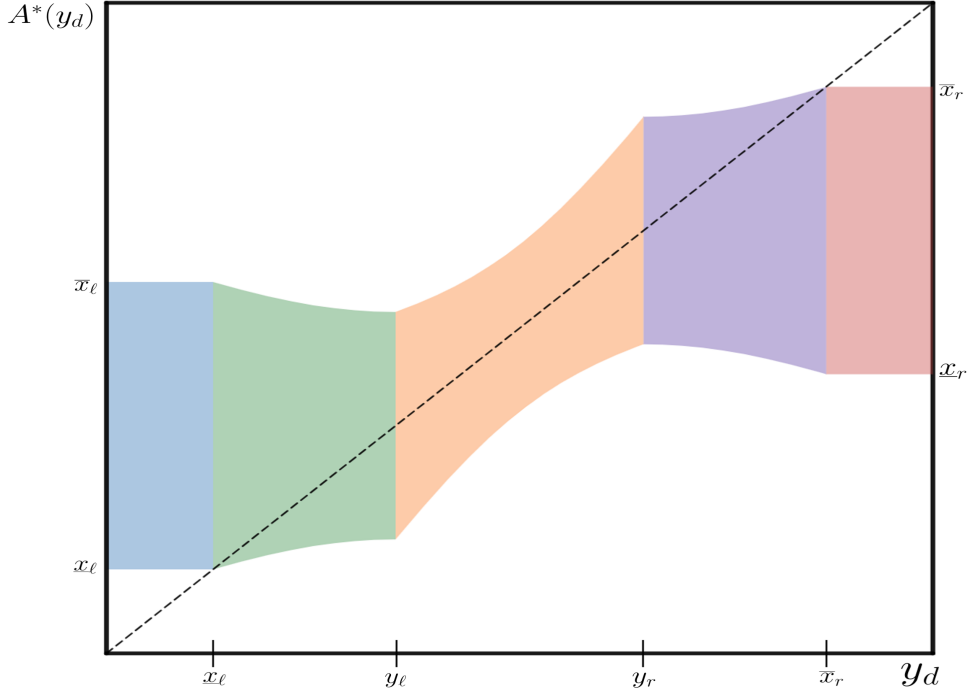
Note: Figure 2 illustrates how $A^*(y_d)$ varies. It is continuous and (i) constant for $y_d \leq \underline{x}_\ell$, (ii) contracting inward over $y_d \in (\underline{x}_\ell, y_\ell)$ at a decreasing rate that goes to zero as y_d approaches y_ℓ, (iii) increasing over (y_ℓ, y_r), (iv) expanding outward over $(y_r, \overline{x}_r)$ at an increasing rate that starts from zero near y_r, and (v) constant over $\overline{x}_r$. Additionally, $\underline{x}_\ell < \underline{x}_r < \overline{x}_\ell < \overline{x}_r$.

Figure 2 also illustrates the policymaking effects of shifting y_d. On each side, all extremists are equivalent since they induce the same acceptance set and propose the same boundary. In contrast, extremists on opposite sides propose different policy *and* generally induce different acceptance sets. Among moderates, those closer to the center shift all proposals weakly inwards by (i) proposing a more centrist policy and (ii) inducing smaller acceptance sets. And among centrists, those who lean farther right shift every proposal weakly rightward by (i) proposing more right-leaning policy and (ii) shifting the acceptance set to the right.

Optimal Representation

We apply our understanding of the representative's various policymaking effects in order to fully characterize optimal representatives. To do so, we study how y_d affects P's equilibrium

value from bargaining. That value is always uniquely defined since the equilibrium policy lottery is unique, so for each pair of y_d and y_p we denote it $U(y_d, y_p)$. Furthermore, the equilibrium characterization implies

$$U(y_d, y_p) = V_p(\underline{x}(y_d), \overline{x}(y_d)). \tag{3}$$

We analyze the set of y_d that maximize P's equilibrium value and how it varies with y_p. Formally, we characterize the optimal representative correspondence $y_d^* : X \rightrightarrows X$, where:

$$y_d^*(y_p) \equiv \operatorname*{argmax}_{y_d \in X} U(y_d, y_p) \tag{4}$$

is non-empty, upper hemicontinuous, and compact valued since $U(y_d, y_p)$ is continuous in y_d.

In general, although equilibrium strategies are unique during bargaining, P could have multiple optimal representatives. That is, y_d^* is not necessarily single-valued, since different representatives may shift and shape A^* in different ways. For instance, two different representatives who induce different policymaking may be equivalent for P by—from P's perspective—trading off worse representative proposals against better extremist proposals, or vice versa. The two key considerations are (i) whether P wants to constrain both extremists, and (ii) if not, which extremist they want to constrain.

With quadratic policy utility, however, convexity properties of the acceptance correspondence ensure that P's objective function is strictly quasi-concave on each interval of moderates and centrists. Thus, in each of those intervals, P has a unique locally optimal representative.

Proposition 1 fully characterizes optimal representatives. There are four primary takeaways. First, a wide interval of principals each have a unique optimal representative. Second, a centrally-located interval wants centrists, flanked by two intervals who want moderates, and the rest want extremists. Third, there is a *dead zone* of representatives around y_ℓ or y_r who are not optimal for anyone. Finally, optimal representatives are ordered: a principal who leans farther leftward wants a more left-leaning representative and vice versa.[19] Figure 3

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[19]We further analyze ordering in the *competitive delegation* extension and show in Proposition 1 that y_d^* is increasing on X under standard assumptions.

illustrates these properties.

Proposition 1. *The optimal representative correspondence y_d^* is increasing. Moreover, there are intervals (E_L, E_R) and $(\underline{y}_p, \overline{y}_p)$ satisfying $(E_L, E_R) \supset (\underline{x}_\ell, \overline{x}_r) \supset [\underline{y}_p, \overline{y}_p] \supset [y_\ell, y_r]$ such that y_d^* is single-valued and continuous on $(E_L, E_R) \backslash \{\underline{y}_p, \overline{y}_p\}$. Additionally,*

1. *$y_p \leq E_L$ implies $y_d^*(y_p) = [0, \underline{x}_\ell]$;*

2. *$y_d^*|_{(E_L, \underline{y}_p)}$ is strictly increasing, with $y_d^*(y_p) \in (y_p, y_\ell)$;*

3. *$y_p^*|_{(\underline{y}_p, \overline{y}_p)}$ is weakly increasing, with $y_d^*(y_p) \in [y_\ell, y_r]$;*

4. *$y_d^*|_{(\overline{y}_p, E_R)}$ is strictly increasing, with $y_d^*(y_p) \in (y_r, y_p)$; and*

5. *$y_p \geq E_R$ implies $y_d^*(y_p) = [\overline{x}_r, 1]$.*

Furthermore, $y_p = \underline{y}_p < y_\ell$ implies $y_d^*(y_p)$ is a doubleton with $\min y_d^*(y_p) < y_\ell < \max y_d^*(y_p)$, and $y_p = \underline{y}_p = y_\ell$ implies $y_d^*(y_p) = y_\ell$; and symmetrically for $y_p = \overline{y}_p$.

Proposition 1 highlights how the principal's taste for centrism and moderation depends on where they are located. First, each centrist has a unique optimal representative, who is also a centrist. Next, each moderate wants either a unique moderate — who is biased inwards — or a unique centrist in (y_ℓ, y_r). Their preference between these options is characterized by two cutpoints, one on each side of the spectrum: those outside want their optimal moderate and those inside want their optimal centrist, with indifference at the cutpoint. Finally, each extremist wants either a unique moderate or any aligned extremist, with two cutpoints similarly partitioning their preference between these two options. Those outside prefer any aligned extremist but those inside prefer their optimal moderate.

Viewed from another angle, Proposition 1 also characterizes *which* types of principal would want a centrist, moderate, or extremist representative. Those preferring a centrist are in a centrally-located interval, $(\underline{y}_p, \overline{y}_p)$, including some moderates, potentially on both sides. Those who prefer a moderate are on either side of that interval, in two intermediate intervals,

Figure 3: Optimal representatives
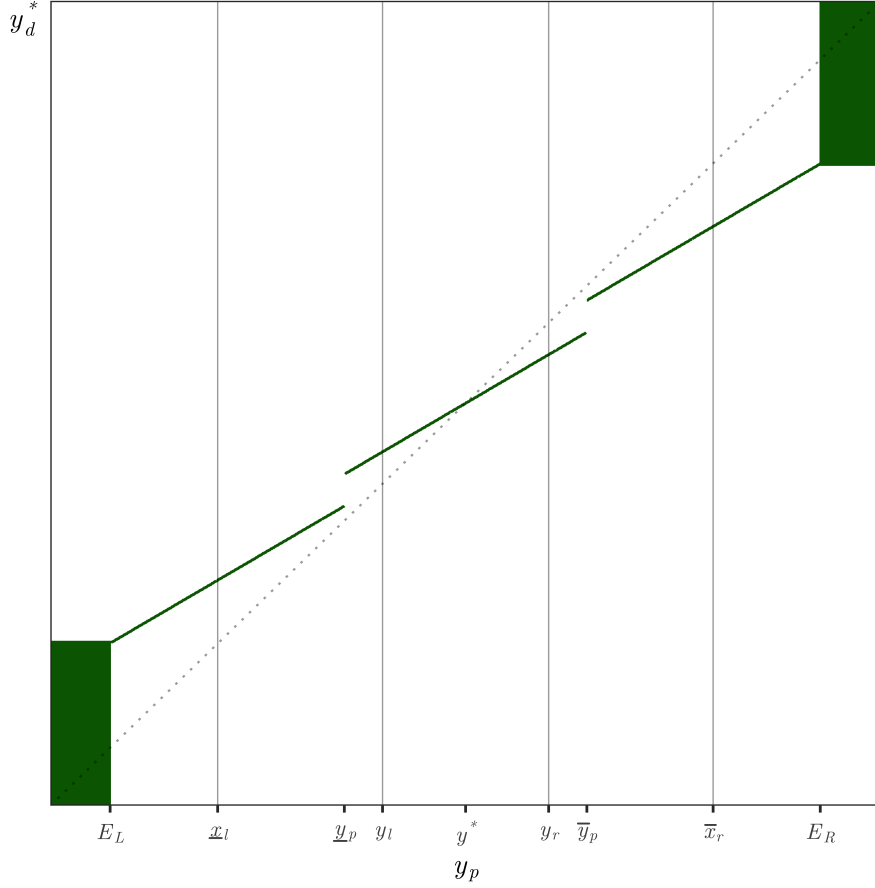
Note: Figure 3 illustrates key properties of the optimal representative characterization in Proposition 1: (i) very extreme principals, $y_p \notin (E_L, E_R)$, prefer any aligned extremist; (ii) intermediate principals, $y_p \in (E_L, \underline{y}_p) \cup (\overline{y}_p, E_R)$ prefer a unique moderate who is strictly more centrist; and (iii) centrist principals, $y_p \in (\underline{y}_p, \overline{y}_p)$, prefer a unique centrist.

$(E_L, \underline{y}_p)$ and $(\overline{y}_p, E_R)$, each including some extremists. Those who prefer an extremist are even further out, in two intervals, $[0, E_L)$ and $(E_R, 1]$, containing only extremists.[20] Finally, P's optimal representative is unique almost everywhere in (E_L, E_R), the potential exceptions being $\underline{y}_p$ or $\overline{y}_p$, where P is indifferent between a centrist and a moderate.[21]

Proposition 1 also reveals that some representatives are *not* optimal for *any* principal.

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[20]Either interval may be empty. For example, all left extremists may strictly prefer a moderate if ρ_R is sufficiently high (see Lemma A7 in the appendix).

[21]Notice also that y_d^* is also continuous a.e. on X. Lower hemi-continuity fails only if $y_p \in \{E_L, \underline{y}_p, \overline{y}_p, E_R\}$, where the optimal representative switches from a moderate to an extremist or centrist.

This *dead zone* consists of open interval(s) around at least one of y_ℓ or y_r, possibly both. Essentially, these representatives are not optimal for any principal because they do not constrain powerful extremists enough relative to other representatives near them. On at least one side, principals near the moderate-centrist boundary want to bias inward in order to constrain their aligned extremist proposers. Then, the indifferent moderate principal wants either a strict centrist or strict moderate because representatives around that boundary do not constrain P's aligned extremists enough to justify their worse proposals.

Furthermore, Proposition 1 also implies that a centrist principal may strictly prefer an ally representative, unlike any moderate or extremist principal. Intuitively, since an ally representative is always best if they propose, a non-ally is optimal only if they induce sufficiently more favorable extremist proposals for P. Thus, moderate principals never want an ally representative because they benefit from moderating slightly in order to constrain extremists on both sides, as discussed above. In contrast, some extremist and centrist principals may want an ally. For extremists, their ally can be optimal (along with all other aligned extremists) because (i) they face a non-negligible proposal cost from moderating enough to affect the acceptance set and (ii) their return from constraining extremists always depends on relative extremist proposal power. For centrists, their ally can be strictly optimal because their negligible proposal cost of biasing slightly in either direction can coincide with a negligible effect on extremist proposals—since one shifts closer and the other shifts away.

Next, we show that only one principal—a unique centrist—strictly prefers an ally representative. Moreover, denoting this principal by y^*, we show that all principals in (E_L, E_R) want to bias their representative towards y^*. Thus, we refer to y^* as the *locus of attraction*. If $y^* \in (y_\ell, y_r)$, then shifting y_d away from $y_p = y^*$ must not change P's expected payoff. In this case, y^* is the location where shifting y_d does not change the expected distance from y^* to the boundaries. Specifically, the marginal effect of y_d on P's value combines the direct effect through d's proposal with indirect effects through the boundary proposals. The direct effect is zero for all $y_p \in [y_\ell, y_r]$. In general, the indirect effects can be positive or negative —

since both boundaries shift in the same direction, P will gain on one side but lose on the other. At $y_p = y^*$, however, these effects must also equal zero. Formally,

$$0 = \frac{\partial U(y_d, y_p)}{\partial y_d}\bigg|_{y_d = y_p} = \rho_L \frac{\partial u(\underline{x}(y_d), y_p)}{\partial y_d}\bigg|_{y_d = y_p} + \rho_R \frac{\partial u(\overline{x}(y_d), y_p)}{\partial y_d}\bigg|_{y_d = y_p}. \tag{5}$$

Essentially, P's marginal gain from shifting one boundary towards y_p must exactly offset her marginal loss from shifting the other boundary away. Furthermore, since d is the median if $y_d = y^* = y_p$, both boundaries will be equally far from y_p, so P is indifferent between them and (5) reduces to:

$$\rho_L \frac{\partial \underline{x}(y_d)}{\partial y_d}\bigg|_{y_d = y^*} - \rho_R \frac{\partial \overline{x}(y_d)}{\partial y_d}\bigg|_{y_d = y^*} = 0. \tag{6}$$

We define a function that represents the effect in (6) as a function of y_p and thus exactly coincides at $y_p = y^*$. Specifically, define $\lambda : [y_l, y_r] \to \mathbb{R}$ as

$$\lambda(y_p) \equiv \rho_L \frac{\partial \underline{x}(y_d)}{\partial y_d}\bigg|_{y_d = y_p} - \rho_R \frac{\partial \underline{x}(y_d)}{\partial y_d}\bigg|_{y_d = y_p} \tag{7}$$

for $y_p \in (y_\ell, y_r)$, then set $\lambda(y_\ell) = \lim_{y_p \to y_\ell^+} \lambda(y_p)$ and $\lambda(y_r) = \lim_{y_p \to y_r^-} \lambda(y_p)$. This function has two properties that together help us characterize y^*. First, it is strictly decreasing in y_p due to the concavity of $\underline{x}(y_d)$ and convexity of $\overline{x}(y_d)$. Second, its sign indicates which way P wants to bias y_d. For example, $\lambda(y_p) > 0$ implies rightward bias is optimal — since shifting y_d rightward from y_p will make P better off by decreasing the expected distance between y_p and boundary proposals.

Proposition 2 uses λ to show that y^* is unique and provides simple conditions to locate it.

Proposition 2. *Over $y_p \in (E_L, E_R)$, the optimal representative correspondence y_d^* has a unique fixed point, y^*. Moreover, (i) $\lambda(y_\ell) \leq 0$ implies $y^* = y_\ell$; (ii) $\lambda(y_r) \geq 0$ implies $y_r = y^*$; and (iii) otherwise, $y^* \in (y_\ell, y_r)$.*

On both sides, fringe centrists want to bias inwards — implying $y^* \in (y_\ell, y_r)$ — if the signs of $\lambda(y_\ell)$ and $\lambda(y_r)$ differ. Otherwise, all centrists want to bias in the same direction, so y^* is on the boundary that does not want to bias inward.

We know from properties of λ that all centrists bias towards y^*. Moreover, all moderates bias inwards towards y^* and extremists never strictly prefer to bias outwards. Thus, P always biases towards y^*.

Corollary 1. *If $y_p \in (E_L, E_R)$, then the principal's optimal representative is biased strictly towards y^*.*

Moreover, principals closer to y^* want a representative who is closer to y^*. This property follows from monotonicity properties of y_d^*.

Proposition 3 uses λ to characterize (i) whether any moderates on either side want a centrist representative and (ii) which side has the *dead zone*, denoted Δ.

Proposition 3. *In equilibrium,*

1. *$\lambda(y_\ell) \leq 0$ implies $\underline{y}_p = y_\ell < y_r < \overline{y}_p$, so $y_r \in \Delta$ but y_ℓ is not;*

2. *$\lambda(y_r) \geq 0$ implies $\underline{y}_p < y_\ell < y_r = \overline{y}_p$, so $y_\ell \in \Delta$ but y_r is not; and*

3. *otherwise, $\underline{y}_p < y_\ell < y_r < \overline{y}_p$, so $\{y_\ell, y_r\} \subset \Delta$.*

The key factor underlying Proposition 3 is whether centrists at y_ℓ or y_r want to bias inwards. If either does, then its nearby moderates also want to bias inward enough to have a centrist representative. Thus, the sign of $\lambda(y_\ell)$ characterizes whether $\underline{y}_p < y_\ell$ and similarly $\lambda(y_r)$ characterizes whether $y_r < \overline{y}_p$. If $\lambda(y_r) < 0 < \lambda(y_\ell)$, then moderates near y_ℓ want to bias rightward into (y_ℓ, y_r) and symmetrically for moderates near y_r, so $y^* \in (y_\ell, y_r)$. If not, then one of y_ℓ or y_r wants an ally representative — i.e., $y^* \in \{y_\ell, y_r\}$ — and none of their nearby moderates want a centrist, but some moderates on the other side will want a centrist.

Proposition 3 implies that representatives at y_ℓ and y_r can be optimal for (i) nobody, (ii) exactly one P, or (iii) an interval of centrist P. Notably, they are the only representatives who can be uniquely optimal for more than one P.

We can further sharpen our characterization of y_d^* if the median is fixed ($y_\ell = y_r$) and the proposal power of extremists is balanced ($\rho_L = \rho_R$). Let $\rho_E \equiv \rho_L + \rho_R$. Under these

conditions, y_d^* is continuous and over (E_L, E_R) and is a convex combination of y_p and y_m, where $\delta \rho_E$ is the weight on y_m.

Corollary 2. *If $y_\ell = y_r$ and $\rho_L = \rho_R$, then $y_d^*(y_p)\big|_{(E_L, E_R)} = (1 - \delta \rho_E)y_p + \delta \rho_E y_m$.*

Using Corollary 2, we know $y_m = y^* = \underline{y}_p = \overline{y}_p$. Moreover, we can shed light on how far P moderates, $|y_d^*(y_p) - y_p|$. The effect of ρ_E highlights the key force for moderation: P moderates to constrain extremists and thus moderates further as extremists gain proposal power. Additionally, more centrist principals do not moderate as far, since biasing y_d towards y_m has a weaker effect on $A^*(y_d)$. Essentially, the "price" of moderating extremist proposals rises as y_d gets closer to y_m. Finally, increasing δ induces P to moderate further. Essentially, there is a drop in the "price" of constraining extremists — increasing δ makes m's expectations about future policymaking more prominent in his voting calculus and thus magnifies the effect of y_d on the acceptance set.

Effects of Extremism. We have shown an incentive to use strategic representation to counteract extremists. We now show how that varies with changes in relative extremism. Specifically, Proposition 4 characterizes how shifting proposal rights between L and R affects: the locus of attraction, y^*; the set principals who choose a moderate, (E_L, E_R); and the set of principals who choose a centrist, $(\underline{y}_p, \overline{y}_p)$. Figure 4 illustrates.

Proposition 4. *Increasing ρ_L:*

1. *weakly increases the locus of attraction, y^*;*

2. *weakly increases the set of principals who strictly prefer a non-extremist, (E_L, E_R); and*

3. *weakly decreases the set of principals who strictly prefer a veto player, $(\underline{y}_p, \overline{y}_p)$.*

Given ρ_E, transferring recognition probability between L and R does not affect the acceptance set on its own, since the median is indifferent between their proposals. Yet, this transfer does affect P's delegation incentives. For example, increasing ρ_L at ρ_R's expense

Figure 4: Optimal representatives vary with extremist proposal rights

(a) $\rho_L \gg \rho_R$

(b) $\rho_L > \rho_R$

(c) $\rho_L < \rho_R$

(d) $\rho_L \ll \rho_R$
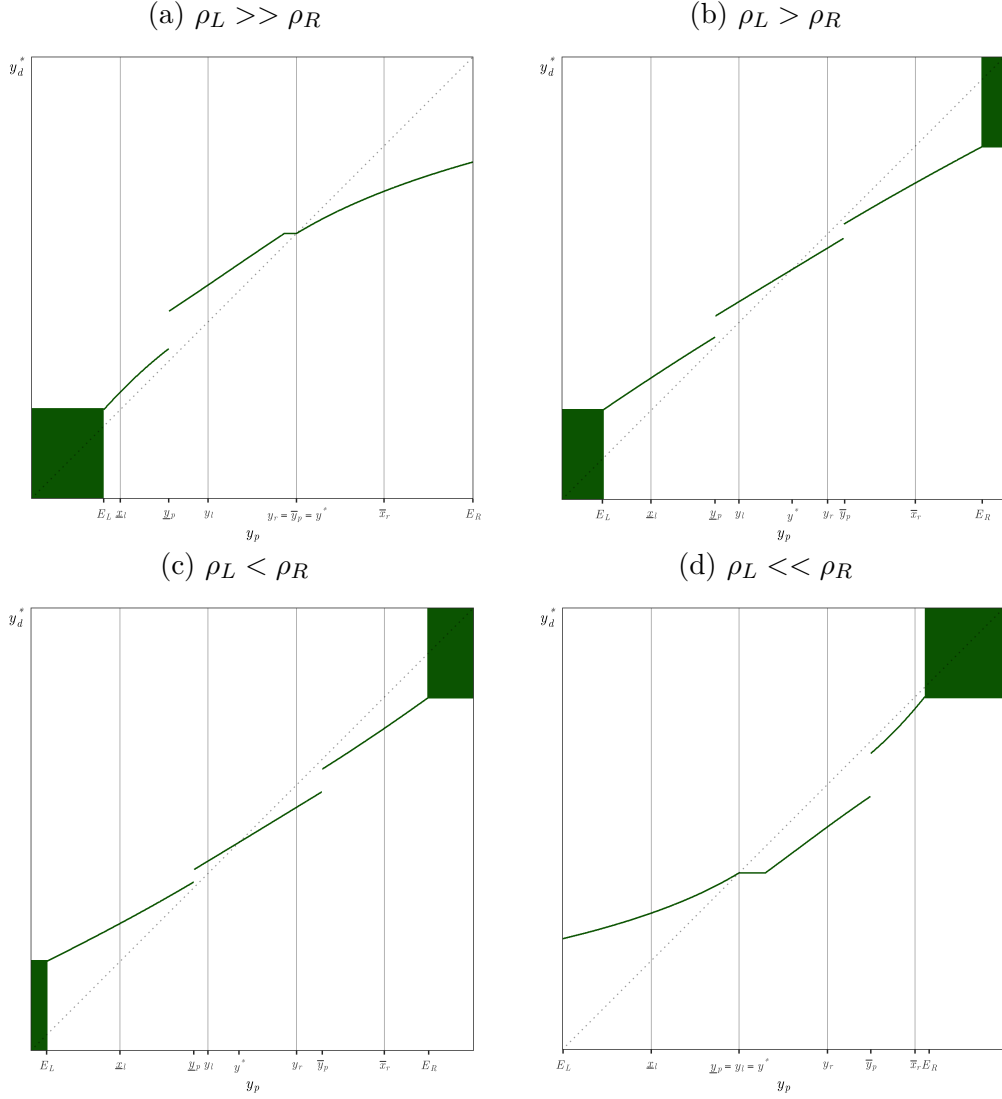
Note: Figure 4 illustrates Proposition 4 to show how optimal representatives change as recognition probability is transferred between the extremist politicians.

amplifies P's sensitivity to constraining left extremists but also dampens her sensitivity to constraining right extremists, and vice versa. Depending on the location of y_p, these effects can change P's optimal representative in different ways.

First, y^* shifts rightward since increasing ρ_L strengthens centrists' desire to constrain left extremists by skewing rightward. Essentially, centrists want to constrain both extremists but grow more concerned about constraining the strengthened side and less concerned about the

weakened side.

Next, E_L and E_R both shift rightward since increasing ρ_L makes right extremists more inclined to moderate and left extremists less inclined. Extremists want to constrain their opposing extremist but not their aligned extremist. Thus, their desire to moderate varies with relative extremism differently if they are on the strengthened side rather than the weakened one. On the strengthened side, moderating is more appealing due to greater return from constraining their opposing extremist and also lower cost of constraining their aligned extremist. On the weakened side, moderating is less appealing since these effects reverse.

Finally, $\overline{y}_p$ and $\underline{y}_p$ both shift leftward since increasing ρ_L makes left moderates more inclined towards centrist representatives and right moderates less inclined towards them. Although moderates experience a similar effect on their strategic calculus as centrists, they weigh whether to choose an aligned moderate or a centrist. For moderates on the weakened side, biasing towards centrists is more appealing than before due to higher return from constraining their opposing extremist, but each centrist is less appealing due to the higher cost of relaxing the far extremist's constraint. On the strengthened side, the forces are analogous but push in the other directions.

Robustness: More General Policy Utility

We generalize the characterization of optimal representatives beyond quadratic policy utility. Specifically, we consider policy utilities of the form $u(x, y)$—where $x \in \mathbb{R}$ denotes a policy and $y \in \mathbb{R}$ denotes an ideal point— that satisfy four standard properties: (i) strict concavity in each argument, i.e., $\frac{\partial^2 u(x,y)}{\partial x^2} < 0$, $\frac{\partial^2 u(x,y)}{\partial y^2} < 0$; (ii) Spence–Mirelees single-crossing, i.e., $\frac{\partial^2 u(x,y)}{\partial x \partial y} > 0$; (iii) symmetry around a single peak, i.e., there exists a decreasing, concave, continuous function g such that $u(x, y) = g(|x - y|)$; and (iv) identical up to translation, i.e., $u(x + \varepsilon, x) = u(y + \varepsilon, y)$ for all $x, y \in [0, 1]$ and ε such that $x + \varepsilon, y + \varepsilon \in [0, 1]$. Results for this general case are shown in Lemma A2 in the appendix.

First, we show the principal never wants a representative who is from the other side of the spectrum, i.e., an opposing moderate or extremist. Specifically, $y_p \leq y_r$ implies $y^*(y_d) \subset [0, y_r]$ and, analogously, $y_p \geq y_\ell$ implies $y^*(y_d) \subset [y_\ell, 1]$. Although this property is quite natural, it is not trivial in our setting. Of course, opposing representatives have clear downside for P since their proposal will be farther away and they relax the constraint on opposing extremists. But they can also have some upside since they relax the constraint on the aligned extremists. Yet, they are always strictly dominated by some aligned representative who equally constrains the aligned extremists but provides less downside from other proposals.

The second general feature is that extremism is not appealing. A more extreme representative is never better than an ally, and for centrist and moderate principals they are strictly worse. Instead, every principal wants the representative to be a centrist or biased in that direction, and never wants someone more extreme than themselves. Thus, only an extremist principal may want an extreme representative.

Specifically, we bound the set of optimal representatives for each y_p and show that optimal representatives are centrist or biased in that direction for a broad set of principals. First, centrist principals may want to bias in either direction but always want centrist representatives. Similar to the logic above, the only appeal of any non-centrist representative can always be matched by a closer centrist. Second, moderate principals want to bias strictly towards the farthest centrist. That provides a tangible gain from moderating extreme proposals at negligible loss from their representative's proposal. Finally, extremists either want any aligned extremist or want to bias towards their closest centrist.

Moreover, we show that extreme principals have limited desire to moderate. First, some extremists do not want to moderate at all. Second, all sufficiently extreme principals—including the most extreme moderates—do not want a centrist representative. For extremists, (i) their nearest centrist is always worse than nearly-centrist aligned moderates, since the acceptance set is approximately equal; and (ii) all farther centrists are even worse, since the acceptance set shifts away. Similar considerations also arise for moderate principals who are

sufficiently extreme.

Altogether, these observations also shed light on which principals might want to have an ally representative. Moderate principals never do, some centrists or extremists can.

We can also shed light on how optimal representatives are ordered. For centrist principals, there is a clear ordering: y_d^* is an increasing correspondence over $[y_\ell, y_r]$.[22] In contrast, for moderate and extremist principals there is not a clear ordering: y_d^* is not necessarily monotone, since the policy lottery induced by y_p is not naturally ordered on X.

Finally, the fixed median case—i.e., $y_\ell = y_r$—sharpens this general characterization in two ways.[23] First, the unique centrist principal, $y_p = y_m$, always wants an ally. Any other representative proposes farther policy *and* also expands the acceptance set on both sides. Second, moderate principals never want a centrist representative. The appeal of a centrist requires that they shift the median, but that is not possible in this case.

Extensions

Thus far, our analysis has focused on addressing the question of "who is best for one principal filling one position in a collective body?" To clarify and expand the scope of our results, we now extend the baseline setting in three ways to consider variants of that question. The first is *mass representation* — addressing "who would a group choose to fill the position?" The second is the *value of representation* —— addressing "how much does the principal gain from optimal representation?" Third, we study *competitive representation* —— addressing "what if multiple principals fill multiple positions?"

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[22]Specifically, U has the single-crossing property on $[y_\ell, y_r] \times X$ because equilibrium proposals are increasing over $y_d \in [y_\ell, y_r]$ and u satisfies the single-crossing property. Then, since $y_p \in [y_\ell, y_r]$ implies $y_d^*(y_p) \subset [y_\ell, y_r]$, we know y_d^* is increasing on $[y_\ell, y_r]$.

[23]See Corollary A2.1 for details.

Mass Representation

Representatives are often chosen by groups —— e.g., voters, parties, etc. —— so we want to understand collective choice over representatives. We show when this collective choice coincides with choice by a single principal. Moreover, we show that a weak condition on extremist proposal rights ensures that coalitions have a natural ordering. Overall, our analysis has implications for electoral competition and voting in collective policymaking bodies.

Proposition 5 shows that preferences over representatives satisfy the single-crossing condition (Milgrom and Shannon 1994) and have a natural ordering as long as extremist proposal rights are not too large. Specifically, any pair of candidate representatives will induce a cutpoint such that all rightward principals prefer the right candidate, and vice versa.

Proposition 5. *If* $\max\{\rho_L, \rho_R\} \leq \frac{1}{2\delta}$, *then (i) the equilibrium value, U, satisfies the single-crossing condition and (ii) sufficiently right-leaning principals prefer the rightmost candidate in any pairwise comparison.*

Proposition 5 implies that the median principal is decisive under majority rule (Cho and Duggan 2003). More generally, any *strong voting rule* will induce a decisive principal (Duggan 2014), so collective choice over representatives reduces to individual choice under a broad class of voting rules. Furthermore, voters who are farther left than the decisive voter will support the leftward candidate, and vice versa.

Underlying Proposition 5, a key property is verifying when collective choice always coincides with the choice of a single, *decisive* principal. Choosing between two representatives is a choice between policy lotteries, so preferences over representatives are not necessarily ordered in a way that yields a decisive principal. But with quadratic policy utility, those preferences are *order restricted* — i.e., for any pair of candidates y_d to y'_d, the set of y_p for which P prefers y_d is an interval (Cho and Duggan 2003; Kartik et al. 2022).[24]

An important wrinkle, however, can arise if the principal is an extremist and her aligned extremists are very likely to propose. Then, in some pairwise comparisons between two

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[24]This property is not guaranteed in our most general setting analyzed in the Appendix.

moderates who are both on the other side of the spectrum, P may prefer the more extreme candidate over the closer candidate. In this scenario, P faces a trade-off. The more allied moderate candidate makes a more favorable proposal and constrains her opposing extremists more than the alternative but also constrains her aligned extremists more. Due to risk aversion, the benefit of constraining her opposing extremists outweighs the cost of constraining aligned extremists unless her aligned extremists have sufficiently high recognition probability. In contrast, whenever P is choosing between two centrist candidates for instance, extremists prefer the nearest candidate since both extreme proposals move in the same direction as y_d within (y_ℓ, y_r).

Value of Representation

Next, we study how much the principal gains from having her optimal representative, relative to having an ally representative who shares her ideal point.

Specifically, we define P's *value of representation* as

$$\nu(y_p) \equiv U(y_d^*(y_p), y_p) - U(y_p, y_p). \tag{8}$$

To sharpen our analysis, we focus on the same conditions as in Corollary 2: a fixed median $(y_\ell = y_r)$ and balanced extremist proposal rights $(\rho_L = \rho_R)$.

We show that extremists benefit more from optimal representation as they become more moderate and, conversely, moderates benefit more as they become more extreme. Thus, on each side of the spectrum, the value of representation is highest for principals on the extremist-moderate boundary — i.e., $y_p \in \{\underline{x}_\ell, \overline{x}_r\}$. Figure 5 illustrates.

Proposition 6. *If $\rho_\ell = \rho_r$ and $\rho_L = \rho_R$, then ν is strictly increasing on $[E_L, \underline{x}_\ell]$, strictly decreasing on $[\underline{x}_\ell, y_m]$ and analogously for $y_p \in [y_m, E_R]$.*

To characterize ν, we exploit the property that P's optimal representative, $y_d^*(y_p)$, always balances her marginal benefit of moderation against her marginal cost. To illustrate more

Figure 5: How the value of representation varies with y_p

(a) $y_p << \underline{x}_\ell$ (b) $y_p < \underline{x}_\ell$

(c) $y_p > \underline{x}_\ell$ (d) $y_p >> \underline{x}_\ell$
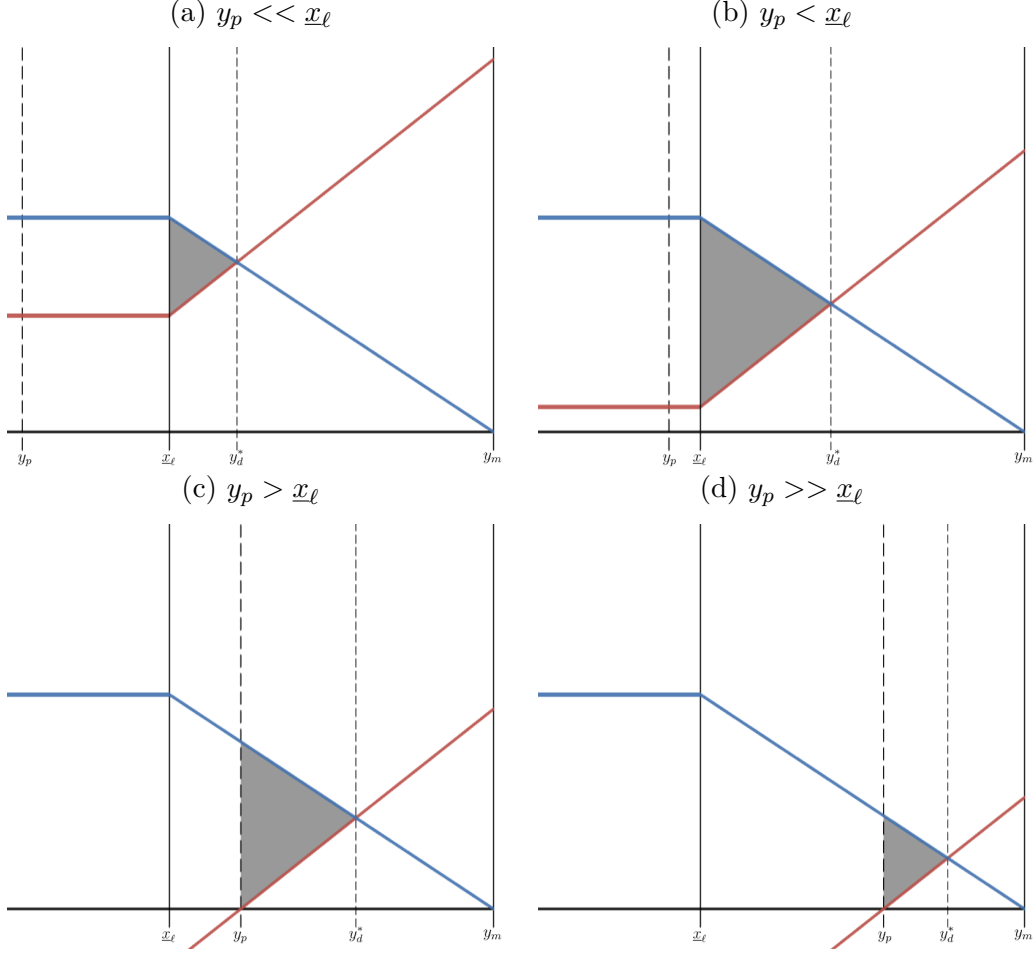
Note: Figure 5 displays the value of representation (ν) for four different values of $y_p \in (E_L, y_m)$. In each panel, $\nu(y_p)$ equals the area of the shaded region between the two curves, which are the marginal benefit (downward sloping) and marginal cost (upward sloping) of moderation as functions of y_d.

precisely, if a left-leaning P moderates further from any $y_d \in (\underline{x}_\ell, y_m)$ then she enjoys marginal benefit $\frac{\delta \rho_E \rho_d (y_m - y_d)}{1 - \delta \rho_E}$, which is her gain from further constraining extremist proposals, and incurs marginal cost $\rho_d(y_d - y_p)$, which is her loss from shifting d's proposal further away. Since (i) P's marginal benefit decreases in y_d over this interval and is independent of y_p, while (ii) P's marginal cost increases in y_d and decreases in y_p,[25] her marginal benefit exceeds marginal cost for $y_d < y_d^*(y_p)$ and vice versa, with the difference increasing in their distance. Thus, $\nu(y_p)$

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[25] As y_p moderates, the marginal benefit from constraining R's proposal decreases relative to constraining L. Since the boundaries of A^* contract at the same rate, if $\rho_L = \rho_R$ then the loss in marginal benefit from shifting $\overline{x}(y_d)$ is exactly equal to the gain from shifting $\underline{x}(y_d)$.

equals the area between P's marginal cost and benefit curves over $y_d \in [\max\{\underline{x}_\ell, y_p\}, y_d^*(y_p)]$.[26]

If P is sufficiently extreme, $y_p \notin (E_L, E_R)$, she nominates an aligned extremist and thus $\nu(y_p) = 0$. As P becomes less extreme, (i) her marginal cost curve shifts down and (ii) y_d^* shifts towards y_m, which increases the difference between marginal benefit and marginal cost at all $y_d \in [\underline{x}_\ell, y_d^*(y_p)]$, so her value of delegation rises. Figure 5a–5b illustrates.

For moderate P, the acceptance set induced by their ally will shrink as y_p approaches y_m, so there is a smaller difference between P's marginal benefit and marginal cost at every $y_d \geq y_p$. Since the extent of P's optimal bias also decreases, the value of representation decreases as y_p approaches y_m. Figure 5c–5d illustrates.

Competitive Representation

Thus far, we have focused on a principal filling one position and fixed the rest of the political environment. This can reflect situations in which other politicians are already in office, but our analysis also highlights that incentives for moderation will arise in situations where multiple positions will be filled simultaneously (as noted by, e.g., Gailmard and Hammond 2011). In this section, we explore whether those incentives will strengthen or weaken by extending our baseline setup so that two principals simultaneously pick their representatives.

We extend the model to have two principals, P_a and P_b, each simultaneously appointing representatives, a and b, to fill two positions in a five-player body. The three other politicians are two extremists, L and R, and a veto player, M, who determines whether any proposal passes. Finally, we assume $y_L < y_{p_a} < y_M < y_{p_b} < y_R$, where y_{p_a} and y_{p_b} denote the principals' ideal points.

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[26]This follows from the fundamental theorem of calculus:

$$\nu(y_p)\big|_{(E_L, \underline{x}_\ell)} = \int_{\underline{x}_\ell}^{y_d^*(y_p)} \left(\frac{\delta \rho_E \rho_d (y_m - y_d)}{1 - \delta \rho_E} - \rho_d(y_d - y_p) \right) dy_d,$$

and

$$\nu(y_p)\big|_{(\underline{x}_\ell, y_m)} = \int_{y_p}^{y_d^*(y_p)} \left(\frac{\delta \rho_E \rho_d (y_m - y_d)}{1 - \delta \rho_E} - \rho_d(y_d - y_p) \right) dy_d.$$

By Lemma 1, each $y_a, y_b \in X^2$ induces a unique distribution over policy outcomes characterized by the equilibrium acceptance set, $A^*(y_a, y_b)$. To streamline key points, we assume (i) y_{p_a} and y_{p_b} are both always inside the acceptance set, while (ii) y_L and y_R are always outside.

Our characterization of optimal representatives in the baseline analysis also characterizes best responses in this competitive setting. Since both principals are moderates, each will bias their representative towards M in equilibrium, so $y_{p_a} < y_a^* < y_M < y_b^* < y_{p_b}$. Furthermore, with quadratic policy utility, Proposition 1 implies that each principal always has a unique best response. Specifically, principal P_a's best response to y_b, denoted $y_a(y_b)$, is the unique $y_a \in (y_{p_a}, y_M)$ satisfying the first-order condition:

$$\frac{\partial \underline{x}(y_a, y_b)}{\partial y_a}[\rho_L(y_{p_a} - \underline{x}(y_a, y_b)) + \rho_R(\overline{x}(y_a, y_b) - y_{p_a})] - \rho_a(y_a - y_{p_a}) = 0, \tag{9}$$

and P_b's best response function is analogous.

Lemma 4 establishes that each principal's best response is monotone. Moreover, the direction is determined by which extremist has greater proposal rights

Lemma 4. *If $\rho_L < \rho_R$, then y_a is strictly decreasing and y_b is strictly increasing; and vice versa if $\rho_L > \rho_R$. If $\rho_L = \rho_R$, then $y_{d_i}(y_{d_{-i}}) = (1 - \delta\rho_E)y_{p_i} + \delta\rho_E y_m$ for all y_{-i}.*

Lemma 4 implies the principals' best responses intersect once. Thus, a unique pair of representatives is mutually optimal and each is strictly more centrist than their principal.

Proposition 7. *There is a unique equilibrium, in which $y_a^* \in (y_{p_a}, y_m)$ and $y_b^* \in (y_m, y_{p_b})$.*

Additionally, Lemma 4 implies that the principal aligned with weaker extremists will moderate further in the competitive setting than in the baseline setting, whereas the principal aligned with stronger extremists will moderate less.

Corollary 3. *In equilibrium: (i) $\rho_L < \rho_R$ implies $y_a(y_b) < y_a^* < y_M < y_b(y_a) < y_b^*$; (ii) $\rho_L = \rho_R$ implies $y_a(y_b) = y_a^* < y_M < y_b(y_a) = y_b^*$; and (iii) $\rho_L > \rho_R$ implies $y_a^* < y_a(y_b) < y_M < y_b^* < y_b(y_a)$.*

Corollary 3 is driven by the two effects of opponent moderation. To fix ideas, consider shifting y_b inwards. One effect is that extremist proposals also shift inwards, which directly benefits P_a and decreases her marginal benefit from shifting y_a inward. Through this effect, moderation by P_b *substitutes* for moderation by P_a. The other effect is that the acceptance set becomes more sensitive to y_a, i.e., $\frac{\partial^2 \underline{x}(y_a, y_b)}{\partial y_a \partial y_b} < 0$. Through this channel, moderation by P_b reduces the "price" of moderating extremist proposals and *complements* moderation by P_a.

Which effect dominates depends on the balance of extremist proposal rights. The complementary effect dominates on the weak side and conversely on the strong side. If $\rho_L = \rho_R$, then the increased marginal elasticity of A^* to y_{d_i} exactly offsets the decreased marginal benefit of moderation. As one extremist gains proposal power at the expense of the other, since each principal's aligned extremist is closer to her ideal point than the non-aligned extremist, the marginal benefit of moderation declines in $y_{d_{-i}}$ at a slower rate for the weak-side principal than the strong-side principal.

Conclusion

We study preferences over representatives who participate in collective policymaking. A key force in our analysis is that a representative's ideology affects legislature-wide expectations about policymaking. This force is present in many contexts and we study its consequences for representation. We show how it has important *anticipation effects* by shaping exactly which policies each politician will support, thereby influencing what would pass and what extremists will propose.

We provide a general logic for why moderate representatives can be appealing. We show that (i) all centrist principals want a centrist representative who will be the median (de facto veto) politician and (ii) all moderate principals want a more centrist representative. Even when they are not the median, their closer alignment improves the median's expectation about proposals and thus narrows what can pass, which constrains extremist politicians.

Moreover, under standard assumptions, principals who are not too extreme have a uniquely optimal representative, who is biased inward towards a centrally located *locus of attraction*. Additionally, we find a *dead zone* of representatives on the centrist-moderate fringe(s) who are not optimal for any principal. Furthermore, the locations of the dead zone and locus of attraction depend on the balance of extremist proposal rights. As that balance changes, the principal grows more concerned with constraining extremists on the gaining side — so the dead zone grows on that side while the locus of attraction shifts away.

We focused on a collective policymaking environment governed by simple majority rule. Our results generalize to any *strong* voting rule, since there will be a single decisive principal who will effectively determine what can pass (Duggan 2014). To illustrate, our analysis with a fixed median ($y_\ell = y_r$) in Corollary 2 is equivalent to the principal appointing a proposer into a dictatorial rule setting with the dictator already in place. Future work could study representation in settings where more than one politician will be decisive. For example, under supermajority rules the acceptance set is determined by two (endogenous) veto players rather than a single median. In those settings, extending our analysis requires characterizing an implicit curve defined by a system of nonlinear equations.

In addition to our main results, which have implications for representation in separation-of-powers systems and congressional committees, our mass representation extension also suggests avenues for new insights into mass representation. It has potential implications for studying behavior by voters and elites in elections to positions in collective bodies. Several possibilities include (i) voting in elections for collective policymaking positions (Kedar 2005, 2009; Duch et al. 2010), (ii) electoral competition over those offices (Austen-Smith and Banks 1988; Krasa and Polborn 2018), and (iii) its representativeness (Austen-Smith and Banks 1991). We shed new light on how expectations about collective policymaking can affect incentives of party leaders and voters (Kedar 2005, 2009; Duch et al. 2010), thereby influencing who gets nominated and their electoral chances. For instance, we show that unless extremist proposal rights are very high, electoral competition in which each party

chooses a candidate representative will feature a unique indifferent voter. Thus, (i) each candidate's win probability is easy to characterize, and (ii) both parties will converge toward the median voter's optimal representative. An interesting complication, however, is that the policymaking environment will not only affect how parties evaluate their own candidates but will also shape their view of opposing candidates. Future work in this direction could build on our foundations in order to explore how elite polarization and extremism affect elections.

Appendix

Contents

A General Setting

In this section, we characterize the equilibrium acceptance in a setting that generalizes the baseline model. In the general setting, there are an arbitrary number of legislators, $k \geq 2$, with arbitrary recognition probabilities, $\rho_i \in (0,1)$. Additionally, the only condition imposed on legislator ideal points is that there are at least two legislators $i \in K$ with $y_i \in (0,1)$.[27] Finally, $u(x,y) : [0,1]^2 \to \mathbb{R}_+$ is $\mathcal{C}^2$, and satisfies the following conditions:

(i) *Strict concavity in each argument*: $\frac{\partial^2 u(x,y)}{\partial x^2} < 0$, and $\frac{\partial^2 u(x,y)}{\partial y^2} < 0$;

(ii) *Spence-Mirrlees single-crossing*: $\frac{\partial^2 u(x,y)}{\partial x \partial y} > 0$;

(iii) *Symmetric around a single peak*: there exists a decreasing, concave, continuous function g such that $u(x,y) = g(|x - y|)$; and

(iv) *Identical up to a translation*: $u(x + \varepsilon, x) = u(y + \varepsilon, y)$ for all $x, y \in [0,1]$ and ε such that $x + \varepsilon, y + \varepsilon \in [0,1]$.

A.1 Proof of Lemma 1.

Follows from Propositions 1–2 in Cardona and Ponsati (2011). $\square$

A.2 Define the cutpoints $\underline{x}_\ell, \overline{x}_\ell, \underline{x}_r,$ and $\overline{x}_r$.

From Cardona and Ponsati (2011), the equation

$$u(x, y_\ell)(1 - \delta\rho_d) - \delta \left(\sum_{i \in K : |y_i - y_\ell| \geq |y_\ell - x|} \rho_i u(x, y_\ell) + \sum_{i \in K : |y_i - y_\ell| < |y_\ell - x|} \rho_i u(y_i, y_\ell) \right) = 0 \quad (10)$$

has exactly two solutions: $\underline{x}_\ell \in (0, y_\ell)$ and $\overline{x}_\ell \in (0, y_\ell)$. Similarly, $\underline{x}_r \in (0, y_r)$ and $\overline{x}_r \in (0, y_r)$ are the only solutions of

$$u(x, y_r)(1 - \delta\rho_d) - \delta \left(\sum_{i \in K : |y_i - y_r| \geq |y_r - x|} \rho_i u(x, y_r) + \sum_{i \in K : |y_i - y_r| < |y_r - x|} \rho_i u(y_i, y_r) \right) = 0. \quad (11)$$

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[27]This ensures that a $\underline{\delta} < 1$ exists such that $A^*(y_d) \subset (0,1)$ for all y_d if $\delta > \underline{\delta}$.

A.3 Proof of Lemma 2.

Part 1 shows that $y_d \in (\underline{x}_\ell, \overline{x}_r)$ implies $y_d \in \text{int } A^*(y_d)$. Part 2 shows that $y_d \in \text{int } A^*(y_d)$ implies $y_d \in (\underline{x}_\ell, \overline{x}_r)$. To show each direction, we use contraposition.

Part 1. Consider $y_d \leq \min A^*(y_d) = \underline{x}(y_d)$. Then $y_d \leq \underline{x}(y_d) < y_m$, so Assumption 1 implies that $\underline{x}(y_d) \in (0, y_\ell)$ and must solve (10). Thus, $\underline{x}(y_d) = \underline{x}_\ell$. Analogously using (11), $y_d \geq \max A^*(y_d) = \overline{x}(y_d)$ implies that $\overline{x}(y_d) = \overline{x}_r$. We have shown that $y_d \notin (\underline{x}(y_d), \overline{x}(y_d))$ implies $y_d \notin (\underline{x}_\ell, \overline{x}_r)$. By contraposition, $y_d \in (\underline{x}_\ell, \overline{x}_r)$ implies $y_d \in (\underline{x}(y_d), \overline{x}(y_d)) = \text{int } A^*(y_d)$.

Part 2. Consider $y_d \leq \underline{x}_\ell$. Then, uniqueness of $A^*(y_d)$ implies that $y_d \notin \text{int } A^*(y_d)$ is equivalent to the lower solution of (10) satisfying $\underline{x}_\ell \geq y_d$. Thus, $y_d \leq \underline{x}_\ell$ implies $y_d \leq \min A^*(y_d)$. An analogous argument shows that $y_d \geq \overline{x}_r$ implies $y_d \geq \max A^*(y_d)$. We have shown $y_d \notin (\underline{x}_\ell, \overline{x}_r)$ implies $y_d \notin \text{int } A^*(y_d)$. By contraposition, $y_d \in \text{int } A^*(y_d)$ implies $y_d \in (\underline{x}_\ell, \overline{x}_r)$. $\square$

A.4 Proof of Lemma 3.

We first establish a useful property in Lemma A1.

Lemma A1. $\underline{x}_\ell \leq \underline{x}_r$ and $\overline{x}_\ell \leq \overline{x}_r$ with both inequalities strict if $y_\ell < y_r$.

Proof. To prove Lemma A1, we construct a function $\zeta(x, y) : [0, 1] \times [y_\ell, y_r] \to \mathbb{R}$ which for each $y \in [y_\ell, y_r]$, has two unique roots, $\underline{x}(y)$ and $\overline{x}(y) = 2y - \underline{x}(y)$. We then establish that $\underline{x}(y)$ is continuous in y with $\underline{x}(y_j) = \underline{x}_j$ for $j \in \{\ell, r\}$. Having characterized $[\underline{x}_\ell, \overline{x}_\ell]$ and $[\underline{x}_r, \overline{x}_r]$ in this way, we prove Lemma A1 by showing that $\underline{x}(y)$ and $\overline{x}(y)$ are strictly increasing.

Let

$$\zeta(x, y) := u(x, y) - \delta \left(\sum_{i \in K : |y - y_i| < |y - x|} \rho_i u(y_i, y) + \left(\rho_d + \sum_{i \in K : |y - y_i| \geq |y - x|} \rho_i \right) u(x, y) \right).$$

Then, (i) $y_d \leq y_\ell$ implies that $\zeta(x, y_\ell) = 0$ is equivalent to (10) and (ii) $y_d \geq y_r$ implies that $\zeta(x, y_r) = 0$ is equivalent to (11). Therefore $\zeta(\underline{x}_\ell, y_\ell) = 0$ and $\zeta(\overline{x}_r, y_r) = 0$. Trivially, $y_\ell = y_r$ implies $[\underline{x}_\ell, \overline{x}_\ell] = [\underline{x}_r, \overline{x}_r]$. To establish the strict inequalities for $y_\ell < y_r$, we use the following

properties of $\zeta(x,y)$: (i) it is continuous in each argument; (ii) it is differentiable in each argument at all (x,y) s.t. $|y_i - y| \neq |y - x|$ for any $i \in K$; (iii) strictly increasing in x and decreasing in y if $x < y$, strictly decreasing in x and increasing in y if $x > y$; (iv) $\zeta(x,y) > 0$ if $x = y$; and (v) $\zeta(0,y), \zeta(1,y) < 0$ (under Assumption 1).

It follows that for every $y \in [y_\ell, y_r]$, unique $\underline{x}(y) \in (0,y)$ and $\overline{x}(y) = 2y - \overline{x}(y) \in (y,1)$ exist such that $\zeta(\underline{x}(y), y) = \zeta(\overline{x}(y), y) = 0$. Moreover, $\underline{x}(y)$ and $\underline{x}(y)$ are continuous and differentiable almost everywhere.[28] At any y such that $\underline{x}(y)$ and $\overline{x}(y)$ are differentiable,

$$\underline{x}'(y) = 1 - \left[\delta \sum_{i \in N : |y - y_i| < |y - \underline{x}|} \rho_i \frac{\partial u(y_i, y)}{\partial y} \right] \left[\left(1 - \delta(\rho_d + \sum_{i \in N : |y - y_i| \geq |y - \underline{x}|} \rho_i) \right) \frac{\partial u(\underline{x}, y)}{\partial y} \right]^{-1} .$$

The strict concavity and symmetry of $u(x,y)$ imply that $\left| \frac{\partial u(x,y)}{\partial y} \right| > \left| \frac{\partial u(y_i, y)}{\partial y} \right|$ for all $y_i \in K$ such that $|y_i - y| < |x - y|$. Therefore $\underline{x}'(y) \in (0,2)$ and $\overline{x}'(y) = 2 - \underline{x}'(y) \in (0,2)$. The continuity of ζ therefore implies that $\underline{x}(y)$ and $\overline{x}(y)$ are strictly increasing. Thus $\underline{x}_\ell < \underline{x}_r$ and $\overline{x}_\ell < \overline{x}_r$. □

Proof of Lemma 3.

Lemma 2 implies Parts 1 and 5. Part 3 is implied by Lemma 5 of Cardona and Ponsati (2011) which shows that $\underline{x}(y_d)$ and $\overline{x}(y_d)$ are strictly increasing in y_m if $A^*(y_d) \subset \text{int} X$ and Lemma A3 which shows that in the quadratic setting analyzed in the main text, the lower bound of $A^*(y_d)$ is strictly concave on each interval described in Parts 2-4 of Lemma 3. Parts 2 and 4 remain to be shown. We provide a proof for Part 2. The proof for Part 4 is analogous.

The concavity result in Part 2 is implied by Lemma A3. To prove the rest of Part 2 for the general setting, we first show that $\underline{x}_\ell(y_d)$ strictly increases and $\overline{x}_\ell(y_d)$ strictly decreases

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[28]Continuity follows from the continuity of ζ. For differentiability a.e., notice that on every non-empty subset of $[y_\ell, y_r]$, symmetry requires that at most one root is constant in y. Now suppose an interval $(a,b) \subset [y_\ell, y_r]$ exists such that $\underline{x}(y)$ is constant. For $y, y' \in (a,b)$ where $y \geq y'$, it follows that $\overline{x}(y') - \overline{x}(y) = 2(y' - y) > 0$. But then $\zeta(\underline{x}(y'), y') < \zeta(\underline{x}(y), y) = 0$. Thus no interval exists where $\underline{x}(y)$ is constant in y. An analogous result holds for $\overline{x}(y)$. Therefore the set of $y \in [y_\ell, y_r]$ such that $y_i \in \{\underline{x}(y), \overline{x}(y)\}$ for some $i \in K$ is finite. The antecedent conditions of the implicit function theorem hold almost everywhere.

on $(\underline{x}_\ell, y_\ell)$. By Lemma 2, $y_d \in (\underline{x}_\ell, y_\ell)$ implies $\underline{x}(y_d)$ is the unique $x \in (\underline{x}_\ell, y_d)$ satisfying

$$\kappa_\ell(x, y_d) := u(x, y_\ell) - \delta \rho_d u(y_d, y_\ell) - \delta \left(\sum_{i \in K : |y_i - y_\ell| \geq |y_\ell - x|} \rho_i u(x, y_\ell) + \sum_{i \in K : |y_i - y_\ell| < |y_\ell - x|} \rho_i u(y_i, y_\ell) \right) = 0,$$

and $\overline{x}(y_d) = 2y_\ell - \underline{x}(y_d)$. Notice that κ_ℓ is strictly increasing over $y_d \geq x$ and strictly decreasing over $y_d \in (x, y_\ell)$. Therefore $\underline{x}(y_d)$ is strictly increasing and $\overline{x}(y_d)$ strictly decreasing.

We now show that the rate of change of $\underline{x}_\ell(y_d)$ approaches zero as $y_d \to y_\ell$. Note that the set of x where $\frac{\partial \kappa_\ell(x;y_d)}{\partial x}\big|_{x^-} \neq \frac{\partial \kappa_\ell(x;y_d)}{\partial x}\big|_{x^+}$ is the finite set of x such that $x = y_i$ for some $i \in K$. It follows that $\underline{x}_\ell(y_d)$ is differentiable almost everywhere on $(\underline{x}_\ell, y_\ell)$. Therefore a non-empty interval $(y_\ell - \varepsilon, y_\ell)$ exists on which $\underline{x}(y_d)$ is continuously differentiable. At any y_d such that $\underline{x}_\ell(y_d)$ is differentiable,

$$\frac{\partial \underline{x}_\ell(y_d)}{\partial y_d} = \left(\frac{\delta \rho_d}{1 - \delta[1 - P(\overline{x}) + P(\underline{x})]} \right) \left(\frac{\partial u(y_d, y_\ell)}{\partial y_d} \right) \left(\frac{\partial u(\underline{x}, y_\ell)}{\partial \underline{x}} \right)^{-1},$$

where $\left(\frac{\delta \rho_d}{1 - \delta[1 - P(\overline{x}) + P(\underline{x})]} \right)$ is a positive constant and $\underline{x}_\ell(y_d) < y_\ell$ implies $\frac{\partial u(\underline{x}, y_\ell)}{\partial \underline{x}} > \frac{\partial u(y_d, y_\ell)}{\partial y_d}$. Therefore

$$\lim_{y_d \to y_\ell^-} \frac{\partial \underline{x}(y_d)}{\partial y_d} \propto \lim_{y_d \to y_\ell^-} \frac{\partial u(y_d, y_\ell)}{\partial y_d} = \frac{\partial u(y_d, y_\ell)}{\partial y_d} \bigg|_{y_d = y_\ell} = 0.$$

Finally, Lemma A1 and Parts 1–5 imply that $A^*(y_d) \subset [\underline{x}_\ell, \overline{x}_r]$ for all y_d. $\square$

A.5   General properties of y_d^*.

We establish general properties of $y_d^*(y_p)$ in Lemma A2.

Lemma A2.

1. $[0, \underline{x}_\ell] \cap y_d^*(y_p) \neq \emptyset \iff [0, \underline{x}_\ell] \in y_d^*(y_p)$ and $[\overline{x}_r, 1] \cap y_d^*(y_p) \neq \emptyset \iff [\overline{x}_r, 1] \in y_d^*(y_p)$;

2. $y_p \leq y_r$ implies $y_d^*(y_p) \cap (y_r, \overline{x}_r] = \emptyset$, and $y_p \geq y_\ell$ implies $y_d^*(y_p) \cap [\underline{x}_\ell, y_\ell) = \emptyset$;

3. $y_p < y_\ell$ implies $y_\ell \notin y_d^*(y_p)$, and $y_p > y_r$ implies $y_r \notin y_d^*(y_p)$;

4. $y_p \in (\underline{x}_\ell, y_\ell)$ implies $[\underline{x}_\ell, y_p] \cap y_d^*(y_p) = \emptyset$, and $y_p \in (y_r, \overline{x}_r)$ implies $[y_p, \overline{x}_r] \cap y_d^*(y_p) = \emptyset$;

5. $y_p \in [0, \underline{x}(y_\ell)] \cup [\overline{x}(y_r), 1]$ *implies* $[y_\ell, y_r] \cap y_d^*(y_p) = \emptyset$.

Proof. The principal solves $\max\limits_{y_d \in [0,1]} U(y_d; y_p)$ where $U : [0,1]^2 \to \mathbb{R}$ is:

$$U(y_d; y_p) := P(\underline{x}(y_d))u(\underline{x}(y_d), y_p) + [1 - P(\overline{x}(y_d))]u(\overline{x}(y_d), y_r) + \sum_{i \in N : y_i \in (\underline{x}(y_d), \overline{x}(y_d)]} \rho_i u(y_i, y_p).$$

For each part 1–5, we prove one side since the other side is analogous.

1. For $y_d \leq \underline{x}_\ell$, Lemma 3 implies $A^*(y_d) = [\underline{x}_\ell, \overline{x}_\ell]$, so $U(\underline{x}_\ell; y_p) = U(y_d; y_p)$.

2. Consider $y_p \leq y_r$ and suppose there exists $y_d' \in (y_r, \overline{x}_r]$ such that $y_d' \in y_d^*(y_p)$. We establish a contradiction by showing there must exist $y_d'' < y_r$ such that $U(y''; y_p) > U(y'; y_p)$. First, Lemma 2 implies $y_p < y_d' < \overline{x}(y_d')$. Because $\underline{x}(y_d)$ is strictly decreasing and $\overline{x}(y_d)$ strictly increasing on $[y_r, \overline{x}_r]$, we know $y_d' \in y_d^*(y_p)$ requires $\underline{x}(y_d) \geq y_p$ (otherwise, $U(y_d - \varepsilon; y_p) > U(y_d; y_p)$ for some $\varepsilon > 0$). Next, by Lemmas A1 and 3, we know that (i) $\underline{x}(y_d)$ is continuous and strictly increasing on $[\underline{x}_\ell, y_r]$ (ii) $\underline{x}_\ell < \underline{x}_r$ and (iii) $y_d \leq \underline{x}(y_d)$ if and only if $y_d \leq \underline{x}_\ell$. Thus, a $y_d'' \in (y_p, y_r)$ exists such that $\underline{x}(y_d'') = \underline{x}(y_d')$. Since $\overline{x}(y_d) = 2y_m - \underline{x}(y_d)$ and $y_p < y_d'' < y_r < y_d'$, it follows that $y_p < \overline{x}(y_d'') < 2y_r - \underline{x}(y') = \overline{x}(y_d')$. We have shown that $|y_p - y_d''| < |y_p - y_d'|$, $|y_p - \overline{x}(y_d'')| < |y_p - \overline{x}(y_d')|$, and $|y_p - \underline{x}(y_d'')| = |y_p - \underline{x}(y_d')|$, which implies $U(y_d', y_p) < U(y_d'', y_p)$. But then $y_p' \in y_d^*(y_p)$, a contradiction.

3. Lemma 3 implies that for $y_p < y_\ell$ we know that $U(y_d; y_p)$ is continuously differentiable on $(y_\ell - \varepsilon, y_\ell)$ with

$$\lim_{y_d \to y_\ell^-} \frac{\partial U(y_d; y_p)}{\partial y_d} = \rho_d \frac{\partial u(y_d, y_p)}{y_d}\bigg|_{y_d = y_\ell^-} < 0.$$

Then, continuity of $U(y_d; y_p)$ implies $y_\ell \notin y_d^*(y_p)$.

4. Consider $y_p \in (\underline{x}_\ell, y_\ell)$. Then, $y_p \in \text{int} A^*(y_d)$ for all $y_d \leq y_p$. By Lemma 3, $\underline{x}(y_d)$ is continuous and strictly increasing on $[\underline{x}_\ell, y_\ell]$, while $\overline{x}(y_d)$ is continuous and strictly increasing. Thus, there exists $\varepsilon > 0$ such that $P(\underline{x}(y_d))\, u(\underline{x}(y_d), y_p) + [1 - P(\overline{x}(y_d))]\, u(\overline{x}(y_d), y_p)$

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is strictly increasing on $[\underline{x}_\ell, y_p + \varepsilon]$. By assumption, $\frac{\partial u(y_d; y_p)}{\partial y_p} > 0$ if $y_p < y_p$ and $\frac{\partial u(y_d; y_p)}{\partial y_p}\Big|_{y_d = y_p} = 0$. By continuity of $\frac{\partial u(y_d; y_p)}{\partial y_p}$ there exists $y'_d \in (y_p, y_p + \varepsilon)$ such that $U(y_d; y_p)$ is strictly increasing over $y_d \in [\underline{x}_\ell, y'_d)$.

5. Lemma 3 establishes that $\underline{x}(y_d)$ and $\overline{x}(y_d)$ are strictly increasing on $[y_\ell, y_r]$. Therefore $U(y_d; y_p)$ is strictly decreasing on $[y_\ell, y_r]$ if $y_p \leq \underline{x}(y_\ell)$. Thus $\text{argmax}_{y_d \in [y_\ell, y_r]} U(y_d; y_p) = y_\ell$ for all $y_p \leq \underline{x}(y_r)$. But by parts 2 and 3, $y_\ell \in y_d^*(y_p)$ only if $y_p \geq y_r$.

□

Finally, Corollary A2.1 sharpens the characterization for the case with $y_\ell = y_r$.

Corollary A2.1. *If $y_\ell = y_r = y_m$, then (i) $y_p = y_m$ implies $y_d^*(y_p) = y_m$, (ii) $y_p \in (\underline{x}_\ell, y_m)$ implies $y_d^*(y_p) \subset (y_p, y_m)$, and (iii) $y_p < \underline{x}_\ell$ implies $y_d^*(y_p) < y_m$.*

B Quadratic Setting

B.1 Proof of Proposition 1.

First, we establish key properties in Lemmas A3–A7. Lemma A3 characterizes $A^*(y_d)$. Lemma A4 provides a sufficient condition for $U(y_d, y_p)$ to satisfy the single-crossing condition on $S \times X$, where $S \subseteq X$ is an arbitrary interval. Lemma A5 characterizes a compact interval in X for which that condition is satisfied. We use this result in our proofs of Propositions 1 and 5 by showing that $y_p^*(y_p)$ is a subset of this interval and must therefore be increasing. Lemmas A6 and A7 characterize local maxima of $U(y_d, y_p)$ for $y_d \in [\underline{x}_\ell, y_\ell]$ and $y_d \in [y_\ell, y_r]$, respectively. An analogous result for $y_d \in [y_r, \overline{x}_r]$ is omitted. We use these local maxima to characterize $y_d^*(y_p)$ in Proposition 1.

Lemma A3. *If policy utility is quadratic, then:*

$$\underline{x}_\ell = y_\ell - \sqrt{\frac{1 - \delta + \delta \rho_r (y_\ell - y_r)^2}{1 - \delta(\rho_E + \rho_d)}}, \quad and \tag{12}$$

$$\overline{x}_r = y_r + \sqrt{\frac{1 - \delta + \delta \rho_\ell (y_\ell - y_r)^2}{1 - \delta(\rho_E + \rho_d)}}. \tag{13}$$

Furthermore, (i) $\underline{x}$ and $\overline{x}$ are $\mathcal{C}^2$ on $(\underline{x}_\ell, y_\ell) \cup (y_\ell, y_r) \cup (y_r, \overline{x}_r)$; (ii) $\underline{x}(y_d)$ is strictly concave and $\overline{x}(y_d)$ strictly convex on each of those intervals; and (iii) $\frac{\underline{x}'(y_d)}{\overline{x}'(y_d)}$ is strictly decreasing over $y_d \in (y_\ell, y_r)$, with $\frac{\underline{x}'(y_d)}{\overline{x}'(y_d)} = 1$ if and only if $y_d = \frac{\rho_\ell y_\ell + \rho_r y_r}{\rho_\ell + \rho_r} \in (y_\ell, y_r)$.

Proof. Direct computations yield $\underline{x}(y_d) = y_m - \sqrt{\phi(y_d)}$ and $\overline{x}(y_d) = y_m + \sqrt{\phi(y_d)}$ for each $y_d \in (\underline{x}_\ell, \overline{x}_r)$, where:

$$\phi(y_d) = \frac{1 - \delta + \delta \rho_\ell (y_\ell - y_m)^2 + \delta \rho_r (y_r - y_m)^2 + \delta \rho_d (y_d - y_m)^2}{1 - \delta \rho_E}.$$

First, (12) follows from solving $y_d = \underline{x}(y_d)$ for $y_d < y_\ell$ and similarly (13) follows from solving $y_d = \overline{x}(y_d)$ for $y_d > y_r$. Next, $\phi(y_d)$ is a quadratic polynomial with a positive leading coefficient on each of the intervals $(\underline{x}_\ell, y_\ell)$, (y_ℓ, y_r), and $(y_r, \overline{x}_r)$, so it is strictly convex on each interval. Thus, on each interval $\underline{x}$ is strictly concave and $\overline{x}$ is strictly convex. Finally, direct computations yield that $\frac{\underline{x}'(y_d)}{\overline{x}'(y_d)}$ is strictly decreasing over $y_d \in (y_\ell, y_r)$, with $\frac{\underline{x}'(y_d)}{\overline{x}'(y_d)} = 1$ if and only if $y_d = \frac{\rho_\ell y_\ell + \rho_r y_r}{\rho_\ell + \rho_r}$. $\square$

Lemma A4. *Each $y_d \in X$ induces a unique policy lottery, with expectation $\mu_x(y_d)$ and variance $\sigma_x^2(y_d)$. If μ_x is weakly increasing on a compact interval $S \subseteq X$, then $U(y_d, y_p)$ satisfies the single-crossing condition on $S \times X$.*

Proof. Lemma 1 implies $\mu_x(y_d)$ and $\sigma_x^2(y_d)$ for all y_d. If $y_d' \geq y_d$, then

$$U(y_d'; y_p) - U(y_d; y_p) = (\mu_x(y_d) - y_p)^2 - (\mu_x(y_d') - y_p)^2 + \sigma_x^2(y_d) - \sigma_x^2(y_d'),$$

so $\frac{\partial}{\partial y_p} U(y_d'; y_p) - \frac{\partial}{\partial y_p} U(y_d; y_p) \geq 0$ if and only if $\mu_x(y_d') - \mu_x(y_d) \geq 0$. Therefore if μ_x is increasing on a compact interval $S \subseteq X$, then U satisfies increasing differences on $S \times X$, which implies that it has the single-crossing property. $\square$

Lemma A5. *There exist unique $\underline{\pi} \in [\underline{x}_\ell, y_\ell)$ and $\overline{\pi} \in [y_r, \overline{x}_r)$ such that μ_x is strictly increasing on $[\underline{\pi}, \overline{\pi}]$. Additionally, (i) $\max\{\rho_L, \rho_R\} \leq \frac{1}{2\delta}$ implies $\underline{x}_\ell = \underline{\pi} < \overline{\pi} = \overline{x}_r$, (ii) $\rho_R > \frac{1}{2\delta}$ implies $\underline{x}_\ell < \underline{\pi} < \overline{\pi} = \overline{x}_r$, and (iii) $\rho_L > \frac{1}{2\delta}$ implies $\underline{x}_\ell = \underline{\pi} < \overline{\pi} < \overline{x}_r$.*

Proof. For almost all y_d, we have $\mu'_x(y_d) = \rho_d + \rho_L \underline{x}'(y_d) + \rho_R \overline{x}'(y_d)$. First, Lemma 3 implies that $\underline{x}'(y_d) > 0$ and $\overline{x}'(y_d) > 0$ for all $y_d \in (y_\ell, y_r)$, so $\mu'(y_d) > 0$ on (y_ℓ, y_r). Next, $y_d \in (\underline{x}_\ell, y_\ell)$ implies $\underline{x}'(y_d) = -\overline{x}'(y_d)$ is strictly increasing and concave with $\lim_{y_d \to \underline{x}_\ell^+} \underline{x}'(y_d) = \frac{\delta \rho_d}{1 - \delta \rho_E}$ and $\lim_{y_d \to y_\ell^-} \underline{x}'(y_d) = 0$. Thus, $\rho_R \leq \frac{1}{2\delta}$ implies $\mu'(y_d) > 0$ on $(\underline{x}_\ell, y_\ell)$, so $\underline{\pi} = \underline{x}_\ell$; and $\rho_R > \frac{1}{2\delta}$ implies existence of a unique $\underline{\pi} \in (\underline{x}_\ell, y_\ell)$ such that $\mu'(y_d) < 0$ on $(\underline{x}_\ell, \underline{\pi})$ and $\mu'(y_d) > 0$ on $(\underline{\pi}, y_\ell)$. Then, a symmetric argument yields analogous characterization for $\overline{\pi}$. Finally, continuity of $\mu(y_d)$ implies μ_x is strictly increasing on $[\underline{\pi}, \overline{\pi}]$. $\square$

Lemma A6. *The mapping $\hat{y}_d(y_p) \equiv \underset{y_d \in [y_\ell, y_r]}{\arg\max} \, U(y_d; y_p)$ is equivalent to an onto function $\hat{y}_d : X \to [y_\ell, y_r]$ that is continuous and weakly increasing. Furthermore, (i) $\hat{y}_d|_{[y_\ell, y_r]}$ has a unique fixed point y_p^*, (ii) $y_p < y_p^*$ implies $\hat{y}_d(y_p) \in (y_p, y_p^*]$, and (iii) $y_p > y_p^*$ implies $\hat{y}_d(y_p) \in [y_p^*, y_p)$.*

Proof. By Lemmas 3 and A3, for all $y_d \in (y_\ell, y_r)$ we have: $\underline{x}''(y_d) < 0 < \underline{x}'(y_d)$ and $0 < \min\{\overline{x}'(y_d), \overline{x}''(y_d)\}$.

Then, (i) $y_p \geq \overline{x}(y_d)$ implies $\frac{\partial U(y_d; y_p)}{\partial y_d} > 0$, (ii) $y_p \in (\underline{x}(y_d), \overline{x}(y_d))$ implies $\frac{\partial^2 U(y_d; y_p)}{\partial y_d^2} < 0$, and (iii) $y_p \leq \underline{x}(y_d)$ implies $\frac{\partial U(y_d; y_p)}{\partial y_d} < 0$, so $U(y_d; y_p)$ is strictly quasi-concave over $y_d \in [y_\ell, y_r]$ for all $y_p \in X$. Additionally, Lemmas A4 and A5 imply that $U(y_d; y_p)$ satisfies the single-crossing condition on $[y_\ell, y_r] \times X$. Thus, $\hat{y}_d(y_p)$ is single-valued, continuous, increasing, and onto. It follows that $\hat{y}_d|_{[y_\ell, y_r]}$ has a fixed point. To show it is unique, first note that: (i) $\hat{y}_d(y_\ell) = y_\ell$ if and only if $\frac{\partial U(y_d; y_\ell)}{\partial y_d}\big|_{y_d = y_\ell^+} \leq 0$, (ii) $\hat{y}_d(y_r) = y_r$ if and only if $\frac{\partial U(y_d; y_r)}{\partial y_d}\big|_{y_d = y_r^-} \geq 0$, and (iii) and $\hat{y}_d(y_p) = y_p \in (y_\ell, y_r)$ if and only if $\frac{\partial U(y_d; y_p)}{\partial y_d}\big|_{y_d = y_p} = 0$. Define $\lambda : (y_\ell, y_r) \to \mathbb{R}$ as

$$\lambda(y_p) := \frac{\partial U(y_d, y_p)}{\partial y_d}\bigg|_{y_d = y_p} = \rho_L \frac{\partial \underline{x}(y_d)}{\partial y_d}\bigg|_{y_d = y_p} - \rho_R \frac{\partial \overline{x}(y_d)}{\partial y_d}\bigg|_{y_d = y_p}.$$

An interior fixed point exists if and only if $\lambda(y_p) = 0$ for some $y_p \in (y_\ell, y_r)$. Strict concavity

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of $\underline{x}(y_d)$ and strict convexity of $\overline{x}(y_d)$ imply $\lambda'(y_p) < 0$. Therefore $\lambda(y_p) = 0$ at most once, which implies that y_p^* is unique. Furthermore, $y_p^* \in \{y_\ell, y_r\}$ if λ does not change sign, and otherwise $y_p^* \in (y_\ell, y_r)$. Finally, since $\hat{y}_d(y_p)$ weakly increasing and $\lambda'(y_p) < 0$, we know that (i) $y_p < y_p^*$ implies $\hat{y}_d(y_p) \in (y_p, y_p^*]$, and (ii) $y_p > y_p^*$ implies $\hat{y}_d(y_p) \in [y_p^*, y_p)$. $\square$

Define $\tilde{y}_d : [0, y_\ell] \to [\underline{x}_\ell, y_\ell]$ as $\tilde{y}_d(y_p) \equiv \mathrm{argmax}_{y_d \in [\underline{x}_\ell, y_\ell]} U(y_d, y_p)$.

Lemma A7. *$\tilde{y}_d$ is single-valued, continuous, increasing, and satisfies $\tilde{y}_d(y_p) \subset [\underline{\pi}, y_\ell]$ for all y_p. Furthermore, $\tilde{y}_d(y_\ell) = y_\ell$ and otherwise $\tilde{y}_d(y_p) \in (y_p, y_\ell)$. A unique $E_L < \underline{x}_\ell$ exists such that (i) $\tilde{y}_d(y_p) = \underline{x}_\ell$ if $y_p < E_L$ (ii) $\tilde{y}_d(y_p) > \underline{x}_\ell$ if $y_p > E_L$, and (iii) $\tilde{y}_d(y_p)$ is strictly increasing on (E_L, y_ℓ). For $y_p \geq y_r$, analogous properties hold for $\mathrm{argmax}_{y_d \in [y_r, \overline{x}_r]} U(y_d; y_p)$.*

Proof. Fix $y_p \leq y_\ell$. To begin, we show that $\tilde{y}_d(y_p) \subset [\underline{\pi}, y_\ell]$. Lemma A5 implies μ is strictly decreasing on $[\underline{x}_\ell, \underline{\pi}]$ and strictly increasing on $[\underline{\pi}, y_\ell]$. The result is trivial if $\underline{\pi} = \underline{x}_\ell$, so suppose $\underline{\pi} > \underline{x}_\ell$ and consider $y_d, y_d' \in [\underline{x}_\ell, y_\ell]$ satisfying $y_d < y_d'$. Note that

$$U(y_d, y_p) - U(y_d', y_p) = (\mu_x(y_d) - y_p)^2 - (\mu_x(y_d') - y_p)^2 + \sigma_x^2(y_d) - \sigma_x^2(y_d').$$

By Lemma A5, we know that $\underline{\pi} > \underline{x}_\ell$ if and only if $\rho_R > \frac{1}{2\delta}$, which implies $\mu(y_d) > \mu(y_d') > y_r$. Therefore $(\mu_x(y_d) - y_p)^2 - (\mu_x(y_d') - y_p)^2 > 0$. Furthermore, $\sigma_x^2(y_d) > \sigma_x^2(y_d')$ since we have:

$$\begin{aligned}
\sigma_x^2(y_d) - \sigma_x^2(y_d') = {} & \rho_R[(\overline{x}(y_d) - \mu(y_d))^2 - (\overline{x}(y_d') - \mu(y_d'))^2] \\
& + \rho_L[(\underline{x}(y_d) - \mu(y_d))^2 - (\underline{x}(y_d') - \mu(y_d'))^2] \\
& + \rho_d[(y_d - \mu(y_d))^2 - (y_d' - \mu(y_d'))^2] \\
& + \rho_r[(y_r - \mu(y_d))^2 - (y_r - \mu(y_d'))^2] \\
& + \rho_\ell[(y_\ell - \mu(y_d))^2 - (y_\ell - \mu(y_d'))^2],
\end{aligned}$$

where (i) the first term is positive because $\overline{x}(y_d) > \mu(y_d)$ always holds and $\overline{x}'(y_d) - \mu'(y_d) = -\rho_d - \rho_L \underline{x}'(y_d) + (1 - \rho_R)\overline{x}'(y_d) < 0$ for all $y_d \in (\underline{x}_\ell, y_p)$, so $(\overline{x}(y_d) - \mu(y_d))^2 > (\overline{x}(y_d') - \mu(y_d'))^2$; (ii) the second term is positive because $\underline{x}(y_d) < \underline{x}(y_d') < \mu(y_d') \leq \mu(y_d)$; and (iii) the last three terms are positive because $y_d < y_d' \leq y_\ell \leq y_r < \mu(y_d') \leq \mu(y_d)$. Altogether, this implies

$U(y'_d, y_p) > U(y_d, y_p)$. It follows that $U(\overline{\pi}, y_p) > U(y_d, y_p)$ for all $y_d \in [\underline{x}_\ell, \overline{\pi})$. Thus, we have shown $\tilde{y}_d(y_p) = \mathrm{argmax}_{y_d \in [\overline{\pi}, y_\ell]} U(y_d, y_p)$.

Next, Lemmas A4–A5 and the theorem of the maximum imply that $\tilde{y}_d(y_p)$ must be non-empty, upper hemicontinuous, compact valued, and increasing. Furthermore, Lemma A2 implies that $\tilde{y}_d(y_\ell) = y_\ell$ and otherwise $\tilde{y}_d(y_p) \subset (y_p, y_\ell)$. Thus, there is a unique $E_L \equiv \inf\{y_p < \underline{x}_\ell : \tilde{y}_d(y_p) \subset (\underline{x}_\ell, y_\ell)\}$. For all $y_p \in (E_L, y_\ell)$, any $y_d \in \tilde{y}_d(y_p)$ must satisfy

$$\frac{\partial U(y_d; y_p)}{\partial y_d} \propto -\rho_d(y_d - y_p) - \underline{x}'(y_d)[\rho_L(\underline{x}(y_d) - y_p) - \rho_R(\overline{x}(y_d) - y_p)] = 0 \qquad (14)$$

and

$$\frac{\partial^2 U(y_d; y_p)}{\partial y_d^2} \propto -\rho_d - \underline{x}''(y_d)[\rho_L(\underline{x}(y_d) - y_p) - \rho_R(\overline{x}(y_d) - y_p)] - [\underline{x}'(y_d)]^2 \rho_E < 0. \qquad (15)$$

Since $y_p < \tilde{y}_d(y_p)$ and $\underline{x}'(y_d) > 0$, the first-order condition in (14) hold only if $\rho_L(\underline{x}(y_d) - y_p) - \rho_R(\overline{x}(y_d) - y_p) < 0$.

To show $\tilde{y}_d(y_p)$ is unique for all $y_p \in (E_L, y_\ell)$, suppose not and let $y_d, y'_d \in \tilde{y}_d(y_p)$ where $y_d < y'_d$. Then, there must be a $y \in (y_d, y'_d)$ satisfying $\rho_L(\underline{x}(y) - y_p) - \rho_R(\overline{x}(y) - y_p)] = -\left(\frac{\rho_d + \rho_E[\underline{x}'(y)]^2}{\underline{x}''(y)}\right) > 0$. And since $\frac{\partial}{\partial y}[\rho_L(\underline{x}(y) - y_p) - \rho_R(\overline{x}(y) - y_p)] = \underline{x}'(y)\rho_E > 0$, we must also have $\rho_L(\underline{x}(y'_d) - y_p) - \rho_R(\overline{x}(y'_d) - y_p) > 0$. But then (14) fails at y'_d which implies that $y'_d \notin \tilde{y}_d(y_p)$, contradicting our assumption that $y'_d \in \tilde{y}_d(y_p)$. Consequently, $\tilde{y}_d(y_p)$ is unique.

Additionally, $\tilde{y}_d(y_p)$ is strictly increasing on $[E_L, y_p]$ because (i) applying the implicit function theorem yields $\tilde{y}'_d(y_p) > 0$ if $\mu'(y_d) > 0$ at $y_d = \tilde{y}_d(y_p)$, and (ii) Lemma A5 shows that $\mu'(y_d) > 0$ on (π, y_ℓ).

Finally, $\tilde{y}_d(y_p) = \underline{x}_\ell$ for all $y_p < E_L$ because $\tilde{y}_d(y_p)$ is continuous and increasing.

Analogous arguments establish the result for $y_p \geq y_r$. $\square$

Proof of Proposition 1.

By Lemma A2, we know: (i) $y_d < E_L$ implies $y_d^*(y_d) = [0, \underline{x}_\ell]$; (ii) $y_p \in (E_L, y_\ell)$ implies $y_d^*(y_d) = \tilde{y}_d(y_p)$; and (iii) $y_p \in [y_\ell, y_r]$ implies $y_d^*(y_d) = \hat{y}_d(y_p)$. Thus, $y_d \in (\underline{x}(y_\ell), y_\ell)$, implies $y_d^*(y_d) \in \{\tilde{y}_d(y_p), \hat{y}_d(y_p)\}$. Lemmas A6 and A7 imply that $\hat{y}_d(y_p)$ is single-valued and $\tilde{y}_d(y_p)$

is single-valued on $(E_L, y_\ell]$. Additionally, Lemma A7 implies that: (i) $y_d^*(y_\ell) = y_\ell$ if and only if $\tilde{y}_d(y_\ell) = \hat{y}_d(y_\ell) = y_\ell$ and (ii) $\tilde{y}_d(y_p) < \hat{y}_d(y_p)$ for all $y_p < y_\ell$. Thus, there exists a $\underline{y}_p \in (\underline{x}(y_\ell), y_\ell]$ such that $y_d^*(\underline{y}_p) = \{\tilde{y}_p(\underline{y}_p), \hat{y}_p(\underline{y}_p)\}$. To show $\hat{y}_p$ is unique, it suffices to verify $y_d^*(y_p)$ is increasing. Lemma A7 implies

$$
y_d^*(y_p) = \begin{cases} \operatorname{argmax}_{y_d \in [0,1]} U(y_p, y_d) & \text{if} \quad \max\{\rho_L, \rho_R\} \le \frac{1}{2\delta} \\[2mm] \operatorname{argmax}_{y_d \in [\underline{\pi},1]} U(y_p, y_d) & \text{if} \quad \rho_R > \frac{1}{2\delta} \\[2mm] \operatorname{argmax}_{y_d \in [0,\overline{\pi}]} U(y_p, y_d) & \text{if} \quad \rho_L > \frac{1}{2\delta}. \end{cases}
$$

Then, Lemmas A4 and A5 imply $y_d^*(y_p)$ is increasing, so $\underline{y}_p$ is unique. Lemmas A6 and A7 then imply y_d^* is single-valued on $(E_L, y_r] \setminus \underline{y}_p$ and strictly increasing on $(E_L, \underline{y}_p)$. An analogous argument establishes the results for $y_d \ge y_\ell$. $\square$

B.2 Proof of Proposition 2.

First, Lemma A2 implies that any fixed point of y_d^* must be in $[y_\ell, y_r]$, where $y_d^*(y_p) = \hat{y}_d(y_p)$. Second, by Lemma A6: $y_p^*|_{[y_\ell, y_r]}$ has a unique fixed point; $y_p < y_p^*$ implies $\hat{y}_d(y_p) \in (y_p, y_p^*]$; and $y_p > y_p^*$ implies $\hat{y}_d(y_p) \in [y_p^*, y_p)$. Third, by Proposition 1: $y_p \in (E_L, \underline{y}_p)$ implies $y_d^*(y_p) \in (y_p, y_\ell)$; $y_p \in (\underline{y}_p, \overline{y}_p)$ implies $y_d^*(y_p) = \hat{y}_d(y_p)$; and $y_p \in (\overline{y}_p, E_R)$ implies $y_d^*(y_p) \in (y_r, y_p)$. Thus, y_p^* is the unique fixed point of $y_d^*|_{(E_L, E_R)}$. $\square$

B.3 Proof of Proposition 3.

By Lemma A2, (i) $y_p < y_\ell$ implies $y_\ell \notin y_d^*(y_p)$ and (ii) $y_p > y_r$ implies $y_r \notin y_d^*(y_p)$. Then, since $y_d^*(y_p)$ is increasing, (i) $\underline{y}_p = y_\ell$ if and only if $y_p^* = y_\ell$ and (ii) $\overline{y}_p = y_r$ if and only if $y_p^* = y_\ell$. Therefore uniqueness of y_p^* implies $y_r \in \Delta$ or $y_\ell \in \Delta$. The characterization using λ follows directly from the characterization of y_p^* in Lemma A6. $\square$

B.4 Proof of Proposition 4.

Fix $\rho_E \equiv \rho_L + \rho_R$. Thus, when we refer to increasing ρ_L throughout the proof, we are implicitly decreasing ρ_R by the same amount. Before proceeding, note that since ρ_E is constant, $A^*(y_d)$ is constant. Therefore $\frac{\partial \lambda(y_d)}{\partial \rho_L} - \frac{\partial \lambda(y_d)}{\partial \rho_R} = \frac{\partial \underline{x}(y_d)}{\partial y_d} + \frac{\partial \overline{x}(y_d)}{\partial y_d} > 0$ for all $y_d \in (y_\ell, y_r)$.

1. Since $\frac{\partial \lambda(y_d)}{\partial \rho_L} - \frac{\partial \lambda(y_d)}{\partial \rho_R} > 0$ for all $y_d \in (y_\ell, y_r)$, we know (i) $\lambda(y_\ell) \leq 0$ implies $y_p^* = y_\ell$, (ii) $\lambda(y_r) \geq 0$ implies $y_p^* = y_r$, and (iii) otherwise $\lambda(y_p^*) = 0$ at $y_p^* \in (y_\ell, y_r)$. Thus, y_p^* is weakly increasing.

2. From Lemma A7, E_L is the smallest $y_p < \underline{x}_\ell$ such that $\left. \frac{\partial U(y_d, E_L)}{\partial y_d} \right|_{y_d = \underline{x}_\ell^+} \geq 0$. Then, computation yields $\left. \frac{\partial^2 U(y_d, E_L)}{\partial y_d \partial \rho_L} \right|_{y_d = \underline{x}_\ell^+} - \left. \frac{\partial^2 U(y_d, E_L)}{\partial y_d \partial \rho_R} \right|_{y_d = \underline{x}_\ell^+} = -\frac{\delta \rho_d [(\underline{x}_\ell - E_L) + (\overline{x}_r - E_L)]}{1 - \delta \rho_E} < 0$, so E_L weakly increases in ρ_L. By an analogous argument, E_R weakly increases in ρ_L.

3. Recall that $\underline{y}_p = y_\ell$ if and only if $y_\ell = y^*$. Since $\frac{\partial \lambda(y_d)}{\partial \rho_L} - \frac{\partial \lambda(y_d)}{\partial \rho_R} > 0$ for all $y_d \in (y_\ell, y_r)$, there is a unique ρ_L' such that $y^* = y_\ell$ if and only if $\rho_L \leq \rho_L'$. Thus $\underline{y}_p = y_\ell$ for all $\rho_L \leq \rho_L'$ and $\underline{y}_p < y_\ell$ otherwise. To complete the proof, we show that $\underline{y}_p$ is decreasing on $\rho_L \in (\rho_L', 1)$. Then, $\underline{y}_p$ is the unique $y_p < y_\ell$ satisfying

$$0 = U(\hat{y}, y_p) - U(\tilde{y}, y_p) \tag{16}$$
$$= \rho_L \left(-(\underline{x}(\hat{y}) - y_p)^2 + (\underline{x}(\tilde{y}) - y_p)^2 \right) + \rho_R \left(-(\overline{x}(\hat{y}) - y_p)^2 + (\overline{x}(\tilde{y} - y_p)^2 \right)$$
$$+ \rho_d \left(-(\hat{y} - y_p)^2 + (\tilde{y} - y_p)^2 \right), \tag{17}$$

where (17) follows by definition. Moreover, $U(\hat{y}, y_p) - U(\tilde{y}, y_p) > 0$ for all $y_p \in (\underline{y}_p, y_\ell)$ and $U(\hat{y}, y_p) - U(\tilde{y}, y_p) < 0$ for all $y_p \in (\underline{x}_\ell, \underline{y}_p)$. Therefore $U(\hat{y}, y_p) - U(\tilde{y}, y_p)$ is increasing at $y_p = \underline{y}_p$. So $\underline{y}_p$ must be decreasing in ρ_L if $U(\hat{y}, y_p) - U(\tilde{y}, y_p)$ is increasing in ρ_L at $y_p = \underline{y}_p$.

Letting $\xi(y) = \frac{\partial U(y, y_p)}{\partial \rho_L} - \frac{\partial U(y, y_p)}{\partial \rho_R}$, the envelope theorem implies

$$\xi(\hat{y}) - \xi(\tilde{y}) = (\underline{x}(\tilde{y}) - y_p)^2 - (\underline{x}(\hat{y}) - y_p)^2 + (\overline{x}(\hat{y}) - y_p)^2 - (\overline{x}(\tilde{y}) - y_p)^2. \tag{18}$$

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To show a contradiction, suppose $\xi(\hat{y}) - \xi(\tilde{y}) < 0$. Then, we must have

$$0 = U(\hat{y}, y_p) - U(\tilde{y}, y_p)$$

$$< \rho_d[-(\hat{y} - y_p)^2 + (\tilde{y} - y_p)^2] + \rho_E[-(\underline{x}(\hat{y}) - y_p)^2 + (\underline{x}(\tilde{y}) - y_p)^2], \qquad (19)$$

where (19) follows from (17) because (i) $U(\hat{y}, y_p) - U(\tilde{y}, y_p)$ is increasing in $(\overline{x}(\tilde{y}) - y_p)^2$, and (ii) $0 > \xi(\hat{y}) - \xi(\tilde{y})$ implies $(\overline{x}(\tilde{y}) - y_p)^2 > (\underline{x}(\tilde{y}) - y_p)^2 - (\underline{x}(\hat{y}) - y_p)^2 + (\overline{x}(\hat{y}) - y_p)^2$. Thus, since adding and subtracting $\rho_R[-(\underline{x}(\hat{y}) - y_p)^2 + (\underline{x}(\tilde{y}) - y_p)^2]$ in (17) yields

$$U(\hat{y}, y_p) - U(\tilde{y}, y_p) = \rho_R[-(\overline{x}(\hat{y}) - y_p)^2 + (\overline{x}(\tilde{y}) - y_p)^2 + (\underline{x}(\hat{y}) - y_p)^2 - (\underline{x}(\tilde{y}) - y_p)^2]$$

$$+ \rho_d[-(\hat{y} - y_p)^2 + (\tilde{y} - y_p)^2] + \rho_E[-(\underline{x}(\hat{y}) - y_p)^2 + (\underline{x}(\tilde{y}) - y_p)^2],$$

we know (19) holds if and only if

$$0 > \rho_R[-(\overline{x}(\hat{y}) - y_p)^2 + (\overline{x}(\tilde{y}) - y_p)^2 + (\underline{x}(\hat{y}) - y_p)^2 - (\underline{x}(\tilde{y}) - y_p)^2]$$

$$\propto \xi(\tilde{y}) - \xi(\hat{y}).$$

Therefore we must have $\xi(\hat{y}) - \xi(\tilde{y}) < 0 < \xi(\hat{y}) - \xi(\tilde{y})$, a contradiction. $\square$

C Extensions

C.1 Proof of Proposition 5.

By Lemma A5, $\max\{\rho_L, \rho_R\} \le \frac{1}{2\delta}$ implies μ_x is increasing on X. Thus, Lemma A4 implies $U(y_d, y_p)$ satisfies the single-crossing property on X^2. $\square$

C.2 Proof of Proposition 6.

From the main text, $y_d^*(y_p)\big|_{(E_L, E_R)} = (1 - \delta\rho_E)y_p + \delta\rho_E y_M$. Applying the envelope theorem yields $\nu'(y_p)\big|_{(\underline{x}_\ell, y_\ell)} = \frac{-(\delta\rho_E)^2 \rho_d(y_m - y_p)}{1 - \delta\rho_E} < 0$ and $\nu'(y_p)\big|_{(E_L, \underline{x}_\ell)} = \rho_d(y_d^*(y_p) - \underline{x}_\ell) > 0$. Thus, the result follows from continuity of ν. Analogously, ν strictly increases on $[y_m, \overline{x}_r]$ and strictly

decreases on $[\overline{x}_r, E_R]$. □

C.3 Proof of Lemma 4.

Direct computations show $\frac{\partial^2 x(y_a, y_b)}{\partial y_a \partial y_b} < 0$. Using this fact, it is straightforward to sign $\frac{\partial y_a(y_b)}{\partial y_b}$ by applying the implicit function theorem to (9). The result for $y_b(y_a)$ is analogous. □

C.4 Proof of Proposition 7.

Proposition 1 implies $y_a^* \in (y_{p_a}, y_M)$ and $y_b^* \in (y_M, y_{p_b})$. Lemma 4 implies uniqueness. □

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