

Access to Proposers and Influence in Collective Policymaking

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Abstract

Access is an important prerequisite for outside influence. I study a model in which targeted access provides interest groups with chances to lobby policy proposals by certain politicians. I show how this prominent form of access can have subtle equilibrium effects on collective policymaking. By raising everyone's anticipation of lobbying, it can alter what pivotal politicians will pass and, in turn, influence other proposers and affect lobbying expenditures. The magnitude of these effects varies with polarization, and their direction depends on the interest group's extremism relative to its target. Furthermore, they can work in the group's favor or against it, potentially even overwhelming the direct benefit of better lobbying prospects. For example, moderate groups crave access to relatively extreme politicians but avoid access to a range of more centrist politicians. The results build our theoretical understanding of access and have implications for various political expenditures.

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One of the primary ways that interest groups influence policy is by lobbying, and one of the best ways to lobby is by shaping bills as they are written in committee (Hall and Wayman, 1990; Powell, 2014). To do so effectively, however, interest groups typically need opportunities to engage with policymakers. That is, they need *access* (Wright, 1996).

Since access is critical for outside influence, it is a central topic for scholars of interest groups.¹ Yet, it is notoriously difficult to study because (i) it is difficult to observe and measure (Miller, 2021a) and (ii) interest groups and politicians are highly strategic, so that multiple explanations are often consistent with observed data.² To suggest new avenues for indirect evidence and shed light on empirical findings, one way to address these obstacles is by refining our theoretical understanding of access.³ That is the goal of this paper.

I address two questions, focusing on targeted access that provides interest groups with chances to lobby certain politicians as they draft proposals. First, what are the consequences of such access? Second, which politicians do interest groups want to target?

The main contribution of this paper is to deepen our theoretical understanding of such access. To get it, interest groups typically must develop relationships or hire lobbyists with relationships well ahead of time, since politicians are busy with many other obligations and carefully allocate their scarce time. I show how those relationships can endogenously affect proposals and votes by other politicians who are not targeted, as well as lobbying expenditures by interest groups. I then study how an interest group's desire to acquire access depends on (i) its own ideology, (ii) the target's ideology and proposal power, as well as (iii) the proposal power of ideologically extreme politicians. The results complement existing theories of access, many of which consider an isolated target and thus cannot study how

¹See Miller (2021a) for an overview of recent work, which has shed light on how groups can acquire access (Kalla and Broockman, 2015; Fournaies and Hall, 2018; McCrain, 2018; Bertrand, Bombardini and Trebbi, 2014; Blanes i Vidal, Draca and Fons-Rosen, 2012), which politicians they want to target (Powell and Grimmer, 2016; Fournaies, 2018; Miller, 2021b; Liu, 2021), and whether access facilitates influence (Ban and You, 2019).

²See, e.g., Powell and Grimmer (2016) or Kalla and Broockman (2015) for more discussion.

³As (Miller, 2021a, p. 297) notes while reviewing empirical studies of targeted access: “though the theoretical linkages between access and other quantities of interest are sometimes unclear, formal theory can help researchers elucidate expectations and guide empirical tests.” For an exemplary empirical study of outside influence that incorporates strategic behavior, see Kang (2015).

non-targeted politicians may react. Additionally, the analysis sheds light on longstanding puzzles about why many groups are less aggressive than expected in pursuing access, and also provides empirical implications for the relationship between access-seeking behaviors and lobbying intensity.

To do so, I study a game-theoretic model of collective policymaking with interest groups. The model has three key features. First, the model reflects the prevailing view that access merely provides opportunities to exert influence (Powell, 2014). Specifically, access and lobbying are linked but distinct: a group with access to a politician can *potentially* lobby her. Second, lobbying occurs only *in committee*, to possibly shape proposals before they are voted on. This focus allows the analysis to isolate the consequences of an important form of outside influence, access that helps groups participate in drafting legislation (Schattschneider, 1960; Kroeger, 2021). Third, there is an explicit political interaction with multiple strategic politicians and a strategic interest group, each of whom anticipates the possibilities of (i) revisiting failed proposals, (ii) changes in agenda control, and (iii) outside influence. Understanding targeted access in a wider political context has been highlighted as an important area for development⁴ (Baumgartner, 2010; Leech, 2010) and, moreover, it allows us to study how access interacts with elite polarization and proposal power.⁵

A key insight of the analysis is that, simply by providing opportunities to lobby the target’s proposal, access can also *endogenously* affect (i) which policies would pass if proposed, (ii) what extreme politicians propose, and (iii) lobbying expenditures. These indirect effects arise because access causes everyone to anticipate the potential subsequent influence in equilibrium, and thereby changes their expectations about future proposals that would follow rejection today. As in other models where bargaining can continue following rejected proposals (e.g., Banks and Duggan, 2006a), those expectations shape the set of policies that

⁴Leech (2010) notes that “[.] organized interests recognize the importance of legislative procedure and agenda setters” and “studies of their targeting choices, and of their lobbying activity more broadly, must consider how procedural context conditions their behavior.”

⁵Additionally, expanding the scope of application for legislative bargaining models has independent theoretical interest.

will pass today. Thus, access can flip decisive votes on certain policies, thereby changing which policies will pass and, in turn, influencing proposals by extreme politicians who are constrained. And furthermore, by indirectly altering those extreme proposals, access can also indirectly affect lobbying expenditures.

I show that the nature of these indirect equilibrium effects depends on whether lobbying would shift the target's proposals towards veto players or away from them. A key finding is that access facilitating lobbying that would shift policy inward will also induce more moderation by extremists, and vice versa. To illustrate, if decisive voters anticipate a *lower* chance of more extreme proposals, then they will be more inclined to continue bargaining and therefore accept a narrower range of proposals today, which forces extreme legislators to pass more moderate policy. Additionally, I show that such access also weakly decreases the amount that *any* group will spend to influence proposals. In contrast, if decisive voters anticipate a higher chance of extreme proposals, then extremists will have more slack and pass more extreme policy, while lobbying expenditures weakly increase.

For interest group, the endogenous effects of access can be good or bad. First, if the group is not too extreme, then it always benefits from access that helps it lobby policy inward towards the center. On the other hand, it always dislikes the endogenous effects of access that helps it lobby outward. Thus, such groups are more inclined to access a relatively extreme politician, but less inclined to access a relatively centrist one. For extreme groups, whether the endogenous effects of access are good or bad depends on the relative proposal power of extremists on both sides of the spectrum. If extreme politicians on the group's side are sufficiently less likely to propose than opposing extremists, then it suffers from the endogenous effects of access that help it lobby policy outward. In contrast, it benefits from such access if its aligned extremists are sufficiently more likely to propose.

I show that the potential downsides of access can outweigh the upside, so that more access to certain politicians makes the group worse off overall. Specifically, groups that are not too extreme forgo access to a range of more centrist legislators because the group's expected loss

from greater policy extremism outweighs its benefit from higher chances of lobbying. This result holds even if access is free and stems from a commitment problem: whenever the group can lobby, it will always pull policy weakly towards itself. Thus, access necessarily causes everyone to *anticipate* that the group will *potentially* shift some of the target’s proposals in its own favor. This anticipation can also work to the group’s advantage, however, as these moderate groups always want access to a broad range of more extreme politicians because such access increases opportunities to lobby and therefore endogenously decreases expected policy extremism in the legislature, which forces extreme politicians to propose more centrist policies that are also more favorable to the group. Sufficiently extreme politicians are not worth targeting, however, since lobbying does not affect their behavior.

The analysis has several implications for our understanding of outside influence and money in politics. First, I shed new light on the prominent view that access is “a precondition for influence, not influence itself” (Wright, 1989, pg. 714). I show how, even by merely providing *opportunities* to lobby, access can *endogenously* influence key votes and proposals by various politicians. Second, the results suggest potential relationships between different types of political expenditures and therefore have implications for how regulations on access-seeking behaviors might “redirect money rather than lessen it” (Powell, 2014). Third, the results shed new light on the empirical regularity that relatively few groups contribute and they contribute surprisingly little, known as *Tullock’s puzzle* (Tullock, 1972; Ansolabehere, de Figueiredo and Snyder Jr., 2003). Finally, the results connect with several other empirical findings, such as that groups often (i) lobby their allies (Ainsworth, 1997; Kollman, 1997; Hojnacki and Kimball, 1998, 1999), (ii) seek access to legislators with substantial agenda power (Powell and Grimmer, 2016; Fourinaies, 2018), and that (iii) contributing groups are overwhelmingly centrist (Bonica, 2013, p. 301).

Related Literature

Our current theoretical understanding of access comes from models that focus on either: one politician (Austen-Smith, 1995; Lohmann, 1995; Hall and Deardorff, 2006), untargeted access in a group of politicians to influence proposals (Levy and Razin, 2013), or targeted access in a group of politicians who are voting on a fixed proposal (Schnakenberg, 2017; Awad, 2020). I study targeted access in a group of politicians and address an important gap by focusing on access that provides chances to influence proposals. This form of access is prominent in practice and, moreover, the analysis suggests that it can have subtle equilibrium effects that can (i) make it more or less desirable than previously expected, and (ii) alter incentives to lobby votes. Additionally, I contribute to a theoretical literature that incorporates lobbying into models of legislative bargaining.

In seminal work, Hall and Deardorff (2006) highlight that interest groups want access to legislators who are their allies in order to assist them in pursuing aligned interests. They study targeted access, as I do, but in contrast they focus on a single politician and therefore abstract from the possibility that targeted access has endogenous indirect effects on behavior by non-targeted politicians. I complement their analysis by explicitly modelling multiple politicians in order to allow for the possibility of such effects and show that they can lead other politicians to act more favorably or less favorably, with the effect potentially differing across politicians. Furthermore, I find that these endogenous effects can discourage access to some allies, as the prospect of other legislators passing less favorable proposals can outweigh the prospect of the target passing a friendlier proposal. Additionally, two other differences are that I explicitly incorporate ideology and political institutions (e.g., proposal rights and voting), which allows me to pursue their suggestion that future work “incorporate the degree of agreement over specific policies” and explore potential tradeoffs between “a legislator’s proximity to their group’s ideal policies and the legislator’s institutional or partisan ability to get things done” (Hall and Deardorff, 2006, p. 80).

The closest analyses of strategically targeted access in a collective body with multiple

strategic politicians are Awad (2020) and Schnakenberg (2017). These papers differ from this analysis in two key ways: the informational environment and the form of lobbying that access facilitates. Specifically, each studies an incomplete information setting in which lobbying provides information about the policy environment in order to influence votes between two exogenous proposals during an interaction that ends after today's vote. In contrast, I study a complete information setting in which lobbying provides resources to influence endogenous policy proposals during an interaction that can continue after failed proposals. The form of lobbying that I study can be interpreted as exchanging resources for more favorable proposals (in the spirit of Grossman and Helpman (1994)), or as providing a legislative subsidy to a likeminded politician in order to help her influence peers (in the spirit of Hall and Deardorff (2006)). These salient forms of lobbying are worth studying in their own right, but studying them also complements the insights from Schnakenberg (2017) and Awad (2020). Future work can study how the effects highlighted in this paper interact with the informational effects they emphasize.⁶

Like this paper, Schnakenberg (2017) and Awad (2020) highlight a strategic incentive to target ally legislators, but they provide a different logic. In Schnakenberg (2017), groups seek access to allies because they are relatively willing to forward favorable unverifiable information to the other politicians and therefore reduce the group's cost of persuading a majority. In Awad (2020), groups target verifiable information at moderate allies who, precisely because they are more moderate, can then provide a public cheap-talk message that is more convincing to a majority of legislators. By doing so, the group can get policies passed that would have failed if it had instead lobbied the legislature publicly. In order to influence the vote, extreme groups always need access but moderate groups may not. Thus, as in this paper, (i) targeted connections can indirectly influence key votes by non-targeted legislators in equilibrium, (ii) interest groups strategically prefer access to allies, and (iii) moderate groups have different access-seeking incentives than extreme groups.

⁶See Grossman and Helpman (2002) for an extensive overview of canonical informational lobbying models.

Additionally, both Schnakenberg (2017) and Awad (2020) find that targeted access can indirectly affect how non-targeted politicians behave, but through a different mechanism than in this paper. There, endogenous indirect effects require targeted politicians to actively communicate with their peers after being lobbied. Here, endogenous indirect effects do not require any action by the targeted politician. Instead, access causes everyone to anticipate the *potential* for future lobbying, and that anticipation can affect how non-targeted politicians act today.

Finally, another key difference from their analyses is that, by incorporating strategic proposals, I highlight how groups can suffer from access that expands what the legislature would pass. Thus, I provide a logic for why groups have incentives to seek access that will narrow what can pass rather than broaden it. This consideration is not present in their analyses and generates different implications for targeting access.

I also contribute to efforts to incorporate lobbying into legislative bargaining models with strategic proposals and votes. Among various differences, they typically study untargeted access (e.g., Levy and Razin, 2013) or do not emphasize access (e.g., Baron, 2019).⁷ Specifically, I extend the legislative interaction in Cho and Duggan (2003) to include ideological interest groups who can potentially transfer resources to influence proposals. I extend their equilibrium concept to account for lobbying, prove existence, and show that equilibrium behavior has a clear connection to their characterization: the distribution of equilibrium proposals with lobbying is equivalent to a slightly modified version of the model without lobbying. Moreover, I show that lobbying does not introduce delay in this setting, which extends well-known *no-delay* properties that bargaining always ends immediately in similar legislative settings without lobbying (e.g., Banks and Duggan, 2006a).

⁷In addition to its different focus, Baron (2019) studies lobbying directed at votes during bargaining over distributive policy that can continue after passage with endogenous status quo. Closer to this paper, Grossman and Helpman (2002) discuss a model in which lobbying can affect a take-it-or-leave-it proposal and the subsequent votes, but their relatively informal analysis does not discuss access and considerations about future bargaining do not play a role.

Model

The model is designed to study access that provides interest groups with opportunities to lobby certain proposers during a collective policymaking interaction in which bargaining can continue after failed proposals. After introducing the details, I discuss several features in the Model Commentary.

Players. The key players are an interest group, denoted g , and a politician, ℓ . Additionally, there are three other politicians: a left partisan L , a moderate M , and a right partisan R .

Timing. Politicians bargain to set policy in the interval $X \subseteq \mathbb{R}$, which is closed and non-empty. Bargaining occurs over an infinite horizon,⁸ with periods discrete and indexed $t \in \{1, 2, \dots\}$. A status quo policy $q \in X$ persists until policy passes. Thereafter, the strategic interaction ends and the passed policy remains forever. During each period t before some proposal passes, bargaining proceeds in the following two stages.

Proposal stage. First, the period- t proposer i_t is drawn from probability distribution $\rho = (\rho_\ell, \rho_L, \rho_M, \rho_R)$, where $\rho_j > 0$ is politician j 's recognition probability. If $i_t \neq \ell$, then g is not active and i_t proposes any $x_t \in X$. If $i_t = \ell$, then g can lobby with probability $\alpha \in [0, 1]$, which parameterizes g 's *access*. If g is unable to lobby, then ℓ simply proposes any $x_t \in X$. Otherwise, g offers ℓ a binding contract (y_t, m_t) consisting of policy $y_t \in X$ and transfer $m_t \geq 0$.^{9,10} After observing g 's offer, ℓ decides whether to accept or reject. If ℓ accepts, then she proposes $x_t = y_t$ and receives m_t from g . If ℓ rejects, then she can propose any $x_t \in X$ and g keeps m_t .

Voting stage. Next, M decides whether to accept the proposal. If M accepts, then bargaining ends with x_t enacted in t and all subsequent periods. If M rejects, then q persists

⁸As usual, this game can alternatively be viewed as having a unknown finite horizon with a constant probability of termination each period.

⁹Assuming that g lobbies whenever possible is without loss of generality, as g can always effectively forgo lobbying by offering ℓ 's default proposal without payment.

¹⁰This lobbying technology is similar to Bils, Duggan and Judd (2021), who study lobbying in a model of repeated elections. They do not study access, as ideology exogenously determines which group can lobby. For other recent work that models lobbying similarly, see Martimort and Semenov (2008) and an extension in Açemoğlu, Egorov and Sonin (2013).

and active bargaining continues in $t+1$. This stage distills the essence of majoritarian voting in a larger interaction where M is a median voter (Banks and Duggan, 2006b).¹¹

Information. All features are common knowledge.

Payoffs. Player i 's per-period policy utility from $x \in X$ is $u_i(x) = -(\hat{x}_i - x)^2$, where \hat{x}_i denotes i 's ideal point. If ℓ accepts g 's offer (y_t, m_t) and x_t is the period- t policy,¹² then g 's period- t payoff is $u_g(x_t) - m_t$ and ℓ 's period- t payoff is $u_\ell(x_t) + m_t$. Thereafter, m_t does not enter per-period payoffs.

Cumulative dynamic payoffs are the sum of streams of discounted per-period payoffs, with all players sharing the common discount factor $\delta \in (0, 1)$. For convenience, I normalize per-period policy payoffs by $(1 - \delta)$. See Appendix B for complete expressions.

To sharpen key tradeoffs, I maintain several additional assumptions that are not essential. First, I assume $\hat{x}_M = 0 \in X$, which is a normalization. Additionally, to model L and R as staunchly ideological and opposing partisans, I assume $\hat{x}_L < 0 < \hat{x}_R$ and $|q| < \min\{|\hat{x}_L|, \hat{x}_R\}$.

Figure 1: A period with lobbying

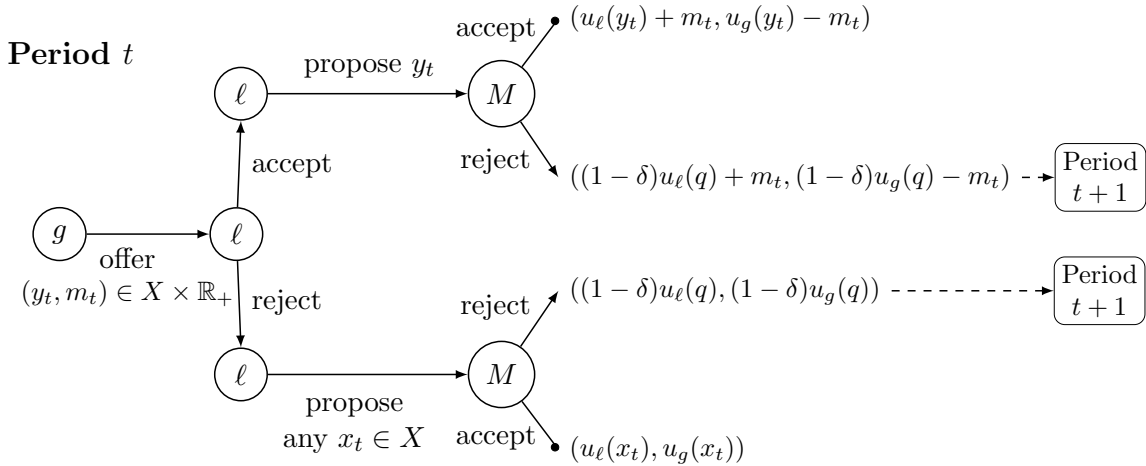


Figure 1 illustrates the within-period interaction if ℓ is recognized and g can lobby. It includes stage payoffs following rejection, and cumulative stage payoffs following acceptance. If ℓ is not recognized or g cannot lobby, the within-period interaction is analogous to Figure 1 after ℓ rejects g 's offer.

¹¹In the appendix, I show that the median is decisive in such a setting and prove the main results.

¹²If y_t passes, then $x_t = y_t$. Otherwise, $x_t = q$.

Equilibrium Concept. I study a refinement of stationary subgame perfect Nash equilibrium that builds on standard equilibrium concepts in the legislative bargaining literature (e.g., Banks and Duggan, 2006a). Informally, a *stationary legislative lobbying equilibrium* satisfies four conditions.¹³ First, M passes a proposal if and only if she weakly prefers to do so rather than reject and continue bargaining. Second, if left to their own devices, each politician proposes policy satisfying M and cannot profitably deviate to any other proposal. Third, politician ℓ accepts a lobby offer if and only if she weakly prefers it over the alternative of making her own proposal. Fourth, g offers a policy that will pass and g cannot profitably deviate to any other offer. By stationarity: (i) M 's voting decision depends only on the current proposal; (ii) politicians other than ℓ propose independently of preceding play; (iii) ℓ accepts or rejects g 's offers based only on the current terms, and ℓ 's proposals in lieu of acceptance are independent of preceding play; and (iv) g 's offers are independent of preceding play. Although players use strategies that are relatively straightforward behavioral rules, no player can profitably deviate to *any* other strategy.

Before proceeding, I note three conditions on strategies that are without loss of generality and streamline the analysis: (i) M passes proposals when indifferent; (ii) ℓ accepts g 's offer when indifferent; and (iii) players use *no-delay* proposal strategies, i.e., each politician proposes passable policy and g offers passable policy. In the appendix, I define *stationary mixed strategy legislative lobbying equilibrium* and show that every such equilibrium is equivalent in outcome distribution to a no-delay stationary pure strategy legislative lobbying equilibrium in which politicians (i) vote in favor of proposals when indifferent and (ii) accept lobby offers when indifferent.¹⁴

¹³See Appendix B for a formal definition.

¹⁴Standard arguments (Banks and Duggan, 2006b) imply that proposal strategies must be no delay. Although related, the no-delay property for interest groups is original to this paper. Essentially, lobbying for delay is always too expensive to be worthwhile in equilibrium. Appendix C provides the technical details.

Model Commentary

A core premise in theories of access is that it weakly increases opportunities to exert influence and, when such opportunities do arise, weakly increases the effectiveness of influence. The baseline model captures the first part of that premise, as access determines the probability that the group can lobby. The model can easily be altered to capture the second part as well, e.g., by allowing access to increase ℓ 's value of transfers from g . Modifying the model in this way does not add substantial insight to the main results. Regardless of how these conceptions of access are combined, the direct consequence of access in the model is that it shifts the target's expected proposal towards the group in equilibrium and thus the indirect effects are qualitatively the same.

The key aspect of lobbying that the model captures is the ability to influence proposals. Groups often lobby in committee to shape the language of bills (Schlozman and Tierney, 1986; Kroeger, 2021) and the policy-for-transfer lobbying technology used here provides a tractable reduced form representation of various ways that such influence could occur (Powell, 2014). Additionally, it allows us to analyze the interaction between access levels and lobbying expenditures. This feature is important when studying incentives to acquire access, as I show that access can increase lobbying expenditures and therefore reduce its appeal.

The exact interpretation of lobbying is not central in this paper, but the model accommodates two prominent forms. First, there is an *exchange* interpretation, which aligns with widespread fears that lobbying is a quid-pro-quo, but can also be interpreted more broadly (Großer, Reuben and Tymula, 2013; Powell, 2014; Baron, 2019).¹⁵ For example, the group could draft language (Schattschneider, 1960) or write a model bill (Kroeger, 2021) to save politicians time or in exchange for various forms of assistance, such as future employment opportunities (Diermeier, Keane and Merlo, 2005) or targeted charitable donations (Bertrand, Bombardini, Fisman and Trebbi, 2020). Second, there is also a *legislative subsidy* interpretation in which the group's lobbying helps a likeminded politician influence her peers on a

¹⁵See Grossman and Helpman (2002) for an extensive overview and discussion.

particular subcommittee whenever it is tasked with writing legislation (Hall and Deardorff, 2006). To streamline discussion, I use the exchange interpretation throughout the analysis.

In order to focus on the effects of access facilitating lobbying that influences policy content, I do not model lobbying that directly influences how politicians vote on proposals. In practice, influencing policy content is particularly appealing for interest groups because it is less visible and more intimate. In contrast, consequential vote buying is relatively difficult because, legality aside, it may require groups to coordinate with several politicians, which is like “herding cats” (Milyo, Primo and Groseclose, 2000). After the analysis, I discuss how the results in this paper complement the large literature on vote buying.

Finally, in the baseline model, access is targeted at one politician and remains constant throughout bargaining. These assumptions streamline the analysis and can be relaxed somewhat. In the appendix, I prove the main results in an extended model allowing more politicians and multiple interest groups that can have access to multiple politicians. And although levels of access could potentially vary over time, stationary access is an analytically convenient way to capture the prevalent view that access is essentially fixed once active policymaking begins (Powell, 2014; Powell and Grimmer, 2016). Studying the finer dynamics of access throughout the policymaking process is an interesting avenue for future work.

Analysis of Equilibrium Legislating and Lobbying

To begin the analysis, I characterize equilibrium behavior in order to introduce how access can affect the strategic calculus for different actors. First, I highlight that equilibrium voting and proposing by politicians has fundamental similarities to related models without lobbying. Then, I characterize equilibrium lobbying and show how it depends on conjectures about voting and non-lobbied proposals. Finally, I combine the preceding qualitative insights in order to sharpen the characterization and more precisely describe how voting, proposing, and lobbying affect each other in equilibrium. Crucially, the characterization explicitly reveals

how access, by determining how strongly players anticipate lobbying, will affect voting, proposing, and lobbying.

Since bargaining continues after rejected proposals, there is a feedback between proposals and legislative voting in equilibrium (as in, e.g., Banks and Duggan, 2006a). To illustrate, consider a politician recognized to propose. To pass policy, they must anticipate the policies M will accept, which depend on M 's expectations about future policymaking, which in equilibrium are consistent with proposal strategies. A key step in the analysis shows how access influences these expectations and thus the acceptance set, thereby affecting proposals that are constrained by the limits of what M will pass.

More precisely, M will pass a proposal if and only if it exceeds her *reservation value* of keeping q for another period and continuing active bargaining. Formally, M 's reservation value is $(1 - \delta)u_M(q) + \delta V_M^*$, where V_M^* denotes M 's equilibrium continuation value immediately after rejecting a proposal.¹⁶ By stationarity, V_M^* is the same each period, so M 's reservation value is constant and thus her voting behavior is the same each period. Specifically, the *acceptance set* is $A^* = [-\bar{x}^*, \bar{x}^*]$, where \bar{x}^* is the positive solution to $u_M(x) = (1 - \delta)u_M(q) + \delta V_M^*$.

Anticipating what M will pass, each politician proposes their favorite policy in A^* whenever recognized (also analogous to Banks and Duggan, 2006a). Clearly, M will simply propose her ideal point, 0. The partisans are constrained by A^* in equilibrium, so L proposes $-\bar{x}^*$ and R proposes \bar{x}^* .¹⁷ Finally, ℓ proposes the policy in A^* closest to \hat{x}_ℓ , denoted z^* , if either g cannot lobby or ℓ rejects g 's offer.

Finally, the interest group, g , wants to shift ℓ 's proposal as far towards \hat{x}_g as is worth paying for, and this strategic calculus depends on its conjectures about voting and non-lobbied proposals. Of course, shifting ℓ 's proposal requires that g compensate her for not instead rejecting and proposing z^* . In equilibrium, g will always make an offer that ℓ accepts, as it can always do weakly better than the trivial acceptable offer of z^* without payment.

¹⁶Appendix B contains explicit expressions of continuation values.

¹⁷This property follows from $|q| < \min\{\hat{x}_L, \hat{x}_R\}$ because standard arguments imply $\bar{x}^* < |q|$.

Additionally, since g knows ℓ 's payoff from proposing z^* , it will compensate her exactly and extract all of the surplus. Stationarity implies that the acceptance set A^* does not depend on today's proposal, so from g 's perspective there is a cost of $u_\ell(z^*) - u_\ell(y)$ associated with each policy $y \in A^*$.

In principle, g could potentially benefit from lobbying for policy outside of A^* if tomorrow's proposer is likely to be an ideological ally who will pass favorable policy for free. Yet, ℓ shares those expectations about future play and therefore must be compensated accordingly in order to propose any policy outside A^* . In equilibrium, the cost of buying delay is never worthwhile for g and therefore it never lobbies for proposals that will be rejected.¹⁸

In sum, g proposes the policy in A^* that provides the best policy payoff given the associated cost. Formally, (y^*, m^*) consists of the policy $y^* = \arg \max_{y \in A^*} u_g(y) + u_\ell(y) - u_\ell(z^*)$ and transfer $m^* = u_\ell(z^*) - u_\ell(y^*)$.¹⁹ Thus, g successfully lobbies ℓ to propose the policy in A^* that maximizes their cumulative policy utility, which is $\hat{y} = \frac{\hat{x}_\ell + \hat{x}_g}{2}$ since they both have quadratic policy utility.

The characterization of equilibrium lobbying implies that the model can be reinterpreted as a one-dimensional bargaining environment in which ℓ has recognition probability $(1 - \alpha)\rho_\ell$ and there is an additional politician at \hat{y} with recognition probability $\alpha\rho_\ell$. After modifying the legislature to include this additional proposer representing the effect of g 's lobbying, politicians propose acceptable bills closest to their ideal point. Applying insights from Cho and Duggan (2003) to this fictitious enlarged legislature implies that this class of equilibria has a unique distribution of equilibrium policies.

Proposition 1 shows that a stationary legislative lobbying equilibrium exists and all such equilibria have the same outcome distribution. Henceforth, I drop qualifiers and say *equilibrium*. Moreover, it collects the preceding observations to characterize a variety of equilibrium behavior: which policies pass and which will be rejected; which policies various politicians will propose; and which policies the interest group will lobby for and how much it will pay.

¹⁸See Appendix C for technical details.

¹⁹Uniqueness of y^* follows because $u_g + u_\ell$ is strictly concave and A^* is a nonempty closed interval.

Figure 2 illustrates a hypothetical equilibrium acceptance set and proposals.

Proposition 1. *A stationary legislative lobbying equilibrium exists and every such equilibrium has the same outcome distribution. In equilibrium,*

- (i) *the acceptance set is $A^* = [-\bar{x}^*, \bar{x}^*]$, where $0 < \bar{x}^* < |q|$;*
- (ii) *M proposes 0 , R proposes \bar{x}^* , and L proposes $-\bar{x}^*$;*
- (iii) *if ℓ is not lobbied, she proposes the policy $z^* \in A^*$ closest to \hat{x}_ℓ ;*
- (iv) *if g can lobby, then it successfully lobbies ℓ to propose the policy $y^* \in A^*$ closest to $\hat{y} = \frac{\hat{x}_\ell + \hat{x}_g}{2}$ using the payment $m^* = u_\ell(z^*) - u_\ell(y^*)$.*

Figure 2: Equilibrium characterization

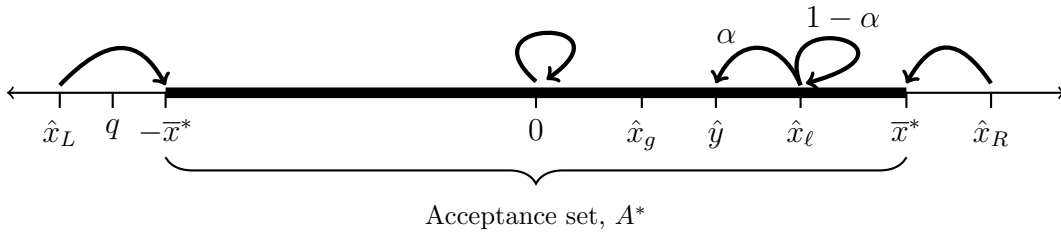


Figure 2 illustrates equilibrium proposals for a hypothetical legislature. Arrows point from politician ideal points to proposals. The bold interval is the acceptance set, A^* . If ℓ is recognized, then with probability α she is lobbied to propose y^* , the policy in A^* closest to $\hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2}$, and otherwise she proposes z^* , the policy in A^* closest to \hat{x}_ℓ . In the depicted legislature, $y^* = \hat{y}$ and $z^* = \hat{x}_\ell$.

Proposition 1 implies that M 's equilibrium continuation value is simply the weighted sum of her policy utility from equilibrium proposals, weighted by their probabilities:

$$V_M^* = \rho_M u_M(0) + \rho_L u_M(-\bar{x}^*) + \rho_R u_M(\bar{x}^*) + \rho_\ell \left(\alpha u_M(y^*) + (1 - \alpha) u_M(z^*) \right). \quad (1)$$

Substituting (1) into M 's indifference condition that defines the boundaries of A^* yields Corollary 1.1, which sharpens our characterization of \bar{x}^* .

Corollary 1.1. *In equilibrium, the boundaries of $A^* = [-\bar{x}^*, \bar{x}^*]$ are characterized by*

$$\bar{x}^* = \left(-\frac{(1 - \delta)u_M(q) + \delta \rho_\ell \left(\alpha u_M(y^*) + (1 - \alpha) u_M(z^*) \right)}{1 - \delta(\rho_L + \rho_R)} \right)^{\frac{1}{2}}. \quad (2)$$

Corollary 1.1 implies that the equilibrium acceptance set expands if: the status quo (q) shifts away from M , patience (δ) decreases, or total partisan recognition probability ($\rho_L + \rho_R$) increases. These effects are familiar from related models without lobbying (e.g., Banks and Duggan, 2006a). Specific to this paper, (2) also reveals that increasing access (α) expands A^* if y^* is farther than z^* from M , but shrinks A^* if y^* is closer than z^* to M . Intuitively, greater access causes M to put more weight on the possibility that g might lobby ℓ in the future if today's proposal fails. If lobbying would make ℓ 's proposal worse for M , then A^* expands because she is less inclined to keep bargaining, and vice versa. Thus, the effect of α on A^* depends critically on how extreme g is relative to ℓ .

Although the effect of access on A^* is original to this paper, it falls under the umbrella of a more general relationship that is familiar from related work without lobbying: the acceptance set expands as the distribution of equilibrium proposals shifts away from M . To be more precise about this general relationship, I next define a notion of changes in *legislative extremism* as a function of α and ρ . The definition compares distributions of unconstrained ideal proposals using first order stochastic dominance, a standard partial order for probability distributions.

Definition 1. For any pair (ρ, α) , let $\Lambda(\rho, \alpha)$ be a lottery that puts probability $\alpha \rho_\ell$ on $|\hat{y}|$, probability $(1 - \alpha) \rho_\ell$ on $|\hat{x}_\ell|$, and probability ρ_j on $|\hat{x}_j|$ for each politician $j \neq \ell$. Say that *legislative extremism increases* if changing (ρ, α) to (ρ', α') is such that: (i) for all $x \in X$, the lottery $\Lambda(\rho', \alpha')$ puts weakly greater probability on x' such that $|x'| \geq |x|$ and (ii) for some $x \in X$, the lottery $\Lambda(\rho', \alpha')$ puts strictly greater probability on x' such that $|x'| \geq |x|$.

Equivalently, legislative extremism increases if $\Lambda(\rho', \alpha')$ first order stochastically dominates $\Lambda(\rho, \alpha)$. Two distinct special cases in which legislative extremism increases are (i) transferring recognition probability from M to other politicians, or (ii) increasing α if \hat{y} is farther than \hat{x}_ℓ from M .

Taking stock, and generalizing our earlier observation, A^* expands as either: legislative extremism increases, δ decreases, or q shifts away from M . By changing the acceptance set, any of these changes will also shift proposals on the boundaries of A^* . Thus, they always affect what L and R will propose. Moreover, they can also shift y^* or z^* if either is constrained by A^* and, if so, they can also affect g 's equilibrium lobby transfer, $m^* = u_\ell(z^*) - u_\ell(y^*)$. Notably, m^* can only vary through changes in A^* since y^* is either \hat{y} or a boundary of A^* , and analogously for z^* . Building on that observation, m^* weakly increases as A^* expands.

Lemma 1. *The interest group's equilibrium payment, m^* , weakly increases with \bar{x}^* .*

Expanding A^* can increase m^* in two distinct ways, depending on whether y^* or z^* is constrained by A^* . If y^* is constrained, then g gets more slack to shift ℓ 's proposal farther and is willing to pay more to do so. If z^* is constrained, then ℓ gets more slack to pass more favorable policy if she rejects g 's offer and is therefore more inclined to reject any lobby offer, but g is willing to pay the additional amount.

Lemma 1 implies that m^* increases as: legislative extremism increases, q shifts away from M , or δ decreases. Next, Proposition 2 expands on that implication by collecting the preceding observations to characterize how equilibrium voting, proposals, and expenditures each depend on legislative extremism (α, ρ), the status quo (q), and patience (δ).

Proposition 2. *If either (i) legislative extremism increases, (ii) the status quo policy becomes more extreme, or (iii) patience decreases, then:*

1. *the acceptance set, A^* , expands;*
2. *proposals constrained by A^* become more extreme; and*
3. *the lobby payment, m^* , weakly increases.*

Consequences of Access

Since access (α) affects legislative extremism, Proposition 2 reveals that it can have a variety of effects in equilibrium. Broadly, the direct effect of α on g 's lobbying chances affects ℓ 's expected proposal, which can then endogenously affect what will pass, what will be proposed, and how many resources will be devoted to lobbying. Corollary 2.1 collects these consequences and shows how they depend on whether lobbying would make ℓ 's proposal more or less extreme.

Corollary 2.1 (Effects of Access). *If $|\hat{y}| > |\hat{x}_\ell|$, then as α increases:*

- (i) **target proposal effect** – ℓ is more likely to propose y^* and less likely to propose z^* ;*
- (ii) **voting effect** – the acceptance set, A^* , expands;*
- (iii) **extreme proposal effect** – proposals constrained by A^* become more extreme; and*
- (iv) **lobbying expenditure effect** – the lobby payment, m^* , weakly increases.*

If $|\hat{y}| < |\hat{x}_\ell|$, then effect (i) is analogous but effects (ii)–(iv) are reversed.

The nature of the indirect effects, (ii) – (iv) depends on how extreme g is relative to ℓ , as that determines whether legislative extremism will increase or decrease in access. For example, if $0 < \hat{x}_\ell < \hat{x}_g$, then increasing α will increase legislative extremism so the acceptance set will expand, constrained proposals will shift farther outward, and lobbying expenditures will weakly increase.

The extreme proposal effect is not limited to the partisans, L and R , as it can also change either the lobby proposal, y^* , or ℓ 's non-lobby proposal, z^* . Yet, it cannot alter both y^* and z^* simultaneously because that would require y^* and z^* to both be constrained. In that case, M would be indifferent between them, but then the target proposal effect would not affect M 's reservation value. Thus, there would be no voting effect and, in turn, no extreme proposal effect on y^* and z^* .

Whom to access?

Thus far, I have shown how access can affect several behaviors by various actors and highlighted how the direction of those effects depends on relative extremism of interest group and target. Since groups appear to have various tools to increase their access in practice, such as campaign contributions or revolving door hiring, I now study who they want to target.

To isolate policy considerations, I allow g to freely choose access.²⁰ The key insights can be conveyed by studying a one-time choice of access prior to bargaining. Substantively, this captures the possibility that interest groups “may make contributions in anticipation that they may need access to a legislator during a legislative term, rather than when the necessity to purchase influence arises” (Powell and Grimmer, 2016, p. 978). Specifically, I analyze how α affects g ’s equilibrium value:

$$\rho_M u_g(0) + \rho_R u_g(\bar{x}^*) + \rho_L u_g(-\bar{x}^*) + \rho_\ell \left[\alpha \left(u_g(y^*) + u_\ell(y^*) - u_\ell(z^*) \right) + (1 - \alpha) u_g(z^*) \right]. \quad (3)$$

Although (3) is similar to (1), it sums over g ’s policy utility and also accounts for g ’s equilibrium lobbying expenditure, $m^* = u_\ell(z^*) - u_\ell(y^*)$.

Inspecting (3) reveals how α can affect g ’s welfare. First, it affects g ’s expected lobbying gain when ℓ is recognized, $\alpha[u_g(y^*) + u_\ell(y^*) - u_\ell(z^*) - u_g(z^*)]$, by changing g ’s lobbying probability and its lobbying surplus. The lobbying surplus changes through (i) the target proposal effect, which alters $u_g(y^*) - u_g(z^*)$, and (ii) the lobbying expenditure effect, which alters m^* . Notably, g ’s lobbying surplus always increases in α : if g is more centrist than ℓ , then g pays weakly less for the same policy; if g is more extreme than ℓ , then g can pass weakly more extreme policy and will do so if that increases lobbying surplus.

Second, α can change g ’s expected policy payoff when a partisan is recognized, $\rho_R u_g(\bar{x}^*) + \rho_L u_g(-\bar{x}^*)$. This effect flows entirely through the extreme proposal effect and can be good

²⁰The core insights are unchanged by including standard convex cost functions for access.

or bad for g , depending on how extreme g is relative to ℓ and potentially also partisan recognition probability, ρ_L and ρ_R . If both extreme proposals shift towards \hat{x}_g , then g benefits. If both shift away, then g is worse off. Finally, if one shifts closer while the other shifts away, then whether g benefits will depend on the relative magnitude of ρ_L and ρ_R .

To begin, I characterize group-legislator pairs for which α has no effect. In addition to highlighting pairs in which the group will not pay for access, it lays a foundation for the subsequent analysis. Note that *both* of the preceding effects of α are zero if and only if lobbying does not change ℓ 's proposal, i.e., $y^* = z^*$, which requires that either (i) $\hat{x}_\ell = \hat{x}_g$, or (ii) \hat{x}_ℓ and \hat{y} are outside the acceptance set in the same direction. In (ii), the acceptance set is $A^* = [-\bar{x}, \bar{x}]$, where

$$\bar{x} = \left(-\frac{(1-\delta)u_M(q)}{1-\delta(\rho_L + \rho_R + \rho_\ell)} \right)^{\frac{1}{2}}. \quad (4)$$

Although \bar{x} resembles (2), it is defined in terms of primitives and, crucially, does not depend on \hat{x}_ℓ, \hat{x}_g , or α . Since \bar{x} does not vary with α , condition (ii) above holds if and only if $\max\{\hat{x}_\ell, \hat{y}\} \leq -\bar{x}$ or $\bar{x} \leq \min\{\hat{x}_\ell, \hat{y}\}$. Thus, a sufficient condition for both effects to be zero is if ℓ leans far enough in either direction because, fixing \hat{x}_g , we have $y^* = z^* = \bar{x}$ if \hat{x}_ℓ leans sufficiently rightward and $y^* = z^* = -\bar{x}$ if \hat{x}_ℓ leans sufficiently leftward.

Very extreme politicians are not worth targeting because lobbying has no effect. Essentially, if ℓ is extreme enough relative to g , then lobbying will not change her proposal, so access is inconsequential and not worth paying for. Proposition 3 formalizes this observation and also uses (4) to place a lower bound on how extreme ℓ must be in order to make lobbying inconsequential.

Proposition 3. *For all \hat{x}_g , there are cutpoints satisfying $\underline{\chi}(\hat{x}_g) \leq -\bar{x}$ and $\bar{\chi}(\hat{x}_g) \geq \bar{x}$ such that g strictly prefers nonzero access only if $\hat{x}_\ell \in (\underline{\chi}(\hat{x}_g), \bar{\chi}(\hat{x}_g))$.*

An implication of the bounds in Proposition 3 is that, regardless of \hat{x}_g , g may want access to ℓ if $\hat{x}_\ell \in (-\bar{x}, \bar{x})$. In this case, sufficiently low α guarantees that ℓ is unconstrained when

proposing, so lobbying would change her proposal and is thus consequential.

If the effects of α are instead nonzero, then they may work in opposite directions or together in g 's favor. For an example in which they work together, consider $0 < \hat{x}_g < \hat{x}_\ell < \bar{x}^*$. Then, increasing α shifts extreme proposals inward towards g from both sides, so g clearly wants access. More broadly, this holds whenever (i) $\hat{x}_g \in \text{int } A^*$ and (ii) A^* shrinks in α , i.e., y^* is more centrist than z^* . Thus, beyond the example above, g also benefits from increasing α if ℓ is in an intermediate range on the opposite side of M . Alternatively, for an example in which the effects oppose, consider $0 < \hat{x}_\ell < \hat{x}_g < \bar{x}^*$. Then, the extreme proposal effect discourages access because both partisan proposals shift outward away from g .

In the two preceding examples, the extreme proposal effect is unambiguous because \hat{x}_g is strictly inside A^* . In that case, varying α either shifts both partisan proposals away from g or shifts both towards g .

In contrast, if \hat{x}_g is *not* strictly inside A^* , then the extreme proposal effect depends on proposal power. Specifically, varying α makes one partisan's proposal more favorable for g but also makes the other partisan's proposal less favorable, so the overall extreme proposal effect depends on the relative recognition probability of L and R .

To distinguish these possibilities in terms of primitives, I show that the extreme proposal effect can be unambiguous if and only if \hat{x}_g lies in an interval around M . Notably, the boundaries of this interval are defined by \bar{x} , introduced earlier in (4). Thus, I first use it to define useful terminology.

Definition 2. Player j is *moderate* if $\hat{x}_j \in (-\bar{x}, \bar{x})$. Otherwise, j is *extremist*.

Lemma 2 shows that moderate groups can be strictly inside the acceptance set, but extremist groups cannot.

Lemma 2. *If g is moderate, then there exists $\tilde{x} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \notin (-\tilde{x}, \tilde{x})$ implies $\hat{x}_g \in \text{int}A^*$ for $\alpha = 0$. If g is extremist, then $\hat{x}_g \notin \text{int}A^*$ for all \hat{x}_ℓ and all α .*

The next two sections leverage the distinction highlighted in Lemma 2 to flesh out a key

insight of this analysis: g 's incentives to acquire access depend on (i) its own extremism and (ii) its extremism relative to ℓ .

Who do moderate groups want to access?

A key implication of Lemma 2 is that, if ℓ is not too centrist, then increasing α from zero has an unambiguous extreme proposal effect for moderate groups. In turn, we can make two broad observations. First, a moderate g wants access to a range of relatively more extreme politicians on its side of the spectrum, as every effect is beneficial. In contrast, access to slightly more centrist politicians has harmful indirect effects that counteract g 's direct benefit from the target proposal effect.

Refining these observations, Proposition 4 shows that moderate groups want to access a range of more extreme politicians *and* an intermediate range of politicians opposite M , but will forgo access to politicians in a relatively more centrist range. Without loss of generality, I analyze $\hat{x}_g > 0$ throughout this section.

Proposition 4. *If $\hat{x}_g \in (0, \bar{x})$, then there are cutpoints satisfying $-\hat{x}_g < x' < x'' < \hat{x}_g$ such that g forgoes access if $\hat{x}_\ell \in (x'', \hat{x}_g)$ but wants access if $\hat{x}_\ell \in (\underline{\chi}(\hat{x}_g), x') \cup (\hat{x}_g, \bar{\chi}(\hat{x}_g))$.*

First, g wants access to ℓ if they are on the same side of M and ℓ is more extreme, but not too extreme. In this case, g benefits from every effect of increasing α . If it lobbies, then it will pay weakly less for the same policy. And even if it does not lobby, M 's reservation value will increase and thereby shrink A^* , with the resulting partisan proposal effect always benefiting g because $\hat{x}_g \in A^*$ for all α in this case.

Additionally, g wants access if ℓ is in an intermediate interval on the opposite side of M . Specifically, if $\hat{x}_\ell \in (\underline{\chi}(\hat{x}_g), -\tilde{x}]$, then g is strictly inside A^* at $\alpha = 0$. Since A^* will shrink as α increases, every effect of increasing α from zero again works in g 's favor. And even if ℓ is slightly more centrist, i.e., $\hat{x}_\ell \in (-\tilde{x}, x')$, then g 's expected gain from the the target proposal effect outweighs any expected loss from the other effects.

Next, g forgoes access if ℓ is on the same side of M and slightly more centrist, i.e., $\hat{x}_\ell \in (x'', \hat{x}_g)$. In this case, g will be strictly inside A^* at $\alpha = 0$ and therefore dislike the extreme proposal effect, which shifts partisan proposals outward as depicted in Figure 3. Crucially, if ℓ and g are close enough, then this negative extreme proposal effect dominates the other effects of access. Intuitively, lobbying will not shift ℓ 's proposal very much and g 's payoff is not very sensitive to those changes, so the direct benefit is small. Meanwhile, M is more sensitive to those changes, and the acceptance set expands enough that the negative extreme proposal effect is relatively larger.²¹ Notably, this case exists for any distribution of proposal power in which L or R is recognized with positive probability. Thus, non-zero partisan proposal power is crucial for g to forgo access, but the magnitude and relative recognition probability of L and R only affect the size of this range.

Finally, g 's preference for access is unclear in general if ℓ is in a centrist range, i.e., $\hat{x}_\ell \in (x', x'')$. In this case, the effects of access conflict, as in the previous case, but now the overall effect depends on partisan recognition probability, specifically either their total or relative magnitude. A stark example is when g is in A^* at $\alpha = 0$. Then, the extreme proposal effect of increasing α from zero depends on the relative magnitude of ρ_L and ρ_R , since one partisan proposal becomes less favorable for g and the other more favorable.

Who do extreme groups want to access?

Like moderate groups, extreme groups have clear preferences over access if ℓ is aligned with them and extremist. Unlike moderate groups, however, extreme groups never want access in that case because lobbying will not change ℓ 's proposal. Formally, $\hat{x}_g \geq \bar{x}$ implies $\bar{\chi}(\hat{x}_g) = \bar{x}$

²¹The indirect effects of access on voting and proposals in this paper have connections with spatial models of dynamic bargaining (Baron, 1996; Buisseret and Bernhardt, 2017; Zápala, 2020). There, the policy in place at the end of today becomes the status quo tomorrow, so proposers weigh how today's proposal can affect what can pass tomorrow when someone else might have proposal rights. In equilibrium, politicians pass more centrist policies today in order to make centrist veto players less inclined to pass policy in the future, thus constraining the scale of policy changes by potential future proposers on the other end of the spectrum. In this paper, policymaking ends once a proposal passes, so a group considering access weighs (i) how it will affect the target's proposal if she is recognized, and (ii) how it will affect what happens if the target is not recognized. Since access can indirectly influence which policies pass in equilibrium, incentives to increase or forgo access are affected by a similar desire to constrain potentially extreme proposers.

Figure 3: Forgoing access to more centrist legislators

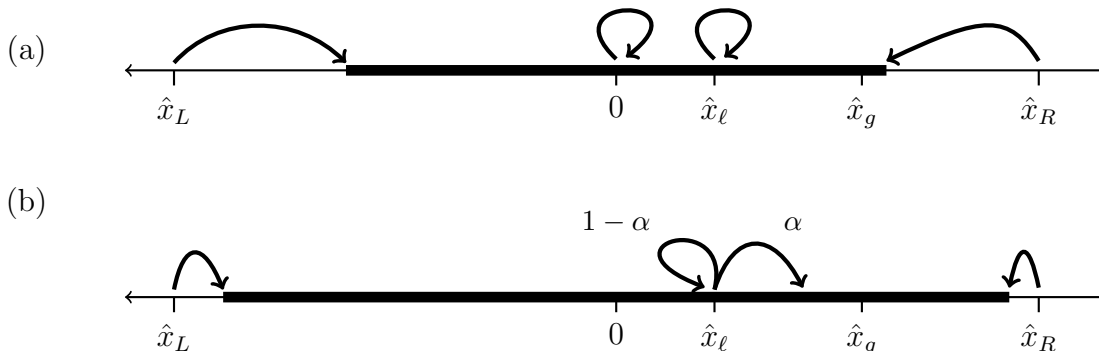


Figure 3 illustrates why a moderate group, g , forgoes access ($\alpha = 0$) if $\hat{x}_\ell \in (x'', \hat{x}_g)$. Part (a) displays equilibrium behavior for $\alpha = 0$. Part (b) illustrates $\alpha > 0$. In each, the bold interval is the acceptance set. Increasing α makes lobbying more likely, which worsens M 's expectations, and expands the acceptance set, as shown in (b). Thus, partisan proposals are more extreme. If \hat{x}_g and \hat{x}_ℓ are close, then the loss from more extreme partisan proposals dominates and g prefers $\alpha = 0$.

in Lemma 3 and analogously $\hat{x}_g \leq -\underline{x}$ implies $\underline{\chi}(\hat{x}_g) = -\bar{x}$.

A key difference is that, since extreme groups are always outside A^* , the direction of the extreme proposal effect always depends on the relative magnitude of ρ_L and ρ_R regardless of \hat{x}_ℓ . To overcome this difficulty and shed some light on who extreme groups want to access, Proposition 5 focuses on cases in which one partisan is sufficiently weak. Substantively, this could reflect partisan gatekeeping in which extremists on one side of the spectrum are largely excluded from writing policy. As in the previous section, I focus on $\hat{x}_g > 0$ without loss of generality.

Proposition 5. *Suppose $\hat{x}_g > \bar{x}$.*

- (i) *If ρ_L is small enough, there exists $x' < 0$ such that g wants access if $\hat{x}_\ell \in (x', \bar{x})$.*
- (ii) *If ρ_R is small enough, there exists $x'' \geq -\bar{x}$ such that g wants access if $\hat{x}_\ell \in (\underline{\chi}(\hat{x}_g), x'')$.*
- (iii) *If $\hat{x}_\ell \geq \bar{x}$, then g does not want access.*

In (i), g 's opposing partisan is unlikely to propose, so g wants access to a range of moderate politicians including all right-leaning moderates and sufficiently centrist opponents.

As long as ℓ does not lean too far leftward, increasing access will worsen M 's expectations about future policy and thus expand A^* . Although L 's proposal gets worse for g , she is unlikely to propose, so that downside is outweighed by the prospect of better proposals by ℓ and R .

In (ii), g 's aligned partisan is unlikely to propose and it wants access to opponents (except those extreme enough to make lobbying trivial) and potentially also to sufficiently centrist aligned moderates if the lobbying surplus is large enough. The logic is symmetric to the previous case.

Proposal power and the value of access

Thus far, I have focused primarily on how ideology affects g 's incentives to acquire access to ℓ , while noting how partisan proposal power can play a role in those incentives. In this section, I focus on the effects of the target's proposal power. Specifically, I study how ℓ 's recognition probability (ρ_ℓ) affects g 's *willingness to pay* (WTP) for access, i.e., the marginal effect of α on g 's equilibrium value in (3).

Empirical evidence suggests that interest groups prioritize access to legislators who have more proposal power²² and it is typically taken for granted that greater proposal power makes access more valuable. Yet, the preceding analysis highlights a potentially important subtlety. Although ρ_ℓ increases g 's expected lobbying benefit from access, it also amplifies the (possibly negative) extreme proposal effect. Despite these potentially competing effects, however, Proposition 6 establishes that the standard intuition holds in this paper: g 's WTP for access weakly increases with ρ_ℓ .

Proposition 6. *All else equal, the interest group is willing to pay more for access if the target politician has higher recognition probability.*

²²This is one of the most prominent stylized facts about outside influence and is supported by two robust empirical regularities: (i) legislators on important committees, especially committee chairmen, attract more contributions (Fournaies, 2018; Berry and Fowler, 2018), and (ii) lobbyists connected to those legislators command a premium (Blanes i Vidal et al., 2012).

Proposition 6 is a stark result, as it does not depend on ℓ or g 's policy preferences, partisan proposal power, patience, or the status quo. Although these other factors can cause ρ_ℓ to have competing effects, the overall effect is always proportional to ρ_ℓ whenever g 's WTP is strictly positive. Thus, g 's WTP either increases in ρ_ℓ or remains at zero. Overall, the starkness of this result fits with the robustness of the empirical finding that groups prioritize politicians with greater proposal power.

Discussion

A key takeaway of the analysis is that access can influence behavior even without materializing into a lobbying opportunity. Instead, merely by increasing the *chances* of lobbying, access alone can endogenously influence votes, proposals, and lobbying expenditures. This equilibrium phenomenon sheds new light on the widespread view that access is necessary for influence, but does not influence behavior on its own (e.g., Wright, 1989). The analysis in this paper highlights how observing no lobbying need not imply that a connected interest group missed out on benefiting from access. Furthermore, it highlights how insider descriptions of what access-seeking behavior (e.g., contributions) “buys” can be misleading, even when they are sincere.

Although some scholars have informally noted the possibility that targeted access can influence behavior by non-targeted politicians (favorably or unfavorably),²³ I formally derive a channel that flows entirely through legislative considerations. By doing so, the results reveal potential obstacles for estimating effects of access. Even if access can be randomized, equilibrium effects can (i) prevent expectations about future proposals from being held constant by such randomization and (ii) generate spillover effects that would violate SUTVA. Thus, attempts to recover causal effects of access must especially clear about their estimand and how they can convincingly estimate it with their data.

²³For example, (Kalla and Broockman, 2015) suggest that other politicians might act differently with the hope of attracting donations from the group as well.

The analysis also offers implications for how lobbying expenditures vary with access. First, I highlight a novel way that access-seeking expenditures can affect lobbying expenditures. By changing what some politicians would propose on their own, access can affect how much lobbying effort *any* group must exert to alter those proposals. Additionally, this channel shows how access-seeking expenditures can affect lobbying effort even without making lobbying more efficient. An empirical implication is that observing a change in lobbying expenditures does not imply a change in lobbying effectiveness. Second, I also highlight how access-seeking expenditures and lobbying expenditures can be complements or substitutes in equilibrium, depending on whether the group is more or less extreme than the politician it targets. Thus, if we do not account for relative extremism, then observing no relationship between measures of access and average lobbying expenditures (across group-legislator pairs) need not imply that there is no effect.

The analysis also offers implications for *Tullock's puzzle*, the empirical regularity that surprisingly few interest groups contribute and they contribute surprisingly low amounts. (Tullock, 1972; Ansolabehere et al., 2003). Given evidence that groups can increase access in various ways (e.g., contributions, revolving door hiring) and the natural expectation that groups want more influence, why do they not spend more? Although there are other explanations based on costs or competition (e.g., Chamon and Kaplan, 2013), this paper provides a new logic that instead emphasizes legislative considerations. A key insight here is that increasing your potential for influence can affect what happens if that potential is not realized. An unfavorable effect discourages access, while a favorable effect increases the bang for the buck. Either way, these effects suggest that groups may spend less than expected on access-seeking behaviors and that they may not spend anything to target slightly more centrist politicians.

Finally, this paper also has implications for our theoretical understanding of direct vote buying. A key difference is that models of vote buying typically consider exogenous or take-it-or-leave-it proposals (e.g., Snyder Jr., 1991; Dekel, Jackson and Wolinsky, 2009),

whereas in this paper politicians make strategic proposals and bargaining continues after failed proposals. Although I abstract from vote buying in order to isolate considerations related to lobbying over policy details in committee, the analysis highlights how access to influence strategic proposals can strengthen or weaken incentives to buy key votes. If such access expands what can pass, then groups are more willing to pay decisive voters for no votes on proposals by opposing extremists, and vice versa.

Conclusion

I analyze a model of legislative policymaking in which access provides interest groups with opportunities to lobby policy proposals. The equilibrium analysis sheds new light on the consequences of this prominent form of access by showing how it can endogenously affect voting, proposals, and lobbying. It does so by changing each legislator's expectations about policymaking, and thereby changing which policies can pass in equilibrium. Essentially, the potential for *future* lobbying can influence *today's* proposal and lobbying expenditures.

The analysis also sheds light on how much access interest groups want to particular legislators who may be involved in writing policy. Moderate groups forgo access to a range of more centrist legislators since such connections endogenously increase policy extremism enough to outweigh the perk of better lobbying prospects. On the other hand, these groups crave access to more extreme legislators because it facilitates lobbying and also *reduces* policy extremism.

By developing our theoretical expectations for the consequences of a link between access and lobbying proposals, which remain largely unexplored, I shed light on how such a link can affect policy and shape observed data. The analysis here emphasizes how such access can have indirect effects due to legislative considerations, i.e., what other politicians will vote for and what they will propose if given the opportunity. Although the channel I emphasize is prominent, other important channels are likely present in various situations. Whenever

we cannot disentangle multiple channels empirically, we need to be aware that they may oppose or complement each other. To understand these relationships and potentially suggest avenues to disentangle various influence tactics, future work should study how the legislative forces highlighted here interact with other channels of outside influence such as vote buying, informational lobbying, and efforts to influence who gets elected.

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Appendix (online only)

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A Proof of Proposition 4 for Baseline Model

First, I prove the key parts of Proposition 4 for the model presented in the main text. The logic is similar to the full proof in Appendix B, but the details are easier to digest. Let subscripts on equilibrium objects denote dependence on α , e.g., A_α^* denotes the equilibrium acceptance set given α .

Consider $\hat{x}_g \in (0, \bar{x})$. I show that (i) there exists $x'' < \hat{x}_g$ such that g strictly prefers $\alpha = 0$ for all $\hat{x}_\ell \in (x'', \hat{x}_g)$, and (ii) if $\hat{x}_\ell \in (\hat{x}_g, \bar{x})$, then g strictly prefers $\alpha > 0$. The argument for the other regions in which g strictly prefers $\alpha > 0$ is similar to (ii) and can be found in the full proof provided in Appendix B.

We can show that there exists $\tilde{x} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (\tilde{x}, \bar{x})$ implies $\hat{x}_g, \hat{x}_\ell \in A_\alpha^*$ for all α (for details, see Lemma 2 in Appendix B and arguments in Appendix D). Then, Proposition 1 implies $m^* = -u_\ell(\hat{y})$ and, moreover, that the equilibrium outcome distribution is equivalent to a lottery λ_α putting probability ρ_M on 0, ρ_R on \bar{x}_α^* , ρ_L on $-\bar{x}_\alpha^*$, $\alpha \rho_\ell$ on $\hat{y} = \frac{\hat{x}_\ell + \hat{x}_g}{2}$, and $(1 - \alpha)\rho_\ell$ on \hat{x}_ℓ . Since u_g is quadratic, we can therefore express g 's equilibrium value from α access using the mean ($\mathbb{E}[\lambda_\alpha]$) and variance ($\mathbb{V}[\lambda_\alpha]$) of λ_α :

$$\mathcal{U}_g(\alpha; \hat{x}_\ell) = u_g(\mathbb{E}[\lambda_\alpha]) - \mathbb{V}[\lambda_\alpha] + \alpha \rho_\ell u_\ell(\hat{y}) \quad (5)$$

$$\begin{aligned} &= -\hat{x}_g^2 + 2\hat{x}_g \left[\alpha \rho_\ell (\hat{y} - \hat{x}_\ell) + \rho_\ell \hat{x}_\ell + (\rho_R - \rho_L) \bar{x}_\alpha^* \right] \\ &\quad - \left[\alpha \rho_\ell (\hat{y}^2 - \hat{x}_\ell^2) + \rho_\ell \hat{x}_\ell^2 + (\rho_R + \rho_L) (\bar{x}_\alpha^*)^2 \right] - \frac{\alpha \rho_\ell}{4} (\hat{x}_\ell - \hat{x}_g)^2. \end{aligned} \quad (6)$$

Differentiating (6) with respect to α yields:

$$\begin{aligned} \frac{\partial \mathcal{U}_g(\alpha; \hat{x}_\ell)}{\partial \alpha} &= 2\hat{x}_g \left[\rho_\ell (\hat{y} - \hat{x}_\ell) + (\rho_R - \rho_L) \frac{\partial \bar{x}_\alpha^*}{\partial \alpha} \right] \\ &\quad - \rho_\ell (\hat{y}^2 - \hat{x}_\ell^2) - 2\bar{x}_\alpha^* (\rho_R + \rho_L) \frac{\partial \bar{x}_\alpha^*}{\partial \alpha} - \frac{\rho_\ell}{4} (\hat{x}_\ell - \hat{x}_g)^2 \end{aligned} \quad (7)$$

$$\propto (\hat{x}_g - \hat{x}_\ell)^2 - \frac{\delta(\hat{x}_g - \hat{x}_\ell)(3\hat{x}_g + \hat{x}_\ell)}{2[1 - \delta(\rho_L + \rho_R)]} \left[\rho_L \left(1 + \frac{\hat{x}_g}{\bar{x}_\alpha^*} \right) + \rho_R \left(1 - \frac{\hat{x}_g}{\bar{x}_\alpha^*} \right) \right], \quad (8)$$

where (8) follows from factoring out ρ_ℓ and simplifying after substituting $\hat{y} = \frac{\hat{x}_\ell + \hat{x}_g}{2}$ and $\frac{\partial \bar{x}_\alpha^*}{\partial \alpha} = \frac{\delta \rho_\ell (\hat{x}_g - \hat{x}_\ell) (3\hat{x}_g + \hat{x}_\ell)}{8\bar{x}_\alpha^* [1 - \delta(\rho_L + \rho_R)]}$. There are two cases.

Case 1: If $\hat{x}_g > \hat{x}_\ell$, then (8) is proportional to

$$(\hat{x}_g - \hat{x}_\ell) - \frac{\delta(3\hat{x}_g + \hat{x}_\ell)}{2[1 - \delta(\rho_L + \rho_R)]} \left[\rho_L \left(1 + \frac{\hat{x}_g}{\bar{x}_\alpha^*} \right) + \rho_R \left(1 - \frac{\hat{x}_g}{\bar{x}_\alpha^*} \right) \right], \quad (9)$$

which converges to

$$-\frac{4\delta\hat{x}_g}{1 - \delta(\rho_L + \rho_R)} \left[\rho_L \left(1 + \frac{\hat{x}_g}{\bar{x}_1^*} \right) + \rho_R \left(1 - \frac{\hat{x}_g}{\bar{x}_1^*} \right) \right] < 0 \quad (10)$$

as $\hat{x}_\ell \uparrow \hat{x}_g$, where the inequality follows because $\frac{\hat{x}_g}{\bar{x}_1^*} \in (0, 1)$. Thus, there exists $x'' < \hat{x}_g$ such that $\mathcal{U}_g(\alpha; \hat{x}_\ell)$ strictly decreases in α for all $\hat{x}_\ell \in (x'', \hat{x}_g)$, as desired.

Case 2: If $\hat{x}_\ell \in (\hat{x}_g, \bar{x})$, then (8) is proportional to

$$(\hat{x}_\ell - \hat{x}_g) + \frac{\delta(3\hat{x}_g + \hat{x}_\ell)}{2[1 - \delta(\rho_L + \rho_R)]} \left[\rho_L \left(1 + \frac{\hat{x}_g}{\bar{x}_\alpha^*} \right) + \rho_R \left(1 - \frac{\hat{x}_g}{\bar{x}_\alpha^*} \right) \right] > 0, \quad (11)$$

where the inequality follows because $\frac{\hat{x}_g}{\bar{x}_\alpha^*} \in (0, 1)$ for all $\alpha \in [0, 1]$. Thus, $\mathcal{U}_g(\alpha; \hat{x}_\ell)$ strictly increases in α .

B Extended Model: More Politicians and Interest Groups

I prove Propositions 1–5 in a model that relaxes restrictions on the number of legislators and interest groups. There are three disjoint sets of players: n^V (finite and odd) voting legislators in N^V ; $n^L \geq 3$ committee members in N^L ; and $n^G \leq n^L$ interest groups in N^G . Let $N = N^V \cup N^L \cup N^G$.

Throughout, voting legislators are denoted by i and called *voters*. I denote committee members by ℓ and interest groups by g . Each $\ell \in N^L$ is associated with only one group, g_ℓ . Each $g \in N^G$ can have access to multiple $\ell \in N^L$ and this set is $N_g^L \subseteq N^L$. Let $\alpha_\ell \in [0, 1]$

denote g_ℓ 's access to ℓ .²⁴

Legislative bargaining occurs over an infinite number of periods $t \in \{1, 2, \dots\}$. The policy space is a non-empty, closed interval $X \subseteq \mathbb{R}$. Let $\rho = (\rho_1, \dots, \rho_{n^L}) \in \Delta([0, 1])^{n^L}$ be the distribution of recognition probability.²⁵ In each period t , bargaining proceeds as follows. If no policy has passed before t , then ℓ proposes with probability $\rho_\ell > 0$. All players observe the period- t proposer, ℓ_t . With probability $1 - \alpha_\ell$, g_{ℓ_t} cannot lobby and ℓ_t freely proposes any $x_t \in X$. With probability α_ℓ , g_{ℓ_t} can lobby and offers ℓ_t a binding contract $(y_t, m_t) \in X \times \mathbb{R}_+$. Next, ℓ_t accepts or rejects. Let $a_t \in \{0, 1\}$ denote ℓ_t 's period- t acceptance decision, where $a_t = 1$ indicates acceptance and $a_t = 0$ if either ℓ_t rejects or g_{ℓ_t} is unable to lobby in t . If ℓ_t accepts, then ℓ_t is committed to propose $x_t = y_t$ in t and g_{ℓ_t} transfers m_t to ℓ_t . If ℓ_t rejects, then she can propose any $x_t \in X$ and g_{ℓ_t} keeps m_t . All players observe x_t . There is a simultaneous vote by $i \in N^V$ using simple majority rule. If x_t passes, then bargaining ends with x_t enacted in t and all subsequent periods. If x_t fails, then q is enacted in t and bargaining proceeds to $t + 1$.

Each player $j \in N$ has quadratic policy utility with ideal point $\hat{x}_j \in X$. To align with the main text, M denotes the median voter. As in the main text, I normalize $\hat{x}_M = 0$ and assume $q \neq 0$. Additionally, I assume there exists $\ell \in N^L$ on the same side of q as M such that: $\alpha_\ell < 1$ or g_ℓ is on the same side of q . For example, if $q > 0$, then some $\ell \in N^L$ satisfies $\hat{x}_\ell < q$ and at least one of the following holds: $\alpha_\ell < 1$ or $\hat{x}_{g_\ell} \leq q$.

Players discount streams of per-period utility by common discount factor $\delta \in (0, 1)$. For convenience, I normalize per-period policy payoffs by $(1 - \delta)$. Let $I_t^\ell \in \{0, 1\}$ equal one if and only if ℓ is the period- t proposer and g_{ℓ_t} can lobby in t . Given a sequence of offers $(y_1, m_1), (y_2, m_2), \dots$, a sequence of proposers ℓ_1, ℓ_2, \dots , a sequence of acceptance decisions a_1, a_2, \dots , and a sequence of independent policy proposals x_1, x_2, \dots such that bargaining

²⁴An independent legislator is accommodated by $\alpha_\ell = 0$.

²⁵Where $\Delta([0, 1])^{n^L}$ denotes the n^L -dimensional unit simplex.

continues until t , the discounted sum of per-period payoffs for $i \in N^V$ is

$$(1 - \delta^{t-1})u_i(q) + \delta^{t-1} \left[(1 - a_t) u_i(x_t) + a_t u_i(y_t) \right];$$

for $\ell \in N^\ell$,

$$\sum_{t'=1}^{t-1} \delta^{t'-1} [(1 - \delta) u_\ell(q) + I_{t'}^\ell a_{t'} m_{t'}] + \delta^{t-1} \left[(1 - a_t) u_\ell(x_t) + a_t \left(u_\ell(y_t) + I_t^\ell m_t \right) \right];$$

and for $g \in N^g$,

$$\sum_{t'=1}^{t-1} \delta^{t'-1} \left[(1 - \delta) u_g(q) - a_{t'} m_{t'} \sum_{\ell \in N_g^\ell} I_{t'}^\ell \right] + \delta^{t-1} \left[(1 - a_t) u_g(x_t) + a_t \left(u_g(y_t) - m_t \sum_{\ell \in N_g^\ell} I_t^\ell \right) \right].$$

The model in the main text is a special case featuring one voter with ideal point \hat{x}_M ; four committee members with ideal points $\hat{x}_L, \hat{x}_M, \hat{x}_\ell$, and \hat{x}_R ; and one group at \hat{x}_g with access $\alpha_\ell \geq 0$ and $\alpha_j = 0$ for all $j \neq \ell$.

Strategies and Equilibrium Concept: I study a refinement of stationary subgame perfect equilibrium. First, I formalize mixed strategies to express continuation values. I then define pure strategies and the equilibrium concept: *no-delay stationary legislative lobbying equilibrium*. In Appendix C, I define *stationary mixed strategy legislative lobbying equilibria* and show that they must be equivalent in outcome distribution to a no-delay stationary pure strategy legislative lobbying equilibrium. Thus, the outcome distribution characterized in Proposition 1 applies even more broadly.

Let $\Delta(X)$ be the set of probability measures on X . Let $W = X \times \mathbb{R}_+$ denote the lobby offer space and $\Delta(W)$ denote the set of probability measures on W . A stationary mixed strategy for $g \in N^G$ is a probability measure $\lambda_g \in \Delta(W)^{|N_g^L|}$ over g 's offers $(y, m) \in W$ to each $\ell \in N_g^L$. A stationary mixed legislative strategy for $\ell \in N_g^L$ is a pair (π_ℓ, φ_ℓ) ; where $\pi_\ell \in \Delta(X)$ specifies a probability measure over ℓ 's independent proposals and $\varphi_\ell : W \rightarrow [0, 1]$ is the probability ℓ accepts each $(y, m) \in W$. Finally, voter i 's stationary mixed strategy

$\nu_i : X \rightarrow [0, 1]$ specifies the probability i votes for each $x \in X$.

Let λ denote a profile of interest group strategies, (π, φ) a profile of committee member strategies, and ν a profile of voter strategies. A stationary strategy profile is $\sigma = (\lambda, \pi, \varphi, \nu)$. Under σ , let $\bar{\nu}_\sigma(x)$ be the probability x passes if proposed.

Let $w = (y, m) \in W$ denote an arbitrary lobby offer. Define

$$\xi_\ell(\alpha, \sigma) = (1 - \alpha_\ell) + \alpha_\ell \int_W [1 - \varphi_\ell(y, m)] \lambda_g^\ell(dw), \quad (12)$$

which is the probability under σ that ℓ makes an independent policy proposal conditional on being recognized. Given σ , $i \in N^V$ has continuation value

$$\begin{aligned} V_i(\sigma) = & \sum_{\ell \in N^L} \rho_\ell \left(\alpha_\ell \int_W \varphi_\ell(y, m) \left[\bar{\nu}_\sigma(y) u_i(y) + [1 - \bar{\nu}_\sigma(y)] [(1 - \delta) u_i(q) + \delta V_i(\sigma)] \right] \lambda_{g_\ell}^\ell(dw) \right. \\ & \left. + \xi_\ell(\alpha, \sigma) \int_X \left[\bar{\nu}_\sigma(x) u_i(x) + [1 - \bar{\nu}_\sigma(x)] [(1 - \delta) u_i(q) + \delta V_i(\sigma)] \right] \pi_\ell(dx) \right), \quad (13) \end{aligned}$$

the continuation value of $\ell \in N^L$ is

$$\begin{aligned} \tilde{V}_\ell(\sigma) = & \sum_{j \neq \ell} \rho_j \left(\alpha_j \int_W \varphi_j(y, m) \left[\bar{\nu}_\sigma(y) u_\ell(y) + [1 - \bar{\nu}_\sigma(y)] [(1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \lambda_{g_j}^j(dw) \right. \\ & \left. + \xi_j(\alpha, \sigma) \int_X \left[\bar{\nu}_\sigma(x) u_\ell(x) + [1 - \bar{\nu}_\sigma(x)] [(1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_j(dx) \right) \\ & + \rho_\ell \left(\alpha_\ell \int_W \varphi_\ell(y, m) \left[\bar{\nu}_\sigma(y) u_\ell(y) + [1 - \bar{\nu}_\sigma(y)] [(1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] + m \right] \lambda_{g_\ell}^\ell(dw) \right. \\ & \left. + \xi_\ell(\alpha, \sigma) \int_X \left[\bar{\nu}_\sigma(x) u_\ell(x) + [1 - \bar{\nu}_\sigma(x)] [(1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma)] \right] \pi_\ell(dx) \right), \quad (14) \end{aligned}$$

and the continuation value of $g \in N^G$ is

$$\hat{V}_g(\sigma) = \sum_{\ell \notin N_g^L} \rho_\ell \left(\alpha_\ell \int_W \varphi_\ell(y, m) \left[\bar{\nu}_\sigma(y) u_g(y) + [1 - \bar{\nu}_\sigma(y)] [(1 - \delta) u_g(q) + \delta \hat{V}_g(\sigma)] \right] \lambda_{g_\ell}^\ell(dw) \right)$$

$$\begin{aligned}
& + \xi_\ell(\alpha, \sigma) \int_X \left[\bar{v}_\sigma(x) u_g(x) + [1 - \bar{v}_\sigma(x)] [(1 - \delta) u_g(q) + \delta \widehat{V}_g(\sigma)] \right] \pi_\ell(dx) \Big) \\
& + \sum_{\ell \in N_g^L} \rho_\ell \left(\alpha_\ell \int_W \varphi_\ell(y, m) \left[\bar{v}_\sigma(y) u_g(y) + [1 - \bar{v}_\sigma(y)] [(1 - \delta) u_g(q) + \delta \widehat{V}_g(\sigma)] - m \right] \lambda_g^\ell(dw) \right. \\
& \left. + \xi_\ell(\alpha, \sigma) \int_X \left[\bar{v}_\sigma(x) u_g(x) + [1 - \bar{v}_\sigma(x)] [(1 - \delta) u_g(q) + \delta \widehat{V}_g(\sigma)] \right] \pi_\ell(dx) \right), \quad (15)
\end{aligned}$$

A stationary pure strategy for $g \in N^G$ is $(y_g, m_g) \in X^{|N_g^L|} \times \mathbb{R}_+^{|N_g^L|}$, where y_g is g 's profile of policy offers and m_g is g 's profile of monetary offers. A pure stationary strategy for $\ell \in N^L$ is (z_ℓ, a_ℓ) ; where $z_\ell \in X$ specifies ℓ 's independent proposal, and $a_\ell : X \times \mathbb{R} \rightarrow \{0, 1\}$ equals one iff ℓ accepts g_ℓ 's offer. Finally, for each $i \in N^V$, $v_i : X \rightarrow \{0, 1\}$ equals one iff i supports the proposal.

Given σ , the set of policies that pass is constant across periods by stationarity and denoted $A(\sigma)$. For $\ell \in N^L$, define

$$\tilde{U}_\ell(x; \sigma) = \begin{cases} u_\ell(x) & \text{if } x \in A(\sigma) \\ (1 - \delta) u_\ell(q) + \delta \tilde{V}_\ell(\sigma) & \text{else.} \end{cases} \quad (16)$$

Formally, $\sigma = (y, m, z, a, v)$ is a *no-delay stationary legislative lobbying equilibrium* if it satisfies five conditions. First, for all $g \in N^G$ and $\ell \in N_g^L$, (y_g^ℓ, m_g^ℓ) satisfies

$$y_g^\ell \in \arg \max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell) \quad (17)$$

and

$$m_g^\ell = u_\ell(z_\ell) - u_\ell(y_g^\ell). \quad (18)$$

Second, for all $\ell \in N^L$ and $(y, m) \in W$, $a_\ell(y, m) = 1$ iff

$$\tilde{U}_\ell(y; \sigma) + m \geq \tilde{U}_\ell(z_\ell; \sigma). \quad (19)$$

Third, for each $\ell \in N^L$,

$$z_\ell \in \arg \max_{x \in A(\sigma)} u_\ell(x). \quad (20)$$

Finally, for each $i \in N^V$, $v_i(x) = 1$ iff

$$u_i(x) \geq (1 - \delta)u_i(q) + \delta V_i(\sigma). \quad (21)$$

B.1 Proof of Proposition 1

Proof. There are four parts. Part 1 shows existence of a fixed point that maps a profile of (i) no-delay stationary lobby offer strategies and (ii) no-delay stationary proposal strategies to itself as the solution to optimization problems for $g \in N^G$ and $\ell \in N^L$. Part 2 uses the fixed point to construct a strategy profile σ . Part 3 verifies that σ satisfies (17) - (21). Part 4 shows there is a unique equilibrium outcome distribution.

Part 1: Let $(y, z) = (y_1, \dots, y_{n^L}, z_1, \dots, z_{n^L}) \in X^{2n^L}$ and for each $j \in N$ define

$$r_j(y, z) = \sum_{\ell \in N^L} \rho_\ell \left(\alpha_\ell u_j(y_\ell) + (1 - \alpha_\ell) u_j(z_\ell) \right). \quad (22)$$

Set $A(r(y, z)) = \{x \in X \mid u_M(x) \geq (1 - \delta)u_M(q) + \delta r_M(y, z)\}$, which is non-empty, compact, and convex because $\delta \in (0, 1)$, $q \neq 0$, and u_M is strictly concave. Moreover, $A(r(y, z))$ is continuous in (y, z) .

For each $\ell \in N^L$, define

$$\tilde{\phi}_\ell(y, z) = \arg \max_{y_\ell \in A(r(y, z))} u_{g_\ell}(y_\ell) + u_\ell(y_\ell), \quad (23)$$

which is unique for all (y, z) because $A(r(y, z))$ is non-empty, compact and convex, and the objective function is strictly concave and continuous. Because $A(r(y, z))$ is continuous, the Theorem of the Maximum implies continuity of $\tilde{\phi}_\ell(y, z)$. Next, define

$$\phi_\ell(y, z) = \arg \max_{z_\ell \in A(r(y, z))} u_\ell(z_\ell), \quad (24)$$

which is unique for all (y, z) and continuous by the Theorem of the Maximum.

Define the mapping $\Phi : X^{2n^L} \rightarrow X^{2n^L}$ as $\Phi(y, z) = \prod_{\ell \in N^L} \tilde{\phi}_\ell(y, z) \times \prod_{\ell \in N^L} \phi_\ell(y, z)$, which is a product of continuous functions and thus continuous. By Brouwer's theorem, a fixed point $(y^*, z^*) = \Phi(y^*, z^*)$ exists because Φ is a continuous function mapping a non-empty, compact, and convex set into itself.

Part 2: Define a stationary pure strategy profile σ as follows. First, for all $g \in N^G$ and $\ell \in N^L$, set $y_g^\ell = y_\ell^*$ and $m_g^\ell = u_\ell(z_\ell^*) - u_\ell(y_\ell^*)$. Next, for $\ell \in N^L$, set $z_\ell = z_\ell^*$ and define

$$a_\ell(y, m) = \begin{cases} 1 & \text{if } u_\ell(y) + m \geq u_\ell(z_\ell^*), \text{ for } y \in A(r(y^*, z^*)) \\ 1 & \text{if } (1 - \delta)u_\ell(q) + \delta(r_\ell(y^*, z^*) + \rho_\ell \alpha_\ell m_g^\ell) + m \geq u_\ell(z_\ell^*), \text{ for } y \notin A(r(y^*, z^*)) \\ 0 & \text{else.} \end{cases} \quad (25)$$

Finally, for each $i \in N^V$ define v_i so that $v_i(x) = 1$ if $u_i(x) \geq (1 - \delta)u_i(q) + \delta r_\ell(y^*, z^*)$ and $v_i(x) = 0$ otherwise.

Part 3: I verify that σ satisfies (17)-(21) and no player has a profitable deviation.

First, I verify (21) to show $A(\sigma) = A(r(y^*, z^*))$. Note that for each $g \in N^G$ and all

$\ell \in N^L$, we have $y_g^\ell \in A(r(y^*, z^*))$ and $a_\ell(y_g^\ell, m_g^\ell) = 1$. Moreover, $z_\ell \in A(r(y^*, z^*))$ for all $\ell \in N^L$. Thus, voter i 's continuation value under σ is $V_i(\sigma) = \sum_{\ell \in N^L} \rho_\ell [\alpha_\ell u_i(y_\ell^*) + (1 - \alpha_\ell) u_i(z_\ell^*)] = r_i(y^*, z^*)$. Thus, each voter i 's strategy satisfies (21). Duggan (2014) implies that M is decisive over lotteries, so $A(\sigma) = A(r(y^*, z^*))$.

To check (17), consider $g \in N^G$ and $\ell \in N^L$. Since $a_\ell(z_\ell, 0) = 1$, focusing on offers that ℓ accepts is without loss of generality. Because $A(\sigma) = A(r(y^*, z^*))$, (23) implies $\tilde{\phi}_\ell(y^*, z^*) = \arg \max_{y_\ell \in A(\sigma)} u_{g_\ell}(y_\ell) + u_\ell(y_\ell) - u_\ell(z_\ell^*)$. Thus, (17) holds because $\tilde{\phi}_\ell(y^*, z^*) = y_\ell^* = y_\ell^g$. Finally, Lemma C.3 implies that g does not have a profitable deviation to any $y \notin A(\sigma)$.

It is immediate that m_g^ℓ satisfies (18) and g does not have a profitable deviation.

To check (19), note that ℓ 's expected dynamic payoff from rejecting g_ℓ 's offer is $\tilde{U}_\ell(z_\ell; \sigma) = u_\ell(z_\ell^*)$. Thus, ℓ weakly prefers to accept any (y, m) satisfying $y \in A(r(y^*, z^*))$ if and only if $u_\ell(y) + m \geq u_\ell(z_\ell^*)$. If $y \notin A(r(y^*, z^*))$, then ℓ weakly prefers to accept (y, m) if and only if $(1 - \delta)u_\ell(q) + \delta(r_\ell(y^*, z^*) + \rho_\ell \alpha_\ell m_g^\ell) + m \geq u_\ell(z_\ell^*)$. Thus, a_ℓ satisfies (19).

To check (20), note that (24) implies $\phi_\ell(y^*, z^*) = \arg \max_{x \in A(\sigma)} u_\ell(x)$ because $A(\sigma) = A(r(y^*, z^*))$. Thus, (20) holds because $\phi_\ell(y^*, z^*) = z_\ell^* = z_\ell$ for each $\ell \in N^L$. The no-delay property implies $x \notin A(\sigma)$ is not a profitable deviation for any $\ell \in N^L$.

Part 4. Let σ and σ' be stationary legislative lobbying equilibria. It suffices to show that $(y_g, m_g) = (y'_g, m'_g)$ for all $g \in N^G$ and $z_\ell = z'_\ell$ for all $\ell \in N^L$. Arguments analogous to Proposition 1 in Cho and Duggan (2003) imply that $y_g = y'_g$ for all $g \in N^G$ and $z_\ell = z'_\ell$ for all $\ell \in N^L$. Thus, $A(\sigma) = A(\sigma')$. Fix ℓ . Since σ and σ' are no-delay, ℓ 's expected dynamic payoff from rejecting g_ℓ 's offer is $u_\ell(z_\ell)$ under both σ and σ' . Because equilibrium lobby offers always make targeted legislators indifferent, g 's equilibrium payment equals $u_\ell(z_\ell) - u_\ell(y_g^\ell)$ in σ and σ' . Therefore $(y_g, m_g) = (y'_g, m'_g)$ for all $g \in N^G$ and $z_\ell = z'_\ell$ for all $\ell \in N^L$. □

B.2 Proof of Lemma 1

Proof. Let $A^* = [-\bar{x}^*, \bar{x}^*]$ denote the equilibrium acceptance set. There are two cases.

Case 1. Suppose $\hat{x}_\ell \in A^*$, which implies $z_\ell = \hat{x}_\ell$. There are two subcases. First, if $\hat{y}_\ell \in A^*$, then $y_g^\ell = \hat{y}_\ell$ and (18) implies $m_g^\ell = u_\ell(\hat{x}_\ell) - u_\ell(\hat{y}_\ell)$, so m_g^ℓ is constant as \bar{x}^* increases because $z_\ell = \hat{x}_\ell$ and $y_g^\ell = \hat{y}_\ell$. Second, consider $\hat{y}_\ell \notin A^*$, which requires $\hat{x}_{g_\ell} \notin [-\bar{x}^*, \bar{x}^*]$ since $\hat{x}_\ell \in A^*$. Without loss of generality, assume $\hat{x}_{g_\ell} > \bar{x}^*$. Thus, $z_\ell = \hat{x}_\ell$ and $y_g^\ell = \bar{x}^*$, so (18) implies $m_g^\ell = u_\ell(\hat{x}_\ell) - u_\ell(\bar{x}^*)$, which increases with \bar{x}^* .

Case 2. Suppose $\hat{x}_\ell \notin A^*$. Without loss of generality, assume $\hat{x}_\ell > z_\ell = \bar{x}^*$. There are three subcases. First, if $\hat{y}_\ell < -\bar{x}^*$, then $y_g^\ell = -\bar{x}^*$ and (18) implies $m_g^\ell = u_\ell(\bar{x}^*) - u_\ell(-\bar{x}^*)$, so m_g^ℓ increases with \bar{x}^* because $-\bar{x}^* < \bar{x}^* < \hat{x}_\ell$. Second, if $\hat{y}_\ell \in A^*$, then $y_g^\ell = \hat{y}_\ell$ is constant as legislative extremism increases, so (18) implies $m_g^\ell = u_\ell(\bar{x}^*) - u_\ell(\hat{y}_\ell)$, which increases with \bar{x}^* . Third, if $\hat{y}_\ell \geq \bar{x}^*$, then $y_g^\ell = \bar{x}^*$, so (18) implies $m_g^\ell = u_\ell(\bar{x}^*) - u_\ell(\bar{x}^*) = 0$, which is constant.

Together, the two cases show that m_g^ℓ weakly increases in \bar{x}^* . \square

B.3 Proof of Proposition 2

Given Lemma 1, it suffices to show that A^* expands as: legislative extremism increases, $|q|$ increases, and δ decreases.

- *Legislative extremism.* Follows from Part 1 of Proposition 8 in Kalandrakis (2021).
- *Status quo exremism, $|q|$.* Follows from Proposition 6 in Kalandrakis (2021).
- *Discount factor, δ .* Letting $C_\ell = \mathbb{I}\{\hat{x}_\ell \in \text{int } A^*\}$ and $\tilde{C}_\ell = \mathbb{I}\{\hat{x}_\ell \in \text{int } A^*\}$ for all $\ell \in N^L$, we have:

$$\bar{x}^* = \left(- \frac{(1 - \delta)u_M(q) + \delta \sum_{\ell \in N^L} \rho_\ell \left[(1 - \alpha_\ell) C_\ell u_M(\hat{x}_\ell) + \alpha_\ell \tilde{C}_\ell u_M(\hat{y}_\ell) \right]}{1 - \delta \sum_{\ell \in N^L} \rho_\ell \left[(1 - \alpha_\ell)(1 - C_\ell) + \alpha_\ell(1 - \tilde{C}_\ell) \right]} \right)^{\frac{1}{2}}. \quad (26)$$

If $\hat{x}_\ell, \hat{y}_\ell \notin \{-\bar{x}^*, \bar{x}^*\}$ for all $\ell \in N^L$, then

$$\begin{aligned} \frac{\partial \bar{x}^*}{\partial \delta} &\propto u_M(q) \left[1 - \sum_{\ell \in N^L} \rho_\ell [(1 - \alpha_\ell)(1 - C_\ell) + \alpha_\ell(1 - \tilde{C}_\ell)] \right] \\ &\quad - \sum_{\ell \in N^L} \rho_\ell \left[(1 - \alpha_\ell) C_\ell u_M(\hat{x}_\ell) + \alpha_\ell \tilde{C}_\ell u_M(\hat{y}_\ell) \right] \end{aligned} \quad (27)$$

$$= \sum_{\ell \in N^L} \rho_\ell \left[(1 - \alpha_\ell) C_\ell [u_M(q) - u_M(\hat{x}_\ell)] + \alpha_\ell \tilde{C}_\ell [u_M(q) - u_M(\hat{y}_\ell)] \right] \quad (28)$$

$$< 0, \quad (29)$$

where (29) follows because $\bar{x}^* < |q|$ implies that $u_M(q) - u_M(\hat{x}_\ell) < 0$ if $C_\ell = 1$ and similarly $u_M(q) - u_M(\hat{y}_\ell) < 0$ if $\tilde{C}_\ell = 1$.

If there exists $\ell \in N^L$ such that \hat{x}_ℓ or \hat{y}_ℓ is in $\{-\bar{x}^*, \bar{x}^*\}$, then (26) has right and left derivatives, which are both negative by an analogous argument.

B.4 Proofs of Lemmas 2 & 3

Consider $\ell \in N^L$ and refer to g_ℓ as g for convenience. Recall

$$\hat{y}_\ell = \arg \max_{y \in X} u_g(y) + u_\ell(y) = \frac{\hat{x}_g + \hat{x}_\ell}{2}. \quad (30)$$

The results fix \hat{x}_g and vary \hat{x}_ℓ . Throughout, $\hat{x}_g > 0$, as the other case is symmetric.

Let $\sigma(\alpha_\ell; \hat{x}_\ell)$ denote an equilibrium given \hat{x}_ℓ and α_ℓ , and denote the corresponding social acceptance set as $A(\alpha_\ell; \hat{x}_\ell)$, with upper bound $\bar{x}(\alpha_\ell; \hat{x}_\ell)$. That is, $A(\alpha_\ell; \hat{x}_\ell)$ corresponds to A^* from the main text but makes explicit the dependence on α_ℓ and \hat{x}_ℓ .

I first state a lemma that partitions whether $\hat{x}_\ell \in \text{int}A(0; \hat{x}_\ell)$ and plays a key role in proving Lemmas 2 and 3.

Lemma B.1. *For all $\ell \in N^L$, there exists $\bar{x}_\ell \in (0, q]$ such that $\hat{x}_\ell \in \text{int}A(0; \hat{x}_\ell)$ if $\hat{x}_\ell \in (-\bar{x}_\ell, \bar{x}_\ell)$ and otherwise $A(0; \hat{x}_\ell) = [-\bar{x}_\ell, \bar{x}_\ell]$.*

The proof of Lemma B.1 proceeds in a series of Lemmas that are provided in Appendix D. A rough outline of the argument is as follows. First, I define a function $\xi^\ell : \mathbb{R}_+ \rightarrow \mathbb{R}$ constructed so that $\xi^\ell(x) > 0$ $x \in \text{int}A(0; x)$. Then, I show that there is a unique $\bar{x}_\ell \in (0, q]$ such that $\xi^\ell(x) > 0$ $x \in [0, \bar{x}_\ell)$. It follows that $\hat{x}_\ell \in (-\bar{x}_\ell, \bar{x}_\ell)$ implies $\hat{x}_\ell \in \text{int}A(0; \hat{x}_\ell)$, and otherwise $A(0; \hat{x}_\ell) = [-\bar{x}_\ell, \bar{x}_\ell]$.

Lemma 2. *If $\hat{x}_g \in (0, \bar{x}_\ell)$, then there exists $x' \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \notin (-x', x')$ implies $\hat{x}_g \in \text{int}A(0; \hat{x}_\ell)$. If $\hat{x}_g \notin (0, \bar{x}_\ell)$, then $\hat{x}_g \notin \text{int}A(\alpha_\ell; \hat{x}_\ell)$ for all \hat{x}_ℓ and α_ℓ .*

Proof. Consider $\hat{x}_g \in (0, \bar{x}_\ell)$. If $\hat{x}_\ell = \hat{x}_g$, then Lemma B.1 implies $\hat{x}_g \in \text{int}A(0; \hat{x}_\ell)$. By symmetry, $\hat{x}_\ell = -\hat{x}_g$ also implies $\hat{x}_g \in \text{int}A(0; \hat{x}_\ell)$. Recall that $A(0; \hat{x}_\ell)$ strictly expands as \hat{x}_ℓ shifts away from 0 over $(-\bar{x}_\ell, \bar{x}_\ell)$. Because there is a unique equilibrium outcome distribution, Theorem 3 of Banks and Duggan (2006a) implies $A(0; \hat{x}_\ell)$ is continuous in \hat{x}_ℓ . Thus, there exists $x' \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \notin (-x', x')$ implies $\hat{x}_g \in \text{int}A(0; \hat{x}_\ell)$.

To complete the proof, consider $\hat{x}_g \geq \bar{x}_\ell$. Lemma B.1 implies $\hat{x}_g \notin \text{int}A(0; \hat{x}_\ell) = (\bar{x}_\ell, \bar{x}_\ell)$ for all $\hat{x}_\ell \geq \hat{x}_g$. Thus, $A(\alpha_\ell; \hat{x}_\ell) \subset A(0; \hat{x}_g)$ for all $(\alpha_\ell, \hat{x}_\ell)$, so $\hat{x}_g \notin \text{int}A(\alpha_\ell; \hat{x}_\ell)$. \square

Using Lemma B.1, I establish a result that lays groundwork for Lemma 3. Let y_0^* denote y^* for $\alpha_\ell = 0$ and define z_0^* analogously.

Lemma B.2. *In equilibrium, $y_0^* \neq z_0^*$ if and only if $\hat{x}_\ell \in (\underline{\chi}(\hat{x}_g), \bar{\chi}(\hat{x}_g))$.*

Proof. If $\hat{x}_\ell \in (-\bar{x}_\ell, \bar{x}_\ell)$, then Lemma B.1 implies $z_0^* = \hat{x}_\ell$. Then, $\hat{x}_\ell \neq \hat{x}_g$ implies $y_0^* \neq z_0^*$.

Otherwise, z_0^* is the boundary of $A_0^* = [-\bar{x}, \bar{x}]$ closer to \hat{x}_ℓ . For $\hat{x}_\ell \leq -\bar{x}$, we have $y_0^* > -\bar{x}$ if and only if $\hat{x}_\ell > \underline{\chi}(\hat{x}_g)$. Analogously for $\hat{x}_\ell \geq \bar{x}$, we have $y_0^* < \bar{x}$ if and only if $\hat{x}_\ell < \bar{\chi}(\hat{x}_g)$. \square

Lemma 3. *Interest group g strictly prefers $\alpha_\ell > 0$ only if $\hat{x}_\ell \in (\underline{\chi}(\hat{x}_g), \bar{\chi}(\hat{x}_g))$.*

Proof. Suppose $\hat{x}_\ell \notin (\underline{\chi}(\hat{x}_g), \bar{\chi}(\hat{x}_g))$. Lemma B.2 implies $y_0^* = z_0^*$. Thus, V_M^{*, α_ℓ} is constant as α_ℓ increases from zero and therefore $A_{\alpha_\ell}^*$ is constant in α_ℓ . It follows that the equilibrium outcome distribution is constant in α_ℓ , so g is indifferent. \square

B.5 Preliminary Results for Propositions 4 & 5

Next, Lemmas B.3–B.6 establish properties used to prove Propositions 4 and 5.

Lemma B.3. *Suppose $\hat{x}_g \in (0, \bar{x}_\ell)$. There exists $\tilde{x} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (\tilde{x}, \hat{x}_g)$ implies $\hat{x}_g \in \text{int}A(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$.*

Proof. Consider $\hat{x}_g \in (0, \bar{x}_\ell)$. By Lemma 2, there exists $x' \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (x', \hat{x}_g)$ implies $\hat{x}_g \in \text{int}A(0; \hat{x}_\ell)$. Then $0 < \hat{x}_\ell < \hat{x}_g$ implies $A(0; \hat{x}_\ell) \subset A(\alpha_\ell; \hat{x}_\ell)$. \square

For each $j \in N^L \setminus \{\ell\}$, define

$$\begin{aligned} E_j^{LB}(\alpha_\ell; \hat{x}_\ell) &= \mathbb{I}\{\hat{x}_j \leq -\bar{x}(\alpha_\ell; \hat{x}_\ell)\}, \\ E_j^{UB}(\alpha_\ell; \hat{x}_\ell) &= \mathbb{I}\{\hat{x}_j \geq \bar{x}(\alpha_\ell; \hat{x}_\ell)\}, \text{ and} \\ C_j(\alpha_\ell; \hat{x}_\ell) &= \mathbb{I}\{\hat{x}_j \in \text{int}A(\alpha_\ell; \hat{x}_\ell)\}. \end{aligned}$$

Define $\tilde{E}_j^{LB}(\alpha_\ell; \hat{x}_\ell)$, $\tilde{E}_j^{UB}(\alpha_\ell; \hat{x}_\ell)$, and $\tilde{C}_j(\alpha_\ell; \hat{x}_\ell)$ analogously using \hat{y}_j . Let $I_j^j \in \{0, 1\}$ indicate whether $j \in N_g^L$.

Assumption B.1. *There exists $j \in N^L \setminus \{\ell\}$ such that $\alpha_j < 1$ and $\hat{x}_j \notin A(0; \hat{x}_g)$.*

Assumption B.2. *There exists $j \in N^L \setminus \{\ell\}$ such that $\alpha_j > 0$ and $\hat{y}_j \notin A^*(0; \hat{x}_g)$.*

Next, define

$$v_1^g(\alpha_\ell; \hat{x}_\ell) = \rho_\ell \left(\alpha_\ell \left[u_g(\hat{y}_\ell) + u_\ell(\hat{y}_\ell) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \right) \quad (31)$$

and

$$\begin{aligned} v_2^g(\alpha_\ell; \hat{x}_\ell) &= \sum_{j \neq \ell} \rho_j \left(\left[\alpha_j \tilde{E}_j^{LB}(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E_j^{LB}(\alpha_\ell; \hat{x}_\ell) \right] u_g(-\bar{x}(\alpha_\ell; \hat{x}_\ell)) \right. \\ &\quad \left. + \left[\alpha_j \tilde{E}_j^{UB}(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E_j^{UB}(\alpha_\ell; \hat{x}_\ell) \right] u_g(\bar{x}(\alpha_\ell; \hat{x}_\ell)) \right) \end{aligned} \quad (32)$$

$$+ \alpha_j \left[\tilde{C}_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{y}_j) - I_g^j m_g^j(\alpha_\ell; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{x}_j) \Bigg).$$

Lemma B.4. *If $\hat{x}_\ell \neq \hat{x}_g$, then $\frac{\partial v_1^g(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} > 0$.*

Proof. Suppose $\hat{x}_\ell \neq \hat{x}_g$. Then $\frac{\partial v_1^g(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} = \frac{\rho_\ell}{2} (\hat{x}_g - \hat{x}_\ell)^2 > 0$, by (31) and $\hat{y}_\ell = \frac{\hat{x}_\ell + \hat{x}_g}{2}$. \square

Lemma B.5. *Suppose $0 \leq \hat{x}_\ell < \hat{x}_g < \bar{x}$. If at least one of Assumption B.1 or B.2 holds, then $v_2^g(\alpha_\ell; \hat{x}_\ell)$ strictly decreases in α_ℓ .*

Proof. It suffices to show that

$$\begin{aligned} & \left[\alpha_j \tilde{E}_j^{LB}(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E_j^{LB}(\alpha_\ell; \hat{x}_\ell) \right] u_g(-\bar{x}(\alpha_\ell; \hat{x}_\ell)) \\ & + \left[\alpha_j \tilde{E}_j^{UB}(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E_j^{UB}(\alpha_\ell; \hat{x}_\ell) \right] u_g(\bar{x}(\alpha_\ell; \hat{x}_\ell)) \\ & + \alpha_j \left[\tilde{C}_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{y}_j) - I_g^j m_g^j(\alpha_\ell; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{x}_j) \end{aligned} \quad (33)$$

decreases in α_ℓ for all $j \in N^L \setminus \{\ell\}$ and strictly decreases for some j .

Without loss of generality, consider $\hat{x}_j \geq 0$. Since $0 \leq \hat{x}_\ell < \hat{x}_g$, we know $\bar{x}(\alpha_\ell; \hat{x}_\ell)$ increases in α_ℓ . There are two implications. First, $\hat{x}_g \in (0, \bar{x})$ implies $\hat{x}_g < \bar{x}(0; \hat{x}_\ell)$ by Lemma 2, so $u_g(\bar{x}(\alpha_\ell; \hat{x}_\ell))$ and $u_g(-\bar{x}(\alpha_\ell; \hat{x}_\ell))$ both decrease in α_ℓ . Second, exactly one of the following holds: $E_j^{UB}(\alpha_\ell; \hat{x}_\ell) = 1$ for all α_ℓ ; $C_j(\alpha_\ell; \hat{x}_\ell) = 1$ for all α_ℓ ; or there is a unique $\bar{\alpha}_\ell^j \in [0, 1]$ such that $\alpha_\ell \in [0, \bar{\alpha}_\ell^j]$ implies $E_j^{UB}(\alpha_\ell; \hat{x}_\ell) = 1$ and $\alpha_\ell \in (\bar{\alpha}_\ell^j, 1]$ implies $C_j(\alpha_\ell; \hat{x}_\ell) = 1$. An analogous observation holds for $\tilde{E}_j^{UB}(\alpha_\ell; \hat{x}_\ell)$ and $\tilde{C}_j(\alpha_\ell; \hat{x}_\ell)$. Thus,

$$E_j^{LB}(\alpha_\ell; \hat{x}_\ell) u_g(-\bar{x}(\alpha_\ell; \hat{x}_\ell)) + E_j^{UB}(\alpha_\ell; \hat{x}_\ell) u_g(\bar{x}(\alpha_\ell; \hat{x}_\ell)) + C_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{x}_j) \quad (34)$$

and

$$\tilde{E}_j^{LB}(\alpha_\ell; \hat{x}_\ell) u_g(-\bar{x}(\alpha_\ell; \hat{x}_\ell)) + \tilde{E}_j^{UB}(\alpha_\ell; \hat{x}_\ell) u_g(\bar{x}(\alpha_\ell; \hat{x}_\ell)) + \tilde{C}_j(\alpha_\ell; \hat{x}_\ell) u_g(\hat{y}_j) \quad (35)$$

both decrease in α_ℓ . Furthermore, because Assumptions B.1 or B.2 holds, at least one of (34) and (35) strictly decreases for some $j \in N^L \setminus \{\ell\}$. Lemma 1 implies $m_g^j(\alpha_\ell; \hat{x}_\ell)$ weakly increases in α_ℓ for all $j \in N_g^L$. Altogether, (33) decreases in α_ℓ for all $j \in N^L \setminus \{\ell\}$ and strictly decreases for some j , as desired. \square

Lemma B.6. *Assume $\hat{x}_g \in (0, \bar{x}_\ell)$. If at least one of Assumption B.1 or B.2 holds, then there exists $x' < \hat{x}_g$ such that $v_1^g(\alpha_\ell; \hat{x}_\ell) + v_2^g(\alpha_\ell; \hat{x}_\ell)$ strictly decreases in α_ℓ for all $\hat{x}_\ell \in (x', \hat{x}_g)$.*

Proof. I show $|\frac{\partial v_2^g(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell}| > \frac{\partial v_1^g(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell}$ for \hat{x}_ℓ sufficiently close to \hat{x}_g .

By Lemma B.3, there exists $\tilde{x} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (\tilde{x}, \hat{x}_g)$ implies $\hat{x}_g \in \text{int}A(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$. Fix $\hat{x}_\ell \in (\tilde{x}, \hat{x}_g)$ and $\alpha_\ell \in [0, 1]$.

First, I characterize a lower bound on $|\frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)}|$. Define

$$\Gamma = \sum_{j \neq \ell} \rho_j \left(\left[\alpha_j \tilde{E}_j^{LB}(\hat{x}_g) + (1 - \alpha_j) E_j^{LB}(\hat{x}_g) \right] \frac{\partial u_g(-\bar{x}(\tilde{x}))}{\partial \bar{x}(\tilde{x})} + \left[\alpha_j \tilde{E}_j^{UB}(\hat{x}_g) + (1 - \alpha_j) E_j^{UB}(\hat{x}_g) \right] \frac{\partial u_g(\bar{x}(\tilde{x}))}{\partial \bar{x}(\tilde{x})} \right). \quad (36)$$

Note $\Gamma < 0$ because (i) $\hat{x}_g \in (-\bar{x}(\tilde{x}), \bar{x}(\tilde{x}))$ implies $\frac{\partial u_g(\bar{x}(\tilde{x}))}{\partial \bar{x}(\tilde{x})} < 0$ and $\frac{\partial u_g(-\bar{x}(\tilde{x}))}{\partial \bar{x}(\tilde{x})} < 0$, and (ii) at least one of Assumptions B.1 and B.2 hold.

I claim $\frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)} < \Gamma$, where

$$\begin{aligned} \frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)} &= \sum_{j \neq \ell} \rho_j \left(\left[\alpha_j \tilde{E}_j^{LB}(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E_j^{LB}(\alpha_\ell; \hat{x}_\ell) \right] \frac{\partial u_g(-\bar{x}(\alpha_\ell; \hat{x}_\ell))}{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)} \right. \\ &\quad + \left[\alpha_j \tilde{E}_j^{UB}(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j) E_j^{UB}(\alpha_\ell; \hat{x}_\ell) \right] \frac{\partial u_g(\bar{x}(\alpha_\ell; \hat{x}_\ell))}{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)} \\ &\quad \left. - I_g^j \alpha_j \frac{\partial m_g^j(\alpha_\ell; \hat{x}_\ell)}{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)} \right). \end{aligned} \quad (37)$$

Three steps show the claim. First, note $\hat{x}_\ell \in (\tilde{x}, \hat{x}_g)$ implies $\bar{x}(\hat{x}_g) \geq \bar{x}(\alpha_\ell; \hat{x}_\ell)$. Thus, we have $\tilde{E}_j^{UB}(\hat{x}_g) \leq \tilde{E}_j^{UB}(\alpha_\ell; \hat{x}_\ell)$, $E_j^{UB}(\hat{x}_g) \leq E_j^{UB}(\alpha_\ell; \hat{x}_\ell)$, $\tilde{E}_j^{LB}(\hat{x}_g) \leq \tilde{E}_j^{LB}(\alpha_\ell; \hat{x}_\ell)$, and $E_j^{LB}(\hat{x}_g) \leq E_j^{LB}(\alpha_\ell; \hat{x}_\ell)$ for all $j \neq \ell$. Second, $\hat{x}_g < \bar{x}(\tilde{x}) < \bar{x}(\alpha_\ell; \hat{x}_\ell)$ implies $\frac{\partial u_g(\bar{x}(\alpha_\ell; \hat{x}_\ell))}{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)} < \frac{\partial u_g(\bar{x}(\tilde{x}))}{\partial \bar{x}(\tilde{x})} < 0$

and symmetrically $\frac{\partial u_g(-\bar{x}(\alpha_\ell; \hat{x}_\ell))}{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)} < \frac{\partial u_g(-\bar{x}(\hat{x}))}{\partial \bar{x}(\hat{x})} < 0$. Third, $\frac{\partial m_g^j(\alpha_\ell; \hat{x}_\ell)}{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)} \geq 0$ for all $j \in N_g^L$ by Lemma 1.

For almost all $\alpha_\ell \in [0, 1]$, $\frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} = \frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)} \frac{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell}$. Define $\mathbf{C}_j(\alpha_\ell; \hat{x}_\ell) = [(1 - \alpha_j)(1 - C_j(\alpha_\ell; \hat{x}_\ell)) + \alpha_j(1 - \tilde{C}_j(\alpha_\ell; \hat{x}_\ell))]$. Then,

$$\frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} < \Gamma \frac{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} \quad (38)$$

$$= \frac{\delta \rho_\ell \Gamma \left[u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell) \right]}{2\bar{x}(\alpha_\ell; \hat{x}_\ell) \left[1 - \delta \left(\sum_{j \in N^\ell} \rho_j \mathbf{C}_j^\ell(\alpha_\ell; \hat{x}_\ell) \right) \right]} \quad (39)$$

$$< \frac{\delta \rho_\ell \Gamma}{2\bar{x}_\ell} \left[\frac{1}{4} (\hat{x}_g - \hat{x}_\ell) (3\hat{x}_\ell + \hat{x}_g) \right], \quad (40)$$

where (38) follows from $\frac{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} > 0$ and $0 > \Gamma > \frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \bar{x}(\alpha_\ell; \hat{x}_\ell)}$; (39) from applying the implicit function theorem to $\bar{x}(\alpha_\ell; \hat{x}_\ell)$, which is possible for almost all $\alpha_\ell \in [0, 1]$; and (40) because $\Gamma [u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell)] < 0$, $\delta \sum_{j \in N^\ell} \rho_j \mathbf{C}_j^\ell(\alpha_\ell; \hat{x}_\ell) \in (0, 1)$, $0 < \bar{x}(\alpha_\ell; \hat{x}_\ell) < \bar{x}_\ell$, and simplifying using $\hat{y}_\ell = \frac{\hat{x}_g + \hat{x}_\ell}{2}$.

By Lemma B.4, $\frac{\partial v_1(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} = \frac{\rho_\ell}{2} (\hat{x}_g - \hat{x}_\ell)^2$. Thus, for generic α_ℓ , (40) implies that $\frac{\partial v_1(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} + \frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} < 0$ if

$$\frac{\rho_\ell}{2} (\hat{x}_g - \hat{x}_\ell)^2 + \frac{\delta \rho_\ell \Gamma}{2\bar{x}_\ell} \left[\frac{1}{4} (\hat{x}_g - \hat{x}_\ell) (3\hat{x}_\ell + \hat{x}_g) \right] < 0,$$

which holds for $\hat{x}_\ell > \hat{x}_g \left(\frac{4\bar{x}_\ell + \delta \Gamma}{4\bar{x}_\ell - 3\delta \Gamma} \right)$. Define $x' = \max \left\{ \tilde{x}, \hat{x}_g \left(\frac{4\bar{x}_\ell + \delta \Gamma}{4\bar{x}_\ell - 3\delta \Gamma} \right) \right\}$. Note $x' < \hat{x}_g$ because (i) $\tilde{x} < \hat{x}_g$ and (ii) $\delta \Gamma < 0$ implies $\frac{4\bar{x}_\ell + \delta \Gamma}{4\bar{x}_\ell - 3\delta \Gamma} < 1$. Thus, $\hat{x}_\ell \in (x', \hat{x}_g)$ implies $\frac{\partial v_1(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} + \frac{\partial v_2(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} < 0$ for generic α_ℓ . Continuity implies $v_1^g(\alpha_\ell; \hat{x}_\ell) + v_2^g(\alpha_\ell; \hat{x}_\ell)$ strictly decreases in α_ℓ for such \hat{x}_ℓ . \square

B.6 Proof of Proposition 4

Proposition 4 *Suppose $\hat{x}_g \in (0, \bar{x}_\ell)$. If either Assumption B.1 or B.2, then there are cutpoints satisfying $-\hat{x}_g < x' < x'' < \hat{x}_g$ such that:*

- (i) $\alpha_\ell^* = 0$ if $\hat{x}_\ell \in (x'', \hat{x}_g)$, and
- (ii) $\alpha_\ell^* > 0$ if $\hat{x}_\ell \in (\underline{\chi}(\hat{x}_g), x') \cup (\hat{x}_g, \bar{\chi}(\hat{x}_g))$.

Proof. Part (i). Consider $\hat{x}_\ell \in [0, \hat{x}_g)$. By Lemma B.3, there exists $\tilde{x} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (\tilde{x}, \hat{x}_g)$ implies $\hat{x}_g \in A(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$. By Lemma B.6, there exists $x' < \hat{x}_g$ such that $\hat{x}_\ell \in (x', \hat{x}_g)$ implies $v_1^g(\alpha_\ell; \hat{x}_\ell) + v_2^g(\alpha_\ell; \hat{x}_\ell)$ strictly decreases in α_ℓ . Let $x'' = \max\{\tilde{x}, x'\}$ and consider $\hat{x}_\ell \in (x'', \hat{x}_g)$. Then $z_\ell = \hat{x}_\ell \in A(\alpha_\ell; \hat{x}_\ell)$ and $y_g^\ell = \hat{y}_\ell \in A(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$. Thus, g 's equilibrium value from α access is $\mathcal{U}_g(\alpha_\ell; \hat{x}_\ell) = v_1^g(\alpha_\ell; \hat{x}_\ell) + v_2^g(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$, so g strictly prefers $\alpha_\ell = 0$. This establishes (i).

Part (ii). First, consider $\hat{x}_\ell \in (\hat{x}_g, \bar{\chi}(\hat{x}_g))$. It suffices to show that g 's ex ante expected utility strictly increases as α_ℓ increases from zero. There are two subcases.

- If $\hat{x}_\ell < \bar{x}_\ell$, then $\mathcal{U}_g(\alpha_\ell; \hat{x}_\ell) = v_1^g(\alpha_\ell; \hat{x}_\ell) + v_2^g(\alpha_\ell; \hat{x}_\ell)$ for sufficiently small α_ℓ . By Lemma 1, $\frac{\partial v_1^g(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} > 0$. To complete this case, I argue that $v_2^g(\alpha_\ell; \hat{x}_\ell)$ increases for sufficiently small α_ℓ . Under the maintained assumptions, $\hat{x}_g \in (-\bar{x}(0; \hat{x}_\ell), \underline{x}(0; \hat{x}_\ell))$ and $\hat{y}_\ell \in (\hat{x}_g, \bar{x}(0; \hat{x}_\ell))$. Thus, $\bar{x}(\alpha_\ell; \hat{x}_\ell)$ strictly decreases for sufficiently small α_ℓ . Therefore $u_g(-\bar{x}(\alpha_\ell; \hat{x}_\ell))$ and $u_g(\bar{x}(\alpha_\ell; \hat{x}_\ell))$ are strictly increasing for such α_ℓ . Lemma 1 implies $m_g^j(\alpha_\ell; \hat{x}_\ell)$ weakly decreases in α_ℓ for all $j \in N_g^L \setminus \{\ell\}$. Thus, $v_2^g(\alpha_\ell; \hat{x}_\ell)$ strictly increases over sufficiently small α_ℓ .
- If $\hat{x}_\ell > \bar{x}_\ell$, then $\bar{x}(0; \hat{x}_\ell) = \bar{x}_\ell$. Thus, $\mathcal{U}_g(0; \hat{x}_\ell)$ is

$$\begin{aligned} & \rho_\ell \left(\alpha_\ell \left[u_g(\hat{y}_\ell) + u_\ell(\hat{y}_\ell) - u_\ell(\bar{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\bar{x}_\ell) \right) \\ & + \sum_{j \neq \ell} \rho_j \left(\left[\alpha_j \tilde{E}_j^{LB}(0; \hat{x}_\ell) + (1 - \alpha_j) E_j^{LB}(0; \hat{x}_\ell) \right] u_g(-\bar{x}_\ell) \right) \end{aligned}$$

$$\begin{aligned}
& + \left[\alpha_j \tilde{E}_j^{UB}(0; \hat{x}_\ell) + (1 - \alpha_j) E_j^{UB}(0; \hat{x}_\ell) \right] u_g(\bar{x}_\ell) \\
& + \alpha_j \tilde{C}_j(0; \hat{x}_\ell) u_g(\hat{y}_j) + (1 - \alpha_j) C_j(0; \hat{x}_\ell) u_g(\hat{x}_j) \\
& - I_g^j \alpha_j m_g^j(0; \hat{x}_\ell) \Big). \tag{41}
\end{aligned}$$

Arguments similar to Case 1 show that (41) strictly increases in α_ℓ at $\alpha_\ell = 0$.

To complete the proof for Part (ii), consider $\hat{x}_\ell < 0$. For $\hat{x}_\ell \in (\underline{\chi}(\hat{x}_g), -\hat{x}_g)$, arguments analogous to Case 2 show that $\mathcal{U}_g(\alpha_\ell; \hat{x}_\ell)$ strictly increases at $\alpha_\ell = 0$. Because g 's ex-ante expected payoff is continuous in \hat{x}_ℓ , there exists $x' > -\hat{x}_g$ such that $\hat{x}_\ell \in (x(\hat{x}_g), x')$ implies $\alpha_\ell^* > 0$. \square

B.7 Proof of Proposition 5

Proposition 5 *Assume $\hat{x}_g \geq \bar{x}_\ell$.*

(i) *If $\sum_{i \in NL} \rho_i \left[(1 - \alpha_i) \mathbb{I}\{\hat{x}_i \leq -\bar{x}\} + \alpha_i \mathbb{I}\{\hat{y}_i \leq -\bar{x}\} \right]$ is sufficiently small, then there exists $x' < 0$ such that $\alpha_\ell^* > 0$ if $\hat{x}_\ell \in (x', \bar{x})$.*

(ii) *If $\sum_{i \in NL} \rho_i \left[(1 - \alpha_i) \mathbb{I}\{\hat{x}_i \geq \bar{x}\} + \alpha_i \mathbb{I}\{\hat{y}_i \geq \bar{x}\} \right]$ is sufficiently small, then there exists $x'' \geq -\bar{x}$ such that $\alpha_\ell^* > 0$ if $\hat{x}_\ell \in (\underline{\chi}(\hat{x}_g), x'')$.*

Proof. I prove (i), as (ii) is analogous. Consider $\hat{x}_\ell \in [0, \bar{x}_\ell)$ and assume $\sum_{i \in NL} \rho_i [(1 - \alpha_i) \mathbb{I}\{\hat{x}_i \leq -\bar{x}\} + \alpha_i \mathbb{I}\{\hat{y}_i \leq -\bar{x}\}] = 0$. I show that g 's ex-ante expected payoff strictly increases at $\alpha_\ell = 0$. The desired result will then follow since g 's ex-ante expected payoff is continuous in $\sum_{i \in NL} \rho_i [(1 - \alpha_i) \mathbb{I}\{\hat{x}_i \leq -\bar{x}\} + \alpha_i \mathbb{I}\{\hat{y}_i \leq -\bar{x}\}]$.

We have $\hat{x}_\ell \in [0, \bar{x}(0; \hat{x}_\ell))$ and $\hat{y}_\ell > \hat{x}_\ell$. Therefore $0 \leq z_\ell(0; \hat{x}_\ell) = \hat{x}_\ell < y_g^\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell$. Furthermore, $-\bar{x}(0; \hat{x}_\ell)$ is not proposed with positive probability because $\sum_{i \in NL} \rho_i [(1 - \alpha_i) \mathbb{I}\{\hat{x}_i \leq -\bar{x}\} + \alpha_i \mathbb{I}\{\hat{y}_i \leq -\bar{x}\}] = 0$. Thus, we have

$$\mathcal{U}_g(0; \hat{x}_\ell) = \rho_\ell \left(\alpha_\ell \left[u_g(y_g^\ell(0; \hat{x}_\ell)) + u_\ell(y_g^\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell) \right)$$

$$\begin{aligned}
& + \sum_{j \neq \ell} \rho_j \left(\left[\alpha_j \tilde{E}_j^{UB}(0; \hat{x}_\ell) + (1 - \alpha_j) E_j^{UB}(0; \hat{x}_\ell) \right] u_g(\bar{x}(0; \hat{x}_\ell)) \right. \\
& \quad \left. + \alpha_j \left[\tilde{C}_j(0; \hat{x}_\ell) u_g(\hat{y}_j) - I_g^j m_g^j(0; \hat{x}_\ell) \right] + (1 - \alpha_j) C_j(0; \hat{x}_\ell) u_g(\hat{x}_j) \right). \quad (42)
\end{aligned}$$

Three steps show (42) strictly increases at $\alpha_\ell = 0$.

- First, $0 \leq \hat{x}_\ell < y_g^\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell$ implies $y_g^\ell(0; \hat{x}_\ell)$ weakly increases in α_ℓ . Therefore $u_g(y_g^\ell(\alpha_\ell; \hat{x}_\ell))$ weakly increases and $u_\ell(y_g^\ell(\alpha_\ell; \hat{x}_\ell))$ weakly decreases. Because u is quadratic and $\hat{x}_\ell < y_g^\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell = \frac{\hat{x}_g + \hat{x}_\ell}{2} < \hat{x}_g$, it follows that $u_g(y_g^\ell(\alpha_\ell; \hat{x}_\ell))$ increases weakly faster than $u_\ell(y_g^\ell(\alpha_\ell; \hat{x}_\ell))$ decreases. Therefore $u_g(y_g^\ell(0; \hat{x}_\ell)) + u_\ell(y_g^\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell)$ weakly increases in α_ℓ . Furthermore, $\hat{x}_\ell < y_g^\ell(0; \hat{x}_\ell) \leq \hat{y}_\ell < \hat{x}_g$ also implies $u_g(y_g^\ell(0; \hat{x}_\ell)) + u_\ell(y_g^\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) - u_g(\hat{x}_\ell) \geq 0$. It follows that $\alpha_\ell \left[u_g(y_g^\ell(0; \hat{x}_\ell)) + u_\ell(y_g^\ell(0; \hat{x}_\ell)) - u_\ell(\hat{x}_\ell) \right] + (1 - \alpha_\ell) u_g(\hat{x}_\ell)$ weakly increases at $\alpha_\ell = 0$.
- Second, $\bar{x}(0; \hat{x}_\ell)$ strictly increases in α_ℓ because $0 \leq z_\ell < y_g^\ell(0; \hat{x}_\ell) \leq \bar{x}(0; \hat{x}_\ell)$. Since $\bar{x}(0; \hat{x}_\ell) < \hat{x}_g$, it follows that $u_g(\bar{x}(0; \hat{x}_\ell))$ increases at $\alpha_\ell = 0$.
- Third, $m_g^j(0; \hat{x}_\ell)$ weakly increases in α_ℓ for all $j \in N_g^L$ by Lemma 1. But it strictly increases only for $j \in N_g^L$ such that $\hat{y}_j > \bar{x}(0; \hat{x}_\ell)$. Thus, g 's lobbying surplus weakly increases in α_ℓ for all $j \in N_g^L$.

□

B.8 Proof of Proposition 6

To state and prove Proposition 6, I modify the baseline model to compare WTP across distinct legislator-group pairs. Specifically, I replace ℓ with two legislators, ℓ_1 and ℓ_2 , and replace g with two groups, g_1 and g_2 . To isolate differences in proposal power, assume $\hat{x}_{g_1} = \hat{x}_{g_2}$ and $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$, but $\rho_{\ell_1} \neq \rho_{\ell_2}$. These modifications do not qualitatively change the equilibrium characterization. Two identical pairs avoid potential complications that can

arise if one group has access to two legislators, where access to one legislator can affect offers to the other.

Proof. Fix $\alpha \in [0, 1]$. It suffices to show that $\frac{\partial \mathcal{U}_{g_1}(\alpha_1; \hat{x}_\ell)}{\partial \alpha_1} \Big|_{\alpha_1=\alpha} \geq 0$ implies $\frac{\partial \mathcal{U}_{g_2}(\alpha_2; \hat{x}_\ell)}{\partial \alpha_2} \Big|_{\alpha_2=\alpha} \geq \frac{\partial \mathcal{U}_{g_1}(\alpha_1; \hat{x}_\ell)}{\partial \alpha_1} \Big|_{\alpha_1=\alpha}$. Because $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$ and $\hat{x}_{g_1} = \hat{x}_{g_2}$, we have $y_{g_1} = y_{g_2}$ and $z_{\ell_1} = z_{\ell_2}$. Thus, $m_{g_1} = m_{g_2}$. Denote $y = y_{g_1}$, $z = z_{\ell_1}$, and $m = m_{g_1}$. Assume $\frac{\partial \mathcal{U}_{g_1}(\alpha_1; \hat{x}_\ell)}{\partial \alpha_1} \Big|_{\alpha_1=\alpha} \geq 0$. There are five cases.

- *Case 1:* Suppose $z = \hat{x}_\ell$ and $y = \hat{y}$. Then,

$$\begin{aligned} \frac{\partial \mathcal{U}_{g_1}(\alpha_1; \hat{x}_\ell)}{\partial \alpha_1} \Big|_{\alpha_1=\alpha} &= \rho_{\ell_1} \left(u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right) \\ &\quad - \frac{\partial \bar{x}_\alpha}{\partial \alpha_1} \left(\rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} - \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right) \\ &= \rho_{\ell_1} \left[u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right. \\ &\quad \left. + \frac{\delta[u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} [1 - \delta(\rho_L + \rho_R)]} \left(\rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} + \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right) \right] \end{aligned} \quad (43)$$

$$\begin{aligned} &\leq \rho_{\ell_2} \left[u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \right. \\ &\quad \left. + \frac{\delta[u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} [1 - \delta(\rho_L + \rho_R)]} \left(\rho_L \frac{\partial u_{g_1}(-\bar{x}_\alpha)}{\partial \bar{x}_\alpha} + \rho_R \frac{\partial u_{g_1}(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} \right) \right] \end{aligned} \quad (44)$$

$$= \frac{\partial \mathcal{U}_{g_2}(\alpha_2; \hat{x}_\ell)}{\partial \alpha_2} \Big|_{\alpha_2=\alpha}, \quad (45)$$

where (43) follows from $\frac{\partial \bar{x}_\alpha}{\partial \alpha_1} = \frac{\delta \rho_{\ell_1} [u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} [1 - \delta(\rho_L + \rho_R)]}$; (44) because (i) $\rho_{\ell_2} > \rho_{\ell_1}$ and (ii) $\frac{\partial \mathcal{U}_{g_2}(\alpha_2; \hat{x}_\ell)}{\partial \alpha_2} \Big|_{\alpha_2=\alpha} \geq 0$ implies that the bracketed expression in (43) is positive; and (45) because $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$, $\hat{x}_{g_1} = \hat{x}_{g_2}$, and $\frac{\partial \bar{x}_\alpha}{\partial \alpha_2} = \frac{\delta \rho_{\ell_2} [u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} [1 - \delta(\rho_L + \rho_R)]}$.

- *Case 2:* Suppose $z = \bar{x}_\alpha$ and $y = \hat{y}$. In this case, $\frac{\partial \bar{x}_\alpha}{\partial \alpha_1} = \frac{\delta \rho_{\ell_1} [u_M(\hat{y}) - u_M(\bar{x}_\alpha)]}{\frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} (1 - \delta[\rho_L + \rho_R + (1 - \alpha)(\rho_{\ell_1} + \rho_{\ell_2})])}$ and $\frac{\partial \bar{x}_\alpha}{\partial \alpha_2} = \frac{\delta \rho_{\ell_2} [u_M(\hat{y}) - u_M(\bar{x}_\alpha)]}{\frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} (1 - \delta[\rho_L + \rho_R + (1 - \alpha)(\rho_{\ell_1} + \rho_{\ell_2})])}$. Arguments analogous to Case 1 show $\frac{\partial \mathcal{U}_{g_2}(\alpha_2; \hat{x}_\ell)}{\partial \alpha_2} \Big|_{\alpha_2 = \alpha} \geq \frac{\partial \mathcal{U}_{g_1}(\alpha_1; \hat{x}_\ell)}{\partial \alpha_1} \Big|_{\alpha_1 = \alpha}$. The argument for $z = -\bar{x}_\alpha$ and $y = \hat{y}$ is symmetric.
- *Case 3:* Suppose $z = \hat{x}_\ell$ and $y = \bar{x}_\alpha$. In this case, $\frac{\partial \bar{x}_\alpha}{\partial \alpha_1} = \frac{\delta \rho_{\ell_1} [u_M(\bar{x}_\alpha) - u_M(\hat{x}_\ell)]}{\frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} (1 - \delta[\rho_L + \rho_R + \alpha(\rho_{\ell_1} + \rho_{\ell_2})])}$ and $\frac{\partial \bar{x}_\alpha}{\partial \alpha_2} = \frac{\delta \rho_{\ell_2} [u_M(\bar{x}_\alpha) - u_M(\hat{x}_\ell)]}{\frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} (1 - \delta[\rho_L + \rho_R + \alpha(\rho_{\ell_1} + \rho_{\ell_2})])}$. Arguments analogous to Case 1 show $\frac{\partial \mathcal{U}_{g_2}(\alpha_2; \hat{x}_\ell)}{\partial \alpha_2} \Big|_{\alpha_2 = \alpha} \geq \frac{\partial \mathcal{U}_{g_1}(\alpha_1; \hat{x}_\ell)}{\partial \alpha_1} \Big|_{\alpha_1 = \alpha}$. The argument for $z = \hat{x}_\ell$ and $y = -\bar{x}_\alpha$ is symmetric.
- *Case 4:* Suppose $z = \bar{x}_\alpha$ and $y = -\bar{x}_\alpha$. In this case, $\frac{\partial \bar{x}_\alpha}{\partial \alpha_1} = \frac{\delta \rho_{\ell_1} [u_M(-\bar{x}_\alpha) - u_M(\bar{x}_\alpha)]}{\frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} [1 - \delta(\rho_L + \rho_R + \rho_{\ell_1} + \rho_{\ell_2})]}$ and $\frac{\partial \bar{x}_\alpha}{\partial \alpha_2} = \frac{\delta \rho_{\ell_2} [u_M(-\bar{x}_\alpha) - u_M(\bar{x}_\alpha)]}{\frac{\partial u_M(\bar{x}_\alpha)}{\partial \bar{x}_\alpha} [1 - \delta(\rho_L + \rho_R + \rho_{\ell_1} + \rho_{\ell_2})]}$. Arguments analogous to Case 1 show $\frac{\partial \mathcal{U}_{g_2}(\alpha_2; \hat{x}_\ell)}{\partial \alpha_2} \Big|_{\alpha_2 = \alpha} \geq \frac{\partial \mathcal{U}_{g_1}(\alpha_1; \hat{x}_\ell)}{\partial \alpha_1} \Big|_{\alpha_1 = \alpha}$. The argument for $z = -\bar{x}_\alpha$ and $y = \bar{x}_\alpha$ is symmetric.
- *Case 5:* Suppose $z = \bar{x}_\alpha$ and $y = \bar{x}_\alpha$. Then, $\frac{\partial \mathcal{U}_{g_2}(\alpha_2; \hat{x}_\ell)}{\partial \alpha_2} \Big|_{\alpha_2 = \alpha} = \frac{\partial \mathcal{U}_{g_1}(\alpha_1; \hat{x}_\ell)}{\partial \alpha_1} \Big|_{\alpha_1 = \alpha} = 0$. The argument for $z = -\bar{x}_\alpha$ and $y = -\bar{x}_\alpha$ is symmetric.

□

C Equivalence of Outcome Distribution

A stationary strategy profile $\sigma = (\lambda, \pi, \varphi, \nu)$ is a *stationary legislative lobbying equilibrium* if it satisfies four conditions. First, for all $g \in N^G$ and $\ell \in N_g^L$, λ_g^ℓ places probability one on

$$\begin{aligned}
& \arg \max_{(y, m)} \bar{\nu}_\sigma(y) u_g(y) + [1 - \bar{\nu}_\sigma(y)] [(1 - \delta) u_g(q) + \delta \widehat{V}_g(\sigma)] - m \\
& \text{s.t. } \bar{\nu}_\sigma(y) u_\ell(y) + [1 - \bar{\nu}_\sigma(y)] [(1 - \delta) u_\ell(q) + \delta \widetilde{V}_\ell(\sigma)] + m \\
& \geq \\
& \int_X \left[\bar{\nu}_\sigma(x) u_\ell(x) + [1 - \bar{\nu}_\sigma(x)] [(1 - \delta) u_\ell(q) + \delta \widetilde{V}_\ell(\sigma)] \right] \pi_\ell(dx). \tag{46}
\end{aligned}$$

Second, for all $\ell \in N^L$ and $(y, m) \in W$,

$$\begin{aligned} & \bar{v}_\sigma(y)u_\ell(y) + [1 - \bar{v}_\sigma(y)][(1 - \delta)u_\ell(q) + \delta\tilde{V}_\ell(\sigma)] + m \\ & > \\ & \int_X \left[\bar{v}_\sigma(x)u_\ell(x) + [1 - \bar{v}_\sigma(x)][(1 - \delta)u_\ell(q) + \delta\tilde{V}_\ell(\sigma)] \right] \pi_\ell(dx). \end{aligned} \quad (47)$$

implies $\varphi_\ell(y, m) = 1$ and the opposite strict inequality implies $\varphi_\ell(y, m) = 0$. Third, for all $\ell \in N^L$,

$$\pi_\ell \left(\arg \max_{x \in X} \bar{v}_\sigma(x)u_\ell(x) + [1 - \bar{v}_\sigma(x)][(1 - \delta)u_\ell(q) + \delta\tilde{V}_\ell(\sigma)] \right) = 1. \quad (48)$$

Finally, for all $i \in N^V$ and $x \in X$, $u_i(x) > (1 - \delta)u_i(q) + \delta V_i(\sigma)$ implies $\nu_i(x) = 1$ and the opposite strict inequality implies $\nu_i(x) = 0$.²⁶

Lemma C.1 shows surplus lobby payments never happen in equilibrium.

Lemma C.1. *In every stationary legislative lobbying equilibrium, for all $\ell \in N^L$ every $(y, m) \in \text{supp}(\lambda_g^\ell)$ satisfies*

$$\begin{aligned} & \bar{v}_\sigma(y)u_\ell(y) + [1 - \bar{v}_\sigma(y)][(1 - \delta)u_\ell(q) + \delta\tilde{V}_\ell(\sigma)] + m \\ & = \\ & \int_X \left[\bar{v}_\sigma(x)u_\ell(x) + [1 - \bar{v}_\sigma(x)][(1 - \delta)u_\ell(q) + \delta\tilde{V}_\ell(\sigma)] \right] \pi_\ell(dx). \end{aligned} \quad (49)$$

The proof of Lemma C.1 is straightforward and omitted.

From (12), recall $\xi_\ell(\alpha; \sigma) = (1 - \alpha_\ell) + \alpha_\ell \int_W [1 - \varphi_\ell(y, m)] \lambda_g^\ell(dw)$. Define

$$\hat{\chi}(X') = \sum_{\ell \in N^L} \rho_\ell \left(\xi_\ell(\alpha; \sigma) \int_{X'} \bar{v}_\sigma(x) \pi_\ell(dx) + \alpha_\ell \int_{X' \times \mathbb{R}_+} \varphi_\ell(y, m) \bar{v}_\sigma(y) \lambda_g^\ell(dw) \right), \quad (50)$$

²⁶Thus, voting strategies are *stage-undominated* (Baron and Kalai, 1993; Banks and Duggan, 2006a).

the probability some $x \in X' \subseteq X$ is passed in a given period under σ . Next, define

$$\tilde{\chi} = \sum_{\ell \in N^L} \rho_\ell \left(\xi_\ell(\alpha; \sigma) \int_X [1 - \bar{v}_\sigma(x)] \pi_\ell(dx) + \alpha_\ell \int_W \varphi_\ell(y, m) [1 - \bar{v}_\sigma(y)] \lambda_g^\ell(dw) \right), \quad (51)$$

the probability of a failed proposal in a given period under σ .

Following Banks and Duggan (2006a), each player's continuation value can be expressed as a function of a common lottery over policy, denoted χ^σ . Using (50) and (51), define χ^σ so that for all measurable $X' \subseteq X$: (i) if $q \notin X'$, then $\chi^\sigma(X') = \frac{\hat{\chi}(X')}{1 - \delta\tilde{\chi}}$, and (ii) if $q \in X'$, then $\chi^\sigma(X') = \frac{\hat{\chi}(X') + (1 - \delta)\tilde{\chi}}{1 - \delta\tilde{\chi}}$.

Set $V^{den}(\sigma) = 1 - \delta\tilde{\chi}$ and define

$$V_i^{num}(\sigma) = \sum_{\ell \in N^L} \rho_\ell \left(\xi_\ell(\alpha; \sigma) \int_X \left[\bar{v}_\sigma(x) u_i(x) + [1 - \bar{v}_\sigma(x)] (1 - \delta) u_i(q) \right] \pi_\ell(dx) \right. \\ \left. + \alpha_\ell \int_W \varphi(y, m) \left[\bar{v}_\sigma(y) u_i(x) + [1 - \bar{v}_\sigma(y)] (1 - \delta) u_i(q) \right] \lambda_g^\ell(dw) \right).$$

For each $i \in N^V$, i 's continuation value defined in (13) satisfies $V_i(\sigma) = \frac{V_i^{num}(\sigma)}{V^{den}(\sigma)}$. Then we can express $V_i(\sigma)$ as a lottery over policy, $V_i(\sigma) = \int_X u_i(x) \chi^\sigma(dx)$.

The policy lottery χ^σ is common to all players, but committee members may receive payment and interest groups may make payments. Define

$$\hat{m}_\ell(\sigma) = \rho_\ell \alpha_\ell \int_W m \varphi_\ell(y, m) \lambda_g^\ell(dw), \quad (52)$$

which is ℓ 's expected lobby payment in each period until passage. For $\ell \in N^L$, re-arranging (14) yields

$$\tilde{V}_\ell(\sigma) = \frac{V_\ell^{num}(\sigma) + \hat{m}_\ell(\sigma)}{V^{den}(\sigma)} \\ = \int_X u_\ell(x) \chi^\sigma(dx) + \frac{\hat{m}_\ell(\sigma)}{V^{den}(\sigma)}. \quad (53)$$

Similarly, for $g \in N^G$ rearranging (15) yields

$$\begin{aligned}\widehat{V}_g(\sigma) &= \frac{V_g^{num}(\sigma) - \sum_{\ell \in N_g^L} \widehat{m}_\ell(\sigma)}{V^{den}(\sigma)} \\ &= \int_X u_g(x) \chi^\sigma(dx) - \sum_{\ell \in N_g^L} \frac{\widehat{m}_\ell(\sigma)}{V^{den}(\sigma)}.\end{aligned}\quad (54)$$

Finally, define

$$\widetilde{U}_\ell(\sigma) = \int_X \left[\bar{v}_\sigma(x) u_\ell(x) + \left(1 - \bar{v}_\sigma(x)\right) \left((1 - \delta) u_\ell(q) + \delta \widetilde{V}_\ell(\sigma) \right) \right] \pi_\ell(dx), \quad (55)$$

which is ℓ 's expected dynamic payoff under σ conditional on being recognized as the proposer and rejecting g_ℓ 's offer.

Lemma C.2. *There does not exist a stationary legislative lobbying equilibrium σ such that χ^σ is degenerate on q .*

Proof. Let σ denote an equilibrium. To show a contradiction, assume $\chi^\sigma(q) = 1$. Thus, $V_M(\sigma) = u_M(q)$, which implies $u_M(q) \geq (1 - \delta)u_M(q) + \delta V_M(\sigma)$ and therefore $q \in A(\sigma)$. Without loss of generality, assume $q > 0$.

By assumption, there exists $\ell \in N^L$ such that $\hat{x}_\ell < q$ and at least one of $\hat{x}_{g_\ell} \leq q$ or $\alpha_\ell < 1$ holds. If $\alpha_\ell < 0$, then it is straightforward to show that ℓ must have a profitable deviation, a contradiction.

For the other case, suppose $\hat{x}_\ell < q$, $\hat{x}_{g_\ell} \leq q$, and $\alpha_\ell = 1$. Note that $u_{g_\ell}(y) + u_\ell(y) - \widetilde{U}_\ell(\sigma)$ is g_ℓ 's expected dynamic payoff from any offer (y, m) such that $\bar{v}_\sigma(y) = 1$, $\varphi_\ell(y, m) = 1$, and ℓ is indifferent between accepting and rejecting. We have $\hat{y}_\ell = \arg \max_{y \in X} u_{g_\ell}(y) + u_\ell(y) - \widetilde{U}_\ell(\sigma)$ and $\hat{y}_\ell < q$. Strict concavity and continuity imply existence of $\varepsilon > 0$ and $y^\varepsilon < q$ such that $\bar{v}_\sigma(y^\varepsilon) = 1$, $\varphi_\ell(y^\varepsilon, \widetilde{U}_\ell(\sigma) - u_\ell(y^\varepsilon) + \varepsilon) = 1$, and

$$u_{g_\ell}(y^\varepsilon) + u_\ell(y^\varepsilon) - \widetilde{U}_\ell(\sigma) - \varepsilon > u_{g_\ell}(q) + u_\ell(q) - \widetilde{U}_\ell(\sigma) \quad (56)$$

$$\geq u_{g_\ell}(q) + u_\ell(q) - \tilde{U}_\ell(\sigma) - \delta \left(\sum_{j \in N_g^L} \frac{\hat{m}_j(\sigma)}{V^{\text{den}}(\sigma)} - \frac{\hat{m}_\ell(\sigma)}{V^{\text{den}}(\sigma)} \right), \quad (57)$$

where (57) follows from $\sum_{j \in N_g^L} \frac{m_j(\sigma)}{V^{\text{den}}(\sigma)} \geq \frac{m_\ell(\sigma)}{V^{\text{den}}(\sigma)}$. The RHS of (56) is weakly greater than g_ℓ 's expected payoff from lobbying ℓ to q if $\bar{v}_\sigma(q) = 1$; and (57) is weakly greater than g_ℓ 's expected payoff from lobbying ℓ to any y' such that $\bar{v}_\sigma(y') = 0$. Thus, g_ℓ must have a profitable deviation, a contradiction. \square

Lemma C.3. *Let σ denote a stationary legislative lobbying equilibrium. For all $\ell \in N^L$ there exists $(y, m) \in X \times \mathbb{R}_+$ such that $\bar{v}_\sigma(y) = 1$ and g_ℓ strictly prefers (y, m) to any (y', m') such that $\bar{v}_\sigma(y') = 0$.*

Proof. Fix an equilibrium σ . Let χ^q denote a probability distribution degenerate on q . Define the continuation distribution following rejection under σ as $\chi = (1 - \delta)\chi^q + \delta\chi^\sigma$, which is non-degenerate because $\delta \in (0, 1)$ and $\chi^\sigma(q) < 1$ by Lemma C.2.

For every player $k \in N$, the expected dynamic policy payoff from a rejected policy proposal satisfies

$$(1 - \delta)u_k(q) + \delta V_k(\sigma) = \int_X u_k(x) \chi(dx).$$

Let x^σ denote the mean of χ . Since u is strictly concave and χ is non-degenerate, Jensen's Inequality implies

$$u_k(x^\sigma) > \int_X u_k(x) \chi(dx) = (1 - \delta)u_k(q) + \delta V_k(\sigma). \quad (58)$$

Consider $\ell \in N^L$. First, assume $\phi_\ell(y, m) = 1$ whenever ℓ is indifferent. The condition for g_ℓ to strictly prefer (y, m) such that $\bar{v}_\sigma(y) = 1$, rather than (y', m') such that $\bar{v}_\sigma(y') = 0$, is

$$u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma) > (1 - \delta)u_{g_\ell}(q) + \delta \widehat{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) - \tilde{U}_\ell(\sigma).$$

Equivalently,

$$u_{g_\ell}(y) + u_\ell(y) > (1 - \delta)u_{g_\ell}(q) + \delta\widehat{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta\widetilde{V}_\ell(\sigma). \quad (59)$$

Notice that

$$\widehat{V}_{g_\ell}(\sigma) + \widetilde{V}_\ell(\sigma) = V_{g_\ell}(\sigma) - \sum_{\ell' \in N_g^L} \frac{\hat{m}_{\ell'}(\sigma)}{V^{den}(\sigma)} + V_\ell(\sigma) + \frac{\hat{m}_\ell(\sigma)}{V^{den}(\sigma)} \quad (60)$$

$$\leq V_{g_\ell}(\sigma) - \frac{\hat{m}_\ell(\sigma)}{V^{den}(\sigma)} + V_\ell(\sigma) + \frac{\hat{m}_\ell(\sigma)}{V^{den}(\sigma)} \quad (61)$$

$$= V_{g_\ell}(\sigma) + V_\ell(\sigma), \quad (62)$$

where (60) follows from substituting for $\widetilde{V}_\ell(\sigma)$ and $\widehat{V}_g(\sigma)$ using (53) and (54); and (61) from

$$\sum_{\ell' \in N_g^L} \frac{\hat{m}_{\ell'}(\sigma)}{V^{den}(\sigma)} \geq \frac{\hat{m}_\ell(\sigma)}{V^{den}(\sigma)}.$$

By (58), $\bar{v}_\sigma(x^\sigma) = 1$ follows because $u_M(x^\sigma) > (1 - \delta)u_M(q) + \delta V_M(\sigma)$. Furthermore, (58) implies $u_{g_\ell}(x^\sigma) > (1 - \delta)u_{g_\ell}(q) + \delta V_{g_\ell}(\sigma)$ and $u_\ell(x^\sigma) > (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma)$. Thus, (62) implies that (59) holds because

$$\begin{aligned} u_{g_\ell}(x^\sigma) + u_\ell(x^\sigma) &> (1 - \delta)u_{g_\ell}(q) + \delta V_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) \\ &\geq (1 - \delta)u_{g_\ell}(q) + \delta\widehat{V}_{g_\ell}(\sigma) + (1 - \delta)u_\ell(q) + \delta\widetilde{V}_\ell(\sigma). \end{aligned}$$

Next, assume $\varphi_\ell(x^\sigma, m) < 1$ for m such that ℓ is indifferent between accepting (x^σ, m) and rejecting. For sufficiently small $\varepsilon > 0$, $\varphi_\ell(x^\sigma, m + \varepsilon) = 1$ and the preceding argument implies g_ℓ strictly prefers $(x^\sigma, m + \varepsilon)$ over any (y', m') such that $\bar{v}_\sigma(y') = 0$. \square

Lemma C.4. *Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential voting.*

Proof. Let σ be an equilibrium. By Duggan (2014), M is decisive. Quadratic utility and $\hat{x}_M = 0 \neq q$ together imply $A(\sigma) = \{x \in X | u_M(x) \geq (1 - \delta)u_M(q) + \delta V_M(\sigma)\}$ is a closed,

non-empty interval symmetric about 0. Let $A(\sigma) = [-\bar{x}(\sigma), \bar{x}(\sigma)]$. Then $x \in (-\bar{x}(\sigma), \bar{x}(\sigma))$ implies $\bar{v}_\sigma(x) = 1$.

Fix $\ell \in N^L$. By Lemma C.2, $\chi^\sigma(q) < 1$. Lemma C.3 implies existence of $(y, m) \in W$ such that $\bar{v}_\sigma(y) = 1$ and g_ℓ strictly prefers (y, m) over all (y', m') with $\bar{v}_\sigma(y') = 0$. Thus, $y \in A(\sigma)$ for all $(y, m) \in \text{supp}(\lambda_{g_\ell})$. Without loss of generality, assume $\bar{v}_\sigma(-\bar{x}(\sigma)) < 1$. It suffices to check two cases.

- *Case 1:* If $\hat{x}_\ell \leq -\bar{x}(\sigma)$ and $u_\ell(-\bar{x}(\sigma)) > (1 - \delta)u_\ell(q) + \delta\tilde{V}_\ell(\sigma)$, then $x \in A(\sigma)$ for all $x \in \text{supp}(\pi_\ell)$. Because u_ℓ is strictly concave and continuous, and $\bar{v}_\sigma(-\bar{x}(\sigma)) < 1$, there exists $\varepsilon > 0$ such that ℓ has a profitable deviation to $-\bar{x}(\sigma) + \varepsilon$, a contradiction.
- *Case 2:* Assume $\hat{y}_\ell \leq -\bar{x}(\sigma)$. Continuity, Lemma C.3, and $\bar{v}_\sigma(-\bar{x}(\sigma)) < 1$ imply existence of $\varepsilon, \varepsilon' > 0$ such that g_ℓ has a profitable deviation to $(y', m') = (-\bar{x}(\sigma) + \varepsilon, \tilde{U}_\ell(\sigma) - u_\ell(-\bar{x}(\sigma) + \varepsilon) + \varepsilon')$, a contradiction.

It follows that either σ must involve deferential voting, or σ is equivalent in outcome distribution to an equilibrium with deferential voting. \square

Lemma C.5. *Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential acceptance strategies.*

Proof. Let σ denote an equilibrium. By Lemma C.4, we can assume $\bar{v}_\sigma(x) = 1$ iff $x \in A(\sigma)$. Fix $\ell \in N^L$ and define $y_{g_\ell}^* = \arg \max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - \tilde{U}_\ell(\sigma)$, which is uniquely defined, and $m_{g_\ell}^* = \tilde{U}_\ell(\sigma) - u_\ell(y_{g_\ell}^*)$.

By Lemma C.2, $\chi^\sigma(q) < 1$. For sufficiently small $\varepsilon > 0$, Lemma C.3 implies g strictly prefers $(y_{g_\ell}^*, m_{g_\ell}^* + \varepsilon)$ over every (y', m') such that $y' \notin A(\sigma)$. Thus, if π_ℓ is not degenerate on $y_{g_\ell}^*$ and $\varphi_\ell(y_{g_\ell}^*, m_{g_\ell}^*) < 1$, then there exists $\varepsilon > 0$ such that g_ℓ has a profitable deviation to $(y_{g_\ell}^*, m_{g_\ell}^* + \varepsilon)$, a contradiction. Thus, σ must satisfy either (i) $\pi_\ell(y_{g_\ell}^*) = 1$, or (ii) $\lambda_g^\ell(y_{g_\ell}^*, m_{g_\ell}^*) = 1$ and $\varphi_\ell(y_{g_\ell}^*, m_{g_\ell}^*) = 1$, as desired. \square

A strategy profile σ is *no-delay* if $\bar{v}_\sigma(x) = 1$ for all $x \in \text{supp}(\pi_\ell)$ and $\bar{v}_\sigma(y) = 1$ for all $(y, m) \in \text{supp}(\lambda_g^\ell)$.

Lemma C.6. *Every stationary legislative lobbying equilibrium is no-delay.*

Proof. Fix an equilibrium σ . By Lemma C.2, $\chi^\sigma(q) < 1$. Thus, Lemma C.3 implies g strictly prefers some $(y, m) \in W$ such that $\bar{v}_\sigma(y) = 1$. Lemma C.4 implies we can assume $\bar{v}_\sigma(x) = 1$ iff $x \in A(\sigma)$. Lemma C.5 implies we can assume all $\ell \in N^L$ use deferential acceptance strategies.

For each $\ell \in N^L$, the preceding observations and Lemma C.1 imply λ_g^ℓ puts probability one on (y^*, m^*) such that $y^* = \arg \max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell; \sigma)$, which is unique. Lemmas C.4 and C.5 imply we can assume $\bar{v}_\sigma(y^*) = 1$ and $\varphi_\ell(y^*, m^*) = 1$.

It remains to verify that $z_\ell \notin A(\sigma)$ cannot be optimal for any $\ell \in N^L$. To show a contradiction, assume proposing $z_\ell \notin A(\sigma)$ is optimal for some $\ell \in N^L$. Let $z^* = \arg \max_{x \in A(\sigma)} u_\ell(x)$. There are two steps. Step 1 establishes useful properties of ℓ 's preferences over lotteries. Step 2 shows a contradiction.

Step 1: Recall the continuation lottery induced by σ , denoted $\chi = (1 - \delta)\chi^q + \delta\chi^\sigma$ with mean x^σ . Jensen's inequality implies $u_i(x^\sigma) > \int_X u_i(x) \chi(dx) = (1 - \delta)u_i(q) + \delta V_i(\sigma)$ for all $i \in N$, so $x^\sigma \in \text{int}A(\sigma)$.

Next, let χ^{z^*} denote the policy lottery nearly equivalent to χ , but shifting probability $\frac{\delta \rho_\ell \alpha_\ell}{V^{den}(\sigma)}$ from y^* to z^* . Let x^{z^*} denote the mean of χ^{z^*} . For all $i \in N$, Jensen's inequality implies

$$u_i(x^{z^*}) > \int_X u_i(x) \chi^{z^*}(dx) = (1 - \delta)u_i(q) + \delta V_i(\sigma) - \frac{\delta \rho_\ell \alpha_\ell u_i(y^*)}{V^{den}(\sigma)} + \frac{\delta \rho_\ell \alpha_\ell u_i(z^*)}{V^{den}(\sigma)}.$$

Moreover, x^{z^*} is located weakly between x^σ and z^* , implying $x^{z^*} \in A(\sigma)$.

Step 2: Since $z_\ell \notin A(\sigma)$ is optimal, Lemma C.1 implies

$$\begin{aligned} m^* &= (1 - \delta)u_\ell(q) + \delta \tilde{V}_\ell(\sigma) - u_\ell(y^*) \\ &= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \hat{m}_\ell(\sigma)}{V^{den}(\sigma)} - u_\ell(y^*). \end{aligned} \tag{63}$$

Using (52), $\hat{m}_\ell(\sigma)$ is expressed recursively as

$$\begin{aligned}\hat{m}_\ell(\sigma) &= \rho_\ell \alpha_\ell \left((1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \hat{m}_\ell(\sigma)}{V^{den}(\sigma)} - u_\ell(y^*) \right) \\ &= \frac{\rho_\ell \alpha_\ell V^{den}(\sigma)}{V^{den}(\sigma) - \delta \rho_\ell \alpha_\ell} \left((1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*) \right).\end{aligned}\quad (64)$$

Because $z_\ell \notin A(\sigma)$ is optimal,

$$u_\ell(z^*) \leq (1 - \delta)u_\ell(q) + \delta \tilde{V}(\sigma) \quad (65)$$

$$= (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta \rho_\ell \alpha_\ell [(1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*)]}{V^{den}(\sigma) - \delta \rho_\ell \alpha_\ell}, \quad (66)$$

where (66) follows from the definition of $\tilde{V}_\ell(\sigma)$ and using (64) to substitute for $\hat{m}_\ell(\sigma)$. Next, we have $V^{den}(\sigma) - \delta \rho_\ell \alpha_\ell \geq 1 - \delta \sum_{j \in N^L} \rho_j (1 - \alpha_j) - \delta \rho_\ell \alpha_\ell > 0$, where the first inequality follows because Lemma C.3 implies all lobby offers are accepted and passed under σ , so $V^{den}(\sigma) \geq 1 - \delta \sum_{j \in N^L} \rho_j (1 - \alpha_j)$; and the second inequality follows from $\delta[\rho_\ell \alpha_\ell + \sum_{j \in N^L} \rho_j (1 - \alpha_j)] < 1$. Rearranging and simplifying (66),

$$\begin{aligned}0 &\leq V^{den}(\sigma) \left((1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) \right) - \delta \rho_\ell \alpha_\ell u_\ell(y^*) - u_\ell(z^*) \left(V^{den}(\sigma) - \delta \rho_\ell \alpha_\ell \right) \\ &\propto (1 - \delta)u_\ell(q) + \delta V_\ell(\sigma) - \frac{\delta \rho_\ell \alpha_\ell [u_\ell(y^*) - u_\ell(z^*)]}{V^{den}(\sigma)} - u_\ell(z^*) \\ &= \int_X u_\ell(x) \chi^{z^*}(dx) - u_\ell(z^*),\end{aligned}$$

a contradiction because $u_\ell(z^*) \geq u_\ell(x^{z^*}) > \int_X u_\ell(x) \chi^{z^*}(dx)$.

□

Lemma C.7. *Every stationary legislative lobbying equilibrium is such that λ_g is degenerate for all $g \in N^G$ and π_ℓ is degenerate for all $\ell \in N^L$.*

Proof. Let σ denote an equilibrium. By Duggan (2014), $A_M(\sigma) = A(\sigma)$, which is nonempty, compact and convex.

First, consider $g \in N^g$ and $\ell \in N^L$. Recall $\tilde{U}_\ell(\sigma)$ from (55). Lemmas C.1 and C.6 imply λ_g^ℓ puts probability one on the unique (y^*, m^*) satisfying $y^* = \arg \max_{y \in A(\sigma)} u_g(y) + u_\ell(y) - \tilde{U}_\ell(\sigma)$, and $m^* = \tilde{U}_\ell(\sigma) - u_\ell(y^*)$.

Second, consider $\ell \in N^L$. Lemma C.6 implies π_ℓ puts probability one on $x^* = \arg \max_{x \in A(\sigma)} u_\ell(x)$, which is unique. \square

Proposition 1.2 *Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay stationary legislative lobbying equilibrium with deferential acceptance and deferential voting.*

Proof. Follows from Lemmas C.4 - C.7. \square

D Partitioning Moderates & Extremists

Consider $\ell \in N^L$. First, I define a function ζ^ℓ that relates to M 's equilibrium voting decision. Then, Lemmas D.3 - D.6 characterize ζ^ℓ . Finally, Lemma 2 delivers a partitional characterization on \hat{x}_g that facilitates Proposition 4.

Preliminaries to define ζ^ℓ . Recall $\bar{x}(0) = \bar{x}(\hat{x}_g)$ for $\hat{x}_g = 0$. Let $\hat{D}^{\ell,y} = \{\hat{y}_j : |\hat{y}_j| > \bar{x}(0), j \neq \ell\}$ and $\hat{D}^{\ell,x} = \{\hat{x}_j : |\hat{x}_j| > \bar{x}(0), j \neq \ell\}$. Next, set $D^{\ell,y} = \{|y| : y \in \hat{D}^{\ell,y}\}$ and $D^{\ell,x} = \{|x| : x \in \hat{D}^{\ell,x}\}$. Define D^ℓ as the unique elements of $D^{\ell,y} \cup D^{\ell,x} \cup \{\bar{x}(0)\}$. Let $K^\ell + 1 = |D^\ell|$. Denote the k -th element of D^ℓ as d_k^ℓ . Index elements $k = 0, \dots, K^\ell$ of D^ℓ in ascending order so that $d_0^\ell = \bar{x}(0)$ and $k' > k$ implies $d_{k'}^\ell > d_k^\ell$.

For each k and $j \neq \ell$, let $C_j^k = \mathbb{I}\{\hat{x}_j \in [-d_k^\ell, d_k^\ell]\}$ and $\tilde{C}_j^k = \mathbb{I}\{\hat{y}_j \in [-d_k^\ell, d_k^\ell]\}$. Define

$$I_j^k = (1 - \alpha_j)C_j^k u_M(\hat{x}_j) + \alpha_j \tilde{C}_j^k u_M(\hat{y}_j)$$

and

$$O_j^k = (1 - \alpha_j)(1 - C_j^k) + \alpha_j(1 - \tilde{C}_j^k),$$

suppressing dependence on ℓ . Let

$$\hat{x}_k^\ell = \left(\frac{1}{\delta \rho_\ell} \left[(1 - \delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_k^\ell) \left(1 - \delta \sum_{j \neq \ell} O_j^k \right) \right] \right)^{\frac{1}{2}}. \quad (67)$$

Because $d_0^\ell = \bar{x}(0)$, rearranging (67) yields $\hat{x}_0^\ell = 0$.

Lemma D.1. *For all $\ell \in N^\ell$ and each $k = 0, \dots, K^\ell$, we have*

$$\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1}) = \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k).$$

Proof. Consider $\ell \in N^L$ and fix $k < K^\ell$. Then,

$$\begin{aligned}
& \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1}) \\
&= \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1}) + \delta u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j O_j^k - \delta u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j O_j^k \\
&= \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j (O_j^{k+1} - O_j^k) \\
&= \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta \sum_{j \neq \ell} \rho_j (I_j^k - I_j^{k+1}) \tag{68}
\end{aligned}$$

$$= \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_{k+1}^\ell) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k), \tag{69}$$

where (68) follows because $u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j (O_j^{k+1} - O_j^k) = \sum_{j \neq \ell} \rho_j (I_j^k - I_j^{k+1})$ by construction. \square

Lemma D.2. *For all $\ell \in N^\ell$, \hat{x}_k^ℓ strictly increases in k .*

Proof. Consider $\ell \in N^L$ and fix $k < K^\ell$. Lemma D.1 and $0 > u_M(d_k^\ell) > u_M(d_{k+1}^\ell)$ together imply

$$\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1}) > \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_k^\ell) (1 - \delta \sum_{j \neq \ell} \rho_j O_j^k). \tag{70}$$

Thus, $\hat{x}_k^\ell < \hat{x}_{k+1}^\ell$ follows from (67). \square

Definition of ζ^ℓ . For $k = 0, \dots, K^\ell$, define $\bar{x}_k^\ell : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ as

$$\bar{x}_k^\ell(x) = \left(\frac{(1 - \delta)u_M(q) + \delta \rho_\ell u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k} \right)^{\frac{1}{2}}, \tag{71}$$

and $\zeta_k^\ell : \mathbb{R}_+ \rightarrow \mathbb{R}$ as

$$\zeta_k^\ell(x) = u_M(x) - \left((1 - \delta)u_M(q) + \delta \rho_\ell u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\bar{x}_k^\ell(x)) \sum_{j \neq \ell} \rho_j O_j^k \right).$$

By construction, $\bar{x}_k^\ell(\hat{x}_k^\ell) = d_k^\ell$ for all k . Adopt the convention $d_{K^\ell+1}^\ell = \infty$. Define the piecewise function $\zeta^\ell : \mathbb{R}_+ \rightarrow \mathbb{R}$ as

$$\zeta^\ell(x) = \zeta_k^\ell(x) \quad \text{if } x \in [d_k^\ell, d_{k+1}^\ell).$$

Lemma D.3. *For all $\ell \in N^L$, $\zeta^\ell(0) > 0$ and $\zeta^\ell(q) \leq 0$.*

Proof. Consider $\ell \in N^L$. First, we have

$$\begin{aligned} \zeta^\ell(0) &= \zeta_0^\ell(0) \\ &= u_M(0) - \left((1 - \delta)u_M(q) + \delta\rho_\ell u_M(0) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(\bar{x}_0^\ell(0)) \sum_{j \neq \ell} \rho_j O_j^0 \right) \\ &= - \left((1 - \delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(d_0^\ell) \sum_{j \neq \ell} \rho_j O_j^0 \right) \\ &> 0, \end{aligned} \tag{72}$$

where (72) follows from $u_M(0) = 0$ and $\bar{x}_0^\ell(0) = \hat{x}_0$.

Next, I show $\zeta^\ell(q) \leq 0$. Let k' denote the largest k such that $\hat{x}_k^\ell \leq q$.

• *Step 1:* Because $\bar{x}^{k'}(\hat{x}_{k'}^\ell) = d_{k'}^\ell$, we have

$$u_M(d_{k'}^\ell) = \frac{(1 - \delta)u_M(q) + \delta\rho_\ell u_M(\hat{x}_{k'}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \tag{73}$$

$$\geq \frac{(1 - \delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \tag{74}$$

$$\geq \frac{(1 - \delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta u_M(d_{k'}^\ell) \sum_{j \neq \ell} \rho_j [(1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'}]}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \tag{75}$$

$$= \frac{(1 - \delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta u_M(d_{k'}^\ell)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'})}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}}, \tag{76}$$

where (73) follows from rearranging (71); (74) from $\hat{x}_{k'}^\ell \leq q$; (75) because for all j the construction of $I_j^{k'}$ implies $I_j^{k'} \geq u_M(d_{k'}^\ell)[(1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'}]$; and (76) because $\sum_{j \neq \ell} \rho_j [(1 - \alpha_j)C_j^{k'} + \alpha_j \tilde{C}_j^{k'}] = 1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}$ by construction.

Rearranging and simplifying (76) yields $u_M(d_{k'}^\ell) \geq \frac{(1-\delta+\delta\rho_\ell)u_M(q)}{1-\delta+\delta\rho_\ell} = u_M(q)$. Thus,

$$\sum_{j \neq \ell} \rho_j I_j^{k'} = \sum_{j \neq \ell} \rho_j \left[(1 - \alpha_j) C_j^{k'} u_M(\hat{x}_j) + \alpha_j \tilde{C}_j^{k'} u_M(\hat{y}_j) \right] \quad (77)$$

$$\geq u_M(d_{k'}^\ell) \sum_{j \neq \ell} \rho_j \left[(1 - \alpha_j) C_j^{k'} + \alpha_j \tilde{C}_j^{k'} \right] \quad (78)$$

$$= u_M(d_{k'}^\ell) (1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}) \quad (79)$$

$$\geq u_M(q) (1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}), \quad (80)$$

where (77) follows from the definition of $I_j^{k'}$; (78) from $u_M(\hat{x}_j) \geq u_M(d_{k'}^\ell)$ if $C_j^{k'} = 1$ and $u_M(\hat{y}_j) \geq u_M(d_{k'}^\ell)$ if $\tilde{C}_j^{k'} = 1$; (79) because $\sum_{j \neq \ell} \rho_j [(1 - \alpha_j) C_j^{k'} + \alpha_j \tilde{C}_j^{k'}] = 1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}$ by construction; and (80) from $u_M(d_{k'}^\ell) \geq u_M(q)$.

- *Step 2:* We have

$$u_M(\bar{x}_{k'}^\ell(q)) = \frac{(1 - \delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \geq \frac{(1 - \delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'})}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \quad (81)$$

$$= u_M(q), \quad (82)$$

where (81) follows from Step 1 and (82) from simplifying.

- *Step 3:* To see $\zeta^\ell(q) \leq 0$, note

$$\begin{aligned} \zeta^\ell(q) &= u_M(q) - \left((1 - \delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'} + \delta u_M(\bar{x}_{k'}^\ell(q)) \sum_{j \neq \ell} \rho_j O_j^{k'} \right) \\ &\leq u_M(q) - \left((1 - \delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}) + \delta u_M(q) \sum_{j \neq \ell} \rho_j O_j^{k'} \right) \end{aligned} \quad (83)$$

$$= 0, \quad (84)$$

where (83) follows from Steps 1 and 2. □

Lemma D.4. *For all $\ell \in N^L$, ζ^ℓ is continuous.*

Proof. Consider $\ell \in N^L$ and fix k . Because $\bar{x}_k^\ell(x)$ is continuous, ζ^ℓ is continuous over $(\hat{x}_k^\ell, \hat{x}_{k+1}^\ell)$. It suffices to show $\zeta_k^\ell(\hat{x}_{k+1}^\ell) = \zeta_{k+1}^\ell(\hat{x}_{k+1}^\ell)$.

First, I establish $d_{k+1}^\ell = \bar{x}_k^\ell(\hat{x}_{k+1}^\ell)$. Rearranging (67) for $k+1$ yields

$$\begin{aligned} 0 &= u_M(d_{k+1}^\ell) \left(1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1} \right) - (1 - \delta)u_M(q) - \delta \rho_\ell u_M(\hat{x}_{k+1}^\ell) - \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} \\ &= u_M(d_{k+1}^\ell) \left(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k \right) - (1 - \delta)u_M(q) - \delta \rho_\ell u_M(\hat{x}_{k+1}^\ell) - \delta \sum_{j \neq \ell} \rho_j I_j^k, \end{aligned} \quad (85)$$

where (85) follows from Lemma D.1. Thus, $u_M(d_{k+1}^\ell) = \frac{(1-\delta)u_M(q) + \delta \rho_\ell u_M(\hat{x}_{k+1}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^k}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k}$, so $d_{k+1}^\ell = \bar{x}_k^\ell(\hat{x}_{k+1}^\ell)$. Then,

$$\begin{aligned} \zeta_k^\ell(\hat{x}_{k+1}^\ell) &= u_M(\hat{x}_{k+1}^\ell) - \left((1 - \delta)u_M(q) + \delta \rho_\ell u_M(\hat{x}_{k+1}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\bar{x}_k^\ell(\hat{x}_{k+1}^\ell)) \sum_{j \neq \ell} \rho_j O_j^k \right) \\ &= u_M(\hat{x}_{k+1}^\ell) - \left((1 - \delta)u_M(q) + \delta \rho_\ell u_M(\hat{x}_{k+1}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} + \delta u_M(\bar{x}_{k+1}^\ell(\hat{x}_{k+1}^\ell)) \sum_{j \neq \ell} \rho_j O_j^{k+1} \right) \end{aligned} \quad (86)$$

$$= \zeta_{k+1}^\ell(\hat{x}_{k+1}^\ell), \quad (87)$$

where (86) follows from Lemma D.1 because $d_{k+1}^\ell = \bar{x}_k^\ell(\hat{x}_{k+1}^\ell)$. □

Lemma D.5. *For all $\ell \in N^L$, ζ^ℓ is strictly decreasing.*

Proof. Consider $\ell \in N^L$ and fix k . The proof shows that the derivative of ζ^ℓ is strictly negative at every $x \in (\hat{x}_k^\ell, \hat{x}_{k+1}^\ell)$. Continuity then implies that ζ^ℓ is strictly decreasing.

Consider $x \in (\hat{x}_k^\ell, \hat{x}_{k+1}^\ell)$. Then,

$$\zeta^\ell(x) = u_M(x) - \left((1 - \delta)u_M(q) + \delta\rho_\ell u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\bar{x}_k^\ell(x)) \sum_{j \neq \ell} \rho_j O_j^k \right),$$

and

$$\frac{\partial \zeta^\ell(x)}{\partial x} = -2x + 2x\delta\rho_\ell + \frac{2x\delta\rho_\ell(\delta \sum_{j \neq \ell} \rho_j O_j^k)}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k} \quad (88)$$

$$\propto \delta\rho_\ell + \delta \sum_{j \neq \ell} \rho_j O_j^k - 1 \quad (89)$$

$$< 0, \quad (90)$$

where (88) follows from $\frac{\partial u_M(\bar{x}_k^\ell(x))}{\partial \bar{x}_k^\ell(x)} \frac{\partial \bar{x}_k^\ell(x)}{\partial x} = -\frac{2x\delta\rho_\ell}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^k}$; and (90) because $\delta \in (0, 1)$ and $\rho_\ell + \sum_{j \neq \ell} \rho_j O_j^k \leq 1$. \square

Lemma D.6. *For all $\ell \in N^\ell$, there is a unique $\bar{x}_\ell \in (0, q]$ such that $\zeta^\ell(x) > 0$ for all $x \in [0, \bar{x}_\ell)$, $\zeta^\ell(\bar{x}_\ell) = 0$, and $\zeta^\ell(x) < 0$ for all $x > \bar{x}_\ell$.*

Proof. Consider $\ell \in N^L$. Lemma D.3 implies $\zeta^\ell(0) > 0$ and $\zeta^\ell(q) \leq 0$. By Lemma D.5, ζ^ℓ is strictly decreasing. Thus, there is a unique $\bar{x}_\ell \in (0, q]$ such that $\zeta^\ell(x) > 0$ for all $x \in [0, \bar{x}_\ell)$ and $\zeta^\ell(x) < 0$ for all $x > \bar{x}_\ell$. Lemma D.4 implies $\zeta^\ell(\bar{x}_\ell) = 0$. \square

Lemma 2. *For all $\ell \in N^L$, $\hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell)$ implies $\hat{x}_g \in \text{int}A(\hat{x}_g)$. Otherwise, $A(\hat{x}_g) = [-\bar{x}_\ell, \bar{x}_\ell]$.*

Proof. Consider $\ell \in N^L$ with associated $g \in N^G$. Assume $\hat{x}_\ell = \hat{x}_g$.

Part 1. First, suppose $\hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell)$ and assume $\hat{x}_g \geq 0$ without loss of generality. I show $\hat{x}_g \in \text{int}A(\hat{x}_g)$. Let k' be the largest k such that $\hat{x}_k^\ell \leq \hat{x}_g$. Define the strategy profile σ' such that it puts probability ρ_ℓ on \hat{x}_g and for each $j \neq \ell$ it (i) puts probability $(1 - \alpha_j)\rho_j$ on: \hat{x}_j if $\hat{x}_j \in [-d_{k'}^\ell, d_{k'}^\ell]$, $\bar{x}_{k'}^\ell(\hat{x}_g)$ if $\hat{x}_j > d_{k'}^\ell$, or $-\bar{x}_{k'}^\ell(\hat{x}_g)$ if $\hat{x}_j < -d_{k'}^\ell$; and (ii) puts probability

$\alpha_j \rho_j$ on: \hat{y}_j if $\hat{y}_j \in [-d_{k'}^\ell, d_{k'}^\ell]$, $\bar{x}_{k'}^\ell(\hat{x}_g)$ if $\hat{y}_j > d_{k'}^\ell$, or $-\bar{x}_{k'}^\ell(\hat{x}_g)$ if $\hat{y}_j < -d_{k'}^\ell$. By construction, $\bar{x}(\sigma') = \bar{x}_{k'}^\ell(\hat{x}_g)$. Furthermore, proposal strategies are optimal given $A(\sigma') = [-\bar{x}(\sigma'), \bar{x}(\sigma')]$.

I now check optimality for M . Because $\hat{x}_g \in [\hat{x}_{k'}^\ell, \hat{x}_{k'+1}^\ell)$, we have $\bar{x}(\sigma') = \bar{x}_{k'}^\ell(\hat{x}_g) \in [d_{k'}^\ell, d_{k'+1}^\ell)$. Thus, M optimally accepts all offers by $j \neq \ell$. Next, I verify $\hat{x}_g \in \text{int}A(\sigma')$. By Lemma D.6, $\hat{x}_g \in (-\bar{x}_\ell, \bar{x}_\ell)$ implies $\zeta(\hat{x}_g) > 0$, which is equivalent to

$$u_M(\hat{x}_g) > \frac{(1 - \delta)u_M(q) + \delta\rho_\ell u_M(\hat{x}_g) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}}.$$

Under σ' , this is equivalent to $\hat{x}_g \in \text{int}A(\sigma')$.

Thus, σ' is equivalent to the equilibrium $\sigma(\hat{x}_g)$ and $\hat{x}_g \in \text{int}A(\hat{x}_g)$, as desired.

Part 2. Assume $\hat{x}_g \notin (-\bar{x}_\ell, \bar{x}_\ell)$ and suppose $\hat{x}_g \geq 0$ without loss of generality. I verify $A(\hat{x}_g) = [-\bar{x}_\ell, \bar{x}_\ell]$ in two steps. Step 1 shows $\bar{x}(\hat{x}_g) \geq \bar{x}_\ell$. Step 2 shows $\bar{x}(\hat{x}_g) \leq \bar{x}_\ell$.

Step 1. Suppose $\bar{x}(\hat{x}_g) < \bar{x}_\ell$. Let k' be the largest k such that $\hat{x}_{k'}^\ell \leq \bar{x}(\hat{x}_g)$. Because $\hat{x}_g > \bar{x}_\ell > \bar{x}(\hat{x}_g)$, it follows that $\sigma(\hat{x}_g)$ puts probability ρ_ℓ on $\bar{x}(\hat{x}_g)$. Thus, $u_M(\bar{x}(\hat{x}_g)) = \frac{(1-\delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta\rho_\ell - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}}$ and rearranging yields $\zeta(\bar{x}(\hat{x}_g)) = 0$. Lemma D.6 implies $\bar{x}(\hat{x}_g) = \bar{x}_\ell$, a contradiction.

Step 2. Suppose $\bar{x}(\hat{x}_g) > \bar{x}_\ell$. If $\hat{x}_g \geq \bar{x}(\hat{x}_g)$, then the argument from Step 1 shows a contradiction. Assume $\hat{x}_g < \bar{x}(\hat{x}_g)$. Let k' be the largest k such that $\hat{x}_{k'}^\ell(\hat{x}_g) \leq \bar{x}(\hat{x}_g)$. Then $\sigma(\hat{x}_g)$ puts probability ρ_ℓ on \hat{x}_g . Next, M optimally accepts \hat{x}_g under $\sigma(\hat{x}_g)$ iff $u_M(\hat{x}_g) \geq \frac{(1-\delta)u_M(q) + \delta\rho_\ell u_M(\hat{x}_g) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}}$. Rearranging, this condition is equivalent to $\zeta(\hat{x}_g) \geq 0$. By Lemma D.6, this requires $\hat{x}_g \leq \bar{x}_\ell$, a contradiction. \square