

Electoral Competition with Voting Costs

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Abstract

How do voting restrictions affect election outcomes? This paper highlights how focusing on turnout or vote shares, as most empirical studies have done, may miss crucial policy effects from new restrictive voting laws. We use a formal model of electoral competition to show how changing voting costs can affect more than just voting behavior. Instead, voters *and* politicians respond, as turnout and platforms affect each other in equilibrium. We show that increasing voting costs for one party's supporters leads that party to choose a more moderate platform and the opposing party to choose a more extreme platform. These effects are magnified as the share of voters in the affected group increases. Our analysis demonstrates that widening our lens beyond turnout and vote shares is important for assessing the impact of voting rights legislation.

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Introduction

A classic concern for democracy is *who participates*, due to fears and evidence that unequal participation begets biased policies (Lijphart 1997). This concern has directed attention towards various laws that appear closely linked with political participation levels. For voting, arguably the central form of democratic participation, such policies include: early voting, mail-in voting, voting hours, pre-registration, and voter ID laws. In many places over the last century, these policies have been introduced and adjusted in order to facilitate voting, but recently there has been prominent examples of policy changes that are widely believed to hinder voting by raising voting costs for certain groups. In the US during 2021 alone, more than thirty restrictive voting laws were passed by states (Center 2021*b*). For example, S.B. 90 in Florida makes obtaining a mail-in ballot more difficult and limits the hours for which voters have access to mail-in ballot drop boxes (Center 2021*a*), whereas S.F. 413 in Iowa and S.B. 1 in Texas would shrink early voting hours (White 2021; Ura 2021).

Scholars have directed considerable effort towards understanding the effects of voting costs on political participation and representation. Most empirical studies focus on turnout and vote share. For example, there is mixed evidence that raising voter costs through voter ID requirements reduces turnout (Grimmer and Yoder 2021; Barreto et al. 2019; Hood and Bullock 2012; Cantoni and Pons 2021; Ansolabehere 2009). Other cost-increasing policies, such as increasing distance to polling locations (Cantoni 2020; Bagwe, Margitic and Stashko 2020) or voter registration costs (Braconnier, Dormagen and Pons 2017), have more consistent empirical support for turnout-reducing effects. In the other direction, empirical evidence suggests that turnout increases following policy changes that lower voting costs, such as same-day registration (Grumbach and Hill 2022), mail-in voting (Gerber, Huber and Hill 2013; Bonica et al. 2021), or early voting (Kaplan and Yuan 2020).

In contrast to accumulating evidence about effects on turnout, we know much less about the empirical effects of voting costs on another outcome of interest, policies. A rare example is Fujiwara (2015), which specifically links the adoption of easier voting technology, electronic voting machines, to higher spending on health policy. Another example is Bertocchi et al. (2020), which

studies how the introduction pre-registration in some US states affects higher education spending by decreasing voting costs for young voters. The lack of work in this vein may be surprising given (i) the importance of policy location and (ii) empirical evidence that turnout affects policies.¹ Of course, measuring policy location is difficult. But, even if it were easy, existing theory provides little guidance about what to expect. Additionally, it may be tempting to interpret unclear evidence about the effects of certain policies on turnout as implying that their effects on voting costs may not be in turn affecting policy.

We provide a game-theoretic analysis to study how voting costs affect turnout, policies and election outcomes in tandem. In doing so, we contribute to developing our theoretical understanding of the effects of voting costs. Our analysis highlights that voting costs do not only affect voting behavior. Instead, we show that even if voting costs *directly* affect only voters, they can have *equilibrium* effects on behavior by voters *and* politicians.

Using a spatial model of elections with voting costs, we show how voting costs can affect not only voter behavior (*turnout*), but also politician behavior (*policy choice*). Essentially, this arises because platforms and turnout influence each other: changing voting costs affects turnout, which affects and is affected by policy platforms. The effect on vote share thus combines two effects.

We show that targeted changes to voting costs shift equilibrium policy platforms towards the party whose voters are less affected. Notably, these platform shifts counteract the direct effect on turnout and, in turn, win probability. Thus, a central takeaway of our analysis is that the clear gain for the non-targeted party is in more favorable policy, rather than turnout or electoral prospects.

Our analysis clarifies how focusing only on turnout and vote shares can obscure the full impacts of laws that alter voting costs. Empirical studies analyzing the effect of policies changing voting costs, such as voter ID laws, reducing polling locations, mail-in voting, and others, should be looking at a wider variety of outcomes than just turnout. Moreover, our analysis highlights why finding no change in relative turnout does not imply that these policies had no effect. We also discuss

¹For a US example, Cascio and Washington (2014) shows that the Voting Rights Act increased distribution to majority black districts. In the European context, evidence suggests that turnout affects government size and performance (Godefroy and Henry 2016; Lo Prete and Revelli 2021; Aggeborn 2016).

several examples of how empirical researchers can link different types of changes in voting costs to their policy consequences, in order to concretely illustrate some of our empirical implications.

To preview our analysis more precisely, we analyze a spatial model of an election with binding campaign platforms and policy motivated parties. There are two groups of citizens, left-leaning citizens and right-leaning citizens. To vote, each citizen must bear a cost. Crucially, we allow voting costs to differ between the two groups. We intentionally associate voting costs and voter ideology to capture the fact that left-leaning voters often face differential costs of voting compared to right-leaning voters. For example, various laws affect specific groups such as urban, Black, or Latino voters (Fraga and Miller 2022) who predominantly vote Democratic.² Additionally, to reflect that voting blocs can differ in size, we allow the groups to have different shares of the population. Our setup provides a unified framework to analyze the effects of voting costs on turnout, campaign platforms, party utility, and other outcomes.

In equilibrium, each citizen's turnout decision depends on whether the policy difference between their preferred party and the opposing party is large enough to offset their voting cost. Thus, aggregate turnout will depend on voting costs, group sizes, and the two party platforms. Anticipating this endogenous turnout, optimal platforms take into account the voting costs and size of each group. These two factors each directly influence which direction platforms are skewed. In general, platforms tilt towards (i) the party aligned with the lower voting cost group, and (ii) the party aligned with the larger group. Of course, (i) and (ii) can be different parties, so that these forces may pull in different directions.

Parties face a tension from moderating their platform. More moderate platforms are farther from the party's ideal point but can mobilize support from moderate citizens who would otherwise abstain. More extreme platforms are more appealing to the party and extreme citizens nearby, but may induce moderate citizens to abstain. Importantly, these turnout considerations also affect citizens in the group that leans towards the other party: more moderate platforms may disengage some of those voters if they no longer see enough of a policy difference to vote.

²While voter suppression laws have mostly come from Republican controlled states, the partisan effects of both relaxing and restricting voting access is contested. See (Burden et al. 2017).

We find that increasing voting costs for one group shifts platforms from both parties towards the other party's ideal point, and similarly, lower voting costs shifts both platforms towards that party's ideal point. For example, if the cost for the left group increases, then platforms for both the left and right party shift rightward. The right party becomes more extreme as a more extreme platform will not induce many left partisans to vote due to the higher cost. The left party becomes more moderate because turning off right group voters becomes relatively more important than appealing to their own, higher cost voters.

This platform shift is larger as the affected voting group grows to contain a larger share of the electorate. Therefore, empirically, we should not expect to see large effects if the group being affected has a small share of the possible electoral support. Even when a policy equally affects both voting groups, the impact on policies is driven by the larger group. For example, mail-in voting does not generally have a partisan slant (Barber and Holbein 2020), and therefore restricting mail-in voting would hit voters who favor both parties. However, the policy effects would be driven by the group with a larger share of the electorate.

These platform shifts do not affect either party's probability of winning in equilibrium. Both parties' winning probabilities are constant in voting costs. However, this does not mean parties do not benefit from lower costs (or higher costs from their opponents). As platforms move closer to one party's ideal point, this party is better off if they win and better off if they lose than before the change in costs. Just observing that both parties win with the same probability before and after voting costs change does not tell you that parties or voters are just as well off as before. The implemented policies matter. These results highlight that empirical findings looking just at vote shares or voter turnout do not tell the entire story of a policy impact.

We also find that turnout decreases for both parties as costs for either voting group increase. The cost increase directly decreases turnout for the side affected, but the policy changes also decrease turnout for the side with the same costs. A more extreme policy decreases turnout for the side where costs stay the same. The overall share of the electorate that turns out is lower, but parties retain the same proportion of voters.

We analyze a game-theoretic model of platform competition with endogenous turnout and costly voting. Most models of costly voting focus on the turnout decision and, more generally, on voter behavior (Borgers 2004; Myatt 2015; Taylor and Yildirim 2010; Krishna and Morgan 2012; Tyson 2016; Arzumanyan and Polborn 2017).³ In contrast, our analysis emphasizes how changes in voting behavior affect politician behavior, and vice versa. Specifically, we highlight the importance of studying how politicians react to changes in voters' costs.⁴

In a similar vein, Hortala-Vallve and Esteve-Volart (2011a) studies a model in which parties choose platforms anticipating how voting costs will affect endogenous turnout, as in this paper. The key difference is that we allow heterogeneous voting costs to be specifically linked to ideology. This allows our model to analyze the effects of specifically targeted changes in voting costs.

The appendix of Bertocchi et al. (2020) presents a model closely related to ours. A key difference is that they study candidates motivated purely by office rents, whereas we study policy-motivated candidates. Due largely to this difference, interesting equilibrium behavior requires an incumbency advantage in their setting. In contrast, our analysis does not rely on incumbency advantage, and we thus set it aside because it is not central to our interest in studying targeted changes to voting costs. Another difference is that they focus on uniformly distributed voting costs, a special case of the log-concave cost distributions that we study. One benefit of our additional generality is that we can analyze the effects of a broader class of changes in voting costs, which allows us to highlight effects on, e.g., expected vote shares, that do not arise in the uniform special case.

Most other models that analyze platform choice with costly voting for policies tend to look at *affinity voting*. In those contexts, abstention occurs due to alienation that arises because all of the candidates are too far from the voter's ideal point (Callander and Wilson 2007; Callander and Carbajal 2022; Adams and Merrill III 2003; Llavador 2006). Our model differs from these by

³The older literature on the rationality of voting, e.g., Palfrey and Rosenthal (1983); Ledyard (1984); Hansen, Palfrey and Rosenthal (1987), is outside the purview of this article. For a summary, see Feddersen (2004). Some models incorporate cost within an ethical voting framework where voters want to match the voting decisions of their group members (Ali and Lin 2013; Feddersen and Sandroni 2006; Bouton and Ogden 2021). For a wider discussion of electoral models, see Dewan and Shepsle (2011); Ashworth (2012); Duggan and Martinelli (2017).

⁴Aldashev (2015) and Hodler, Luechinger and Stutzer (2015) also endogenize policy choice, but only in the specific case of public good expenditures.

focusing on *exogenous* costs of voting that are more interpretable as policies, such as voter id laws or mail-in voting. In the appendix, we show that in a model with affinity voting, all of our qualitative results hold.

Model

We analyze a one-period model of an election that features platform competition and endogenous turnout with costly voting. The election decides which platform is enacted. The policy space is one-dimensional and we normalize it to be $X = [-1, 1]$.

Players. There are two parties, L and R . Party L has ideal point -1 , whereas R 's ideal point is 1 .

Additionally, there is a unit mass of citizens. They are split into two groups, G_L and G_R . The share of citizens in G_L is α , so the share in G_R is $1 - \alpha$. Each citizen has (i) an associated ideal point in X , and (ii) a voting cost. In G_R , citizen ideal points are uniformly distributed on $[0, 1]$, whereas in G_L they are uniformly distributed on $[-1, 0]$.

Crucially, citizen ideology and voting cost are related. Specifically, every citizen in G_R has voting cost c_R , while every citizen in G_L has voting cost c_L . We assume that $c_R \geq 0$ is fixed and that c_L is a random variable distributed according to a distribution function F that has support $[\underline{c}, \bar{c}]$, where $\underline{c} \geq 0$, and associated density f that is log-concave.⁵ Let \tilde{c} denote the median of F .⁶

Timing. First, the parties L and R simultaneously commit to policy platforms, i.e., each party j commits to $x_j \in X$. Second, the voting cost c_L is realized for citizens in G_L . Third, each citizen chooses whether to vote and, if so, which party to vote for. Finally, the party with the greater vote share wins the election and enacts their platform.⁷

Preferences. Both parties are purely policy motivated. Specifically, each party evaluates the winning

⁵Log-concavity of f implies that F is log-concave. Many common distributions such as normal, exponential, uniform, and others are log-concave. Note that if a distribution is log-concave, a truncation of that distribution is also log-concave (Bagnoli and Bergstrom 2005).

⁶The uncertainty over costs plays a similar role in inducing divergence as uncertainty over the median voter's ideal point does in models such as Wittman (1983); Calvert (1985) and Groseclose (2001).

⁷For simplicity, we assume L wins in the event of a tie. This has no effect on our results.

policy with linear loss, i.e., party j 's utility from elected platform x is

$$U_j(x) = -|x - \hat{x}_j|. \quad (1)$$

Citizens are policy motivated but voting is costly to them. Specifically, they (i) receive utility from the elected platform and (ii) incur their personal voting cost if they turn out. Formally, if x is the winning candidate's platform, then citizen i 's payoff is

$$U_i(x) = -|x - \hat{x}_i| - c \cdot \mathbb{I}(i \text{ votes}), \quad (2)$$

where $\mathbb{I}(i \text{ votes})$ indicates whether i voted.

Strategies and Equilibrium Concept. A strategy for each party is a platform location. A strategy for each citizen is a mapping from their voting cost and the platform locations into a decision of (i) whether to turn out and (ii) who to vote for.

We study what we refer to as *partisan voting equilibria* (PVE). These are a refinement of pure strategy Subgame Perfect Nash Equilibrium (SPNE). The first additional requirement is that each citizen's vote and turnout choices are *sincere*: each citizen votes for the closer platform if they turn out, and turns out if and only if their voting cost is less than their difference in policy utility between the platforms. Intuitively, each citizen's voting behavior is as if they are pivotal.⁸ Second, they feature *partisan voting*: on the equilibrium path (i) no citizen in G_L will vote for R and vice versa, and (ii) each party has positive turnout. Our analysis of PVE is strategically equivalent to analyzing SPNE of an otherwise equivalent model in which citizens receive expressive utility from voting and paying their turnout cost to vote, which has empirical support (Jessee 2009, 2010; Hortala-Vallve and Esteve-Volart 2011b; Shor and Rogowski 2018).⁹

⁸Conditions (i) and (ii) have the spirit of eliminating undominated strategies in the voting subgame. Since we have a continuum of citizens, no citizen is ever pivotal and thus strategies violating (i) and (ii) are not dominated.

⁹Alternatively, related models with endogenous turnout have studied expressive voting behavior driven by proximity to the nearest candidate (e.g., Callander and Wilson 2007; Callander and Carbajal 2022) rather than the difference between candidates. In the Appendix, we study a setting in that vein and show that our key qualitative points carry over.

Analysis

We proceed by backwards induction. First, we fix a pair of policy platforms and study the election stage by analyzing which citizens vote and, if so, who they vote for. Then, anticipating the electoral consequences, we study which platforms parties choose. From there, we analyze how changing voting costs affects equilibrium platforms, turnout, electoral prospects, and party welfare. Throughout, we also study how those effects can depend on relative group size, i.e., how citizens are split between the two groups.

Election Stage

Individual Citizen Behavior: Turnout & Vote Choice

Each citizen has a simple voting calculus. They each prefer the platform closer to their ideal point and will turn out if that platform is sufficiently closer that voting is worthwhile. We refer to citizens who turn out to vote as *voters*.

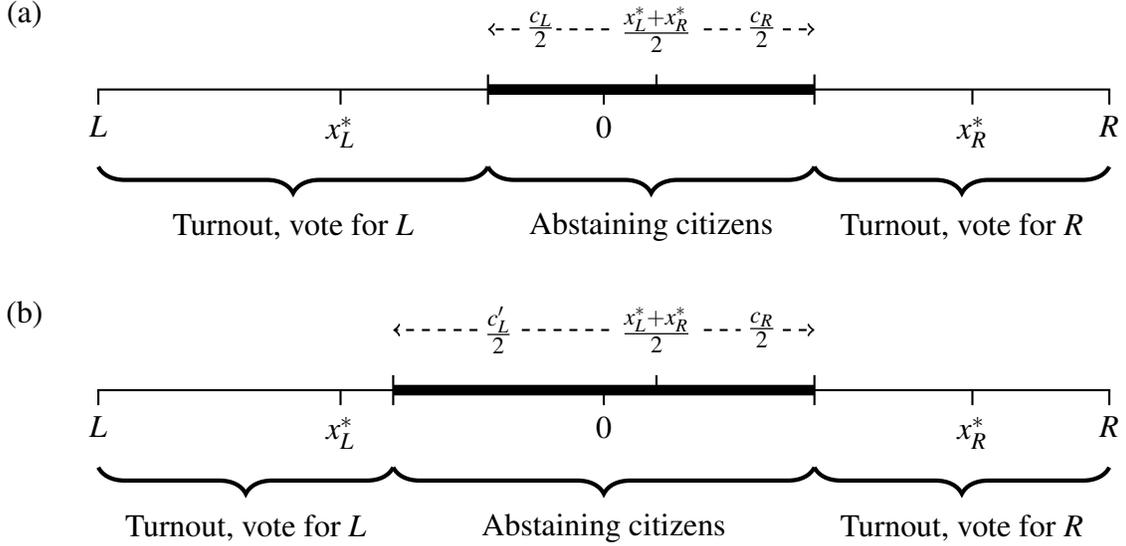
To illustrate more precisely, fix platforms $x_L < x_R$. There are three qualitatively distinct cases that are consequential. First, each citizen with $\hat{x}_i \notin (x_L, x_R)$ will vote for the nearest candidate if $c_i \leq x_R - x_L$, and otherwise they will abstain. Second, each citizen with $\hat{x}_i \in (x_L, \frac{x_R+x_L}{2})$ will vote for candidate L if $-|x_L - \hat{x}_i| + |x_R - \hat{x}_i| \geq c_i$, which reduces to $\hat{x}_i \leq \frac{x_R+x_L}{2} - \frac{c_i}{2}$, and otherwise they abstain. Finally, each citizen with $\hat{x}_i \in (\frac{x_R+x_L}{2}, x_R)$ will vote for candidate R if $\hat{x}_i \geq \frac{x_R+x_L}{2} + \frac{c_i}{2}$ and abstain otherwise.

Aggregate Citizen Behavior: Voters, Turnout, & Election Outcomes

With partisan voting, the set of voters in G_L is $[-1, \frac{x_R+x_L}{2} - \frac{c_L}{2}]$ if $c_L \leq x_R - x_L$ and is empty otherwise. We denote this set as \mathcal{V}_L , suppressing the dependence on x_L, x_R , and c_L to streamline presentation. Analogously, the set of voters in G_R , denoted \mathcal{V}_R , is $[\frac{x_R+x_L}{2} + \frac{c_R}{2}, 1]$ if $c_R \leq x_R - x_L$ and is empty otherwise. The entire set of voters is $\mathcal{V}_L \cup \mathcal{V}_R$. Thus, under partisan voting, abstention comes from centrist citizens.¹⁰ Figure 1 illustrates how different voting costs affect which citizens will turn out.

¹⁰This is not essential for our key points, as illustrated in the Appendix by our extension with affinity voting, where abstention can come from citizens who are centrist or extreme.

Figure 1: Equilibrium voters & abstainers



Note: Figure 1 illustrates which citizens turnout and, if so, who they vote for at two different realizations of group G_L voting cost, where $c_L < c'_L < x_R - x_L$. In 1(a), the citizens who abstain are in $(\frac{1}{2}[x_L^* + x_R^* - c_L], \frac{1}{2}[x_L^* + x_R^* + c_R])$. In 1(b), where G_L 's realized voting cost is $c'_L > c_L$, the abstaining set expands leftward to $(\frac{1}{2}[x_L^* + x_R^* - c'_L], \frac{1}{2}[x_L^* + x_R^* + c_R])$.

Having characterized who votes, we can then easily characterize turnout and vote shares. First, letting $\mu(\mathcal{V}_j)$ denote the Lebesgue measure¹¹ of \mathcal{V}_j for $j \in \{L, R\}$ and suppressing dependence on (x_L, x_R, c_L, c_R) , total turnout is

$$\tau \equiv \alpha \mu(\mathcal{V}_L) + (1 - \alpha) \mu(\mathcal{V}_R).$$

Next, the share of citizens who vote for L is $\tau_L \equiv \alpha \mu(\mathcal{V}_L)$ and the share voting for R is $\tau_R \equiv (1 - \alpha) \mu(\mathcal{V}_R)$. Finally, party L 's vote share is $\frac{\tau_L}{\tau}$ and party R 's vote share is $\frac{\tau_R}{\tau}$.

It is straightforward that party L wins if and only if $\tau_L \geq \tau_R$.¹² Next, we unpack this observation and characterize electoral outcomes in terms of c_L , the voting cost for citizens in G_L .

Equivalently, Party L wins if and only if c_L is low enough, which we will characterize precisely. This follows from a few observations. First, party R must have positive turnout in equilibrium, i.e., $\tau_R^* > 0$, because otherwise R would have a profitable deviation to avoid losing for sure. Thus,

¹¹For our purposes, it suffices to know that $\mu(\emptyset) = 0$ and $\mu([a, b]) = b - a$.

¹²Recall that L wins if there is a tie.

for L to win, c_L must be sufficiently small since $c_L > x_R - x_L$ implies no turnout in G_L . Using that observation, we can be more precise, as party L wins only if

$$\tau_R = \alpha \left| \left(\frac{x_R + x_L}{2} - \frac{c_L}{2} \right) - (-1) \right| \geq (1 - \alpha) \left| 1 - \left(\frac{x_R + x_L}{2} + \frac{c_R}{2} \right) \right| = \tau_L, \quad (3)$$

where the left side of the inequality is L 's vote share and the right side is R 's vote share. Rearranging (3) in terms of c_L , the voting costs for citizens in G_L , we get

$$c_L \leq \frac{1}{\alpha} \left[x_R + x_L + (1 - \alpha) c_R + 2(2\alpha - 1) \right] \equiv \hat{c}. \quad (4)$$

Condition (4) highlights that group-specific costs (c_L and c_R) and relative group size (α) can play a role in deciding the election winner, along with both parties' chosen platforms.

To fully characterize when L wins as a function of c_L , we must account for two additional considerations: (i) $\hat{c} < x_R - x_L$ is possible and (ii) L loses if $c_L < x_R - x_L$. After doing so, we have that L wins if and only if

$$c_L \leq \min\{\hat{c}, x_R - x_L\} \equiv \check{c},$$

and otherwise R wins. Since c_L is a random variable that follows the distribution F , the probability that L wins is $F(\check{c})$ and the probability that R wins is $1 - F(\check{c})$.

Party Behavior: Platform Choice

We now turn to party-level behavior — choosing platforms. First, we characterize equilibrium policy platforms in partisan voting equilibria. After that, we characterize conditions under which partisan voting equilibria exist and are unique. Then, we study how voting costs and group size affect platform location.

When choosing its platform, each party balances a few competing incentives. A party wins if their vote share is larger than the other party's vote share. On one hand, choosing a more moderate policy attracts more voters from the party's own side and turns off voters from other side. This makes winning more likely. On the other hand, such policy is farther from the party's ideal point,

making winning less beneficial.

More precisely, L 's expected utility from a platform pair (x_L, x_R) is

$$\mathbb{E}_{c_L}[u_L(x_L; x_R)] = -|x_L - \hat{x}_L| \cdot Pr(L | x_L, x_R) - |x_R - \hat{x}_L| \cdot (1 - Pr(L | x_L, x_R)) \quad (5)$$

$$= -(1 + x_R) + (x_R - x_L) \cdot F(\check{c}), \quad (6)$$

and R 's expected utility is

$$\mathbb{E}_{c_L}[u_R(x_R; x_L)] = -(1 - x_R) + (x_L - x_R) \cdot F(\check{c}). \quad (7)$$

From these expected utilities, we can see that for any policy pair $x_R \geq x_L$, each party is better off from winning the election. Moreover, the benefit of winning is increasing in the difference between the two platforms.

First, we characterize several equilibrium features, including platforms. Proposition 1 takes advantage of the requirement in PVE that each must have positive expected vote share, which implies that $\check{c} = \hat{c}$ in equilibrium.

Proposition 1. *In any partisan voting equilibrium:*

(i) *party platforms are*

$$x_L^* = (1 - 2\alpha) + \frac{1}{2}[\alpha\check{c} - (1 - \alpha)c_R] - \frac{\alpha}{4f(\check{c})} \quad (8)$$

$$x_R^* = (1 - 2\alpha) + \frac{1}{2}[\alpha\check{c} - (1 - \alpha)c_R] + \frac{\alpha}{4f(\check{c})}; \text{ and} \quad (9)$$

(ii) *the parties win with equal probability.*¹³

Equilibrium platforms are a combination of three distinct forces. The first two are common between both platforms. First, $(1 - 2\alpha)$ is a direct effect of group size alone: enlarging a voting

¹³Due to factors outside the scope of this model, we do not expect all real world elections to be 50/50. However, this stark result allows us to very clearly illustrate our main results about endogenous response to changing voting costs (see Propositions 3 and 4).

group shifts platforms further towards that group's aligned party. Second, $\frac{1}{2}[\alpha\tilde{c} - (1 - \alpha)c_R]$ is a direct effect of voting costs and group size, in tandem. When platforms are chosen, there is a 50/50 chance that the realized citizen-wide average voting cost is below this value.¹⁴ Later on, we go into more detail about voting costs. The third force, $\frac{\alpha}{4f(\tilde{c})}$, differs between the parties and therefore drives equilibrium platform divergence. The key component is $f(\tilde{c})$, which reflects party-level uncertainty about the left group's voting cost on the electoral margin, i.e., around \tilde{c} . Group size again plays a supporting role, since it affects the magnitude of this force and, in turn, scales platform divergence.

In addition to characterizing party platforms, Proposition 1 establishes that each party wins half the time in equilibrium. We explore this in more detail later, but want to emphasize that this *does not* imply that both parties are equally well off. Since parties are policy motivated, their equilibrium expected payoff is equal only if the equilibrium platforms are symmetric around 0. If platforms are not symmetric around 0, then $\frac{x_L^* + x_R^*}{2}$ is closer to one party's ideal point, so that party has a higher expected payoff.

Thus far, we have characterized what happens in a PVE if it exists. For partisan voting to be consistent with our equilibrium voting calculus, however, centrist citizens must not prefer to vote for the party on the other side. That is, citizens slightly left of 0 must not prefer to turn out and vote for R 's candidate, and vice versa. Next, Proposition 2 provides sufficient conditions on α, \tilde{c} , and c_R for this property to hold and, moreover, there to exist a unique partisan voting equilibrium.¹⁵ To streamline and sharpen the rest of the analysis, we henceforth maintain these conditions. Accordingly, we hereafter drop the qualifiers and simply say *equilibrium*.

Proposition 2. *There is a unique partisan voting equilibrium if (i) G_L 's share of the electorate (α) is not too large, (ii) the median voting cost for G_L voters (\tilde{c}) is intermediate, and (iii) the voting cost for G_R voters (c_R) is intermediate.*

¹⁴More precisely, this term is the median of the distribution of average voting cost over all citizens, i.e., $G_L \cup G_R$. This follows from \tilde{c} being the median of F , the distribution of G_L voting costs.

¹⁵See the appendix for explicit definitions of the cost cutpoints.

Comparative Statics on Platforms

Effect of Costs. Next, we study how equilibrium platforms depend on voting costs. We consider targeted cost changes, where costs only change for only one group, as well as untargeted cost changes affecting both groups. These two types of cost changes have different interpretations. A targeted policy aimed specifically at urban voters has a different impact than a universal policy that hits the entire electorate. We parameterize the untargeted change by adding ε to both costs.¹⁶

Proposition 3 follows from Proposition 1 and illustrates how equilibrium platforms depend on voting costs. First, increasing the median cost for one voting group shifts policies away from that group. Second, an untargeted increase in voting costs shifts platforms away from the larger group and towards the smaller group.

Proposition 3. *In equilibrium, each party's platform:*

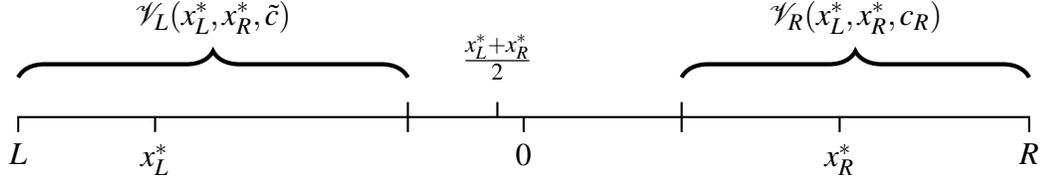
1. *increases by $\frac{\alpha}{2}$ as \tilde{c} increases,*
2. *decreases by $\frac{1-\alpha}{2}$ as c_R increases, and*
3. *changes by $\alpha - \frac{1}{2}$ if all voting costs increase equally.*

To see why a cost increase for one group shifts platforms towards the opposing party, consider a pair of equilibrium platforms x_L^* and x_R^* with associated costs \tilde{c} and c_R . To provide intuition, we will describe the logic in steps even though parties of course act simultaneously. To see the direct effect of voting costs, let the median left-group voting cost increase from \tilde{c} to \tilde{c}' while holding the platforms constant. Right-group turnout does not change but the median of left-group turnout decreases, so R would win more than half the time. This improvement in R 's electoral prospects alters each party's electoral calculus: R is emboldened to moderate less towards L , whereas L is pushed to moderate more towards R . Thus, the equilibrium effect on platforms is to shift both rightward, which alters turnout in turn.

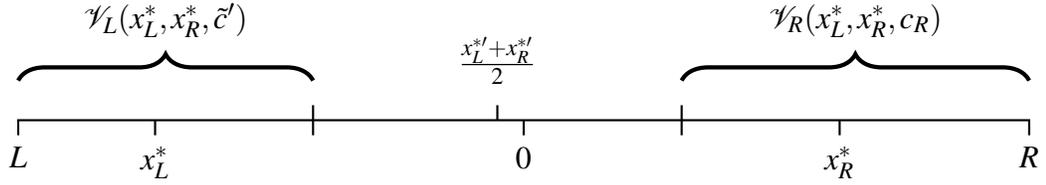
¹⁶We consider a change in group G_L voting costs, c_L , to be a shift of the cost distribution. If the original density was $f(c)$ with support $[\underline{c}, \bar{c}]$, the support of shifted density $f'(c)$ would instead be $[\underline{c} + \varepsilon, \bar{c} + \varepsilon]$. This would then give the new median as $\tilde{c} + \varepsilon \equiv \tilde{c}'$. Note that $f(\tilde{c}) = f'(\tilde{c}')$. We abuse notation and refer to this change as $\partial \tilde{c}$.

Figure 2: Effects of changing \tilde{c} , the median voting cost for G_L

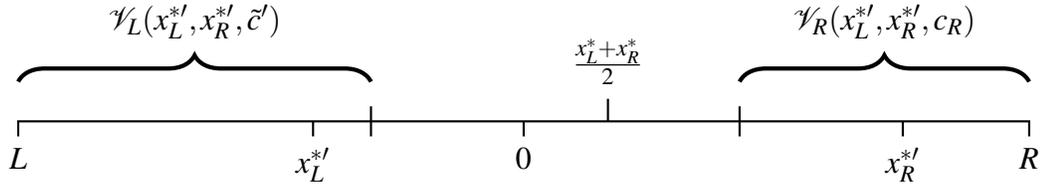
(a) Equilibrium behavior given \tilde{c} & c_R :



(b) Direct effect of $\uparrow \tilde{c}$ to \tilde{c}' on voting behavior:



(c) Equilibrium effects of $\uparrow \tilde{c}$ to \tilde{c}' on platforms and voting behavior:



Note: Figure 2 illustrates how increasing \tilde{c} affects equilibrium platforms and voting behavior. Figure 2(a) depicts a baseline case with voting cost c_R for G_R and median cost \tilde{c} for G_L . The effects of increasing \tilde{c} to \tilde{c}' are depicted in Figure 2(b) and 2(c). First, given the platforms in 2(a), Figure 2(b) illustrates the direct effect on voting behavior: less turnout in G_L . Second, (c) illustrates the overall effects as platforms and voting behavior adjust in equilibrium.

Proposition 3 reveals that group size plays a role in the effects of voting costs, whether targeted or untargeted. As one group grows, both parties react more to a cost change. For example, if the left group is larger, then changing its voting cost will shift each party's platform further than if the change were targeted at the right group. Similarly, as the left group grows, i.e., α increases, an untargeted cost increase shifts both platforms leftward. Broadly, our results find that voting costs and group size are complementary in affecting platform location.

Effect of Group Size. Next, we study how group size affects equilibrium platforms. Proposition 4 follows easily from Proposition 1.

Proposition 4. *In equilibrium,*

1. $\frac{\partial x_L^*}{\partial \alpha} = \frac{\tilde{c}+c_R}{2} - 2 - \frac{1}{4f(\tilde{c})} < 0$, and
2. $\frac{\partial x_R^*}{\partial \alpha} = \frac{\tilde{c}+c_R}{2} - 2 + \frac{1}{4f(\tilde{c})}$.

Notice that changing α has a total effect on equilibrium platforms that combines two effects. First, and common to both platforms, is the *direct effect* of group size, $\frac{\tilde{c}+c_R}{2} - 2$. Since $\frac{\tilde{c}+c_R}{2} < 2$, this effect shifts both equilibrium platforms leftward as α increases, and vice versa. When the left group (G_L) grows, there is a larger proportion of citizens willing to vote for a relatively extreme left platform. And conversely, there will be a smaller share of right-leaning citizens are willing to vote for a relatively extreme right platform. Thus, both platforms shift left. Moreover, this effect is magnified by larger voting costs, which again highlights the complementarity between group size and group costs.

The second effect of α , felt by both groups but in opposite directions, is the *uncertainty effect* of group size. This effect shifts L 's platform leftward by $\frac{1}{4f(\tilde{c})}$ and shifts R 's platform rightward by that same distance. All of the uncertainty over voting costs is about G_L 's costs. Therefore as G_L becomes a larger share of the electorate, there is more uncertainty over costs and, in turn, electoral outcomes.¹⁷

In addition to studying how group size affects the party platforms individually, we can also study how it affects equilibrium *divergence* ($x_R^* - x_L^*$). By doing so, we highlight the difference between the direct and uncertainty effects of α . Furthermore, we isolate the uncertainty effect. Defining $\Delta_x^* = x_R^* - x_L^* = \frac{\alpha}{2f(\tilde{c})}$, Proposition 4 yields the following corollary.

Corollary 1. *In equilibrium, platform divergence increases in α , i.e., as G_L 's population share grows.*

Since divergence is the distance between party platforms, the common direct effect of group size on platforms drops out. Thus, divergence arises solely due to electoral uncertainty. As is

¹⁷Divergence increasing in α does not require G_R to have no uncertainty over costs, just lower uncertainty than G_L 's costs. If there was more uncertainty over G_R 's costs, then divergence would decrease in α , but the midpoint shift would remain exactly the same.

common in spatial electoral models, more electoral uncertainty means greater policy divergence. Although group size (α) does not generate divergence, it does affect the *magnitude* of equilibrium divergence by magnifying the uncertainty effect. Increasing α widens the gap between equilibrium platforms even as the midpoint between the two platforms moves left.

Turnout

Next we focus on equilibrium turnout. Let τ_L^* realized equilibrium turnout for G_L and define τ_R^* analogously. For a given realization of c_L , we have:

$$\tau_L^* = \alpha \left[2(1 - \alpha) + \frac{1}{2}(\alpha \tilde{c} - (1 - \alpha)c_R - c_L) \right]. \quad (10)$$

Using (10), Proposition 5 characterizes equilibrium turnout for R and expected equilibrium turnout for L .

Proposition 5. *In equilibrium, L 's expected turnout is*

$$\mathbb{E}[\tau_L^*] = \alpha \left[2(1 - \alpha) + \frac{1}{2}(\alpha \tilde{c} - (1 - \alpha)c_R - \mathbb{E}(c_L)) \right] \quad (11)$$

and R 's turnout is

$$\tau_R^* = \alpha(1 - \alpha) \left[2 - \frac{1}{2}(\tilde{c} + c_R) \right]. \quad (12)$$

Note that if G_L 's expected voting cost equals G_L 's median voting cost, i.e., $\mathbb{E}(c_L) = \tilde{c}$, then L 's expected equilibrium turnout is equal to R 's equilibrium turnout, i.e., $\tau_R^* = \mathbb{E}[\tau_L^*]$. This equivalence always holds for symmetric, single peaked distributions. Otherwise, the parties have different expected equilibrium turnout even though they share the same probability of winning.

Zeroing in on G_R 's turnout, notice that turnout decreases whenever voting costs increase. This is due to changes in equilibrium platforms. If platforms remained constant, changing \tilde{c} would

not affect G_R 's turnout. However, because a higher voting cost for G_L emboldens R to adopt a more extreme platform, turnout decreases for both R and L .

In fact, if voting costs remained constant, than adopting a more moderate policy would increase turnout. But, although a more moderate platform mitigates some of the effects of higher voting costs on turnout, in equilibrium the group of voters still shrinks.

Vote Shares & “Representativeness.” In addition to turnout, many scholars are interested in understanding the relationship between voting costs and two measures that are each a straightforward function of turnout: (i) vote shares and (ii) the *representativeness* of voters. First, *vote shares*, which are often viewed as reflecting electoral competitiveness. Second, the *representativeness* of turnout — the similarity between the composition of voters versus the composition of eligible voters — which is seen as an indicator of how policy will align with public interests. A widespread intuition is that increasing voting costs will decrease competitiveness and decrease policy alignment.

To discuss how our analysis thus far can shed light on these relationships, we fix ideas by focusing on average vote shares and average representativeness. First, shifting the voting cost distribution does not necessarily change either measure, due to equilibrium responses in party platforms. To illustrate, suppose $\mathbb{E}[c_L] = \tilde{c}$ and let F denote the initial distribution of c_L . Then, expected vote share is .5 for both parties and, moreover, uniformly shifting the cost distribution F rightward has no effect on average vote share or average representativeness. Instead, to see changes in those quantities, any shift in F must change $|\mathbb{E}[c_L] - \tilde{c}|$. This observation is a stark illustration of a more general point: shifting the cost distribution is not sufficient to observe changes in these measures, as it is important to understand *how* that distribution changes.

Second, changes in average vote share or representativeness can occur without any change in expected policy payoffs. To illustrate, consider the initial cost distribution F , which has $\mathbb{E}_F[c_L] = \tilde{c}$, and two different shifts in the cost distribution, F' and F'' , that both increase the median cost to \tilde{c}' but differ in that only F' preserves the equality of expected cost and median cost. Formally, $\text{median}(F') = \text{median}(F'') = \tilde{c}' > \tilde{c}$, $\mathbb{E}_{F'}[c_L] = \tilde{c}'$, and $\mathbb{E}_{F''}[c_L] \neq \tilde{c}'$. Then, expected policy payoff changes in the same way after either shift, but average vote share and average representativeness

change only after the shift to F'' . Thus, changes in either of those measures, or the lack thereof, are not necessarily informative about welfare effects (as measured in our model by policy payoffs).

Party Welfare

We now study how each party's equilibrium payoff changes with voting costs and group size. This section will again highlight how parties are affected by changing voting costs even when their probability of winning does not change. By doing so, we shed light on how strongly each party would want to change voting costs under different conditions.

First, L 's equilibrium value is

$$U_L^* = -(1 + x_R^*) + \frac{1}{2}[x_R^* - x_L^*] \quad (13)$$

$$= \frac{1}{2}[(1 - \alpha)c_R - \alpha\tilde{c}] - 2(1 - \alpha). \quad (14)$$

For R , we have

$$U_R^* = -(1 - x_R^*) + \frac{1}{2}[x_L^* - x_R^*] \quad (15)$$

$$= \frac{1}{2}[\alpha\tilde{c} - (1 - \alpha)c_R] - 2\alpha. \quad (16)$$

Group size affects both parties directly and indirectly, by weighting on the effects of voting costs. Thus, group size influences how salient costs are for party welfare, as it did for equilibrium platforms. This is highlighted in the proposition below.

Proposition 6. *In equilibrium, we have the following effects on party welfare:*

$$\frac{\partial U_L^*}{\partial \tilde{c}} = -\frac{\partial U_R^*}{\partial \tilde{c}} = -\frac{\alpha}{2}, \quad (17)$$

$$\frac{\partial U_L^*}{\partial c_R} = -\frac{\partial U_R^*}{\partial c_R} = \frac{1 - \alpha}{2}, \text{ and} \quad (18)$$

$$\frac{\partial U_L^*}{\partial \alpha} = -\frac{\partial U_R^*}{\partial \alpha} = 2 - \frac{1}{2}(c_R + \tilde{c}). \quad (19)$$

It is not surprising that each party benefits when their side's voting cost decreases or the other side's voting cost increases. Yet, Proposition 6 also shows that the *magnitude* of these gains or losses will depend on the size of the affected group. For example, if G_L 's median voting cost increases, then R 's equilibrium welfare increases by a larger amount as G_L grows, i.e., α increases.

Finally, we can study the difference in equilibrium party welfare, which we denote Δ_u^* . This difference simplifies to equal the sum of the equilibrium platforms:

$$\Delta_u^* \equiv U_R^* - U_L^* = x_R^* + x_L^*. \quad (20)$$

This difference in party welfare is positive when the right platform is more extreme and negative when the left platform is more extreme. It provides another illustration that parties are better off when they are able to campaign on extreme rather than moderate platforms. From Proposition 6, we have the effects of voting costs and group size on the party welfare difference.

Corollary 2. *In equilibrium, we have the following effects on the difference in party welfare, Δ_u^* :*

$$\frac{\partial \Delta_u^*}{\partial \alpha} = \tilde{c} + c_R - 4 < 0, \quad (21)$$

$$\frac{\partial \Delta_u^*}{\partial \tilde{c}} = \alpha > 0, \text{ and} \quad (22)$$

$$\frac{\partial \Delta_u^*}{\partial c_R} = -(1 - \alpha) < 0. \quad (23)$$

The party welfare difference shrinks as G_L grows (α increases). Yet, the amount it shrinks will depend on the voting costs of both groups. As expected, increasing the median voting cost for G_L will widen the welfare difference, whereas increasing the voting cost for G_R will shrink it. The magnitude of these effects depends on group size. If G_L is small (low α), then increasing \tilde{c} will not widen the welfare difference by that much. Similarly, if G_R is small (high α), then increasing c_R will not shrink the difference by that much.

Empirical Discussion

In this section, we explore how the results from our model can be used to expand upon existing studies and help guide future empirical work. We use concrete examples for both voting cost increases and decreases and illustrate how these papers can be extended to look at more outcomes than just turnout. To be clear, this is not a criticism of these papers; on the contrary, we believe these papers provide compelling evidence for turnout effects. We want to encourage researchers to try and link possible policy effects with their documented turnout effects.

Same Day Registration. Grumbach and Hill (2022) looks at the effect of Same Day Registration (SDR) policies and their effect on turnout. They have data on SDR policies by state over time. This allows them to show that SDR has a differentially positive effect on turnout for younger voters. As the paper notes, younger voters tend to lean Democratic. Our model predicts that policies or platforms would be more left leaning after SDR implementation. A very similar research design could be used to look at how platforms or candidate ideology changed given the introduction of Same Day Registration.

Voter ID. Fraga and Miller (2022) uses very detailed microdata from the 2014 and 2016 Texas elections to show that Black and Latino voters were more affected than other groups by Texas's voter ID law. Our model predicts that policy effect sizes will be larger when the group affected is larger. Combined with the results from Fraga and Miller (2022), we should expect to see bigger effects on policy or candidate ideology in districts with a larger proportion of Black and Latino voters.

Pre-registration. In the US, citizens must register before they can vote. *Pre-registration* allows young US citizens to register at a variety of convenient locations *before* they are old enough to be eligible to vote. Thus, pre-registration reduces voting costs for young voters without affecting voting costs for older voters. Bertocchi et al. (2020) show that pre-registration decreases the average turnout gap between young and old voters. Through the lens of our analysis, we can make two points about this finding. First, the existence of a gap in average turnout indicates that young voters have

a right-skewed distribution of voting costs, i.e., $\mathbb{E}[c_Y] > \tilde{c}$. Second, the decrease in average turnout gap indicates that pre-registration decreased that skewness, i.e., $|\mathbb{E}[c_Y] - \tilde{c}|$ decreased.

Mail-in Voting. Bonica et al. (2021) finds that all-mail voting (AMV) in Colorado increased turnout by about 8 percentage points. However, they also find that the turnout increase was similar for Republicans and Democrats. In a state where voter registration is evenly split between the two parties (of State 2022), we should not expect this voting cost decrease to affect policy even as turnout increases.

Polling Place Location. Cantoni (2020) shows that people are less likely to vote the farther they live from a polling location. This research design does not neatly map onto our model where everyone of a certain voting group has the same cost. However, we can still use this result about distance to polling location and turnout to think about policy effects. For example, eliminating polling locations in Democratic leaning precincts, thus increasing average distance to polling locations, would shift policy rightward.

Backlash. A prominent explanation for the unclear relationship between restrictive voting laws and turnout is *backlash*: new laws anger members of targeted groups, increasing their motivation to turn out (Valentino and Neuner 2017). We do not model this psychological channel, in order to focus on fleshing out the interaction between platforms and turnout. Our results reveal a countervailing effect on aggregate turnout that is similar to backlash. Thus, it is important to empirically disentangle backlash from platform shifts. To do so, it is especially useful to have variation in individual-level turnout and perceived voting costs. For example, Biggers and Smith (2020) use individual-level data of (i) targeting during an aborted purge of Florida’s voter rolls and (ii) turnout.

Extreme Candidates. Our results rely on the fact that parties can choose any available platform location. However, parties may nominate extreme candidates who will be unwilling to choose moderate policies (Hall 2019; Nielson and Visalvanich 2017). consider a situation where L could only choose a platform more extreme than some cutoff platform κ_L . Further, assume that κ_L is more extreme than the optimal platform L would choose without such a constraint (that is, $\kappa_L \leq x_L^*$). In the

absence of possible moderation from L , an increase in c_L would push R to move her platform to an even more extreme location than she would otherwise. If parties nominated extreme candidates who would not consider full moderation, then the effects of voting cost increases on campaign platforms may be exacerbated.

Conclusion

In this paper, we use a formal model to show that the equilibrium effects of increasing voting costs are not solely the province of the voters. Rather, they can influence both voters *and* politicians, since turnout and platforms affect each other. Our analysis sheds light on why looking solely at turnout may miss crucial effects of new restrictive voting laws. Although these laws may have a variety of effects, we highlight how and why policy shifts are a potential downside.

We particularly urge empiricists to look at whether platforms and policy change after restrictive or expansive voting laws have been introduced. Since we know that restrictive voting laws do reduce turnout, and that changes in turnout affect policy, there should also be a policy effect linkage between voting laws and policy. Are more conservative policies implemented after restrictive voting laws targeting urban voters passed? Do candidates change their rhetoric to try and appeal to broader or narrower sets of voters when voting costs change? Understanding the full impacts of voting laws requires a wider lens than has been used to date.

While our model does not make direct predictions about when governments will enact new restrictive voting laws, we can still use the logic of the model to explore this issue. We show that increasing voting costs for the opposing group is more beneficial if the size of that voting group increases. For example, it is no surprise that as Texas becomes more purple, the Republican government has instituted a sweeping restrictive voting law.

Similarly, increasing voting access of supporters by lowering costs will also have the most benefit when the opposing group is relatively small. This is true even when lowering costs is non-targeted and affects citizens of all voting blocs. From this perspective, While SB 1202 in Connecticut expands access to voting in numerous ways for the whole state, we expect this to benefit Democrats

because they are a larger share of the Connecticut electorate.

We focus on voting costs, but those are only one class of restrictive voting laws. While the models would be different, assessing the policy impacts and not focusing exclusively on turnout impacts of gerrymandering, voter purges, and other forms of restrictive voting laws should be a high priority.

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Appendix

Omitted Proofs

Lemma 1. *In equilibrium, $x_L^* < x_R^*$.*

Proof. Suppose not. If $x_L^* < 1 - c_R$, then there is sufficiently small $\varepsilon > 0$ such that R has a profitable deviation to $x_R = x_L^* + c_R + \varepsilon$. Otherwise, L has a strictly profitable deviation to $x_R^* - c_R$ since it will win for sure and get more favorable policy. Since we have a contradiction in both cases, the desired result follows. ■

Lemma 2. *In equilibrium, $x_R^* - x_L^* \geq c_R$.*

Proof. Suppose not. Then, R loses the election with probability one. There are two cases. First, if $x_L^* < 1 - c_R$, then there is sufficiently small $\varepsilon > 0$ such that R has a profitable deviation to $x_L^* + c_R + \varepsilon$. Otherwise, L has a profitable deviation to $x_L^* - \varepsilon$ for small enough $\varepsilon > 0$, since it will still win for sure and also get more favorable policy. ■

Corollary 3. *Given x_R , Party L 's best response is weakly less than $x_R - c_R$. Given x_L , Party R 's best response is weakly greater than $x_L + c_R$.*

Lemma 3. *In equilibrium, if there is partisan voting over platforms satisfying $x_R - x_L > c_R$, then Party L wins if and only if*

$$c_L \leq \frac{1}{\alpha} \left(x_R + x_L + (1 - \alpha)c_R + 2(2\alpha - 1) \right) \equiv \hat{c}(x_L, x_R). \quad (24)$$

Proof. Fix (x_L, x_R) such that equilibrium citizen behavior produces partisan voting. Then, L wins if

and only if:

$$(1 - \alpha) \left| 1 - \left(\frac{x_R + x_L}{2} + \frac{c_R}{2} \right) \right| \leq \alpha \left| -1 - \left(\frac{x_R + x_L}{2} - \frac{c_L}{2} \right) \right| \quad (25)$$

$$c_L \leq \frac{1}{\alpha} \left(x_R + x_L + (1 - \alpha) c_R + 2(2\alpha - 1) \right). \quad (26)$$

Thus, (26) implies that L wins if and only if $c_L \leq \hat{c}(x_L, x_R)$, as desired. \blacksquare

Proof of Proposition 1. Let (x_L^*, x_R^*) be platforms in a partisan voting equilibrium. Since turnout must be positive for both parties, we know $x_R^* - x_L^* > c_R$. Thus, Lemma 3 implies that party L wins if and only if $c_L \leq \hat{c}(x_L^*, x_R^*)$. To streamline notation, let $\hat{c}^* = \hat{c}(x_L^*, x_R^*)$. Therefore the probability that L wins, i.e., (4) holds, is

$$Pr(L | x_L^*, x_R^*) = F(\hat{c}^*). \quad (27)$$

Then, the expected utility for L is

$$\mathbb{E}[u_L(x_L^*; x_R^*)] = -|x_L^* - \hat{x}_L| \cdot Pr(L | x_L^*, x_R^*) - |x_R^* - \hat{x}_L| \cdot (1 - Pr(L | x_L^*, x_R^*)) \quad (28)$$

$$= -(1 + x_R^*) + (x_R^* - x_L^*) \cdot F(\hat{c}^*), \quad (29)$$

and the expected utility for R is

$$\mathbb{E}[u_R(x_R^*; x_L^*)] = -(1 - x_R^*) + (x_L^* - x_R^*) \cdot F(\hat{c}^*). \quad (30)$$

The derivative of (29) with respect to x_L is

$$\frac{\partial \mathbb{E}[u_L(x_L^*; x_R^*)]}{\partial x_L} = -F(\hat{c}^*) + \frac{x_R^* - x_L^*}{\alpha} f(\hat{c}^*). \quad (31)$$

A similar derivation yields

$$\frac{\partial \mathbb{E}[u_R(x_R^*; x_L^*)]}{\partial x_R} = [1 - F(\hat{c}^*)] - \frac{x_R^* - x_L^*}{\alpha} f(\hat{c}^*). \quad (32)$$

Then, the FOCs require

$$\frac{x_R^* - x_L^*}{\alpha} \frac{f(\hat{c}^*)}{F(\hat{c}^*)} = 1, \text{ and} \quad (33)$$

$$\frac{x_R^* - x_L^*}{\alpha} \frac{f(\hat{c}^*)}{1 - F(\hat{c}^*)} = 1. \quad (34)$$

Log concavity of f implies that (i) the LHS of (33) is strictly decreasing in x_L and (ii) the LHS of (34) is strictly increasing in x_R . Thus, L and R always have unique best responses, and their respective best response functions are characterized by (33) and (34). Combining (33) and (34), we know that the following must hold in equilibrium:

$$F(\hat{c}^*) = \frac{1}{2}. \quad (35)$$

Thus, (i) elections are 50/50 in equilibrium, and (ii) we must have $\hat{c}^* = \tilde{c}$, which is the median of F . Since $\hat{c}^* = \tilde{c}$, we know that the midpoint of the equilibrium platforms must satisfy

$$\frac{x_R^* + x_L^*}{2} = \frac{1}{2}[\alpha \tilde{c} - (1 - \alpha)c_R] + (1 - 2\alpha). \quad (36)$$

Additionally, since $F(\hat{c}^*) = \frac{1}{2}$ implies $\hat{c}^* = \tilde{c}$, we know the divergence between equilibrium platforms must be

$$x_R^* - x_L^* = \frac{\alpha}{2f(\tilde{c})}. \quad (37)$$

Then, combining (36) and (37) pins down equilibrium platforms:

$$x_L^* = \frac{1}{2}[\alpha\tilde{c} - (1 - \alpha)c_R] + (1 - 2\alpha) - \frac{\alpha}{4f(\tilde{c})} \quad (38)$$

$$x_R^* = \frac{1}{2}[\alpha\tilde{c} - (1 - \alpha)c_R] + (1 - 2\alpha) + \frac{\alpha}{4f(\tilde{c})}. \quad (39)$$

■

To rigorously state Proposition 2, first define:

$$\begin{aligned} \tilde{c}_\ell &= \frac{1 - \alpha}{4f(\tilde{c})} - \frac{2}{\alpha}(1 - 2\alpha) - \frac{3 - \alpha}{2\alpha}\underline{c} \\ \tilde{c}'_\ell &= \frac{3 - 24f(\tilde{c}) + \alpha\left(6 + 56f(\tilde{c}) - \alpha(13 + 16f(\tilde{c}) - 8\alpha)\right)}{4f(\tilde{c})(3 + \alpha)} \\ \tilde{c}''_\ell &= \frac{1 + \alpha}{4f(\tilde{c})} \\ \tilde{c}_u &= \frac{1 - \alpha}{4f(\tilde{c})} + \frac{2}{\alpha}(1 - 2\alpha) \\ \tilde{c}'_u &= \frac{\alpha}{2f(\tilde{c})} \\ \underline{c}_R &= \frac{1}{3 - \alpha}\left(\alpha\tilde{c} + 2(1 - 2\alpha)\frac{\alpha}{2f(\tilde{c})}\right) \\ \underline{c}'_R &= \frac{2 - \alpha(4 - \tilde{c})}{1 - \alpha} \\ \bar{c}_R &= \frac{\alpha}{4f(\tilde{c})} \\ \bar{c}'_R &= \frac{2 + \underline{c} - \alpha(4 - \tilde{c})}{1 - \alpha} \\ \bar{c}''_R &= \frac{4f(\tilde{c})(2 + \tilde{c} - \alpha(4 - \tilde{c})) - (1 + \alpha)}{4f(\tilde{c})(1 - \alpha)}. \end{aligned}$$

Then, let

$$\tilde{c}_\ell^\dagger = \max\{\tilde{c}_\ell, \tilde{c}'_\ell, \tilde{c}''_\ell\} \quad (40)$$

$$\tilde{c}_u^\dagger = \max\{\tilde{c}_u, \tilde{c}'_u, \tilde{c}''_u\} \quad (41)$$

$$\underline{c}_R^\dagger = \max\{\underline{c}_R, \underline{c}'_R\} \quad (42)$$

$$\bar{c}_R^\dagger = \min\{\bar{c}_R, \bar{c}'_R, \bar{c}''_R\}. \quad (43)$$

It can be verified that there exists $\bar{\alpha} > 0$ such that $\alpha < \bar{\alpha}$ implies $\tilde{c}_\ell^\dagger < \tilde{c}_u^\dagger$ and $\underline{c}_R^\dagger < \bar{c}_R^\dagger$.

Proposition 2. *If $\alpha < \bar{\alpha}$, $\tilde{c} \in (\tilde{c}_\ell^\dagger, \tilde{c}_u^\dagger)$, and $c_R \in (\underline{c}_R^\dagger, \bar{c}_R^\dagger)$, then there is a unique partisan voting equilibrium.*

Proof of Proposition 2. Let (x_L^*, x_R^*) denote a platform pair that solves (33) and (34). Suppose $\alpha < \bar{\alpha}$, $\tilde{c} \in (\tilde{c}_\ell^\dagger, \tilde{c}_u^\dagger)$ and $c_R \in (\underline{c}_R^\dagger, \bar{c}_R^\dagger)$.

We have already shown that the platform pair (x_L^*, x_R^*) is necessary for a PVE. We will show that there exists a PVE under the maintained conditions. Then, uniqueness is immediate.

First, we verify that there is partisan voting in equilibrium. For (x_L^*, x_R^*) , there is partisan voting if and only if (i) $\frac{x_R^* + x_L^*}{2} + \frac{c}{2} > 0$ and (ii) $\frac{x_R^* + x_L^*}{2} - \frac{c_R}{2} < 0$. Simplifying, conditions (i) and (ii) are equivalent to $c_R \in (\underline{c}'_R, \bar{c}'_R)$, which holds.

Second, we check that each party has positive expected vote share in equilibrium. For R , a straightforward derivation shows that $c_R < x_R^* - x_L^*$ if and only if $c_R < \frac{\alpha}{2f(\tilde{c})}$. Since $c_R < \bar{c}_R^\dagger \leq \bar{c}_R < \frac{\alpha}{2f(\bar{c})}$, it follows that R has positive vote share in equilibrium. For L , we have $\check{c} = \hat{c}(x_L^*, x_R^*) = \tilde{c}$ by Lemma 3 and Proposition 2 together. A straightforward derivation shows that $\tilde{c} < \tilde{c}_u^\dagger \leq \tilde{c}'_u$ implies $\hat{c}(x_L^*, x_R^*) < x_R^* - x_L^*$. Together, these observations imply that $\tilde{c} < x_R^* - x_L^*$, so L 's expected vote share is positive in equilibrium.

To finish the proof, we verify that no player has a profitable deviation. Since voters are infinitesimal, we only need to consider deviations by parties.

We start with party R . Earlier results imply that all $x_R < x_L^* + c_R$ are not profitable. There are three types of deviations to check: those that induce partisan voting, those for which some voters in

G_L vote for R , and those for which some voters in G_R vote for L .

First, consider any x_R such that (x_L^*, x_R) induces partisan voting and positive expected turnout for both parties. This cannot be profitable because R 's maximization problem over such x_R is uniquely solved by x_R^* .

Second, consider x_R such that some voters in G_R support L 's candidate. Then, R 's expected payoff is

$$\mathbb{E}[u_R(x_R; x_L^*)] \leq \mathbb{E}[u_R(x_R; x_L^*) \mid \text{partisan votes}] \quad (44)$$

$$< \mathbb{E}[u_R(x_R^*; x_L^*) \mid \text{partisan votes}] = \mathbb{E}[u_R(x_R^*; x_L^*)], \quad (45)$$

where (44) follows because R is weakly more likely to win if only partisan votes are counted in this case, and (45) because x_R^* solves R 's maximization problem given x_L^* and partisan voting, which $c_R \in (\underline{c}'_R, \bar{c}'_R)$ guarantees at (x_L^*, x_R^*) .

Next, consider x_R such that some voters in G_L support R 's candidate. More precisely, these are x_R for which there exists $c^\ddagger > \underline{c}$ such that $c_L < c^\ddagger$ implies that sufficiently centrist voters in G_L vote for R . Importantly, all $c_L \in [\underline{c}, c^\ddagger]$ generate the same electoral margin and thus lead to the same winner. There are two subcases. First, if $c^\ddagger < \hat{c}$, then L wins if and only if $c_L < \hat{c}$. Thus, R 's expected payoff is equivalent to a setting in which only partisan votes count and therefore x_R is strictly worse than x_R^* . Second, if $c^\ddagger \geq \hat{c}$, then R wins for sure. Thus, in this subcase R strictly prefers the rightmost x_R , which is such that $c^\ddagger = \hat{c}(x_L^*, x_R)$. We denote this platform as x_R^\dagger and solve to obtain:

$$x_R^\dagger = \frac{\alpha^2 \left(1 - 2f(\tilde{c})(\tilde{c} + c_R - 4) \right) - 2\alpha f(\tilde{c})(\tilde{c} - 2c_R + 6) + \alpha - 2f(\tilde{c})(c_R - 2)}{4f(\tilde{c})(1 + \alpha)}.$$

A straightforward derivation shows that $c_R < \bar{c}_R^\dagger \leq \bar{c}_R''$ implies that R does not strictly prefer deviating to x_R^\dagger .

Finally, we check for profitable deviations by Party L . First, clearly L has no profitable deviation to any x_L that induces partisan voting and positive expected turnout for both candidates.

Second, L does not want to deviate to any x_L that induces some voters in G_L to vote for R 's candidate, for the same reason as above in the analogous case for R in which some G_R voters support L . Third, note that $c_R > \underline{c}_R$ implies $x_R^* < c_R$, which ensures that L cannot profitably induce voters in G_R to vote for L . Finally, $c_R < \bar{c}_R^\dagger \leq \bar{c}_R$ implies that L does not strictly prefer deviating to $x_L = x_R^* - c_L$, the leftmost platform which would guarantee victory for L .

We have shown that neither party has a profitable deviation, which completes the proof. ■

Proof of Proposition 3. The comparative statics are:

$$\begin{aligned}\frac{\partial x_L^*}{\partial c_R} &= \frac{\partial x_R^*}{\partial c_R} = -\frac{1-\alpha}{2} < 0. \\ \frac{\partial x_L^*}{\partial \tilde{c}} &= \frac{\partial x_R^*}{\partial \tilde{c}} = \frac{\alpha}{2} > 0 \\ \frac{\partial x_L^*}{\partial \varepsilon} &= \frac{\partial x_R^*}{\partial \varepsilon} = \alpha - \frac{1}{2}.\end{aligned}$$

■

Proof of Proposition 4. The comparative statics of equilibrium platforms x_L^* and x_R^* are:

$$\frac{\partial x_L^*}{\partial \alpha} = \frac{\tilde{c} + c_R}{2} - 2 - \frac{1}{4f(\tilde{c})} < 0 \quad (46)$$

$$\frac{\partial x_R^*}{\partial \alpha} = \frac{\tilde{c} + c_R}{2} - 2 + \frac{1}{4f(\tilde{c})}. \quad (47)$$

■

Note that $\frac{\partial x_L^*}{\partial \alpha} < \frac{\partial x_R^*}{\partial \alpha}$ and $|\frac{\partial x_L^*}{\partial \alpha}| > |\frac{\partial x_R^*}{\partial \alpha}|$.

Proof of Corollary 1. Defining $\Delta_x^* = x_R^* - x_L^* = \frac{\alpha}{2f(\tilde{c})}$, we have

$$\frac{\partial \Delta_x^*}{\partial \alpha} = \frac{1}{2f(\tilde{c})} > 0, \quad (48)$$

$$\frac{\partial \Delta_x^*}{\partial \tilde{c}} = -\frac{\alpha f'(\tilde{c})}{2f(\tilde{c})^2}. \quad (49)$$

■

Proof of Proposition 5. For L 's equilibrium turnout given a realization of c_L , we have:

$$\tau_L^* = \alpha \left[2(1 - \alpha) + \frac{1}{2}(\alpha \tilde{c} - (1 - \alpha)c_R - c_L) \right], \quad (50)$$

and thus L 's expected turnout in equilibrium is

$$\mathbb{E}[\tau_L^*] = \alpha \left[2(1 - \alpha) + \frac{1}{2}(\alpha \tilde{c} - (1 - \alpha)c_R - \mathbb{E}(c_L)) \right]. \quad (51)$$

Next, R 's equilibrium turnout is

$$\tau_R^* = \alpha(1 - \alpha) \left[2 - \frac{1}{2}(\tilde{c} + c_R) \right]. \quad (52)$$

■

Proof of Proposition 6. For L , we have

$$U_L^* = -(1 + x_R^*) + \frac{1}{2}[x_R^* - x_L^*] \quad (53)$$

$$= \frac{1}{2}[(1 - \alpha)c_R - \alpha \tilde{c}] - 2(1 - \alpha). \quad (54)$$

For R , we have

$$U_R^* = -(1 - x_R^*) + \frac{1}{2}[x_L^* - x_R^*] \quad (55)$$

$$= \frac{1}{2}[\alpha \tilde{c} - (1 - \alpha)c_R] - 2\alpha. \quad (56)$$

For comparative statics on each party's equilibrium value, we have

$$\frac{\partial U_L^*}{\partial \tilde{c}} = -\frac{\partial U_R^*}{\partial \tilde{c}} = -\frac{\alpha}{2} \quad (57)$$

$$\frac{\partial U_L^*}{\partial c_R} = -\frac{\partial U_R^*}{\partial c_R} = \frac{1-\alpha}{2} \quad (58)$$

$$\frac{\partial U_L^*}{\partial \alpha} = -\frac{\partial U_R^*}{\partial \alpha} = 2 - \frac{1}{2}(c_R + \tilde{c}). \quad (59)$$

■

Affinity Voting

Suppose there are two groups of voters, G_L and G_R , each with associated voting costs λ_L and λ_R . Let $\lambda_R \geq 0$ be fixed and common knowledge, whereas λ_L is a random variable drawn from a log-concave probability distribution F that has support on the interval $[\underline{\lambda}, \bar{\lambda}]$, where $\underline{\lambda} \geq 0$, and associated density function f .

The timing is analogous to the baseline model: parties make binding campaign commitments, then uncertainty over λ_L is realized, then voters vote.

For each voter in G_R , suppose they turn out and vote for candidate R if

$$|\hat{x}_i - x_R| \leq \lambda_R$$

and otherwise they abstain. Analogous for voters in G_L . Thus, voters support a candidate only if she is from their affiliated party.

Analysis

The condition for L to win election with platforms (x_L, x_R) is

$$(1 - \alpha)(1 - x_R + \lambda_R) \leq \alpha(1 + x_L + \lambda_L). \quad (60)$$

Thus, L wins the election if and only if

$$\lambda_L \geq \frac{1-\alpha}{\alpha}(1+\lambda_R) - 1 - \left[\frac{1-\alpha}{\alpha}x_R + x_L\right] \equiv \hat{\lambda}. \quad (61)$$

It follows that $Pr(L \text{ wins} \mid x_L, x_R) = 1 - F(\hat{\lambda})$.

Then, given a platform pair (x_L, x_R) , we can express L expected payoff as

$$U_L(x_L, x_R) = -(1 - F(\hat{\lambda}))(1 + x_L) - F(\hat{\lambda})(1 + x_R) \quad (62)$$

$$= -(1 + x_L) - F(\hat{\lambda})(x_R - x_L), \quad (63)$$

and R 's expected payoff as

$$U_R(x_L, x_R) = -(1 - F(\hat{\lambda}))(1 - x_L) - F(\hat{\lambda})(1 - x_R) \quad (64)$$

$$= -(1 + x_L) + F(\hat{\lambda})(x_R - x_L). \quad (65)$$

The FOCs are:

$$0 = \frac{\partial U_L(x_L, x_R)}{\partial x_L} = -(1 - F(\hat{\lambda})) + f(\hat{\lambda})(x_R - x_L) \quad (66)$$

$$0 = \frac{\partial U_R(x_L, x_R)}{\partial x_R} = F(\hat{\lambda}) - \frac{1-\alpha}{\alpha}f(\hat{\lambda})(x_R - x_L). \quad (67)$$

Log-concavity of F implies that each FOC has a unique solution.

To solve for equilibrium platforms x_L^* and x_R^* , first let $\hat{\lambda}_\alpha$ denote the unique solution to $\hat{\lambda} = H(1 - \alpha)$, where $H = F^{-1}$ denotes the inverse cdf. The FOCs together imply $F(\hat{\lambda}_\alpha) = 1 - \alpha$ and, using that observation, they also imply $x_L^* - x_R^* = \frac{f(\hat{\lambda}_\alpha)}{\alpha}$. From there, a straightforward derivation yields:

$$x_L^* = (1 - \alpha)\lambda_R - \alpha\hat{\lambda}_\alpha + (1 - 2\alpha) - \frac{1-\alpha}{\alpha}f(\hat{\lambda}_\alpha) \quad (68)$$

$$x_R^* = (1 - \alpha)\lambda_R - \alpha\hat{\lambda}_\alpha + (1 - 2\alpha) + f(\hat{\lambda}_\alpha). \quad (69)$$

Next, we characterize each party's equilibrium value, which simplify to the following:

$$U_R^* = (1 - \alpha)\lambda_R - \alpha(2 + \hat{\lambda}_\alpha) \quad (70)$$

$$U_L^* = \alpha\hat{\lambda}_\alpha - (1 - \alpha)(2 + \lambda_R). \quad (71)$$

Then, we have the following comparative statics:

$$\frac{\partial U_R^*}{\partial \lambda_R} = -\frac{\partial U_L^*}{\partial \lambda_R} = (1 - \alpha) > 0 \quad (72)$$

$$\frac{\partial U_R^*}{\partial \hat{\lambda}_\alpha} = -\frac{\partial U_L^*}{\partial \hat{\lambda}_\alpha} = -\alpha < 0 \quad (73)$$

$$\frac{\partial U_R^*}{\partial \alpha} = -\frac{\partial U_L^*}{\partial \alpha} = (2 + \lambda_R + \hat{\lambda}_\alpha) - \alpha h(1 - \alpha), \quad (74)$$

where $H' = h$. From there, we can characterize

$$\Delta^* = U_R^* - U_L^* = 2 \left((1 - 2\alpha) + (1 - \alpha)\lambda_R - \alpha\hat{\lambda}_\alpha \right)$$

and comparative statics are immediate.

For equilibrium turnout, we have

$$\tau_R^* = (1 - \alpha) \left(\alpha(2 + \lambda_R + \hat{\lambda}_\alpha) - f(\hat{\lambda}_\alpha) \right) \quad (75)$$

$$\tau_L^* = \alpha \left(2(1 - \alpha) + (1 - \alpha)\lambda_R + \lambda_L - \alpha\hat{\lambda}_\alpha - \frac{1 - \alpha}{\alpha} f(\hat{\lambda}_\alpha) \right). \quad (76)$$

From there, we have the expected turnout differential in equilibrium:

$$\tau_R^* - \mathbb{E}[\tau_R^*] = \alpha(\hat{\lambda}_\alpha - \mathbb{E}[\lambda_L]). \quad (77)$$