# Nominating Legislative Representatives*

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**Abstract**

Which candidates do parties nominate to run for legislative office? I analyze a game-theoretic model where parties balance two concerns: winning elections and influencing legislative policy. Unless agenda power is concentrated among centrists, party incentives differ from canonical electoral settings where legislative considerations are absent. Parties strategically prefer more centrist representatives, even without electoral incentives, because of their anticipated effect on equilibrium policy. Such representatives narrow the set of passable policies, indirectly constraining the party's ideological opponents in the legislature. As usual, elections encourage parties to skew candidates towards their opponent, complementing the moderation incentive under empirically prevalent conditions. In more polarized legislatures, majority-party incumbents win re-election less often because challenger parties nominate more competitive candidates. Stronger majority-party agenda control has similar consequences.

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In democracies, many political offices are filled through elections and require officeholders to work together in well-established institutions. It is well known that collective policymaking is sensitive to these institutional details, along with politician attributes and potentially the broader political environment.[1] When evaluating a potential representative, sophisticated political actors are thus likely to account for how the representative's preferences interact with broader legislature-level considerations.

This observation raises basic questions about preferences over a representative's ideology, how they compare to fundamental policy preferences, and how they depend on legislative conditions such as polarization or agenda rights. To pursue these questions, I explicitly account for the reality that individual representatives lack full control over legislative policy. Then, building upon this foundation, I explore various electoral consequences. How do legislative conditions shape which candidates are nominated? In turn, how do those conditions affect electoral regularities such as reelection rates?

I combine ingredients from canonical models of legislative policymaking and electoral competition. Specifically, I analyze a game-theoretic model where policy-motivated parties choose candidates for a legislative position. The model is rich enough to disentangle several legislative fundamentals, but tractable enough to characterize how those fundamentals shape policymaking and, in turn, candidate selection.

The electoral stage accommodates two widely noted features from US congressional elections. First, parties often wield substantial influence over district-level nominations (Desmarais, La Raja and Kowal, 2015; Hassell, 2016). Second, to curry favor with district interests, parties can support candidates who are quite distinct from national party platforms (McCarty, Poole and Rosenthal, 2006).

The legislative interaction is a non-cooperative multilateral bargaining game that descends from Romer and Rosenthal (1978) and Baron and Ferejohn (1989), in the tradition of Banks and Duggan (2000a, 2006). There are four key ingredients. First, legislators strategically craft policy whenever they control the agenda. Second, policymaking continues until a proposal receives sufficient support to pass, with status quo policy persisting until agreement. Third, legislators are forward-looking and anticipate subsequent legislative interaction following failed proposals. Fourth, the identity of future proposers is not perfectly predictable.

The analysis yields four broad takeaways. First, I provide comparative statics on the electoral consequences of several legislative conditions, including the extent of po-

[1]For recent overviews, see Diermeier (2014); Clark and Gandhi (2015) and Svolik (2015).

larization or majority-party agenda power. Second, these comparative statics arise because a representative's ideology has equilibrium spillover effects, which are driven by other legislators anticipating strategic legislative interaction. Third, these spillovers cause party preferences over representatives to diverge from fundamental policy preferences and produce a preference for moderation. Finally, a technical takeaway is that, even with single-peaked policy preferences, strategic legislative considerations can make preferences over representatives multi-peaked.

I now elaborate on these main contributions.

*Electoral consequences of legislative considerations:* Unless agenda control is sufficiently concentrated on centrist legislators, incentives differ from canonical electoral models. Candidates must appeal to voters in their district to be competitive, but party members also want candidates who favorably influence policymaking if elected. I show that electoral and legislative forces are complementary under empirically prevalent conditions, pulling optimal candidates towards both the legislative center and the opposing candidate.

I also characterize how several political conditions affect electoral outcomes and behavior. First, strengthening majority-party agenda power leads the minority party to nominate more competitive candidates in majority-incumbent districts. This effect reduces majority-party re-election rates and the ideological gap between candidates in those districts. In contrast, the majority party nominates less competitive candidates in minority-incumbent districts. Thus, the ideological gap grows in these districts as well, but majority-party election rates decrease with majority-party strength. Overall, stronger majority-party agenda power decreases majority-party candidate election rates, but the ideological gap can increase or decrease depending on the incumbent's party affiliation.

Greater polarization has similar effects, as does more patient legislators. More extreme status quo policies produce more competitive challengers regardless of majority status, decreasing re-election rates and the ideological gap. These results arises from party incentives and none requires overly sophisticated voters.

*Microfoundation for spillover effects of representative's ideology:* The electoral consequences of legislative considerations emerge because the representative's ideology has equilibrium spillover effects. It can directly affect the representative's own proposals, of course. But it can also indirectly affect proposals by other legislators.

The intuition is as follows. Because legislators are forward-looking, they account for expected future policy proposals when deciding whether to support policy today. Anticipated proposals thus enter each legislator's expectations. These expectations determine which policies each legislator is willing to vote for and therefore pin down which policies can pass. Shifting the set of passable policies then changes proposals of ideologically extreme legislators who are constrained by legislative voting.

This microfoundation is useful for several reasons. It reveals how spillovers can arise from well-studied strategic considerations even if legislators have no intrinsic concern about each other's ideology. Furthermore, it helps unpack how these spillovers depend on various legislative conditions. For example, their direction and magnitude depend on where the representative lies in the ideological distribution of all legislators. They also depend on legislative fundamentals such as legislative polarization, party strength, and status quo policy. Finally, it allows us to study several electoral consequences that these legislative conditions have through their effect on preferences over candidates.

*Characterizing preferences over representatives:* Independent of electoral considerations, spillover effects cause preferences over representatives to diverge endogenously from primitive policy preferences. Furthermore, they can shift optimal representatives. Broadly, the characterization has two main insights: parties have a general preference for moderation and their preferences over representatives can be multi-peaked. I now elaborate on each.

First, the endogenous spillovers can create a tradeoff in how ideologically similar a party wants to be with its representative. A representative close to the party proposes favorable policy if recognized, benefiting the party. But such a representative may also indirectly enable other legislators to enact less favorable policies.

The second effect can dominates so that parties do not want a doppelgänger representative. I show that parties prefer representatives skewed weakly towards the center. Moreover, parties that are not too extreme have uniquely optimal representatives, who are strictly more centrist under broad conditions.

Incentives for strategic moderation arise because forward-looking legislators bargain over spatial policy, active bargaining continues after rejected proposals, agenda control is unpredictable, and centrist legislators retain effective veto power throughout. Intertemporal considerations create a moderation incentive similar to that of dynamic, spatial legislative settings with endogenous status quo (Baron, 1996; Forand, 2014; Buisseret and Bernhardt, 2017; Zápal, 2014). There, bargaining continues after successful proposals and proposers sometimes skew policy towards a persistent median to constrain future proposers, who may be ideologically distant. Here, bargaining ends after any proposal passes and parties use more centrist representatives as a commitment device to indirectly constrain extreme legislators.

Second, I show that preferences over representatives can be multi-peaked even though policy preferences are single-peaked. Thus, aggregating preferences over representatives can be difficult even if fundamental policy preferences satisfy canonical conditions favorable for preference aggregation over policy. Moreover, I characterize legislative conditions producing violations of weak single-peakedness to highlight when we should expect such difficulties.

*Theoretical contributions:* There are several theoretical contributions. First, I further efforts aimed at endogenizing the set of legislators in legislative bargaining models. Second, in a model of district-level electoral competition, I provide legislative conditions sufficient to ensure existence of pure strategy equilibria and characterize equilibrium nominees. Finally, I highlight sufficient conditions for electoral competition featuring legislative considerations to coincide with canonical models without legislative considerations. The conditions are precisely those under which spillover effects are inconsequential. Beyond their theoretical interest, these conditions are also relevant for empirical work using legislative election data to evaluate implications from canonical models of electoral competition (e.g., Fowler and Hall, 2016).

## Related Literature

*Legislative Elections:* Many scholars have modeled electoral competition. To apply their insights to elections where legislative forces are important, a key consideration is how they allow for potential spillover effects. Scholars make varying assumptions to structure on how policy outputs depend on legislators, institutions, and conditions.

The simplest approach is to abstract from legislative considerations altogether. In these models, relevant outputs depend only on the elected legislator. Canonical models of electoral competition fall in this class (e.g., Downs, 1957; Wittman, 1983; Calvert, 1985). They focus on first-order considerations about candidate characteristics, abstracting from potential spillovers generated by legislature-level conditions.

To incorporate considerations beyond an isolated election, models of competition in legislative elections date to Hinich and Ordeshook (1974) and Austen-Smith (1981). Typically, parties choose platforms that apply to all districts and there is no explicit post-election bargaining over policy. Austen-Smith (1984) gives each candidate full discretion over her platform. He studies a two-party multi-district model and assumes candidate platforms are aggregated within each party to determine their respective legislative platforms.[2] Callander (2005) considers a similar model to study candidate entry and deterrence. Unlike the policy-motivated parties in this paper, parties and candidates are office motivated in these papers.

Closer to this paper are several studies explicitly modeling the legislative interaction.[3] Klumpp (2007) studies a single-member district election game where legislators play a version of Romer and Rosenthal (1978) where each legislator has equal proposal probability. When selecting candidates, parties forecast the distribution of policy based on the full legislator preference profile, status quo, and distribution of agenda power. The game reduces to each district's median voter choosing her representative's ideology. They prefer delegating to status quo biased representatives.

Unlike Klumpp (2007), I focus on candidate selection in one district, fixing the rest of the legislature. Furthermore, I model a richer legislative setting, as recognition probability can vary across legislators and forward-looking legislators anticipate future bargaining after rejected proposals. There is no status quo bias here, but instead incentives to skew towards the center.[4]

Another approach uses reduced-form functions mapping preference profiles to legislative outputs. An example is Krasa and Polborn (2018), which, like this paper, studies district-level party competition in single-member, majoritarian district elections with policy-motivated parties. In each district, voters derive utility from the platform of their representative and the national platform of the legislature's majority party. National conditions may advantage one party at the district level, producing more extreme candidates. They allow national platforms to depend on elected legislator

[2]Also see Austen-Smith (1986).

[3]I focus on plurality-rule elections, but there are also studies of elections and legislative bargaining in proportional representation electoral systems (e.g. Austen-Smith and Banks, 1988; Baron, 1993; Baron and Diermeier, 2001; Cho, 2014). Baron, Diermeier and Fong (2012) extends the analysis to a dynamic setting.

[4]An extension in Beath, Christia, Egorov and Enikolopov (2016) also studies a model where multiple districts each elect a representative to serve in a majoritarian legislature. In contrast to this paper, districts are ordered ideologically and partition the policy space so that districts are constrained with respect to their representative's ideology. Legislators are forward looking, as in this paper, but they only analyze arbitrarily patient players, unlike this paper. Altogether, these features ensure that only the median district(s) influences policy. The main focus of Beath et al. (2016) is to study how electoral rules affect the tradeoff between competence and ideology that voters often face when electing representatives. They provide evidence from a field experiment in Afghanistan supporting their results.

ideologies, but do not explicitly model legislative policymaking.

My analysis complements theirs. They consider a coarser legislative setting, but model a more complex electoral environment with simultaneous elections in multiple districts. To unpack the effects of specific legislative features, I study a richer legislature that is fixed outside of one legislator's ideology. Additionally, they model sophisticated voters who account for both local and national considerations. Here, voters do not directly account for legislature-level conditions and simply compare candidate ideologies. Thus, sophisticated parties drive the results in this paper.

*Delegation:* Analyzing preferences over representatives contributes to studies of delegation to agents serving in a collective body (Persson and Tabellini, 1992; Besley and Coate, 2003; Loeper, 2015). Specifically, it relates to delegation models where agents participate in policymaking under particular bargaining protocols.

A classic result is the *ally principle*: principals prefer to delegate to agents sharing their ideological preferences (Ferejohn and Shipan, 1990; Epstein and O'Halloran, 1999; Bendor and Meirowitz, 2004).[5] Violations of the ally principle have been shown in various delegation settings (e.g., Bendor and Meirowitz, 2004).[6] I add to this body of results by identifying a new setting where it fails, pinning down when it fails, and describing the nature of violations. Here, legislative considerations produce a preference for more centrist representatives. This result contrasts with Gailmard and Hammond (2011), where violations of the ally principle are always towards the status quo.

The analysis of preferences over representatives here complements Patty and Penn (2019), which characterizes preferences over a representative's voting behavior on an exogenous agenda. The two models consider different types of principals with different legislative interests. Patty and Penn (2019) captures citizens who focus on roll call records, whereas I study party elites concerned with policy specifics. Developing the implications of both settings is worthwhile. In their setting, the ally principle always holds, but induced preferences are asymmetric and favor extremism. Given two representatives equidistant from the principal, she prefers the more extreme option. In contrast, I focus on players evaluating their representative's effect on policy outputs in a legislature where the agenda arises from equilibrium play. Here, the ally principle can fail and I find a preference for moderation rather than extremism.

[5] Also see, e.g., Bendor et al. (2001); Jo (2010).

[6] Also see, e.g., Brueckner (2000); Klumpp (2007); Harstad (2010); Gailmard and Patty (2012) and Christiansen (2013).

# Model of Legislative Policymaking

I first describe policymaking in a fixed legislature, deferring analysis of electoral considerations and preferences over representatives. The setting follows canonical game-theoretic models of legislative bargaining. Specifically, the legislative model is a streamlined descendant of Banks and Duggan (2006) that facilitates the subsequent analysis of legislative candidates while preserving qualitatively important features.

To parsimoniously reflect a larger legislative body, there is a centrist legislator $m$, a left-leaning legislator $L$, and a right-leaning legislator $R$. Each player $i$ is associated with ideal point $\hat{x}_i$ in a convex policy space $X \subset \mathbb{R}$, where $\hat{x}_L < \hat{x}_m < \hat{x}_R$. Throughout the analysis, I normalize $\hat{x}_m = 0$. $L$ and $R$ are modeled to represent ideologically extreme members of each party and, for brevity, I refer to them as *partisans*. Additionally, there is a legislator who represents a legislative district $d$. This representative has ideal point $\hat{x}_d \in X$. We take $\hat{x}_d$ as fixed for now, implicitly arising from an unmodeled election. Later in the analysis, $\hat{x}_d$ will arise endogenously from parties' candidate selection decisions and electoral outcomes.

Legislators bargain to set a collective policy until a proposal passes. Policymaking occurs over an infinite horizon, with periods discrete and indexed by $t \in \{1, 2, \dots\}$. In each period $t$, bargaining proceeds as follows. If no policy has passed before $t$, then each legislator $i$ is recognized as the period-$t$ proposer with probability $\rho_i \in (0, 1)$. The distribution of proposal power is $\rho = (\rho_m, \rho_d, \rho_L, \rho_R)$, where the individual probabilities sum to 1.[7] The recognized legislator proposes a policy $x \in X$. All legislators observe the proposal, and the centrist legislator, $m$, chooses whether to accept or reject. If $m$ accepts, then $x$ passes and bargaining ends with $x$ enacted in period $t$ and all subsequent periods. If $m$ rejects, then the status quo $q \in \mathbb{R}$ is enacted in period $t$ and bargaining proceeds to period $t + 1$.[8]

Players are purely policy motivated and weight future payoffs by the common dis-

[7]The random agenda protocol can represent a reduced-form representation of competition over agenda control within each period, as proposal opportunities are always valuable in the moment and will be sought by all. Asymmetries could result from institutional restrictions, such as seniority or committee rank, or other exogenous power imbalances affecting each legislator's capacity to compete for agenda control. See McCarty (2000) and Kalandrakis (2006) for more thorough discussions.

[8]In the stated setup, $m$'s voting power has the spirit of a model in which a larger legislative body uses majority rule and $m$ is the legislative median. Although the median legislator could vary with $\hat{x}_d$ in this alternative setup, it is a robust statistic in large legislatures. When analyzing optimal representatives and candidates in such settings, incentives to move the median are thus less central than when delegating to smaller collective bodies, such as courts (See, e.g., Krehbiel, 2007; Cameron and Kastellec, 2016). I abstract from this incentive.

count factor $\delta \in (0,1)$. For convenience, I normalize stage payoffs by $(1-\delta)$. If policy proposal $x_t \in X$ passes in $t$, then player $i$'s payoff is

$$(1-\delta^{t-1})u_i(q) + \delta^{t-1}u_i(x_t), \tag{1}$$

where $u_i(x) = -|\hat{x}_i - x|$.[9]

This stylized environment features formal rules seen in practice, including closed agenda and majority voting. Although sparse, it permits rich strategic interaction, as legislators endogenously formulate proposals and make voting decisions. Its sparsity allows for flexible interpretation.[10] For example, legislators can be interpreted as individuals on relevant committees, or homogeneous blocs. Additionally, the one-dimensional policy space can be interpreted as ideological, regulation levels, a tax rate, or a collective policy with externalities.

## Equilibrium Policymaking

The equilibrium concept in the legislative subgame closely follows Banks and Duggan (2006). To preview, there is an interval of passable policies symmetric about $m$'s ideal point. Legislators in this interval propose their ideal point and those outside propose the nearest boundary. Thus, in equilibrium, proposals always pass and bargaining ends immediately. But the prospect of continued bargaining affects which policies can pass and, in turn, can indirectly affect proposals.

In principle, legislators can use intricate strategies during policymaking, perhaps conditioning behavior on previous play. Thus, I impose standard refinements from the legislative bargaining literature to study a selection of the institution's subgame perfect equilibria. First, I focus on stationary legislative strategies.[11] For each legislator, a stationary proposal strategy specifies that she proposes the same policy in any legislative period she is recognized. Additionally, $m$'s stationary voting strategy specifies whether she votes for any policy $x \in X$ in each legislative period. This strategy induces an *acceptance set*, which is the set of policies that pass. By stationarity, the acceptance set is constant over time. The second refinement is that legislators use no-delay proposal

[9]Many results are qualitatively similar if players have other canonical forms of policy utility, such as quadratic. I discuss this alternative after the analysis, in the Model Commentary.

[10]Various interpretations, and the scope of application, have been thoroughly discussed elsewhere. See, e.g., Baron and Ferejohn (1989); Baron (1991); Banks and Duggan (2000b); McCarty (2000); Kalandrakis (2006).

[11]See Baron and Kalai (1993) for discussion of stationarity in legislative bargaining games.

strategies, which specify only acceptable policy proposals.[12]

Informally, legislative strategies in any stationary subgame perfect equilibrium must be such that (i) proposal strategies are optimal given $m$'s acceptance set and (ii) $m$'s acceptance set is optimal given her expectations about future legislative behavior prescribed under the strategy profile.[13] For brevity, I refer to *equilibria* and leave the refinements implicit.

Equilibrium existence and upper-hemicontinuity follow from Banks and Duggan (2006) and uniqueness follows from Cardona and Ponsati (2011). Uniqueness and upper-hemicontinuity together imply the equilibrium is continuous in legislative parameters, including $\hat{x}_d$. Lemma 1 states these observations.

**Lemma 1.** *For every ideology of the district $d$ representative, $\hat{x}_d$, there is a unique equilibrium. Furthermore, the equilibrium is continuous with respect to $\hat{x}_d$.*

Given the district-$d$ representative's ideology, $\hat{x}_d$, known results provide a sharp characterization of equilibrium legislative behavior (Banks and Duggan, 2006). The *acceptance set* is denoted $A^*(\hat{x}_d)$ to make explicit the dependence on $\hat{x}_d$. Because $X$ is one-dimensional and players have absolute-value policy utility, $A^*(\hat{x}_d)$ is a closed interval symmetric that is about 0. Legislators use pure proposal strategies, as each in $A^*(\hat{x}_d)$ proposes her ideal point and each outside proposes the policy in $A^*(\hat{x}_d)$ closest to her ideal point.

To illustrate with an example, Figure 1 displays equilibrium behavior in a hypothetical legislature. The representative proposes and passes her ideal point if recognized because $\hat{x}_d \in A^*(\hat{x}_d)$.[14] The upper bound of $A^*(\hat{x}_d)$ in Figure 1 is

$$\overline{x}^*(\hat{x}_d) = -\frac{(1-\delta)\, u_m(q) + \delta\, \rho_d\, u_m(\hat{x}_d)}{1 - \delta\,(\rho_L + \rho_R)}. \tag{2}$$

Symmetry of $u_m$ implies the lower bound is $-\overline{x}^*(\hat{x}_d)$. Thus, $A^*(\hat{x}_d) = [-\overline{x}^*(\hat{x}_d), \overline{x}^*(\hat{x}_d)]$.

Throughout the analysis, I assume that the status quo, $q$, is closer to 0 than each partisan. Formally, $\min\{\hat{x}_R, -\hat{x}_L\} > |q|$. This assumption ensures $L$ and $R$ are always

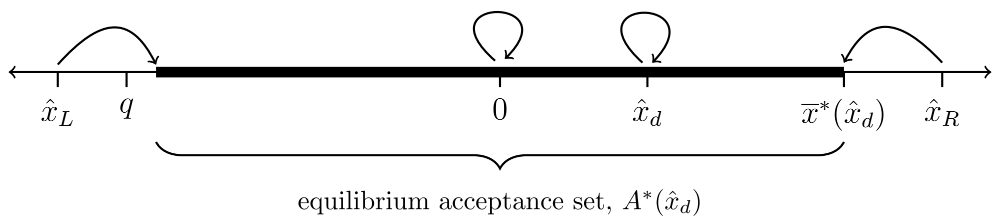
[12]Focusing on no-delay proposal strategies is without loss of generality, as it is straightforward to verify that $\delta > 0$ and $\min\{\rho_R, \rho_L\} > 0$ ensure that every stationary SPE of the legislative subgame is no-delay. Alternatively, if players have strictly concave policy utility, then all stationary SPE are no-delay quite generally (Banks and Duggan, 2006).

[13]See Banks and Duggan (2006) for a formal description and further discussion.

[14]In the next section, Lemma 2 shows existence of $\overline{x} > 0$ such that $\hat{x}_d \in A^*(\hat{x}_d)$ if and only if $\hat{x}_d \in [-\overline{x}, \overline{x}]$.

constrained by legislative voting in equilibrium, so that neither can pass its ideal policy. Specifically, $L$ proposes $-\overline{x}^*(\hat{x}_d)$ and $R$ proposes $\overline{x}^*(\hat{x}_d)$ for all $\hat{x}_d \in X$.

Figure 1: Characterization of equilibrium proposals




Note: Figure 1 depicts equilibrium legislative proposals for a hypothetical legislature. Arrows point from legislators to proposals if recognized. The equilibrium acceptance set is the bold interval. Each legislator proposes the acceptable policy closest to her ideal point.

Uniqueness pins down each player's expectations about the legislative subgame as a function of $\hat{x}_d$. Player $i$'s expected equilibrium payoff from a representative with ideal point $\hat{x}_d$, denoted $\mathcal{U}_i(\hat{x}_d)$, is the weighted sum of $i$'s utility from each legislator's equilibrium proposal, using recognition probabilities as weights. For the example in Figure 1,

$$\mathcal{U}_i(\hat{x}_d) = \rho_d\, u_i(\hat{x}_d) + \rho_m\, u_i(0) + \rho_L\, u_i(-\overline{x}^*(\hat{x}_d)) + \rho_R\, u_i(\overline{x}^*(\hat{x}_d)). \tag{3}$$

# Preferences over Representatives

With a grasp on legislative play, I analyze preferences over the representative's ideology, $\hat{x}_d$. These preferences depend on legislative conditions and can differ from fundamental policy preferences. I provide a general characterization revealing an endogenous preference for moderation. This section continues to abstract from electoral concerns, but lays the foundation for the subsequent electoral analysis.

The analysis focuses on a policy-motivated party, $j$. I characterize $j$'s expected equilibrium payoff, $\mathcal{U}_j$, as a function of $\hat{x}_d$. As displayed in (3), $\mathcal{U}_j$ depends directly on anticipated proposals and $\rho$, the distribution of proposal power. These features, along with the status quo $q$ and legislator patience $\delta$, also enter $\mathcal{U}_j$ through their effect on $\overline{x}^*(\hat{x}_d)$ shown in (2).

Varying $\hat{x}_d$ can alter the proposals of several legislators: the representative, of course, but also $L$ and $R$ through changes in $\overline{x}^*(\hat{x}_d)$. Representatives who induce the same equilibrium proposals are equivalent. Lemma 2 provides a partition on $\hat{x}_d$ identifying which representatives are equivalent and, alternatively, where changes in $\hat{x}_d$ matter. All proofs are in the appendix.

**Lemma 2.** *There exists $\overline{x}$ satisfying $0 < \overline{x} < |q|$ such that if the representative's ideal point is $\hat{x}_d \in (-\overline{x}, \overline{x})$, then the equilibrium acceptance set is $[-\overline{x}^*(\hat{x}_d), \overline{x}^*(\hat{x}_d)]$. Otherwise, it is $[-\overline{x}, \overline{x}]$.*

Using Lemma 2, I define terminology to categorize player ideologies independently of equilibrium play.

**Definition 1.** Player $i$ is an *ideologue* if $\hat{x}_i \notin (-\overline{x}, \overline{x})$. Otherwise, $i$ is a *non-ideologue.*

Lemma 2 has two key implications. First, all ideologue representatives induce the same acceptance set, which they are outside of. Second, all non-ideologue representatives are inside the acceptance set they induce, the boundaries of which depend on the representative's ideology.

Building on Lemma 2, Proposition 1 characterizes general features of $\mathcal{U}_j$. Say that party $j$ and its representative are *aligned* if and only if their respective ideal points are on the same side of the centrist legislator, $\hat{x}_m = 0$.

**Proposition 1.** *Party $j$'s preferences over representatives are (i) differentiable almost everywhere and continuous; (ii) constant over $j$'s aligned ideologues and non-aligned ideologues, respectively; (iii) single-peaked over $j$'s aligned and non-aligned representatives, respectively; and (iv) maximized by a representative aligned with $j$.*

All ideologues on the same side of 0 are equivalent. They are constrained by majority voting in the same fashion, propose identical policy, and thus induce the same equilibrium outcome distribution. But it is not true that all ideologues are necessarily equivalent, as ideologues on opposite sides of 0 propose different policies.

Next, I describe qualitative features of preferences over non-ideologue representatives. To ease discussion and align with the electoral analysis presented subsequently, suppose $j$ is a left-leaning ideologue party.[15] Thus, $j$ is always left of all equilibrium proposals. As $\hat{x}_d$ increases over $(-\overline{x}, \overline{x})$, the overall effect on $j$'s expected payoff is

$$\frac{\partial \mathcal{U}_j(\hat{x}_d)}{\partial \hat{x}_d} = -\rho_d + \frac{\overline{x}^*(\hat{x}_d)}{\partial \hat{x}_d}(\rho_L - \rho_R). \tag{4}$$

[15] Formally, assume $\hat{x}_j \leq -\overline{x}$, which implies $\hat{x}_j \leq -\overline{x}^*(\hat{x}_d)$ for all $\hat{x}_d \in X$. Key forces are not specific to this case. The appendix contains a full analysis.

The direct effect, $-\rho_d < 0$, is $j$'s marginal loss from shifting the representative's proposal rightward, weighted by the representative's recognition probability.

The indirect effect, $\frac{\overline{x}^*(\hat{x}_d)}{\partial \hat{x}_d}(\rho_L - \rho_R)$, is the cumulative marginal effect of changing $L$ and $R$'s proposals by shifting the boundaries of the acceptance set, $A^*(\hat{x}_d)$. It depends on the proposal power of each partisan and whether $\hat{x}_d$ is left or right of 0. As $\hat{x}_d$ increases over $(-\overline{x}, 0)$, $m$'s equilibrium continuation value increases because the representative's proposal is more centrist. Thus, $A^*(\hat{x}_d)$ shrinks and partisans thus propose more centrist policy. From $j$'s perspective, $R$'s proposal improves and $L$'s worsens. If $R$ has greater proposal power than $L$, so $\rho_R > \rho_L$, then $j$ benefits and vice versa.[16] As $\hat{x}_d$ increases over $(0, \overline{x})$, these forces reverse symmetrically. The indirect effect changes sign, but the magnitude does not change.

By Proposition 1, $\mathcal{U}_j$ is single-peaked on each side of 0 and therefore at most double-peaked overall. Single-peakedness can fail over centrist representatives. Specifically, violations occur if the indirect effect of moving $x_d$ away from $\hat{x}_j$ towards 0 harms $j$, but the indirect effect of moving $\hat{x}_d$ rightward past 0 benefits $j$ and dominates the direct loss.[17] Informally, when evaluating potential representatives on the other side of 0, under some conditions $j$ expects to gain more from relaxing $L$'s proposal constraint than it loses from the prospect of the representative proposing policies farther right. Thus, $j$ can favor representatives whose presence generates more favorable proposals by $L$ even though the representative's proposals are less favorable.

Proposition 2 establishes useful properties of optimal representatives.

**Proposition 2.** *At least one of the party's optimal representatives is weakly more centrist. For non-ideologue parties, all optimal representatives are weakly more centrist.*

Proposition 2 implies that parties never strictly prefer more extreme representatives. Under broad and empirically prevalent conditions, legislative forces thus discourage parties from nominating candidates skewed away from the legislative center. To reiterate, the result concerns the party's optimal choice if it can freely appoint the representative, without regard for electability.

[16] This follows because policy utility is linear and ideologue proposals change symmetrically.

[17] Lemma A2 in the appendix fully characterizes when $\mathcal{U}_j$ is single-peaked. In general, single-peakedness fails if and only if the party is sufficiently extreme, its aligned partisan has sufficient proposal power and legislators are patient enough.

## District Election

Building upon the preceding analysis of preferences over representatives, I introduce electoral incentives. The legislative environment remains fixed except for the district representative's ideology, which now depends on the district's winning candidate. I focus on an election pitting an incumbent against a challenger to fill the open position. Such elections are prevalent in the US. Moreover, there is substantial interest in understanding how various legislative conditions affect incumbent re-election rates and the size of ideological swings when incumbents are replaced.[18]

There is an election in district $d$, which is home to a continuum of voters and potential candidates equal to the policy space, $X$. An *incumbent*, $I$, is up for re-election against a challenger candidate nominated by the non-incumbent, *challenger* party, $C$. Each citizen $i$ has ideal point $\hat{x}_i \in X$, as do $C$ and $I$. Notably, $\hat{x}_I$ is common knowledge, as are the ideologies of other legislators ($\hat{x}_L, \hat{x}_R$, and $\hat{x}_m = 0$) and legislative conditions ($\rho, \delta$, and $q$).

Isolating ideological competition, the distribution of agenda power is independent of the winning candidate's ideology or party affiliation. Formally, the district's representative always has recognition probability $\rho_d$ in the legislature and all other legislators have fixed proposal power as well. Substantively, $\rho_d$ can be interpreted as the share of agenda power the representative retains after expropriation and centralization of agenda power by either her party or the majority party.[19] By assuming the winner's party does not alter the broader distribution of agenda power, I capture an election that is not pivotal in deciding majority status. Such elections predominate in large legislative bodies.

To begin the electoral game, $C$ nominates a candidate. The pool of potential candidates is unrestricted, so $C$ can nominate any citizen in $X$.[20] Voters observe each

[18] Although incumbent-contested elections predominate in the US, understanding how legislative considerations shape open-seat electoral competition also arouses interest (e.g., Krasa and Polborn, 2018). For such elections, existence of a pure strategy equilibrium is not always guaranteed because party preferences need not be quasi-concave for all possible opposing candidates. In the appendix, I establish sufficient conditions for existence of a pure strategy equilibrium in an open-seat election and provide characterization. The conditions are substantively reasonable and equilibrium behavior has similarities to Wittman (1983). Mixed strategy equilibria exist generally by joint continuity of each party's objective function.

[19] Under this interpretation, the assumption is that the residual agenda power left over as $\rho_d$, however small, is independent of the representative's majority status and partisan attachment.

[20] Allowing $C$ to choose any $x \in X$ is not crucial. But without strong reasons for restricting $C$'s choice, I err against arbitrary constraints.

candidate's ideology and vote. During the election, citizens are not fully strategic and simply vote for the closer candidate.[21] The winner becomes the district's representative. To set policy, legislators bargain by majority rule as described above.

During the election, the median citizen is decisive and elects the closer candidate. To capture uncertainty about the electorate's preferences, parties do not know the median voter's ideology, as in Wittman (1983) and Calvert (1985). Both parties share common beliefs represented by the cumulative distribution function (cdf) $F : X \to [0, 1]$. This distribution has associated probability density function (pdf) $f$ that is differentiable and strictly positive on $X$.

Without loss of generality, suppose $I$ is right-leaning, $\hat{x}_C < 0 < \hat{x}_I$. Given this ordering, assume $F$ is log-concave. Log-concavity is satisfied by many well-known distributions and is a standard assumption in electoral competition models with uncertainty about voter preferences (Roemer, 1994; Bernhardt et al., 2009).[22] Analyzing the symmetric ordering, $\hat{x}_I < 0 < \hat{x}_C$, is analogous under the assumption that $1 - F$ is log-concave.

Throughout much of the analysis, I focus on the substantively sensible scenario in which district-level party brokers are closely aligned with their national party leaders. Specifically, I study the case where $C$ shares the ideology of the left-leaning partisan, i.e. $\hat{x}_C = \hat{x}_L$. This corresponds to national-level partisanship percolating down to district-level party decision makers.[23] Local-national alignment is not crucial for all of the results, however, and at the end of this section I describe how the results change under other assumptions about $\hat{x}_C$.

## Optimal Challenger Candidates

As in canonical settings without legislative considerations, electoral competition pushes $C$ to skew its candidate towards $I$. Policy motivation encourages office-seeking behavior

[21] Baron (1993) makes an analogous assumption in a model of parliamentary elections. This assumption reflects citizens having limited time to acquire information about legislative minutiae and simply compare the respective ideologies of candidates in their district. To the extent that citizens know about legislative politics, they likely know more about their local candidates. Furthermore, voting based on expected legislative policy outcomes requires sophisticated calculations that many citizens may be unlikely to perform. The stated setup provides a benchmark where sophisticated parties drive the results without requiring an overly sophisticated electorate. Technically, it also avoids potential difficulties arising from violations of single-peakedness.

[22] For more discussion of log-concavity, see Bagnoli and Bergstrom (2005).

[23] In the US context, this could reflect local party leaders adhering to their national party platform or, alternatively, national party operatives influencing the selection of local candidates (Hassell, 2016).

because $C$ wants a representative who influences legislative policy more favorably than $I$. Unlike canonical settings that abstract from legislative policymaking, however, $C$ anticipates the legislative process and evaluates prospective candidates on that basis.[24] A key difference from canonical settings is that $\hat{x}_C$ may not be an optimal representative for $C$.

Proposition 2 implies that $C$ never nominates a candidate more extreme than itself. Under weak conditions, optimal challenger candidates are skewed towards $I$. Beyond understanding where $C$'s optimal candidate stands relative to $I$, where does this candidate stand relative to the legislature? By Proposition 1, optimal representatives never cross over to the opposite side of the ideological spectrum. Consistent with prevailing wisdom, electoral motivations are thus necessary for crossing over. Roughly, $C$ will not cross over if either: its aligned partisan has substantial agenda power, or the district's voting fundamentals do not overwhelmingly favor $I$.

Under broad conditions, $C$ has a unique optimal candidate, who is located between $\hat{x}_C$ and 0. This observation suggests that legislative considerations typically encourage moderation and facilitates subsequent comparative statics. Moreover, there are clear connections to results in canonical settings with uncertainty about voter preferences. Throughout the following analysis, I implicitly maintain the required assumptions.[25] An important condition requires the probability the district's median voter is closer to $\hat{x}_C$ than $\hat{x}_I$ not be too low. I refer to this condition as $C$ being *electorally viable*.

In addition to $C$ not crossing over, I maintain conditions ensuring that $C$ does not nominate an aligned ideologue. Roughly, the median voter cannot be too likely to lean far left.[26] This assumption is reasonable for districts with right-leaning incumbents and eases the discussion. At the end of this section, I discuss the consequences of other assumptions.

**Effects of Legislative Conditions on Challenger Candidate Ideology**

How do optimal challenger candidates depend on legislative features? Under empirically prevalent conditions, I show that challenger parties in the legislative minority nominate more competitive candidates as the majority consolidates agenda power. In

[24]See the appendix for formal statements of the definitions and results studying optimal challengers.

[25]Formally, $C$ has a unique optimal candidate, who is aligned with $C$ and weakly more centrist than $C$'s most centrist optimal representative, if either: $\delta\rho_L < \frac{1}{2}$ and $F(\frac{\hat{x}_I}{2})$ is large enough, or $\delta\rho_L \geq \frac{1}{2}$ and $F(\frac{\hat{x}_C+\hat{x}_I}{2})$ is large enough. See Proposition A4 in the appendix.

[26]Formally, this holds if and only if $F(\frac{\hat{x}_I-\overline{x}}{2})$ is not too large.

contrast, majority-party challengers are less competitive. I also characterize the effects of the status quo and legislator patience. All of the results have direct implications for reelection rates.

Legislative features shape $C$'s optimal candidate through two channels, a *representative effect* and a *competition effect*. The representative effect changes $C$'s preferences over representatives. It is weighted by $C$'s electoral prospects and indicates whether $C$ wants a more extreme or more centrist representative. The competition effect changes $C$'s value of winning, that is the difference between $C$'s expected legislative utility from its own candidate relative to having $I$ as the representative. It reflects whether $C$'s election motivations grow or shrink as legislative conditions change.

**Polarization and Party Strength:** The first two comparative statics, Propositions 3 and 4, study how the distribution of agenda power affects $C$'s optimal candidate. Specifically, I analyze the effect of transferring agenda power from the centrist, $m$, to the partisans, $L$ and $R$. I refer to such a transfer as *consolidating agenda power.*

The results fix the district $d$ representative's agenda power. Residual agenda power moves between centrist and partisan legislators, reflecting a legislature-level shift in the balance of power between centrist and extremist copartisans. Substantively, this shift can be interpreted as legislative party leaders reallocating agenda influence away from more centrist members to their staunchest legislators.

To sharpen the results, I focus on a substantively motivated restriction where one partisan is in the legislative minority and will not be recognized.[27] This assumption is stronger than necessary, as the results only require sufficiently low minority partisan recognition probability, but it eases discussion without losing any key insights.

**Definition 2.** The legislature is under *minority-partisan agenda exclusion* if either $\rho_R = 0$ or $\rho_L = 0$.

Minority-party agenda exclusion reflects the prominent outlook that majority parties monopolize the agenda in the US and typically control the composition of key committees to exclude staunch opponents (Cox and McCubbins, 2005, 2007). Yet, because the majority partisan is not always recognized to propose, it also fits with evidence that individual legislators enjoy some autonomy from their party (Fouirnaies,

[27]Technically, an equilibrium with delay can exist if one partisan has zero recognition probability. But a no-delay equilibrium also exists and is the equilibrium approached as minority recognition probability goes to zero. Additionally, equilibrium delay never occurs in the substantively reasonable case where $q$ is skewed away from the majority partisan.

2018). Under minority-partisan agenda exclusion, consolidating agenda power amounts to increasing the agenda power of the majority partisans at the expense of the centrist legislator, $m$.

**Proposition 3.** *Suppose there is minority-partisan agenda exclusion. As the legislative majority consolidates agenda power, a minority-party challenger nominates a weakly more competitive candidate.*

Let $v^* = \frac{\hat{x}^* + \hat{x}_I}{2}$ denote the voter who is indifferent between $I$ and $C$'s optimal candidate, $\hat{x}^*$. If $C$ is in the legislative minority, then the change to $\hat{x}^*$ as the majority consolidates agenda power is proportional to

$$\frac{f(v^*)}{2}\Bigg(u_m(\hat{x}^*) - u_m(\hat{x}_I)\Bigg) + \delta\rho_d F(v^*). \tag{5}$$

The representative effect in (5), $\delta\rho_d F(v^*)$, is positive and encourages $C$ to nominate a more centrist candidate. The logic is as follows. Consolidating agenda power shifts more weight to partisan proposals, worsening $m$'s policy expectations. Thus, the acceptance set expands and partisans propose more extreme policy. Crucially, the ideology of the district's representative affects the size of this shift, and more centrist representatives dampen the change. To dampen increased extremism by majority partisans, minority challenger parties thus want more centrist representatives.

The direction of the competition effect, $\frac{f(v^*)}{2}\Bigg(u_m(\hat{x}^*) - u_m(\hat{x}_I)\Bigg)$, depends on the extremism of $C$'s candidate relative to $I$. As noted, the acceptance set expands with majority agenda consolidation regardless of who holds office. But it expands faster if the representative is more extreme. Minority challenger parties want to dampen this expansion. Consolidating agenda power thus increases their attraction to more centrist representatives. If $C$'s optimal candidate is more centrist than $I$, then the competition effect increases $C$'s value of office and encourages moderation. Otherwise, it decreases $C$'s value of office and discourages moderation.

What is the overall effect? If $C$ is more centrist than $I$, then both effects pull $C$'s candidate towards the center and thus more competitive. But if $C$ is more extreme, then the effects counteract: the representative effect pulls towards the center, but the competition effect pulls away. When $C$ is electorally viable, however, the representative effect prevails and $C$'s optimal candidate is more competitive.

An immediate implication of Proposition 3 is that a majority-party incumbent's re-election prospects worsen as her party consolidates agenda power. This effect arises even though her own agenda power is unchanged.

**Corollary 3.1.** *Suppose there is minority-partisan agenda exclusion. Consolidating agenda power weakly decreases a majority-party incumbent's re-election probability.*

Next, I turn the tables and consider majority-party challengers. In this case, $C$ is aligned with majority partisans and thus benefits from expanding the acceptance set.

**Proposition 4.** *Suppose there is minority-partisan agenda exclusion. As the legislative majority consolidates agenda power, a majority-party challenger nominates a weakly less competitive candidate.*

Now, right partisans have no agenda power under majority-party agenda control. Thus, $C$ wants to expand the acceptance set because there is no threat of more extreme policy proposals by $R$. Under the maintained assumptions, however, electoral incentives ensure that $C$ never nominates an ideologue. Therefore it is not guaranteed that consolidating agenda power causes $C$ to nominate a more extreme candidate, as these candidates are less competitive.

In Proposition 4, agenda consolidation reduces the appeal of nominating centrist candidates because the acceptance set is more sensitive to moderation. For a majority-party challenger, the change to $\hat{x}^*$ as the majority consolidates agenda power is proportional to

$$\frac{f(v^*)}{2}\bigg(u_m(\hat{x}_I) - u_m(\hat{x}^*)\bigg) - \delta\rho_d F(v^*). \tag{6}$$

The first term of (6) is the competition effect and the second term is the representative effect. The representative effect discourages moderation because $C$ now wants to expand the acceptance set. In general, the competition effect can go either direction and works symmetrically to Proposition 3. Because $C$ is electorally viable, however, the representative effect always prevails whenever it is countered by the competition effect. Thus, $C$'s optimal candidate is less competitive as the majority consolidates agenda power.

Similar to Proposition 3, Proposition 4 speaks to the re-election prospects of minority party incumbents under minority-partisan agenda exclusion. They are more likely to win re-election as the majority consolidates agenda power.

**Corollary 4.1.** *Suppose there is minority-partisan agenda exclusion. Consolidating agenda power weakly increases a minority-party incumbent's re-election probability.*

**Legislator Patience:** Next, I study how legislator patience, $\delta$, affects optimal challenger candidates. The results do not assume minority-partisan agenda exclusion. Patience can be interpreted as a measure of legislator sophistication. Alternatively, it could capture legislature efficiency, as more efficient legislatures may be more likely to revisit a policy area after a failed vote.

**Proposition 5.** *If the challenger party's aligned partisan has less agenda power than the other partisan, then the challenger nominates a weakly more competitive candidate as legislator patience increases. Otherwise, it nominates a weakly less competitive candidate.*

The overall effect of increasing $\delta$ is proportional to

$$(\rho_R - \rho_L)\left[\frac{f(v^*)}{2}\left(u_m(\hat{x}^*) - u_m(\hat{x}_I)\right)\right] + (\rho_R - \rho_L)F(v^*). \tag{7}$$

The first term of (7) is the competition effect and the second term is the representative effect.

The representative effect depends entirely on relative partisan power because $F(v^*) > 0$. If patience increases, then the acceptance set shrinks symmetrically. Furthermore, it is more sensitive to changes in representative ideology. If $\rho_R > \rho_L$, then $C$ gains more from constraining $R$ than it loses from constraining $L$. Increasing $\delta$ makes the acceptance set more sensitive, and thus moderation is more attractive for $C$. These incentives are reversed if $\rho_L > \rho_R$ and cancel out if $\rho_L = \rho_R$.

Two features drive the competition effect, relative partisan power and relative candidate extremism. As above, relative partisan power determines whether $C$ is more concerned about $L$'s constraint or $R$'s. Relative candidate extremism matters because the acceptance set's sensitivity to $\delta$ depends on the representative's ideology, as in the previous comparative statics. If the representative is more centrist, then the acceptance set contracts faster as $\delta$ increases.

To see how these features combine in an example, suppose $\rho_L > \rho_R$ and $C$'s optimal candidate is more centrist than $I$. First, $\rho_L > \rho_R$ implies that $C$ benefits more from relaxing $L$'s constraint than it loses from relaxing $R$'s. Second, the acceptance set contracts slower as $\delta$ increases if $I$ holds office, relative to $C$'s nominee. Combining these observations, the competition effect favors a less competitive challenger candidate.

In general, the representative and competition effects again work together if $C$'s optimal candidate is more centrist than $I$. Moreover, the overall effect is proportional to $\rho_R - \rho_L$. Otherwise, the two effects counteract. But the overall effect is again proportional to $\rho_R - \rho_L$ if $C$ is electorally viable. In each case, relative partisan agenda power determines whether $C$ nominates a more or less competitive candidate.

Corollary 5.1 states an immediate implications for incumbent re-election prospects. Unlike Corollaries 3.1 and 4.1, the incumbent's majority status plays no role.

**Corollary 5.1.** *If the challenger party's aligned partisan has less agenda power than the other partisan, then the incumbent's re-election probability weakly decreases as legislator patience increases. Otherwise, the incumbent's re-election probability weakly increases.*

**Status Quo Extremism:** To conclude the comparative static analysis, I vary the status quo. The results do not require $\hat{x}_C \leq -\overline{x}$, minority agenda exclusion, or that $C$ is electorally viable. More extreme status quo produce weakly more competitive challengers under broad conditions.

Proposition 6 imposes a condition on $F(0)$, the probability that the median voter is aligned with $C$. This quantity reflects how favorable the district is for $C$ from a legislature-level perspective. Crucially for the proposition, it helps pin down whether $C$'s optimal candidate is more centrist than $I$. The district is *incumbent friendly* if $F(0)$ is sufficiently large that $C$'s candidate is more centrist than $I$, which is well-defined.[28]

**Proposition 6.** *Suppose either (i) the district is incumbent-friendly or (ii) the prospect of partisan agenda control is high, $\delta(\rho_L + \rho_R) > \frac{1}{2}$. As the status quo gets more extreme, the challenger party nominates a weakly more competitive candidate.*

The overall effect as the status quo becomes more extreme is proportional to

$$\frac{\partial u_C(-\overline{x}^*(\hat{x}^*))}{\partial \overline{x}^*(\hat{x}^*)} - \frac{\partial u_C(-\overline{x}^*(\hat{x}_I))}{\partial \overline{x}^*(\hat{x}_I)}, \tag{8}$$

which is purely the competition effect. Unlike Propositions 3 and 4, there is no representative effect. Although more extreme status quo expand the acceptance set, thus altering $C$'s expected payoff, the magnitude does not depend on representative ideology. That is, marginal changes to $q$ do not alter how $C$ resolves the tradeoff between the ideology of its own representative and indirectly constraining ideologues.

[28]See the appendix for details.

For the competition effect, the preceding observation implies that the acceptance set changes with $q$ at a rate that is independent of which candidate prevails. The competition effect is therefore non-zero if and only if $C$ satisfies $\hat{x}_C \in (-\overline{x}(\hat{x}_I), -\overline{x}(\hat{x}^*))$. In this case, $C$'s value of office increases as $q$ becomes more extreme and therefore $C$'s optimal candidate is more centrist. If $C$ is sufficiently extreme or sufficiently centrist, however, then shifting the status quo has no effect.

Proposition 6 implies that, quite generally, $I$'s re-election prospects weakly decrease as $q$ becomes more extreme. Corollary 6.1 formally states the result, which arises purely through changes in $C$'s competitive calculus. Notably, it does not require that voters suffer any adverse effect directly from changes to $q$.

**Corollary 6.1.** *Suppose either (i) the district is incumbent-friendly or (ii) the prospect of partisan agenda control is high, $\delta(\rho_L + \rho_R) > \frac{1}{2}$. The incumbent's re-election probability weakly decreases as the status quo gets more extreme.*

### Comparative Statics under Alternative Assumptions

The preceding results maintain assumptions ensuring that $C$ neither crosses over nor nominates an aligned ideologue. Moreover, Propositions 3-5 assume $\hat{x}_C = \hat{x}_L \leq -\overline{x}$. I now discuss how these assumptions affect the comparative statics.

First, if $C$ is an ideologue and crosses over, then optimal candidates remain unaffected by small changes to $q$, but other comparative statics become less clear. The key difference is that crossing over reverses the representative effect due to the pull of the legislative median. When $C$ does not cross over, marginally more competitive candidates shrink the acceptance set. But the reverse is true is $C$ crosses over.

Next, if the distribution of the district median is favorable enough that $C$ optimally nominates an aligned ideologue, then small changes to legislative conditions affect $C$'s optimal candidate entirely though representative effects. Thus, optimal challenger candidates are less competitive if agenda polarization increases, $q$ is more extreme, or $\delta$ decreases.

Finally, relaxing $\hat{x}_C = \hat{x}_L$ in Propositions 3-5 can qualitatively change the results only if $C$ becomes sufficiently more centrist. In that case, a key difference is that $C$ wants to constrain its aligned partisan. Under minority-partisan agenda exclusion, however, this difference is inconsequential. More generally, if $C$ is sufficiently centrist, then consolidating agenda power always produces more competitive challenger candidates because (i) the representative effect increases $C$'s benefits from constraining both partisans in the legislature and (ii) the competition effect increases $C$'s benefit from defeating $I$. Otherwise, agenda consolidation has ambiguous effects without more conditions on the voter distribution.

## Model Commentary

Overall, the main setup has two important virtues. First, the electoral analysis yields reasonably crisp comparative statics. Second, the setting reveals a preference for moderate representation in a canonical legislative institution. These virtues can hold more generally, but I present the most convenient setting featuring both. With the formal analysis in place, I discuss two features of the model: absolute-value policy utility and the infinite-horizon legislative stage.

Absolute-value policy utility provides clear relationships between legislative features and electoral behavior. For example, quadratic policy utility muddies the electoral comparative statics by introducing second-order effects that are direct and indirect, through $m$'s acceptance condition. When analyzing optimal representatives, however, the main qualitative takeaways do not require absolute-value utility. Instead, strictly concave utility strengthens preferences for moderation.

An endogenous preference for moderation with absolute-value policy utility can also arise in a finite-horizon legislative stage with $T \geq 3$ periods. But this virtue weakens if $T \leq 2$. Preferences over representatives can diverge from fundamental policy preferences in that case, but the ally principle always holds. I present the infinite-horizon setting rather than $T \geq 3$ because it provides analogous logic and a cleaner equilibrium characterization.

## Conclusion & Implications

I study which candidates parties nominate for legislative positions. The analysis explores the interaction between two party-level concerns: influencing legislative outputs and winning elections. A key contribution is studying the effects of specific legislature-level conditions, including party strength and polarization. To do so, I model a rich legislative setting where legislators formulate policy and anticipate future policymaking efforts if policy fails today.

Analyzing preferences over representatives isolates the first concern, influencing legislative policy. Anticipating effects on equilibrium policy, parties strategically prefer more centrist representatives even without electoral incentives. Such representatives indirectly constrain the party's ideological opponents in the legislature by narrowing the set of passable policies. Various legislative conditions can alter preferences over representatives by changing the representative's anticipated effect on legislative policy.

Elections introduce the second party-level concern. As usual, they encourage parties to nominate candidates skewed towards their opponent. Under empirically prevalent conditions, this force complements the moderation incentive: skewing towards the center has the added benefit of skewing towards the opponent, and vice versa. The analysis thus suggests that legislative elections can provide stronger incentives for convergence than executive elections. Recent progress in estimating ideologies of incumbent and non-incumbent candidates in both types of elections on a common scale (see, e.g., Bonica, 2014; Hall and Snyder, 2013) provides a potential tool to explore this possibility. Scholars could compare ideological gaps between incumbents and non-incumbents in legislative elections against those in executive elections.

The analysis also identifies legislative conditions under which party incentives are equivalent to canonical settings without legislative considerations, such as Wittman (1983) and Calvert (1985). A general sufficient condition is that legislative voting does not constrain any potential agenda setters, which eliminates indirect effects of representative ideology. This condition can arise in three ways: agenda power concentrated among centrists, extreme status quo policy, or impatient legislators. A special case of the third condition is complete myopia, $\delta = 0$. In this case, which is equivalent to Romer and Rosenthal (1978), legislators place no value on future bouts of active policymaking and simply pass any policy that improves upon the status quo. This final possibility highlights the insight gained by studying forward-looking legislators. Otherwise, the indirect effects of representative ideology are absent and party competition adheres to the canonical form.

Empirical exploration of canonical electoral competition models frequently uses data from legislative elections (Ansolabehere, Snyder and Stewart III, 2001; Burden, 2004; Montagnes and Rogowski, 2015; Fowler and Hall, 2016). This practice provides a useful empirical benchmark and legislative election data is more abundant than executive election data. I do not aim to discourage this vein of empirical work. Instead, I aim to highlight which circumstances are best-suited to this practice.

I analyze electoral consequences of several specific legislative factors, including party strength and ideological polarization. These features affect incumbent re-election rates and the size of ideological swings between successive officeholders. In the analysis, these effects do not require voters to observe changes in legislative conditions. Instead, they arise from party incentives to influence legislative outputs through their representative's ideology. Additionally, they are policy-driven, which is a theoretical strength but also important to remember for empirical exploration. For example, the model abstracts from possibility that stronger parties are more effective at supporting vulnerable incumbents.

The analysis suggests that seats held by majority incumbents should experience more turnover in legislatures with stronger majority control. Furthermore, ideological swings should be smaller when incumbents lose re-election in those seats. Opposite effects are expected in minority-incumbent districts. Analogous results arise in more polarized legislatures. These results align with recent evidence documenting declining incumbency advantage (Jacobson, 2015) and increasing polarization (McCarty et al., 2006; Bonica, 2014; Gentzkow et al., 2018) in Congress.[29] They also suggest a party-level explanation for *midterm slumps*, which are usually attributed to *partisan balancing* by voters (Erikson, 1988; Alesina and Rosenthal, 1996; Folke and Snyder, 2012).

There are several possible avenues to explore the preceding implications empirically. For the turnover implications, scholars could use data on either re-election rates or incumbent vote shares. To evaluate the ideological implications, recent efforts to estimate the ideologies of incumbents and challengers on a common scale (e.g., Bonica, 2014) provide a potential tool.

The analysis shows that legislative considerations can endogenously generate preferences over representatives violating single-peakedness even though policy preferences are single-peaked. Thus, even if policy preferences are well-behaved, aggregating preferences over representatives can be fundamentally more difficult than aggregating policy preferences. In particular, difficulties may arise when agenda setting power is concentrated on one end of the ideological spectrum. A separate empirical implication is that parties aligned with powerful partisan agenda setters are especially averse to centrists. These parties prioritize giving aligned partisans slack to enact more extreme policy.

Future work could alter the analysis in several ways. First, I abstract from the

[29]Many measures of polarization struggle to disentangle party discipline from legislator ideology (see, e.g., Snyder and Groseclose (2000); Cox et al. (2010) and Canen et al. (2018) for attempts to address this issue). Because these two features work through the same channel in the model here, however, there is little strain in interpreting this empirical evidence through the model's lens.

possibility that different representatives affect the distribution of agenda power, policy issue selected, or policy expertise. My approach isolates ideology, but these considerations are important in many contexts. Second, I consider a widely studied legislative institution where particular legislators temporarily monopolize the agenda, capturing key features of committee-based systems. Scholars could also consider institutions where ideologically diverse interests contest policy before it reaches the floor. Finally, I analyze an election followed by potentially extended bargaining without modeling the possibility of another election. In a stationary setting, qualitatively similar forces exist if there are periodic elections provided there can be multiple rounds of bargaining within each term. Future work could more explicitly study the dynamic feedback between elections and legislating.

# Appendix

Assume $\hat{x}_m = 0$ and $q > 0$ without loss of generality. Define $\rho_E = \rho_R + \rho_L$. Let $\hat{x}$ denote the district-$d$ representative's ideal point. Define

$$\overline{x}^*(x) = \frac{(1-\delta)\, q + \delta\, \rho_d\, |x|}{1 - \delta\, \rho_E} \tag{9}$$

and

$$\overline{x} = \frac{(1-\delta)\, q}{1 - \delta\, (\rho_E + \rho_d)}. \tag{10}$$

**Proof of Lemma 2.**

*Proof.* Fix the representative's ideal point $\hat{x}$. In equilibrium, the representative proposes $\hat{x}$ iff $u_m(\hat{x}) \geq \frac{(1-\delta)u_m(q)+\delta\rho_d u_m(\hat{x})}{1-\delta\rho_E}$. Otherwise, she proposes the nearest boundary of the acceptance set. First, note that

$$u_m(0) > -\frac{(1-\delta)q}{1-\delta\rho_E} \tag{11}$$

$$= \frac{(1-\delta)u_m(q) + \delta\rho_d u_m(0)}{1-\delta\rho_E}, \tag{12}$$

where (11) follows from $\delta < 1$ and $q > 0$. Next, for $\hat{x} \geq 0$, we have

$$\frac{\partial u_m(x)}{\partial x} = -1 \tag{13}$$

$$< -\frac{\delta\rho_d}{1-\delta\rho_E} = \frac{\partial}{\partial x}\frac{(1-\delta)u_m(q) + \delta\rho_d u_m(\hat{x})}{1-\delta\rho_E}, \tag{14}$$

where (14) follows because $\delta(\rho_d + \rho_E) < 1$. Finally, $u_m(\hat{x}) = \frac{(1-\delta)u_m(q)+\delta\rho_d u_m(\hat{x})}{1-\delta\rho_E}$ iff

$$\hat{x} = \frac{(1-\delta)\, q}{1-\delta\,(\rho_E + \rho_d)} = \overline{x}. \tag{15}$$

It follows that the representative proposes $\hat{x} \geq 0$ iff $\hat{x} \in [0, \overline{x}]$. A symmetric argument show that the representative proposes $\hat{x} \leq 0$ iff $\hat{x} \in [-\overline{x}, 0]$. Thus, the representative proposes her ideal point in equilibrium iff $\hat{x} \in [-\overline{x}, \overline{x}]$. Otherwise, she proposes $x \in \{-\overline{x}, \overline{x}\}$. The desired result follows. □

**Corollary 1.** *The upper bound of the equilibrium acceptance set is strictly decreasing* *over $\hat{x} \in [-\overline{x}, 0]$, strictly increasing over $\hat{x} \in [0, \overline{x}]$, and constant in $\hat{x}$ otherwise.*

Because $\overline{x}^*(x)$ is continuous, Corollary 1 implies $\arg\min_{x\in X} \overline{x}^*(x) = 0 = \hat{x}_m$. Define $\overline{x}^c = \overline{x}(0) = \frac{(1-\delta)\,q}{1-\delta\rho_E}$.

**Corollary 2.** *The upper bound of the acceptance set increases as either (i) agenda polarization increases, (ii) status quo quality decreases, or (iii) legislator patience decreases.*

**Lemma A1.** *A representative is optimal for party $j$ only if she is aligned with $j$. Furthermore, if $\hat{x}_j \in (-\overline{x}, \overline{x})$, then $j$'s optimal representative is located weakly between $\hat{x}_j$ and 0.*

*Proof.* Consider a party $j$ and assume $\hat{x}_j > 0$ without loss of generality. Let $\hat{x}^*$ denote an optimal representative for $j$.

*Part 1:* First, I prove $\hat{x}^* \geq 0$. It suffices to show $\mathcal{U}_j(\hat{x}) \geq \mathcal{U}_j(-\hat{x})$ for all $\hat{x} \geq 0$. Suppose $\hat{x} \leq \overline{x}$. We have

$$\begin{aligned}
\mathcal{U}_j(\hat{x}) &= \rho_d u_j(\hat{x}) + \rho_m u_j(0) + \rho_R u_j(\overline{x}^*(\hat{x})) + \rho_L u_j(-\overline{x}^*(\hat{x})) && (16)\\
&= \rho_d u_j(\hat{x}) + \rho_m u_j(0) + \rho_R u_j(\overline{x}^*(-\hat{x})) + \rho_L u_j(-\overline{x}^*(-\hat{x})) && (17)\\
&\geq \rho_d u_j(-\hat{x}) + \rho_m u_j(0) + \rho_R u_j(\overline{x}^*(-\hat{x})) + \rho_L u_j(-\overline{x}^*(-\hat{x})) && (18)\\
&= \mathcal{U}_j(-\hat{x}), && (19)
\end{aligned}$$

where (17) follows because $\overline{x}^*(\hat{x}) = \overline{x}^*(-\hat{x})$; and (18) because $\hat{x}_j \geq 0$ and $\hat{x} \geq 0$. If $\hat{x} > \overline{x}$, then replacing $\overline{x}^*(\hat{x})$ with $\overline{x}$ in (16)-(19) yields an analogous result. Thus, $\mathcal{U}_j(\hat{x}) \geq \mathcal{U}_j(-\hat{x})$ for all $\hat{x} \geq 0$, as desired.

*Part 2:* By Lemma 2, $\hat{x} \in A(\hat{x})$ iff $\hat{x} \in (-\overline{x}, \overline{x})$. Assume $\hat{x}_j < \overline{x}$. For all $\hat{x} \in (\hat{x}_j, \overline{x})$, we have

$$\begin{aligned}
\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} &= \rho_d \frac{\partial u_j(\hat{x})}{\partial \hat{x}} + \rho_R \frac{\partial u_j(\overline{x}^*(\hat{x}))}{\partial \overline{x}^*(\hat{x})} \frac{\partial \overline{x}^*(\hat{x})}{\partial \hat{x}} + \rho_L \frac{\partial u_j(-\overline{x}^*(\hat{x}))}{\partial \overline{x}^*(\hat{x})} \frac{\partial \overline{x}^*(\hat{x})}{\partial \hat{x}} && (20)\\
&= \rho_d \frac{\partial u_j(\hat{x})}{\partial \hat{x}} - \rho_R \frac{\partial \overline{x}^*(\hat{x})}{\partial \hat{x}} - \rho_L \frac{\partial \overline{x}^*(\hat{x})}{\partial \hat{x}} && (21)\\
&= -\rho_d - (\rho_R + \rho_L) \frac{\delta \rho_d}{1 - \delta \rho_E} < 0, && (22)
\end{aligned}$$

where (21) follows because $\hat{x}_j \in (-\overline{x}^*(\hat{x}), \overline{x}^*(\hat{x}))$ implies $\frac{\partial u_j(-\overline{x}^*(\hat{x}))}{\partial \overline{x}^*(\hat{x})} = \frac{\partial u_j(\overline{x}^*(\hat{x}))}{\partial \overline{x}^*(\hat{x})} = -1$; and (22) because $\frac{\partial \overline{x}^*(\hat{x})}{\partial \hat{x}} = \frac{\delta \rho_d}{1-\delta\rho_E}$. Therefore $\mathcal{U}_j(\hat{x})$ strictly decreases over $[\hat{x}_j, \overline{x}]$.

Next, assume $\hat{x} \geq \overline{x}$. We have

$$\begin{aligned} \mathcal{U}_j(\hat{x}_j) &= \rho_d u_j(\hat{x}_j) + \rho_m u_j(0) + \rho_R u_j(\overline{x}^*(\hat{x}_j)) + \rho_L u_j(-\overline{x}^*(\hat{x}_j)) && (23)\\ &> \rho_d u_j(\overline{x}) + \rho_m u_j(0) + \rho_R u_j(\overline{x}) + \rho_L u_j(-\overline{x}) && (24)\\ &= \mathcal{U}_j(\hat{x}), && (25) \end{aligned}$$

where (24) follows because $u_j(\hat{x}_j) > u_j(\overline{x})$, $u_j(\overline{x}^*(\hat{x}_j)) > u_j(-\overline{x})$, and $u_j(-\overline{x}^*(\hat{x}_j)) > u_j(-\overline{x})$.

The preceding cases imply that $\mathcal{U}_j(\hat{x}_j) > \mathcal{U}_j(\hat{x})$ for all $\hat{x} > \hat{x}_j$. Together with Part 1, this observation implies $\hat{x} \notin [0, \hat{x}_j]$ cannot be optimal for $\hat{x}_j \in [0, \overline{x})$. □

Define

$$\tilde{x}_j = \frac{(1-\delta)q + (1-\delta\rho_E)|\hat{x}_j|}{\delta\rho_d}. \quad (26)$$

Simplifying (26) yields $\tilde{x}_j \in (0, |\hat{x}_j|)$. For $\hat{x}_j > 0$, it is the unique $\hat{x} > 0$ such that $\hat{x}_j = \overline{x}^*(\hat{x})$, as $\hat{x} \in [0, \tilde{x}_j)$ implies $\overline{x}^*(\hat{x}) < \hat{x}_j$ and $\hat{x} \in (\tilde{x}_j, \hat{x}_j]$ implies $\hat{x}_j < \overline{x}^*(\hat{x})$. Symmetric properties hold for $\hat{x}_j < 0$ with respect to $-\tilde{x}_j$.

For all $\hat{x} \in (-\overline{x}, \overline{x})$, Lemma 2 implies

$$\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} = \rho_d \frac{\partial u_j(\hat{x})}{\partial \hat{x}} + \frac{\overline{x}^*(\hat{x})}{\partial \hat{x}}\left(\rho_R \frac{\partial u_j(\overline{x}^*(\hat{x}))}{\partial \overline{x}^*(\hat{x})} + \rho_L \frac{\partial u_j(-\overline{x}^*(\hat{x}))}{\partial \overline{x}^*(\hat{x})}\right). \quad (27)$$

**Lemma 3.** *Assume $\hat{x}_j \in (-\overline{x}, \overline{x})$. If $\delta\rho_E < \frac{1}{2}$, then $\mathcal{U}_j(\hat{x})$ increases over $\hat{x} \in (-\overline{x}, \hat{x}_j)$ and decreases over $\hat{x} \in (\hat{x}_j, \overline{x})$.*

*Proof.* Fix $\hat{x}_j \in [0, \overline{x})$ without loss of generality. Suppose $\delta\rho_E > \frac{1}{2}$. There are two cases.

*Case 1:* Assume $\hat{x}_j \in (0, \overline{x}^c]$. First, $\hat{x} \in (-\overline{x}, 0)$ implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} = \rho_d + \frac{\delta\rho_d}{1-\delta\rho_E}(\rho_R + \rho_L) > 0$. Second, $\hat{x} \in (0, \hat{x}_j)$ implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} = \rho_d - \frac{\delta\rho_d}{1-\delta\rho_E}(\rho_R + \rho_L) \propto 1 - 2\delta\rho_E > 0$. Third, $\hat{x} \in (\hat{x}_j, \overline{x})$ implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} < 0$ by (20)–(22).

*Case 2:* Assume $\hat{x}_j \in (\overline{x}^c, \overline{x})$. Subcase 1 of Case 1 implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} > 0$ for $\hat{x} \in (-\overline{x}, -\tilde{x}_j)$. Subcase 2 of Case 1 implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} > 0$ for $\hat{x} \in (\tilde{x}_j, \hat{x}_j)$. Subcase 3 of Case 1 implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} < 0$ for $\hat{x} \in (\hat{x}_j, \overline{x})$. There are two remaining cases. First, $\hat{x} \in (-\tilde{x}_j, 0)$

implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} = \rho_d - \frac{\delta\rho_d}{1-\delta\rho_E}(\rho_R - \rho_L) \propto 1 - 2\delta\rho_R > 0$. Second, $\hat{x} \in (0, \tilde{x}_j)$ implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} = \rho_d + \frac{\delta\rho_d}{1-\delta\rho_E}(\rho_R - \rho_L) \propto 1 - 2\delta\rho_L > 0$. □

**Lemma 4.** *Assume $\hat{x}_j \in [-\overline{x}^c, \overline{x}^c]$. If $\delta\rho_E > \frac{1}{2}$, then $\mathcal{U}_j(\hat{x})$ increases over $\hat{x} \in (-\overline{x}, 0)$ and decreases over $\hat{x} \in (0, \overline{x})$.*

*Proof.* Fix $\hat{x}_j \in [0, \overline{x}^c]$. Suppose $\delta\rho_E > \frac{1}{2}$. The proof of Lemma 3 implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} > 0$ for $\hat{x} \in (-\overline{x}, 0)$ and $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} < 0$ for $\hat{x} \in (\hat{x}_j, \overline{x})$. To complete the proof, $\hat{x} \in (0, \hat{x}_j)$ implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} = -\rho_d - \frac{\delta\rho_d}{1-\delta\rho_E}(\rho_R + \rho_L) \propto 2\delta\rho_E - 1 < 0$. Symmetric arguments apply for $\hat{x}_j \in [-\overline{x}^c, 0]$. □

**Lemma 5.** *Assume $\hat{x}_j \in (-\overline{x}, -\overline{x}^c)$.*

1. *If $\delta\rho_L > \frac{1}{2}$, then $\mathcal{U}_j(\hat{x})$ increases over $\hat{x} \in (-\overline{x}, -\tilde{x}_j) \cup (0, \tilde{x}_j)$ and decreases over $\hat{x} \in (-\tilde{x}_j, 0) \cup (\tilde{x}_j, \overline{x})$.*

2. *If $\delta\rho_E > \frac{1}{2} > \delta\rho_L$, then $\mathcal{U}_j(\hat{x})$ increases over $\hat{x} \in (-\overline{x}, -\tilde{x}_j)$ and decreases over $\hat{x} \in (-\tilde{x}_j, \overline{x})$.*

*Symmetric results hold for $\hat{x} \in (\overline{x}^c, \overline{x})$.*

*Proof.* Fix $\hat{x}_j \in (-\overline{x}, -\overline{x}^c)$. Symmetric arguments apply for $\hat{x}_j \in (\overline{x}^c, \overline{x})$

*1.* Suppose $\delta\rho_L > \frac{1}{2}$. Arguments symmetric to those in Lemma 3 imply $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} > 0$ for $\hat{x} \in (-\overline{x}, -\tilde{x}_j)$. Second, $\hat{x} \in (-\tilde{x}_j, 0)$ implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} = -\rho_d + \frac{\delta\rho_d}{1-\delta\rho_E}(\rho_R - \rho_L) \propto 2\delta\rho_R - 1 < 0$. Third, $\hat{x} \in (0, \tilde{x}_j)$ implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} = -\rho_d - \frac{\delta\rho_d}{1-\delta\rho_E}(\rho_R - \rho_L) \propto 2\delta\rho_L - 1 > 0$. Finally, $\hat{x} \in (\tilde{x}_j, \overline{x})$ implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} < 0$ by a derivation equivalent to (20)–(22).

*2.* Suppose $\delta\rho_E > \frac{1}{2} > \delta\rho_L$. Arguments from Part 1 apply for $\hat{x} \notin (0, \tilde{x}_j)$. If $\hat{x} \in (0, \tilde{x}_j)$, then $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} \propto 2\delta\rho_L - 1 < 0$. □

**Lemma 6.** *Assume $\hat{x}_j \leq -\overline{x}$.*

1. *If $\delta\rho_R > \frac{1}{2}$, then $\mathcal{U}_j(\hat{x})$ increases over $(-\overline{x}, 0)$ and decreases over $\hat{x} \in (0, \overline{x})$.*

2. *If $\delta\rho_L > \frac{1}{2}$, then $\mathcal{U}_j(\hat{x})$ decreases over $(-\overline{x}, 0)$ and increases over $\hat{x} \in (0, \overline{x})$.*

3. *Otherwise, $\mathcal{U}_j(\hat{x})$ decreases over $\hat{x} \in (-\overline{x}, \overline{x})$.*

*Symmetric results hold for $\hat{x} \geq \overline{x}$.*

*Proof.* Assume $\hat{x}_j \le -\overline{x}$.

*1.* Suppose $\delta\rho_R > \frac{1}{2}$. First, $\hat{x} \in (-\overline{x}, 0)$ implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} = -\rho_d + \frac{\delta\rho_d}{1-\delta\rho_E}(\rho_R - \rho_L) \propto 2\delta\rho_R - 1 > 0$. Next, $\hat{x} \in (0, \overline{x})$ implies $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} = -\rho_d - \frac{\delta\rho_d}{1-\delta\rho_E}(\rho_R - \rho_L) \propto 2\delta\rho_L - 1 < 0$.

*2.* Suppose $\delta\rho_L > \frac{1}{2}$. Then $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} \propto 2\delta\rho_R - 1 < 0$ for $\hat{x} \in (-\overline{x}, 0)$ and $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} \propto 2\delta\rho_L - 1 > 0$ for $\hat{x} \in (0, \overline{x})$.

*3.* Suppose $\max\{\delta\rho_R, \delta\rho_L\} < \frac{1}{2}$. Then $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} \propto 2\delta\rho_R - 1 < 0$ for $\hat{x} \in (-\overline{x}, 0)$ and $\frac{\partial \mathcal{U}_j(\hat{x})}{\partial \hat{x}} \propto 2\delta\rho_L - 1 < 0$ for $\hat{x} \in (0, \overline{x})$. □

**Proof of Proposition 1.**

*Proof.* Fix $\hat{x}_j \ge 0$ without loss of generality.

*(i)* Continuity follows from Lemma 1. The following three observations are straightforward to show. If $\hat{x}_j \in (-\overline{x}, -\overline{x}^c) \cup (\overline{x}^c, \overline{x})$, then $\mathcal{U}_j(\hat{x})$ is differentiable for all $\hat{x} \notin \{-\overline{x}, -\tilde{x}_j, 0, \tilde{x}_j, \overline{x}, \hat{x}_j\}$. If $\hat{x}_j \in [-\overline{x}^c, \overline{x}^c]$, then $\mathcal{U}_j(\hat{x})$ is differentiable for all $\hat{x} \notin \{-\overline{x}, 0, \overline{x}, \hat{x}_j\}$. Finally, if $\hat{x}_j \notin (-\overline{x}, \overline{x})$, then $\mathcal{U}_j(\hat{x})$ is differentiable for all $\hat{x} \notin \{-\overline{x}, 0, \overline{x}\}$. In each case, $\mathcal{U}_j(\hat{x})$ is differentiable a.e.

*(ii)* Lemma 2 implies $\overline{x}^*(\hat{x}) = \overline{x}$ for all $\hat{x} \notin (-\overline{x}, \overline{x})$. Thus, $\mathcal{U}_j(\hat{x}) = \mathcal{U}_j(\hat{x}')$ for all $\hat{x}, \hat{x}' \le -\overline{x}$. Analogously, $\mathcal{U}_j(\hat{x}) = \mathcal{U}_j(\hat{x}')$ for all $\hat{x}, \hat{x}' \ge \overline{x}$.

*(iii)* Follows from Lemmas 3 – 6.

*(iv)* Follows from Lemma A1. □

**Lemma A2.** *Suppose party $j$ is left-leaning.Then $j$'s preferences over representatives are double-peaked if and only if $\hat{x}_j < -\overline{x}^c$ and $\delta\rho_L > \frac{1}{2}$. Otherwise, they are single-peaked. A symmetric result holds if $j$ is right-leaning.*

*Proof.* Follows from Lemmas 3 – 6. □

**Lemma A3.** *The ally principle holds for party $j$ if and only if either: (i) $\hat{x}_j = 0$, or (ii) the prospect of partisan agenda control is low, $j$ is a partisan, and the prospect of opposing partisan agenda control is low. Otherwise, the party has a unique optimal representative, who is strictly more centrist.*

*Proof.* Follows from Lemma A1 and Lemmas 3 – 6. □

**Proof of Proposition 2.**

*Proof.* Follows from Lemma A3. □

## Electoral Competition with Fixed Incumbent

Throughout the following analysis, suppose $\hat{x}_C = \hat{x}_L < 0 < \hat{x}_I$. Let $v^C = \frac{\hat{x}_C + \hat{x}_I}{2}$. Define

$$\underline{F} = \frac{\mathcal{U}_C(\overline{x}) - \mathcal{U}_C(\hat{x}_I)}{\mathcal{U}_C(-\overline{x}) - \mathcal{U}_C(\hat{x}_I)} \tag{28}$$

and

$$\underline{F}' = \frac{f(v^C)[u_m(\hat{x}_I) - u_m(\hat{x}_C)]}{2\delta\rho_d}. \tag{29}$$

Say $C$ is *electorally viable* if $F(v^C) \geq \underline{F}^* \equiv \max\{\underline{F}, \underline{F}'\}$. Set $v^m = \frac{\hat{x}_I}{2}$ and recall that $1 - F(v^m)$ is the *incumbent's base* in the district. Finally, let $x^r$ denote $C$'s most centrist optimal representative.

**Lemma 7.** *If $F(v^C) > \underline{F}$, then $C$'s optimal candidate satisfies $\hat{x} \in [x^r, \hat{x}_I)$.*

*Proof.* Suppose $F(v^C) > \underline{F}$. Because $\hat{x}_C = \hat{x}_L < -\overline{x}$, Lemma 6 implies $x^r \in [-\overline{x}, 0]$. Assume $\hat{x}_I < \overline{x}$. The proof for $\hat{x}_I \geq \overline{x}$ is similar.

There are two parts. Part 1 shows that nominating $\hat{x} \geq \hat{x}_I$ is not optimal for $C$. Part 2 shows that optimal candidates satisfy $\hat{x} \geq x^r$.

*Part 1.* There are two cases.

- First, suppose $\delta\rho_L < \frac{1}{2}$, which implies $\mathcal{U}_C(\hat{x})$ strictly decreases over $\hat{x} \in [0, \overline{x}]$ and is constant over $\hat{x} \geq \overline{x}$. Thus, $\mathcal{U}_C(x^r) > \mathcal{U}_C(\hat{x}_I) \geq \mathcal{U}_C(\hat{x})$ for all $\hat{x} \geq \hat{x}_I$. Because $F(\frac{x^r + \hat{x}_I}{2}) > 0$, nominating $\hat{x} \geq \hat{x}_I$ is not optimal for $C$.

- Second, suppose $\delta\rho_L \geq \frac{1}{2}$. Then $x^r = -\overline{x}$. Note that $\underline{F} < 1$ because $\mathcal{U}_C(-\overline{x}) > \mathcal{U}_C(\overline{x})$. For all $\hat{x} \geq \hat{x}_I$,

$$\widetilde{\mathcal{U}}_C(\hat{x}) = \left(1 - F\left(\frac{\hat{x} + \hat{x}_I}{2}\right)\right)\mathcal{U}_C(\hat{x}) + F\left(\frac{\hat{x} + \hat{x}_I}{2}\right)\mathcal{U}_C(\hat{x}_I) \tag{30}$$

$$\leq \mathcal{U}_C(\overline{x}) \tag{31}$$

$$< F\left(\frac{\hat{x}_I - \overline{x}}{2}\right)\mathcal{U}_C(-\overline{x}) + \left(1 - F\left(\frac{\hat{x}_I - \overline{x}}{2}\right)\right)\mathcal{U}_C(\hat{x}_I) = \widetilde{\mathcal{U}}_C(-\overline{x}), \tag{32}$$

where (31) follows because $\delta\rho_L \geq \frac{1}{2}$ implies $\mathcal{U}_C(\overline{x}) \geq \mathcal{U}_C(\hat{x})$ for all $\hat{x} \geq 0$; and (32) from $F(\frac{\hat{x}_I - \overline{x}}{2}) > F(v^C) > \underline{F}$. Thus, $\hat{x} \geq \hat{x}_I$ is not optimal.

*Part 2.* For $\hat{x} \leq \hat{x}_I$,

$$\widetilde{\mathcal{U}}_C(\hat{x}) = F\left(\frac{\hat{x}+\hat{x}_I}{2}\right)\mathcal{U}_C(\hat{x}) + \left(1 - F\left(\frac{\hat{x}+\hat{x}_I}{2}\right)\right)\mathcal{U}_C(\hat{x}_I). \tag{33}$$

Because $F(\frac{\hat{x}+\hat{x}_I}{2})$ strictly increases with $\hat{x}$ and $\mathcal{U}_C(\hat{x}) \leq \mathcal{U}_C(x^r)$ for all $\hat{x} < x^r$, we have $\widetilde{\mathcal{U}}_C(\hat{x}) < \widetilde{\mathcal{U}}_C(x^r)$ for all $\hat{x} < x^r$. Thus, any optimal candidate must satisfy $\hat{x} \geq x^r$. Together with Part 1, this implies optimal candidates satisfy $\hat{x} \in [x^r, \hat{x}_I)$. □

**Lemma A4.** *Suppose either: $F(v^m)$ is sufficiently large and $\delta\rho_L < \frac{1}{2}$, or $F(v^C) \geq \underline{F}^*$ and $\delta\rho_L \geq \frac{1}{2}$. The challenger has a uniquely optimal candidate $\hat{x}^* \in [x^r, 0]$.*

*Proof.* Suppose $\hat{x}_I < \overline{x}$. The proof for $\hat{x}_I \geq \overline{x}$ is similar. There are two parts. Part 1 shows $\hat{x}^* \in [x^r, 0]$ if $\delta\rho_L < \frac{1}{2}$ and $F(v^m)$ is sufficiently large. Part 2 shows $\hat{x}^* \in [x^r, 0]$ if $\delta\rho_L \geq \frac{1}{2}$ and $F(v^C) \geq \underline{F}^*$.

*Part 1.* Suppose $\delta\rho_L < \frac{1}{2}$ and

$$F(v^m) \geq \frac{f(v^m)}{2}\frac{\mathcal{U}_C(0) - \mathcal{U}_C(\hat{x}_I)}{\frac{\partial^+ \mathcal{U}_C(x)}{\partial x}|_{x=0}}. \tag{34}$$

Because $\hat{x}_C = \hat{x}_L$, there are two subcases: $x^r = -\overline{x}$ and $x^r = 0$.

- First, suppose $x^r = -\overline{x}$. The proof proceeds in two steps. Step 1 shows $C$'s expected utility over candidates, $\widetilde{\mathcal{U}}_C$, is strictly quasi-concave over $[x^r, 0]$ and $[0, \hat{x}_I]$, respectively. Step 2 establishes the desired result.

  *Step 1.* Because $\mathcal{U}_C$ is differentiable at all $\hat{x} \in (x^r, 0)$, $\widetilde{\mathcal{U}}_C$ is differentiable over $(x^r, 0)$. Any $\hat{x} \in (x^r, 0)$ maximizing $\widetilde{\mathcal{U}}_C$ must satisfy the following first-order condition:

$$\frac{\partial \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x}} = \frac{1}{2}f\left(\frac{\hat{x}+\hat{x}_I}{2}\right)\left(\mathcal{U}_C(\hat{x}) - \mathcal{U}_C(\hat{x}_I)\right) + F\left(\frac{\hat{x}+\hat{x}_I}{2}\right)\frac{\partial \mathcal{U}_C(\hat{x})}{\partial \hat{x}} = 0. \tag{35}$$

  Let $\hat{x}^* \in (x^r, 0)$ be a solution to (35), and define $v^* = \frac{\hat{x}^*+\hat{x}_I}{2}$. The second derivative of $\widetilde{\mathcal{U}}_C$ at $\hat{x}^*$ is

$$\frac{\partial^2 \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x}^2}|_{\hat{x}=\hat{x}^*} = \frac{f'(v^*)}{4}\left(\mathcal{U}_C(x^*) - \mathcal{U}_C(\hat{x}_I)\right) + F(v^*)\frac{\partial^2 \mathcal{U}_C(\hat{x})}{\partial \hat{x}^2}|_{\hat{x}=\hat{x}^*} + f(v^*)\frac{\partial \mathcal{U}_C(\hat{x})}{\partial \hat{x}}|_{\hat{x}=\hat{x}^*} \tag{36}$$

$$= \frac{f'(v^*)}{4}\Big(\mathcal{U}_C(\hat{x}^*) - \mathcal{U}_C(\hat{x}_I)\Big) + F(v^*)\frac{\partial^2 \mathcal{U}_C(\hat{x})}{\partial \hat{x}^2}|_{\hat{x}=\hat{x}^*}$$
$$- \frac{f(v^*)^2}{2F(v^*)}\Big(\mathcal{U}_C(\hat{x}^*) - \mathcal{U}_C(\hat{x}_I)\Big) \tag{37}$$
$$= \Big(\mathcal{U}_C(\hat{x}^*) - \mathcal{U}_C(\hat{x}_I)\Big)\Big(\frac{f'(v^*)}{4} - \frac{f(v^*)^2}{2F(v^*)}\Big) + F(v^*)\frac{\partial^2 \mathcal{U}_C(\hat{x})}{\partial \hat{x}^2}|_{\hat{x}=\hat{x}^*} \tag{38}$$
$$= \Big(\mathcal{U}_C(\hat{x}^*) - \mathcal{U}_C(\hat{x}_I)\Big)\Big(\frac{f'(v^*)}{4} - \frac{f(v^*)^2}{2F(v^*)}\Big) \tag{39}$$
$$\propto \frac{f'(v^*)}{2} - \frac{f(v^*)^2}{F(v^*)} \tag{40}$$
$$< 0, \tag{41}$$

where (37) follows from using (35) to substitute for $\frac{\partial \mathcal{U}_C(\hat{x})}{\partial \hat{x}}|_{\hat{x}=\hat{x}^*}$; (38) from rearranging; (39) because $\frac{\partial^2 \mathcal{U}_C(\hat{x})}{\partial \hat{x}^2}|_{\hat{x}=\hat{x}^*} = 0$; (39) is proportional to (40) because $\mathcal{U}_C(\hat{x}^*) > \mathcal{U}_C(\hat{x}_I)$; and (41) because log-concavity of $F$ implies $f'(v^*) < \frac{f(v^*)^2}{F(v^*)}$. Thus, any $\hat{x}^* \in (x^r, 0)$ also satisfies the second-order condition and is a strict local maximizer. Therefore $\widetilde{\mathcal{U}}_C$ is strictly quasi-concave on $[x^r, 0]$. Analogous arguments show $\widetilde{\mathcal{U}}_C$ is strictly quasi-concave over $[0, \hat{x}_I]$.

*Step 2.* We know $\hat{x} \geq \hat{x}_I$ is not optimal because $\mathcal{U}_C(\hat{x})$ is constant over $\hat{x} \geq \overline{x}$ and $\delta\rho_L < \frac{1}{2}$ implies that $\mathcal{U}_C(\hat{x})$ strictly decreases over $\hat{x} \in [0, \overline{x}]$. Next, because $\frac{\partial^+ \mathcal{U}_C(\hat{x})}{\partial \hat{x}}|_{\hat{x}=0} < 0$, rearranging (35) reveals that (34) implies $\frac{\partial^+ \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x}}|_{\hat{x}=0} \leq 0$. Because $\widetilde{\mathcal{U}}_C$ is strictly quasi-concave over $[0, \hat{x}_I]$, continuity implies that optimal candidates must be in $[x^r, 0]$. Finally, strict quasi-concavity of $\widetilde{\mathcal{U}}_C$ over $[x^r, 0]$ implies uniqueness of $\hat{x}^*$.

- Second, suppose $x^r = 0$. An analogous argument shows $\widetilde{\mathcal{U}}_C$ is strictly quasi-concave over $[x^r, \hat{x}_I]$. Thus, (34) implies $\widetilde{\mathcal{U}}_C$ strictly decreases over $[x^r, \hat{x}_I]$ and $\hat{x}^* = 0$.

*Part 2.* Suppose $\delta\rho_L \geq \frac{1}{2}$ and $F(v^C) > \underline{F}$. Because $\delta\rho_L \geq \frac{1}{2}$, we have: $x^r = -\overline{x}$, $\mathcal{U}_C(\hat{x})$ strictly decreases over $[x^r, 0]$, and $\mathcal{U}_C(\hat{x})$ strictly increases over $\hat{x} \in [0, \overline{x}]$. Moreover, Lemma 7 implies $\hat{x}^* \in [x^r, \hat{x}_I)$. Because $\mathcal{U}_C(x^r) > \mathcal{U}_C(\hat{x}_I)$, continuity implies existence of a unique $x' \in (x^r, 0)$ satisfying $\mathcal{U}_C(x') = \mathcal{U}_C(\hat{x}_I)$. Furthermore, $\mathcal{U}_C(\hat{x}) < \mathcal{U}_C(\hat{x}_I)$ for all $\hat{x} \in (x', \hat{x}_I)$. Thus, $\widetilde{\mathcal{U}}_C(x') = \widetilde{\mathcal{U}}_C(\hat{x}_I)$ and $\widetilde{\mathcal{U}}_C(x') > \widetilde{\mathcal{U}}_C(\hat{x})$ for all $\hat{x} \in (x', \hat{x}_I)$.

Consequently, $\hat{x}^* \in [x^r, x')$. An argument analogous to Part 1 shows $\widetilde{\mathcal{U}}_C$ is strictly quasi-concave over $[x^r, 0]$. Thus, $\hat{x}^*$ is unique. $\square$

## Comparative Statics on Optimal Candidate

To ensure $C$'s optimal candidate is interior, the following results assume $F(\frac{\hat{x}_I - \overline{x}}{2})$ is sufficiently small, so that the district is not too favorable for $C$.

**Proposition 3.** *Suppose $F(v^C) \geq \underline{F}^*$, $F(v^m)$ is sufficiently large, there is minority-party agenda exclusion, and $C$ is in the legislative minority. Then $C$ nominates a weakly more competitive candidate as the majority consolidates agenda power.*

*Proof.* Suppose $F(v^C) \geq \underline{F}^*$, $F(v^m)$ satisfies (34), and $\rho_R \geq \rho_L = 0$. By Proposition A4, $C$ has a uniquely optimal candidate $\hat{x}^* \in [x^r, 0]$. The proof is based on signing the derivative of $\hat{x}^*$ as $\rho_R$ increases at the expense of $\rho_m$. There are two cases: $\hat{x}_I \in (0, \overline{x})$ and $\hat{x}_I \geq \overline{x}$.

*Case 1:* Suppose $\hat{x}_I < \overline{x}$. Recall $v^* = \frac{\hat{x}^* + \hat{x}_I}{2}$. Then $C$'s expected payoff from nominating $\hat{x}^*$ is $\widetilde{\mathcal{U}}_C(x^*) = F(v^*)\,\mathcal{U}_C(\hat{x}^*) + [1 - F(v^*)]\,\mathcal{U}_C(\hat{x}_I)$.

Because $F(\frac{\hat{x}_I - \overline{x}}{2})$ is sufficiently small, $\hat{x}^* > x^r$. By Proposition A4, there is a uniquely optimal $\hat{x}^* \in (x^r, 0)$ satisfies (35) and the second-order sufficient condition. Let $\frac{\partial \mathcal{U}_C(\hat{x})}{\partial \rho_R}$ denote the marginal change in $\mathcal{U}_C(\hat{x})$ as $\rho_R$ increases at the expense of $\rho_m$ and define $\frac{\partial^2 \mathcal{U}_C(\hat{x})}{\partial \hat{x} \partial \rho_R}$ similarly. Applying the implicit function theorem,

$$\frac{\partial \hat{x}^*}{\partial \rho_R} = -\frac{\frac{\partial^2 \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x} \partial \rho_R}|_{\hat{x}=x^*}}{\frac{\partial^2 \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x}^2}|_{\hat{x}=x^*}} \tag{42}$$

$$\propto \frac{f(v^*)}{2}\left(\frac{\partial \mathcal{U}_C(\hat{x}^*)}{\partial \rho_R} - \frac{\partial \mathcal{U}_C(\hat{x}_I)}{\partial \rho_R}\right) + F(v^*)\frac{\partial^2 \mathcal{U}_C(\hat{x})}{\partial \hat{x} \partial \rho_R}|_{\hat{x}=\hat{x}^*}, \tag{43}$$

where (42) is proportional to (43) because $\frac{\partial^2 \widetilde{\mathcal{U}}_C(\hat{x}}{\partial \hat{x}^2}|_{\hat{x}=x^*} < 0$, and (43) expresses $\frac{\partial^2 \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x} \partial \rho_R}|_{\hat{x}=x^*}$ explicitly. First, for all $\hat{x} \in X$,

$$\frac{\partial \mathcal{U}_C(\hat{x})}{\partial \rho_R} = u_C(\overline{x}^*(\hat{x})) - u_C(0) + \rho_R \frac{\partial \overline{x}^*(\hat{x})}{\partial \rho_R}. \tag{44}$$

Next,

$$\frac{\partial \mathcal{U}_C(\hat{x}^*)}{\partial \rho_R} - \frac{\partial \mathcal{U}_C(\hat{x}_I)}{\partial \rho_R} = \overline{x}^*(\hat{x}_I) - \overline{x}^*(\hat{x}^*) + \rho_R\left(\frac{\partial \overline{x}(\hat{x}^*)}{\partial \rho_R} - \frac{\partial \overline{x}(\hat{x}_I)}{\partial \rho_R}\right) \tag{45}$$

$$= \overline{x}^*(\hat{x}_I) - \overline{x}^*(\hat{x}^*) + \frac{\delta^2\,\rho_R\,\rho_d}{(1-\delta\rho_R)^2}\left(u_m(\hat{x}^*) - u_m(\hat{x}_I)\right) \tag{46}$$

$$= u_m(\hat{x}^*) - u_m(\hat{x}_I) + \frac{\delta\rho_R}{1-\delta\rho_R}\left(u_m(\hat{x}^*) - u_m(\hat{x}_I)\right) \tag{47}$$

$$= \frac{u_m(\hat{x}^*) - u_m(\hat{x}_I)}{1-\delta\rho_R}, \tag{48}$$

where (45) follows because $u_C$ is Euclidean; (46) from $\frac{\partial \overline{x}^*(\hat{x})}{\partial \rho_R} = \frac{\delta[(1-\delta)u_m(q) - \delta\rho_d u_m(x)]}{(1-\delta\rho_R)^2}$ and rearranging; (47) from the definition of $\overline{x}^*(\hat{x})$ and simplifying.

Additionally, for all $\hat{x} \in X$,

$$\frac{\partial^2 \mathcal{U}_C(\hat{x})}{\partial \hat{x}\partial \rho_R} = \frac{\partial u_C(\overline{x}^*(\hat{x}))}{\partial \overline{x}^*(\hat{x})}\left(\frac{\partial \overline{x}^*(\hat{x})}{\partial \hat{x}} + \rho_R\frac{\partial^2 \overline{x}^*(\hat{x})}{\partial \hat{x}\partial \rho_R}\right) \tag{49}$$

$$= -\left(\frac{\partial \overline{x}^*(\hat{x})}{\partial \hat{x}} + \rho_R\frac{\partial^2 \overline{x}^*(\hat{x})}{\partial \hat{x}\partial \rho_R}\right) \tag{50}$$

$$= \frac{\delta\rho_d}{1-\delta\rho_R}, \tag{51}$$

where (50) follows because $\hat{x}_C < 0 < \overline{x}^*(\hat{x})$ implies $\frac{\partial u_C(\overline{x}^*(\hat{x}))}{\partial \overline{x}^*(\hat{x})} = -1$; and (51) from $\frac{\partial \overline{x}^*(\hat{x})}{\partial \hat{x}} = -\frac{\delta\rho_d}{1-\delta\rho_R}$, $\frac{\partial^2 \overline{x}^*(\hat{x})}{\partial x\partial \rho_R} = -\frac{\delta^2\rho_d}{(1-\delta\rho_R)^2}$, and simplifying.

Using (48) and (51) to re-express (43) yields

$$\frac{\partial \hat{x}^*}{\partial \rho_R} \propto \frac{f(v^*)}{2}\left(u_m(\hat{x}^*) - u_m(\hat{x}_I)\right) + \delta\rho_d F(v^*). \tag{52}$$

If $u_m(\hat{x}^*) \geq u_m(\hat{x}_I)$, then (52) is strictly positive and therefore $\hat{x}^*$ increases.

Otherwise, (52) is strictly positive iff

$$\frac{f(v^*)}{F(v^*)} < \frac{2\delta\rho_d}{u_m(\hat{x}_I) - u_m(\hat{x}^*)}. \tag{53}$$

We have

$$\frac{f(v^*)}{F(v^*)} < \frac{f(v^C)}{F(v^C)} \tag{54}$$

$$\leq \frac{2\delta\rho_d}{u_m(\hat{x}_I) - u_m(\hat{x}_C)} \tag{55}$$

$$< \frac{2\delta\rho_d}{u_m(\hat{x}_I) - u_m(x^*)}, \tag{56}$$

where (54) holds because $v^C < v^*$ and $F$ is log-concave; (55) from $F(v^C) \geq \underline{F}'$ and rearranging (29); and (56) from $u_m(\hat{x}_I) > u_m(\hat{x}^*) > u_m(\hat{x}_C)$. Thus, $\frac{\partial \hat{x}^*}{\partial \rho_R} > 0$, as desired.

*Case 2:* If $\hat{x}_I \geq \overline{x}$, then $u_m(\hat{x}^*) \geq u_m(\hat{x}_I)$ and thus (52) implies $x^*$ increases. □

**Proposition 4.** *Suppose $F(v^C) \geq \underline{F}^*$, $F(v^m)$ is sufficiently large, there is minority-party agenda exclusion and $C$ is in the legislative majority. Then $C$ nominates a weakly less competitive candidate as the majority consolidates agenda power.*

*Proof.* Assume $F(v^C) \geq \underline{F}^*$, $F(v^m)$ satisfies (34), and $\rho_L \geq \rho_R = 0$. Proposition A4 implies $\hat{x}^*$ is unique and satisfies $\hat{x}^* \in [x^r, 0]$. The proof signs the derivative of $\hat{x}^*$ as $\rho_L$ increases at the expense of $\rho_m$.

- First, if $\hat{x}^*$ satisfies (35), then by Proposition A4 it also satisfies the second-order sufficient condition. Applying the implicit function theorem, a derivation similar to (45)-(52) yields

$$\frac{\partial \hat{x}^*}{\partial \rho_L} = -\frac{\frac{\partial^2 \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x} \partial \rho_L}|_{\hat{x}=x^*}}{\frac{\partial^2 \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x}^2}|_{\hat{x}=x^*}} \tag{57}$$

$$\propto \frac{f(v^*)}{2}\Big(u_m(\hat{x}_I) - u_m(\hat{x}^*)\Big) - \delta\rho_d F(v^*). \tag{58}$$

  If $u_m(\hat{x}^*) \geq u_m(\hat{x}_I)$, then (58) is strictly negative and therefore $\hat{x}^*$ decreases.

  Otherwise, (58) is strictly negative iff

$$\frac{f(v^*)}{F(v^*)} < \frac{2\delta\rho_d}{u_m(\hat{x}_I) - u_m(\hat{x}^*)}. \tag{59}$$

  As (59) is equivalent to (53), (54) - (56) implies that (58) is strictly negative. Thus, $x^*$ decreases.

- Second, if $\hat{x}^*$ does not satisfy (35), then $\hat{x}^* = -\overline{x}$. Because $\hat{x}_C < -\overline{x}$, $x^*$ decreases.

$\square$

**Proposition 5.** *Suppose $F(v^C) \geq \underline{F}^*$ and $F(v^m)$ is sufficiently large. If $\rho_L < \rho_R$, then $C$ nominates a weakly more competitive candidate as $\delta$ increases. Otherwise, $C$ nominates a weakly less competitive candidate as $\delta$ increases.*

*Proof.* Suppose $F(v^C) \geq \underline{F}^*$ and $F(v^m)$ satisfies (34). Proposition A4 implies $\hat{x}^*$ is unique and satisfies $\hat{x}^* \in [x^r, 0]$. Because $F(\frac{\hat{x}_I - \overline{x}}{2})$ is sufficiently small, $x^* > \underline{x}$. It follows that either $x^*$ satisfies (35) or $x^* = 0$. The proof signs the derivative of $\hat{x}^*$ with respect to $\delta$.

If $\hat{x}^* = 0$, then $\frac{\partial \hat{x}^*}{\partial \delta} = 0$.

If $\hat{x}^*$ satisfies (35), then by Proposition A4 it also satisfies the second-order sufficient condition. Applying the implicit function theorem,

$$\frac{\partial \hat{x}^*}{\partial \delta} = -\frac{\frac{\partial^2 \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x} \partial \delta}|_{\hat{x}=x^*}}{\frac{\partial^2 \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x}^2}|_{\hat{x}=x^*}} \tag{60}$$

$$\propto \frac{f(v^*)}{2}\left(\frac{\partial \mathcal{U}_C(\hat{x}^*)}{\partial \delta} - \frac{\partial \mathcal{U}_C(\hat{x}_I)}{\partial \delta}\right) + F(v^*)\frac{\partial^2 \mathcal{U}_C(\hat{x})}{\partial \hat{x} \partial \delta}|_{\hat{x}=\hat{x}^*}, \tag{61}$$

$$= \frac{f(v^*)}{2}\left(\frac{\rho_d(\rho_R - \rho_L)}{(1-\delta\rho_E)^2}[u_m(\hat{x}^*) - u_m(\hat{x}_I)]\right) + F(v^*)\frac{\rho_d(\rho_R - \rho_L)}{(1-\delta\rho_E)^2} \tag{62}$$

$$\propto (\rho_R - \rho_L)\left(\frac{f(v^*)}{2}\Big(u_m(\hat{x}^*) - u_m(\hat{x}_I)\Big) + F(v^*)\right) \tag{63}$$

$$\propto \rho_R - \rho_L, \tag{64}$$

where (62) follows from substituting for $\frac{\partial \mathcal{U}_C(\hat{x}^*)}{\partial \delta} - \frac{\partial \mathcal{U}_C(\hat{x}_I)}{\partial \delta}$ and $\frac{\partial^2 \mathcal{U}_C(\hat{x}^*)}{\partial \hat{x}^* \partial \delta}$ using derivations similar to (45)-(52) and (49)-(49), respectively; and (64) because a derivation analogous to (54) - (56) implies $\frac{f(v^*)}{2}[u_m(\hat{x}^*) - u_m(\hat{x}_I)] + F(v^*) > 0$. Thus, $\rho_R > \rho_L$ implies $\frac{\partial \hat{x}^*}{\partial \delta} > 0$ and $\frac{\partial \hat{x}^*}{\partial \delta} < 0$ otherwise. $\square$

**Proposition 6.** *If either:*

1. *$F(v^C) \geq \underline{F}^*$ and $\delta\rho_L \geq \frac{1}{2}$; or*

2. *$F(0)$ is sufficiently small, $F(v^m)$ is sufficiently large, and $\delta\rho_L < \frac{1}{2}$;*

*then $C$ nominates a weakly more competitive candidate as the status quo becomes more extreme.*

*Proof.* Suppose $\hat{x}_C < 0 < \hat{x}_I$ and $q > 0$ without loss of generality. Both parts of the proof sign the derivative of $\hat{x}^*$ with respect to $q$.

*1.* Assume $F(v^C) \geq \underline{F}^*$ and $\delta\rho_L \geq \frac{1}{2}$. Proposition A4 implies $\hat{x}^*$ is unique and satisfies $\hat{x}^* \in [x^r, 0]$. There are three cases.

- First, consider $\hat{x}_C \in (-\overline{x}^c, 0)$. Because $\delta\rho_E > \frac{1}{2}$, we have $\hat{x}^* = 0$. Thus, $\frac{\partial \hat{x}^*}{\partial q} = 0$.
- Second, consider $\hat{x}_C \in (-\overline{x}, -\overline{x}^c)$. Define $\tilde{x}_C$ as the lower analogue of (26). Because $\delta\rho_E \geq \delta\rho_L > \frac{1}{2}$, we have $x^* \in [\tilde{x}_C, 0]$.

  If $\hat{x}^* = \tilde{x}_C$, then inspection of (26) reveals $\frac{\partial \hat{x}^*}{\partial q} > 0$.

  Next, suppose $\hat{x}^*$ satisfies (35). By Proposition A4, $\hat{x}^*$ satisfies the second-order sufficient condition. Applying the implicit function theorem,

$$\frac{\partial \hat{x}^*}{\partial q} = -\frac{\frac{\partial^2 \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x}\partial q}|_{\hat{x}=x^*}}{\frac{\partial^2 \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x}^2}|_{\hat{x}=x^*}} \tag{65}$$

$$\propto \frac{f(v^*)}{2}\left(\frac{\partial \mathcal{U}_C(\hat{x}^*)}{\partial q} - \frac{\partial \mathcal{U}_C(\hat{x}_I)}{\partial q}\right) + F(v^*)\frac{\partial^2 \mathcal{U}_C(\hat{x})}{\partial \hat{x}\partial q}|_{\hat{x}=\hat{x}^*} \tag{66}$$

$$\propto \rho_R\left(\frac{\partial u_C(\overline{x}(\hat{x}^*))}{\partial \overline{x}(\hat{x}^*)}\frac{\partial \overline{x}(\hat{x}^*)}{\partial q} - \frac{\partial u_C(\overline{x}(\hat{x}_I))}{\partial \overline{x}(\hat{x}_I)}\frac{\partial \overline{x}(\hat{x}_I)}{\partial q}\right) + \rho_L\left(\frac{\partial u_C(-\overline{x}(\hat{x}^*))}{\partial \overline{x}(\hat{x}^*)}\frac{\partial \overline{x}(\hat{x}^*)}{\partial q} - \frac{\partial u_C(-\overline{x}(\hat{x}_I))}{\partial \overline{x}(\hat{x}_I)}\frac{\partial \overline{x}(\hat{x}_I)}{\partial q}\right) \tag{67}$$

$$= \frac{1-\delta}{1-\delta\rho_E}\left(\rho_R\left(\frac{\partial u_C(\overline{x}(\hat{x}^*))}{\partial \overline{x}(\hat{x}^*)} - \frac{\partial u_C(\overline{x}(\hat{x}_I))}{\partial \overline{x}(\hat{x}_I)}\right) + \rho_L\left(\frac{\partial u_C(-\overline{x}(\hat{x}^*))}{\partial \overline{x}(\hat{x}^*)} - \frac{\partial u_C(-\overline{x}(\hat{x}_I))}{\partial \overline{x}(\hat{x}_I)}\right)\right) \tag{68}$$

$$= \frac{(1-\delta)\rho_L}{1-\delta\rho_E}\left(\frac{\partial u_C(-\overline{x}(\hat{x}^*))}{\partial \overline{x}(\hat{x}^*)} - \frac{\partial u_C(-\overline{x}(\hat{x}_I))}{\partial \overline{x}(\hat{x}_I)}\right) \tag{69}$$

$$\propto \frac{\partial u_C(-\overline{x}(\hat{x}^*))}{\partial \overline{x}(\hat{x}^*)} - \frac{\partial u_C(-\overline{x}(\hat{x}_I))}{\partial \overline{x}(\hat{x}_I)}, \tag{70}$$

  where (67) follows from $\frac{\partial^2 \mathcal{U}_C(\hat{x})}{\partial \hat{x}\partial q}|_{\hat{x}=\hat{x}^*} = 0$ and $f(v^*) > 0$; (68) from $\frac{\partial \overline{x}(\hat{x}^*)}{\partial q} = \frac{\partial \overline{x}(\hat{x}_I)}{\partial q} = \frac{1-\delta}{1-\delta\rho_E}$; (69) because $\hat{x}_C < \hat{x}_m$ implies $\frac{\partial u_C(\overline{x}(\hat{x}^*))}{\partial \overline{x}(\hat{x}^*)} = \frac{\partial u_C(\overline{x}(\hat{x}_I))}{\partial \overline{x}(\hat{x}_I)}$; and (69) is proportional to (70) because $\frac{(1-\delta)\rho_L}{1-\delta\rho_E} > 0$.

If $\hat{x}_C \in (-\overline{x}(\hat{x}_I), \overline{x}^c)$, then $\underline{x}(\hat{x}_I) < \hat{x}_C < -\overline{x}(\hat{x}^*)$, where the second inequality follows from $\hat{x}^* \in (\tilde{x}_C, 0]$. Therefore (70) implies $\frac{\partial \hat{x}^*}{\partial q} > 0$. If $\hat{x}_C \leq -\overline{x}(\hat{x}_I)$, then $\hat{x}_C \leq \min\{-\overline{x}(\hat{x}_I), -\overline{x}(\hat{x}^*)\}$ and thus $\frac{\partial \hat{x}^*}{\partial q} = 0$.

- Finally, consider $\hat{x}_C \leq -\overline{x}$, which implies either $\hat{x}^* = 0$ or $\hat{x}^*$ satisfies (35). Because $\hat{x}_C \leq \min\{-\overline{x}(\hat{x}_I), -\overline{x}(\hat{x}^*)\}$, Case 2 implies $\frac{\partial \hat{x}^*}{\partial q} = 0$.

*2.* Suppose $F(v^m)$ satisfies (34) and $\delta\rho_L < \frac{1}{2}$. By the proof of Proposition A4, $\hat{x}^*$ is unique and satisfies $\hat{x}^* \in [x^r, 0]$. There are three cases.

- First, consider $\hat{x}_C \leq -\overline{x}(\hat{x}_I)$. There are two steps. Step 1 shows existence of $\overline{F}$ such that $\hat{x}^* \geq \hat{x}_I$ iff $F(0) \leq \overline{F}$. Step 2 establishes the result.

  *Step 1.* An argument analogous to the proof of Proposition A4 implies $\hat{x}^* \geq -\hat{x}_I$ iff $\frac{\partial^- \widetilde{\mathcal{U}}_C(\hat{x})}{\partial \hat{x}}|_{\hat{x}=-\hat{x}_I} \geq 0$. Equivalently,

$$\frac{f(0)}{2}\Big(\mathcal{U}_C(-\hat{x}_I) - \mathcal{U}_C(\hat{x}_I)\Big) + F(0)\frac{\partial^- \mathcal{U}_C(\hat{x})}{\partial \hat{x}}|_{x=-\hat{x}_I} \geq 0 \tag{71}$$

$$F(0)\frac{\partial^- \mathcal{U}_C(\hat{x})}{\partial \hat{x}}|_{\hat{x}=-\hat{x}_I} \geq \frac{f(0)}{2}\Big(\mathcal{U}_C(\hat{x}_I) - \mathcal{U}_C(-\hat{x}_I)\Big) \tag{72}$$

$$F(0) \leq \frac{f(0)}{2}\frac{\mathcal{U}_C(\hat{x}_I) - \mathcal{U}_C(-\hat{x}_I)}{\frac{\partial^- \mathcal{U}_C(\hat{x})}{\partial \hat{x}}|_{\hat{x}=-\hat{x}_I}}, \tag{73}$$

  where (73) follows from $\frac{\partial^- \mathcal{U}_C(\hat{x})}{\partial \hat{x}}|_{\hat{x}=-\hat{x}_I} < 0$, which holds because $\delta\rho_L < \frac{1}{2}$, $\hat{x}_C < -\overline{x}(\hat{x}_I)$, and $\hat{x}_I < \overline{x}$. Setting $\overline{F} = \frac{f(0)}{2}\frac{\mathcal{U}_C(\hat{x}_I) - \mathcal{U}_C(-\hat{x}_I)}{\frac{\partial^- \mathcal{U}_C(\hat{x})}{\partial \hat{x}}|_{\hat{x}=-\hat{x}_I}}$ yields the desired result.

  *Step 2.* Suppose $F(0) \leq \overline{F}$. Either $\hat{x}^* = 0$ or $\hat{x}^*$ satisfies (35). Because $\hat{x}^* \geq -\hat{x}_I$, we have $\hat{x}_C \leq \min\{-\overline{x}(\hat{x}_I), -\overline{x}(\hat{x}^*)\}$. Thus, Case 3 of Part 1 implies $\frac{\partial \hat{x}^*}{\partial q} = 0$.

- Second, consider $\hat{x}_C \in (-\overline{x}(\hat{x}_I), -\overline{x}^c)$. If $\hat{x}^*$ satisfies (35), then inspection of (70) reveals that $\frac{\partial \hat{x}^*}{\partial q} \geq 0$ because (i) $\frac{\partial u_C(-\overline{x}(\hat{x}^*))}{\partial \overline{x}(\hat{x}^*)} \geq -1$ and (ii) $\hat{x}_C > -\overline{x}(\hat{x}_I)$ implies $\frac{\partial u_C(-\overline{x}(\hat{x}_I))}{\partial \overline{x}(\hat{x}_I)} = -1$. If $\hat{x}^* \in \{0, \tilde{x}_C\}$, then $\frac{\partial \hat{x}^*}{\partial q} \geq 0$ also holds.

- Third, consider $\hat{x}_C \in (-\overline{x}^c, 0)$. Either $\hat{x}^*$ satisfies (35) or $\hat{x}^* \in \{0, \hat{x}_C\}$. If $\hat{x}^*$ satisfies (35), then applying the implicit function theorem yields (70). Because $\hat{x}^* \geq -\hat{x}_I$ and $\hat{x}_C \geq \max\{-\overline{x}(\hat{x}_I), -\overline{x}(\hat{x}^*)\}$, we have $\frac{\partial \hat{x}^*}{\partial q} = 0$. It follows that $\frac{\partial \hat{x}^*}{\partial q} \geq 0$.

□

## Open-seat Election: Strategies and Equilibrium Definition

Suppose the legislative partisans, $L$ and $R$, each nominate a candidate to contest an open-seat in the legislature. As in the main text, parties are unconstrained when selecting candidates, so that each can nominate any citizen in $X$. To continue to isolate ideological competition, the winner's agenda power is $\rho_d$ regardless of ideology or party affiliation. Voters observe candidate ideologies before the election. Finally, I add the assumption that $f$, the density of the median voter distribution $F$, is log-concave.[30]

A pure nomination strategy for party $j$ is $\kappa_j \in X$, which specifies the ideology of party $j$'s candidate. A legislative proposal strategy for a citizen with ideal point $\hat{x}$ is $\pi_{\hat{x}} \in X$. Next, a legislative proposal strategy for $i \in \{L, R, m\}$ is a function $\pi_i : X \to X$ that maps from the district-$d$ representative's ideal point to policy. As in the main analysis, I focus on no-delay legislative proposal strategies. Finally, a legislative voting strategy for $m$ is a function $\alpha : X^2 \to \{0, 1\}$ mapping from proposal-representative pairs to approval decisions. Let $\sigma = (\kappa, \pi, \alpha)$ denote a profile of pure strategies.

I now provide further notation to formalize the equilibrium conditions. As in the main analysis, legislative strategies are pinned down by $\hat{x}_d$. Thus, let $V_m^*(\hat{x}_d)$ denote $m$'s equilibrium continuation value from rejecting a proposal if $\hat{x}_d$ is the district-$d$ representative's ideal point. Similarly, let $A^*(\hat{x}_d) \subseteq X$ be the acceptance set given $\hat{x}_d$. Finally, let $U_j(x; y)$ denote the expected payoff to party $j$ from nominating a candidate with ideal point $x$ if the other party's candidate has ideal point $y \in X$.

Formally, $\sigma$ is an *equilibrium* of the open-seat election game if it satisfies four conditions. First, for all $\hat{x}_d \in X$, the acceptance set satisfies

$$A(\hat{x}_d) = \{x \in X | u_m(x) \geq (1 - \delta)u_m(q) + \delta V_m^*(\hat{x}_d)\}. \tag{74}$$

Second, legislative proposal strategies for citizens with ideal point $\hat{x}$ solve

$$\pi_{\hat{x}} = \max_{x \in A^*(\hat{x}_d)} u_{\hat{x}}(x), \tag{75}$$

[30] Log-concavity of $f$ implies that the distributions $F$ and $1 - F$ are both log-concave.

and for each $\hat{x}_d \in X$, legislative proposal strategies for $i \in \{L, R, m\}$ satisfy

$$\pi(\hat{x}_d) = \max_{x \in A^*(\hat{x}_d)} u_i(x). \tag{76}$$

Finally, given an opposing candidate with ideal point $y$, party $j$'s nomination strategy solves

$$d_j = \max_{x \in X} U_j(x; y). \tag{77}$$

Altogether, $\sigma$ is an equilibrium if the nomination strategy of each party member satisfies (77); and (75), (76), and (74) hold for every $\hat{x}_d \in X$. Note that, given $\hat{x}_d$, the first three conditions are analogous to those specifying a no-delay stationary pure strategy equilibrium in a fixed legislature.

**Lemma A5.** *If $\delta\rho_L > \frac{1}{2}$, then there exists a unique $x^\dagger \in (-\overline{x}, 0)$ satisfying $\mathcal{U}_L(x^\dagger) = \mathcal{U}_L(\overline{x})$.*

*Proof.* Suppose $\delta\rho_L > \frac{1}{2}$. Lemma 6 implies that $L$'s most centrist optimal representative is $x^r = -\underline{x}$. Furthermore, $\mathcal{U}_L(\hat{x})$ strictly decreases over $[-\underline{x}, 0]$, strictly increases over $[0, \overline{x}]$ and is constant over $\hat{x} \geq \overline{x}$. Because $\mathcal{U}_L(-\underline{x}) > \mathcal{U}_L(\overline{x})$, continuity delivers the result. □

Proposition 7 studies open-seat competition for a position in a legislature with strong legislative parties and patient legislators. Formally, either $\delta\rho_L > \frac{1}{2}$ or $\delta\rho_R > \frac{1}{2}$, so that one partisan has high prospect of agenda control. I discuss $\delta\rho_L > \frac{1}{2}$ without loss of generality. Under these conditions, $L$ wants to relax the acceptance constraint on the left partisans, as noted previously. Thus, $L$ prefers representatives farther from 0 in either direction. By Lemma A5, however, $L$'s induced preferences over representatives are asymmetric in favor left-leaning representatives. Specifically, there is a unique $x^\dagger \in (-\overline{x}, 0)$ such that $L$ is indifferent between a representative at $x^\dagger$ and one at $\overline{x}$.

**Definition 3.** The district's *electoral hostility* towards $L$ is $1 - F(\frac{x^\dagger - \overline{x}}{2})$.

Proposition 7 shows that a pure strategy equilibrium exists when $L$ faces sufficiently low electoral hostility. Additionally, it characterizes the candidate ordering in any such equilibrium. Both candidates are skewed towards $L$, but $L$'s candidate is strictly more extreme. The result does not require minority agenda exclusion.

The logic is as follows. Coupled with the asymmetry noted above, low electoral hostility ensures $L$ never crosses over, regardless of $R$'s candidate. In turn, $R$ does not select an aligned candidate. Because $\delta\rho_L > \frac{1}{2}$, $R$ wants to maximally constrain left-leaning partisans and thus her optimal representative is $\hat{x} = 0$. Because $L$ does not cross over, $R$'s must nominate a candidate weakly left of 0. Consequently, $L$'s induced preference for extremism produces a candidate strictly left of $R$'s.

**Proposition 7.** *Suppose $f$ is log-concave, $\delta\rho_L > \frac{1}{2}$, and $F(\frac{x^\dagger - \overline{x}}{2})$ is sufficiently large. A pure strategy Nash equilibrium exists. In every such equilibrium, $x_L^* < x_R^* \leq 0$.*

*Proof.* Assume $f$ is log-concave and $\delta\rho_L > \frac{1}{2}$. Lemma A5 implies existence of a unique $x^\dagger \in (-\overline{x}, 0)$ such that $\mathcal{U}_L(x^\dagger) = \mathcal{U}_L(\overline{x})$. Furthermore, $\delta\rho_L > \frac{1}{2}$ implies $\mathcal{U}_C(-\overline{x}) > \mathcal{U}_C(x^\dagger)$. Suppose

$$F\left(\frac{x^\dagger - \overline{x}}{2}\right) > \frac{\mathcal{U}_C(x^\dagger) - \mathcal{U}_C(0)}{\mathcal{U}_C(-\overline{x}) - \mathcal{U}_C(0)}. \tag{78}$$

There are four parts. Part 1 shows that each player's best-response correspondence is a continuous function and, furthermore, best responses are located in $[-\overline{x}, 0]$. Part 2 uses the best-response functions from Part 1 to establish existence of a fixed point. Part 3 shows the fixed point satisfies the equilibrium conditions. Part 4 provides characterization.

*Part 1.* There are two steps. Step 1 shows $x_L^* \leq 0$ for any $x_R$. Step 2 shows $x_R^* \leq 0$ for all $x_L \leq 0$.

- *Step 1:* Fix any $x_R \in X$. Because (78) holds, arguments similar to the proof of Proposition A4 imply $x_L^*$ is unique and satisfies $x_L^* \in [-\overline{x}, 0]$. Thus, we can represent $L$'s best-response correspondence $b_L(x) : [-\overline{x}, 0] \to [-\overline{x}, 0]$ as $b_L(x) = \arg\max_{x \in [-\overline{x}, 0]} \widetilde{U}_L(x)$, which is nonempty and single-valued. Continuity follows from the Theorem of the Maximum.

- *Step 2:* Fix $x_L \leq 0$. By arguments similar to those of Proposition A4, $x_R^*$ is unique and satisfies $x_R^* \in [-\overline{x}, 0]$. Thus, for $x_L \leq 0$, $R$'s best-response correspondence $b_R(x) : [-\overline{x}, 0] \to [-\overline{x}, 0]$ can be represented as $b_R(x) = \arg\max_{x \in [-\overline{x}, 0]} \widetilde{U}_R(x)$, which is nonempty and single-valued, and continuous by the Theorem of the Maximum.

*Part 2.* Define the mapping $b : [-\overline{x}, 0]^2 \to [-\overline{x}, 0]^2$ as $b(x_L, x_R) = b(b_L(x_R), b_R(x_L))$. Properties of $b_L$ and $b_R$ imply $b$ is a continuous function mapping a compact, convex set to itself. Brouwer's theorem implies existence of a fixed point.

*Part 3.* To check that a fixed point $(x_L^*, x_R^*) = b(x_L^*, x_R^*)$ is an equilibrium, the construction of $b$ ensures that it suffices to show neither player has a profitable deviation to $x \notin [-\overline{x}, 0]$. By Part 1, $L$'s best response is always in $[-\overline{x}, 0]$ and $R$'s best response to any $x_L \in [-\overline{x}, 0]$ is always in $[-\overline{x}, 0]$. Therefore $(x_L^*, x_R^*)$ is an equilibrium.

*Part 4.* That $x_R^* \leq 0$ follows from the construction of $b$. Next, $x_L^* < x_R^*$ follows because $f$ is strictly positive and $\delta\rho_L > \frac{1}{2}$ implies $\mathcal{U}_L$ strictly decreases over $[-\overline{x}, 0]$. □

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