# Learning by Lobbying

Emiel Awad* Gleason Judd† Nicolás Riquelme‡

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**Abstract**

Effective lobbying requires understanding politicians' preferences, while lobbying itself provides useful information about those preferences. How does this link between lobbying and learning shape relationships between interest groups and politicians? We develop a game-theoretic model where an interest group can lobby a politician while learning about their ideological alignment. Our analysis highlights strategic tensions where interest groups balance information-gathering against policy influence in their lobbying, while forward-looking politicians manage their reputations to shape future interactions. These forces shape dynamics: policies and transfers shift over time as uncertainty resolves, with early-career politicians showing greater policy variance and extracting larger benefits through reputation management than veterans. Politicians with secure positions receive more favorable treatment due to their stronger incentives to appear less aligned than they truly are. Our results address empirical regularities and provide a theoretical foundation for understanding how lobbying relationships evolve across political careers and institutional contexts.

*Analyst, Oxera Consulting LLP, emielawad@gmail.com.

†Assistant Professor, Department of Politics, Princeton University, gleason.judd@princeton.edu.

‡Assistant Professor, School of Business and Economics, Universidad de los Andes, Chile, nriquelme@uandes.cl.

# Introduction

Information is crucial for effective lobbying. Understanding key policymakers is particularly valuable, even beyond knowledge about policies and procedures.[1] Interest groups gather this information in various ways—researching backgrounds, consulting staff, hiring connected lobbyists—but lobbying itself is particularly useful. Politicians' responses to specific lobbying overtures can reveal preferences that public statements or voting records might not capture.[2] Thus, lobbying and learning are linked: understanding politicians' preferences facilitates effective lobbying, while lobbying can reveal those preferences.

Parsing this link is important for understanding political influence. In lobbying relationships, it creates strategic tensions for interest groups and politicians. Interest groups must decide whether to leverage current information or pursue strategies that yield more information about politicians to lobby them more effectively later on. Meanwhile, politicians may recognize that their policy choices reveal information about their preferences, potentially affecting future lobbying approaches or revolving-door opportunities (McChesney 1997; Egerod and Tran 2023).[3] These considerations are especially pronounced with unfamiliar politicians.[4] And more broadly, the potential for *learning-by-lobbying* also affects interest groups' broader strategic decisions about which politicians to target, how to learn about them, and how much to invest in relationship-building.

Our central questions address these fundamental tensions. How does the link between lobbying and learning shape relationships between interest groups and politicians? How do

[1] A seasoned lobbyist emphasizes: "it is not about what you want; it is about what the other person needs" (Levine 2008, p. 163).

[2] Kerr, Lincoln and Mishra (2014, p. 344) note: "firms may gain from learning about policymakers' private dispositions, which may not be fully reflected in their public positions."

[3] Politicians sometimes attempt to induce more favorable lobbying by posturing via "milker"/"juicer"/"fetcher" bills threatening unfavorable taxes, permitting, price controls, etc. (McChesney 1997, pgs. 29–32).

[4] Kerr, Lincoln and Mishra (2014, p. 344) note: "the costs of learning and establishing relationships with policymakers are likely to be the highest in a firm's first several years of lobbying."

politicians' reputational concerns affect this dynamic? How does this learning mechanism interact with other aspects of political influence such as the value of information about politicians obtained through other means, access that facilitates lobbying, constraints politicians face during policymaking, and revolving-door considerations?

We develop a game-theoretic model in which an interest group lobbies and learns about a politician's preferences over time. In each of two periods, the group lobbies to influence the politician's policy choice. Initially uncertain about the politician's ideology—which might be relatively aligned with the group's goals (*ally*) or more distant (*adversary*)—the group updates its beliefs by observing responses to first-period lobbying before engaging again in the second period. Meanwhile, the politician strategically manages their reputation, aware that the group may learn from their choices. Our model highlights the link between learning and lobbying: the first period captures initial interactions under uncertainty, while the second shows how behavior evolves with learning.

Our model captures lobbying's frequently granular nature, as interest groups attempt to shape specific policy details.[5] The group lobbies by offering a menu of policy-transfer pairs, where "transfers" represent various forms of political support—campaign contributions, charitable donations, or legislative assistance—making the framework applicable across diverse scenarios.[6] The politician sees the offered menu and can either select one pair or set policy independently.

To illustrate, consider a hypothetical scenario. Advanced Medical Technology Association (AdvaMed) approaches a new member of the House Ways and Means Health Subcommittee regarding Medicare reimbursement policies. The association must determine how to lobby this legislator whose true stance on industry regulation is unclear. AdvaMed could offer pol-

[5] A veteran lobbyist notes, "What matters is getting stuff put in the bill, a line here, a line there... What you're looking to do is put a line in a law, get something tweaked. You're looking to change this line in subsection B. You just need one person to make that change" (Drutman 2015, p. 31).

[6] For more discussion and examples, see McChesney (1997, pgs. 45–54).

icy packages with varying degrees of regulatory change paired with different levels of industry support. Meanwhile, a legislator who privately supports industry-friendly policies may face a strategic dilemma: accepting the most favorable package might reveal their alignment too clearly, potentially reducing leverage in future negotiations. In its lobbying, AdvaMed balances its short-term policy gains against what it might learn about the legislator's true preferences to lobby her more effectively later. These strategic tensions—between information revelation and concealment, between immediate and future influence—fundamentally shape how lobbying relationships evolve over time.

Our setting highlights two key strategic considerations. First, the interest group can face a tension between influence and learning: foregoing immediate policy gains may help them learn about politicians' true preferences for more effective future lobbying. Second, the politician can have reputational concerns: even when aligned with the interest group, they may strategically act more adversarial to extract better terms later. We analyze how the interplay between these factors shapes distinctive patterns in lobbying and influence.

The interest group's equilibrium lobbying strategy varies based on its prior beliefs about the politician. Under broad conditions, first-period lobbying distinguishes whether the politician is an ally or adversary. The group's particular screening approach depends on its beliefs about the politician. If it believes the politician is likely an ally, the first-period menu pairs terms attractive to allies with modest demands for potential adversaries. Conversely, when it suspects an adversary, the first-period menu pairs modest demands for adversaries paired with terms that are especially generous for allies. The group foregoes learning, however, if it is not very forward-looking and the politician is likely an adversary—it simply offers terms that would be efficient with an adversary, which the politician would accept regardless of whether they are an ally or adversary.

As politicians become more concerned with their long-term reputation, screening efforts grow increasingly costly for interest groups. This occurs because politicians have stronger incentives to misrepresent their preferences—allies may strategically mimic adversaries to extract better future terms. To maintain effective screening, interest groups must either further moderate demands intended for adversaries or offer increasingly generous terms for allies, depending on their prior beliefs about alignment. In some cases, the politician might even be lobbied to choose first-period policy that appears less favorable to interest groups than if no lobbying occurred—a counterintuitive outcome driven by the combination of screening and reputation concerns.

These dynamics generate two key insights about lobbying relationships.

First, policies and transfers shift as interest groups learn about politicians. If the group initially expects an ally, first-period lobbying includes an underaggressive option intended for adversaries, and if the politician chooses that option, then second-period lobbying is more aggressive (i.e., paying more to shift policy further). If the group initially expects an adversary, first-period lobbying includes an option that is overly generous to potential supporters, and if that is chosen then second-period lobbying is less generous.

This pattern creates observable implications. Policy predictability varies across career stages, with greater uncertainty about newcomers' policy choices. We also expect a convergence in late-career lobbying behavior as the role of screening diminishes. Politicians who receive aggressive lobbying early in their careers might face milder approaches later, and vice versa—not because their preferences changed, but because interest groups know them more precisely.

Second, politicians with secure positions enjoy more favorable early-career lobbying. This is driven by their stronger reputational concerns. They have stronger incentives to appear less aligned than they truly are to get favorable future lobbying, forcing groups to offer more generous terms to screen them. Thus, different political positions can create distinct patterns in how lobbying relationships evolve. Our results suggest that politicians with more secure offices (committee chairs, favorable constituencies) might receive disproportionately favorable lobbying terms compared to those facing electoral uncertainty. Additionally, they also suggest that politicians in positions with greater policy discretion might receive more favorable early-career lobbying due to more intensive screening by interest groups.

Our extensions study four key aspects of relationships. First, analyzing the value of early-career information reveals that understanding politicians' preferences is more valuable when politicians are patient (as this increases learning costs) but less valuable when interest groups are forward-looking (as they become more willing to learn the politician's preferences). Second, early-career access can be undesirable and interest groups may prefer to forego access to certain politicians rather than incur high learning costs. Third, incorporating revolving-door incentives shows that politicians' reputational incentives are less adversarial when post-career employment opportunities are valuable, making learning-by-lobbying less costly for interest group. Fourth, our analysis of policymaking constraints demonstrates that learning politicians' preferences is less valuable when they have less policy discretion.

Our analysis speaks to empirical patterns in interest group activities. The learning-by-lobbying mechanism helps understand why experience with specific policymakers is valuable, why early interactions are particularly formative, why allies may receive especially favorable terms, and how lobbying relationships evolve. In extensions, we show how this mechanism's role in some of those patterns can vary with other aspects of the broader political landscape through revolving-door considerations or policymaking constraints. Moreover, our extensions also offer implications for lobbying-adjacent activities like background research (for information), campaign contributions (for access), or hiring staffers (for both).

# Relationship to Existing Literature

We contribute to understanding lobbying relationships and political influence.[7] Experience with specific policymakers is valuable for lobbying (Kerr, Lincoln and Mishra 2014; Drutman

[7] For empirical work on the dynamics of lobbying, see Kerr, Lincoln and Mishra (2014). For broader overviews of theoretical and empirical work, see Grossman and Helpman (2001), Bombardini and Trebbi (2020), and Schnakenberg and Turner (2023).

2015), and we illuminate this relationship by modeling how lobbying relationships evolve as interest groups learn about politicians through repeated lobbying. We depart from prior work by combining two key features: incomplete information about politicians and the potential for repeated lobbying. Classic theories with complete information in one-shot exchanges (Hall and Wayman 1990; Austen-Smith and Wright 1994; Besley and Coate 2001) illuminate static influence but cannot address dynamics or learning. Static models with incomplete information about politicians' preferences (Martimort and Semenov 2008; Kolotilin et al. 2017; Schnakenberg 2017; Minaudier 2022) capture uncertainty but not its evolution. Dynamic models with complete information (Iaryczower and Oliveros 2017; Chen and Zápal 2022; Bils, Duggan and Judd 2021) address temporal aspects but not learning or reputation.

Our *learning-by-lobbying* mechanism provides a distinct perspective complementing established approaches. While policy or procedural expertise and relationship building can each involve information (LaPira and Thomas III 2017), our analysis examines how interest groups learn about politicians' preferences through the lobbying process. This sheds new light on why early interactions with politicians are so formative and why lobbying patterns evolve systematically as uncertainty resolves (Kerr, Lincoln and Mishra 2014). Furthermore, we illuminate how learning-by-lobbying can impact other activities by affecting the value of different relationships and how they should be managed.

Our focus on interest groups learning about politicians provides a novel perspective on information flows in lobbying relationships. Existing work has studied how politicians learn about interest groups (Groll and Ellis 2014, 2017; Ellis and Groll 2024) or use lobbyists as gatekeepers to screen interest groups (Hirsch et al. 2023), but our focus addresses the other direction of learning. Our setup aligns with observations that beliefs about legislators' preferences are updated through lobbying (Austen-Smith and Wright 1992), but provides a formal account of how this updating affects strategic behavior on both sides. By modeling this two-sided relationship, we capture how interest groups' learning incentives interact with politicians' reputational concerns (McChesney 1997; Egerod and Tran 2023).

We model lobbying as an *exchange* of policy for transfers (Grossman and Helpman 1994) in the menu-auction tradition (Bernheim and Whinston 1986; Grossman and Helpman 2001). Our setup captures the granular nature of real-world influence and encompasses various lobbying tactics. While lobbying can be modeled in various other ways—e.g., legislative subsidies (Hall and Deardorff 2006) or information transmission (Schnakenberg 2017; Awad 2020)—a common theme is that interest groups aim to influence politicians' behavior, with tactics depending on both parties' preferences.[8] Our approach reflects this theme and is particularly suited for analyzing how interest groups adjust their strategies as they learn about politicians' preferences.

Our extensions contribute to broader understanding of interest group politics by parsing how learning-by-lobbying forces interact with other influence-oriented activities. First, our extension on the value of information sheds new light on why revolving-door lobbyists are valuable and how their value can vary depending on which politicians they know about, complementing the role of lobbyists as a certification tool (Hirsch et al. 2023). Second, our extension on value of access highlights how interest groups' desire to cultivate relationships that facilitate lobbying depend on politicians' job security, career stage, or familiarity, complementing the role of relative ideology (Austen-Smith 1995; Schnakenberg 2017; Awad 2020; Judd 2023) or proposal power (Judd 2023). Third, our revolving-door extension clarifies how lobbying and reputation can interact to shape politicians' observed in-office behavior before potentially revolving, complementing politician-centric theories (Che 1995; Bar-Isaac and Shapiro 2011; Hübert, Rezaee and Colner 2023). Finally, our policy-constraints extension addresses how voting rules and collective policymaking can shape how learning and reputation affect lobbying, complementing complete-information theories of lobbying in collective bodies (Grossman and Helpman 2001; Judd 2023).

[8]See Schnakenberg and Turner (2023) for an overview.

Finally, our model of learning, reputation, and lobbying influence descends from traditional models of repeated contracting (Hart and Tirole 1988; Laffont and Tirole 1990; Salanié 2005). These models typically feature a *ratchet effect* (Freixas, Guesnerie and Tirole 1985), where players have incentives to conceal their true characteristics to avoid future exploitation of this information. This incentive also arises in our setting, where appearing adversarial can induce more favorable lobbying. Yet, two key aspects of our lobbying and policymaking setting lead to different equilibrium conditions than existing models of repeated contracting. First, the spatial policy preferences introduce complex interdependencies when offering menus and screening different types that preclude standard single-crossing conditions.[9] Second, the politician can set policy independently if she rejects the group's menu, whereas elsewhere the agent either cannot act after rejection or the game ends. Since these are two prominent political features, our analysis may facilitate future work on political influence.

# Model

**Players.** There are two players: an interest group, $G$, and a politician, $P$.

**Timing.** There are two periods of policymaking. In each period $t \in \{1, 2\}$, $P$ will enact a policy and $G$ can lobby by offering $P$ a menu $M_t$ of policy-transfer pairs, with $G$ choosing both (i) how many pairs to include and (ii) for each pair $(x, T)$, the exact policy $x$ and transfer $T$. Next, $P$ observes $M_t$ and then either selects one pair or rejects all of them. If $P$ selects a pair $(x, T)$ from $M_t$, then the enacted policy is $x_t = x$ and $G$ transfers $T$ to $P$. Otherwise, if $P$ rejects the menu, then $P$ chooses $x_t$ freely without receiving a transfer $T$. Since $P$'s selection determines the realized policy and transfer—that is, we abstract from short-term commitment issues—we follow the literature on menu auctions and refer to each pair in a menu as a *contract*. Accordingly, we define an arbitrary contract as $c = (x, T)$.

**Payoffs.** In each period, $G$'s utility function is $\Pi(x, T) := -(x-1)^2 - T$, where $x$ is the

[9] In seller-buyer or firm-worker environments, this single-crossing condition is typically implicitly assumed, as in, Beccuti and Möller (2018), Gerardi and Maestri (2020) and Breig (2022).

implemented policy, 1 is $G$'s ideal point, and $T$ is the transfer (if accepted). Similarly, $P$'s utility function is $U(x,T) := -(x-\theta)^2 + T$, where $\theta$ denotes $P$'s ideal point. Notably, $P$'s ideal point can be either $\underline{\theta}$ or $\overline{\theta}$, so we also refer to $\theta$ as $P$'s *type*. We assume $\underline{\theta} < \overline{\theta} < 1$, so that $\underline{\theta}$ is the *adversary* type and $\overline{\theta}$ is the *ally* type, closer to $G$.

Each player's cumulative payoff is the sum of their utility across both periods. Each player discounts second-period utility with (potentially different) discount factors: $\delta_P, \delta_G \in [0,1]$.

**Information.** The interest group $G$ does not know $P$'s ideal point, $\theta \in \{\underline{\theta}, \overline{\theta}\}$. At the beginning of the game, $G$'s prior belief puts probability $\mu_0 \in [0,1]$ on $\theta = \overline{\theta}$. After observing $P$'s first-period behavior, $G$'s updated belief about $P$ is denoted $\mu_1$. All other features of the game are common knowledge.

**Strategies and Equilibrium Concept.** We study Perfect Bayesian Equilibria (PBE) in pure strategies.[10] Thus, strategies are sequentially rational at every information set and Bayes' rule is applied whenever possible. As off-path information sets arise in equilibrium, we apply the Never-a-Weak-Best-Response (NWBR) refinement.[11] This refinement ensures that off-equilibrium-path beliefs assign higher probability to types more inclined to deviate.

Our results in the main text focus on the players' equilibrium strategies. Formal statements of beliefs and all proofs are provided in the Appendix.

# Analysis

Our analysis proceeds in several stages. We first analyze two complete-information benchmark cases: with and without lobbying. We then begin our main analysis, studying lobbying

[10]We focus on pure strategies for tractability. Although mixed strategies could yield different equilibrium outcomes (see Bester and Strausz 2001, for more details), the pure-strategy analysis captures strategic considerations in lobbying relationships. Moreover, our approach follows the precedent set by canonical models using menu auctions (Bernheim and Whinston 1986).

[11]Either $P$ may reject an on-path offer, leading to an off-path information set, or $G$ may make an off-path offer. In either case, applying NWBR constrains $G$'s off-path beliefs. For an overview of refinements in signaling games, see Fudenberg and Tirole (1991) and Manelli (1997).

in a static setting with incomplete information, which corresponds to the second period of our model. Working backward, we study the first-period interaction, where lobbying occurs in a dynamic setting with incomplete information. To conclude our main analysis, we analyze the dynamics of how policies and transfers shift over time. Finally, in extensions we study how early-career information, access, revolving-door incentives, and policymaking constraints affect our main insights.

## Two benchmarks

To set the stage for our main analysis, we characterize behavior in two benchmark settings: (i) no lobbying and (ii) lobbying with complete information.

**Benchmark 1** (No lobbying)**.** *If the interest group cannot lobby, then $P$ will set policy at her ideal point, $\theta$, in both periods.*

Without lobbying, $P$ obtains a payoff of zero in both periods, regardless of type. However, $P$'s type does impact $G$'s payoff, $-(1+\delta_G)(1-\theta)^2$. Specifically, $G$'s payoff is lower if $P$ is an adversary ($\theta = \underline{\theta}$) than if $P$ is an ally ($\theta = \overline{\theta}$).

**Benchmark 2** (Lobbying with complete information)**.** *If $G$ can lobby and has complete information about $P$, then in each period $P$ enacts policy $x_\theta := \frac{1}{2}(1+\theta)$ and receives transfer $t_\theta := \frac{1}{4}(1-\theta)^2$.*

If $G$ knows $P$'s ideal point, $\theta$, then it can perfectly calibrate its lobbying. Notably, $P$'s ideal point acts as a reservation policy that she will enact if she rejects $G$'s menu. Thus, $\theta$ determines both (i) the minimal transfer required for $P$ to deviate from her ideal policy and (ii) $G$'s willingness to provide that minimal transfer. To induce $P$ to enact a policy $x$, $G$ must offer a transfer of at least $T_\theta(x) := (\theta - x)^2$.

In each period, $G$ optimally balances its marginal benefit of more favorable policy against its marginal cost of increasing the transfer. If $P$ is an ally, then $G$ induces a mild policy shift at a moderate cost. Otherwise, if $P$ is an adversary, $G$ has a larger marginal gain from shifting $P$'s policy and therefore induces a larger shift at a higher cost.

The politician never receives surplus, since she is indifferent between accepting and rejecting the offered contract in both periods. Thus, we refer to $c_{\underline{\theta}} := (x_{\underline{\theta}}, t_{\underline{\theta}})$ as the *adversary-efficient contract*, and define $c_{\overline{\theta}}$ analogously as the *ally-efficient contract*.[12] Accordingly, we define $\pi_{\underline{\theta}} := -(x_{\underline{\theta}} - 1)^2 - t_{\underline{\theta}}$ as $G$'s *adversary-efficient payoff* and define $\pi_{\overline{\theta}} := -(x_{\overline{\theta}} - 1)^2 - t_{\overline{\theta}}$ as $G$'s *ally-efficient payoff*.

## Lobbying with incomplete information

We now begin our main analysis. The interest group does not know the politician's type (i.e., her ideal point). We study how $G$'s influence is shaped by *uncertainty* about the legislator's alignment and the effect of strategic *dynamic* considerations. Since we focus on pure strategies, the Revelation Principle implies that it is without loss of generality to focus on menus with at most three options: two 'type-specific' contracts and an 'empty' contract. The type-specific contracts are distinct: an *adversary contract*, $\underline{c} = (\underline{x}, \underline{T})$, intended for an adversary, $\underline{\theta}$, and an *ally contract*, $\overline{c} = (\overline{x}, \overline{T})$, intended for an ally, $\overline{\theta}$. We say a menu is *separating* if $\underline{c} \neq \overline{c}$ and *pooling* if $\underline{c} = \overline{c}$.

## Second-period lobbying and policymaking

In the second period, $G$'s sole focus is influence. This parallels Benchmark 2 but now $G$ may not know $P$'s ideal point, $\theta$. Instead, $G$ has updated belief $\mu_1 \in [0, 1]$ about $\theta$, where $\mu_1 := \mu(\theta = \overline{\theta}|h)$ represents the posterior probability that $P$ is an ally given history $h$.

The interest group wants to induce favorable policy without overpaying or under-lobbying. However, without knowing $\theta$, $G$'s equilibrium menu cannot include both efficient contracts $c_{\underline{\theta}}$ and $c_{\overline{\theta}}$. If such a menu were offered, then an ally would choose the adversary-contract $c_{\underline{\theta}}$, so $G$ would infer they are potentially overpaying an ally to enact *worse* policy: $\underline{x}_2$ instead of $\overline{x}_2 > \underline{x}_2$. More broadly, $G$'s menu must ensure *incentive compatibility* for $P$ to ensure that each type selects its intended contract.

[12] That is, $c_{\underline{\theta}}$ and $c_{\overline{\theta}}$ are the two possible full-information contracts. Throughout, we refer to equilibrium contracts that differ from these full-information contracts as having *distortions* from the perspective of $G$.

Since $G$ cannot prevent $P$ from misrepresenting their preference, $G$ will proactively distort the offered menu. Specifically, $G$'s second-period menu will not include both $c_{\underline{\theta}}$ and $c_{\overline{\theta}}$. A distortion (a move away from the efficient ally- and adversary-contracts) could in principle arise as either a *separating menu* with two options or a *pooling menu* with a single option. A separating menu can become prohibitively expensive for $G$, given the distortions required to each type selects the contract intended for them, and as a result $G$ may prefer to offer just one contract. However, in the second period of any equilibrium, we show that $G$ will offer a separating menu.

Thus, in $G$'s optimal separating menu, at least one contract will be distorted so that the ally and adversary types choose the contract intended for them. To deter an ally from selecting the adversary-contract, $G$ offers a menu that either (i) overpays an ally to ensure it chooses the ally-efficient policy, $x_{\overline{\theta}}$, or (ii) makes an underaggressive adversary-offer (i.e., $\underline{x}_2 < x_{\underline{\theta}}$ with $\underline{t}_2 = T_{\underline{\theta}}(\underline{x}_2)$), extracting smaller policy concessions from an adversary. The particular distortion $G$ chooses depends on $\mu_1$, $G$'s updated belief about $P$.

If $G$ believes $P$ is probably an ally (i.e., $\mu_1$ is sufficiently high), then $G$ offers a separating menu with only one distortion: an underaggressive adversary-offer. This menu pairs (i) a ally-efficient contract and (ii) an overly conservative adversary-contract. Essentially, $G$ makes the adversary-contract less appealing to an ally to ensure it would choose the ally-efficient contract. Although this sacrifices efficiency in the adversary-contract, that inefficiency is relatively unlikely to materialize. To minimize distortions, the equilibrium contracts are calibrated so that (i) an ally is indifferent between them (and rejecting) and (ii) an adversary is indifferent between accepting the adversary-contract and rejecting $G$'s menu.

Otherwise, if $\mu_1$ is lower, $G$ prioritizes favorable terms in the adversary-contract and offers a separating menu with two distortions: an excessive ally-transfer *and* an underaggressive adversary-offer. This menu pairs (i) an ally-contract that overpays for the ally-efficient policy $x_{\overline{\theta}}$ with (ii) an adversary-contract that is again overly conservative but to a lesser degree.

The size of these distortions is inversely related to $\mu_1$. As $\mu_1$ decreases, making an adversary more likely, $G$ increasingly prioritizes efficiency in the adversary-contract while offering an increasingly excessive (but less likely) ally-payment to ensure incentive compatibility.

Lemma 1 characterizes second-period policy and transfers. A key factor is whether $G$'s belief $\mu_1$ is relatively high or low, distinguished by the cutpoint $\tilde{\mu}_1 := \frac{1-\overline{\theta}}{1-\underline{\theta}} \in (0,1)$.

**Lemma 1.** *In any second-period history and for every belief $\mu_1$, the interest group's optimal menu in equilibrium is separating.*

1. *If $\mu_1 \leq \tilde{\mu}_1$, then (i) an ally enacts $\overline{x}_2 = x_{\overline{\theta}}$ and receives $\overline{t}_2 = T_{\overline{\theta}}(\overline{x}_2) + \frac{(\overline{\theta}-\underline{\theta})(1-\overline{\theta}-\mu_1(1-\underline{\theta}))}{1-\mu_1}$, and (ii) an adversary enacts $\underline{x}_2 = x_{\underline{\theta}} - \frac{\mu_1}{1-\mu_1}\frac{\overline{\theta}-\underline{\theta}}{2}$ and receives $\underline{t}_2 = T_{\underline{\theta}}(\underline{x}_2)$.*
2. *If $\mu_1 > \tilde{\mu}_1$, then (i) an ally enacts policy $\overline{x}_2 = x_{\overline{\theta}}$ and receives transfer $\overline{t}_2 = t_{\overline{\theta}}$, and (ii) an adversary enacts $\underline{x}_2 = x_{\underline{\theta}} - \frac{1-\overline{\theta}}{2}$ and receives $\underline{t}_2 = T_{\underline{\theta}}(\underline{x}_2)$.*

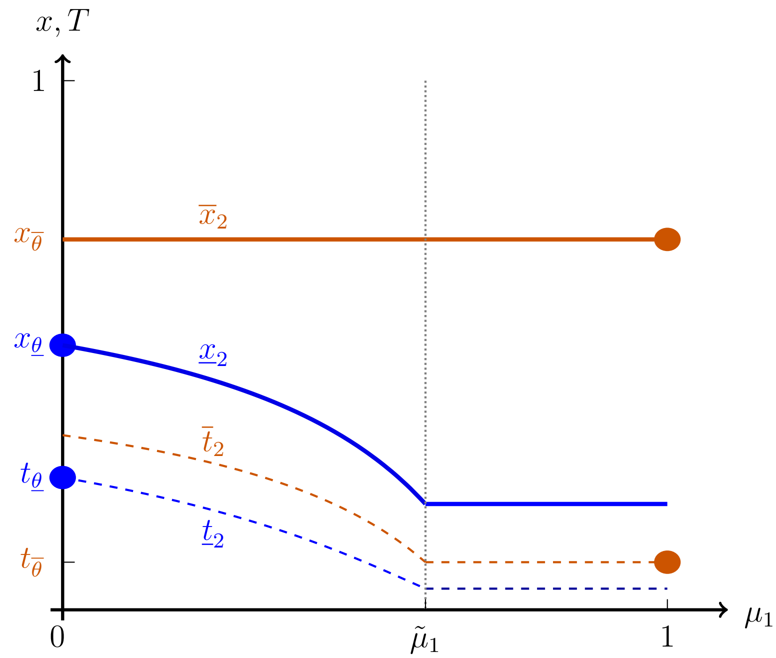
Figure 1 illustrates the findings of Lemma 1 by displaying equilibrium policies and transfers as functions of $G$'s updated belief $\mu_1$. The blue and orange dotted contracts illustrate the efficient contracts $c_{\underline{\theta}}$ and $c_{\overline{\theta}}$ that $G$ would offer under complete information ($\mu_1 = 0$ and $\mu_1 = 1$). The ally-contract always contains the same policy offer, but its transfer either decreases with $\mu_1$ or remains constant. In contrast, the adversary's policy- and transfer-offer are always sensitive to $G$'s belief. $G$ can only ensure that it offers efficient contracts if it perfectly learns $P$'s alignment. If not, $G$ distorts lobbying, and the degree to $G$ does so is generally increasing in the level of uncertainty about $P$'s alignment.

## First-period lobbying and policymaking

We now analyze first-period lobbying and policymaking, unpacking how the prospect of future lobbying opportunities shapes these activities.

Both $P$ and interest group balance first-period incentives against forward-looking considerations about second-period consequences. $G$ wants to influence policy favorably today while also learning about $P$'s preferences to facilitate future lobbying. Meanwhile, $P$ wants to re-

Figure 1: Second-Period Policies and Transfers




*Note: The figure illustrates equilibrium policies and transfers listed in Lemma 1. Blue lines represent the equilibrium adversary-contract as a function of* $\mu_1$*, where the dashed line is the adversary-transfer* ($\underline{t}_2$) *and the solid line the adversary-policy* ($\underline{x}_2$)*. Orange lines represent the equilibrium ally-contract, where the dashed line is the ally-transfer* ($\overline{t}_2$) *and the solid line the ally-policy* ($\overline{x}_2$)*. The dots indicate the efficient contracts:* $c_{\underline{\theta}} = (x_{\underline{\theta}}, t_{\underline{\theta}})$ *for* $\mu_1 = 0$ *and* $c_{\overline{\theta}} = (x_{\overline{\theta}}, t_{\overline{\theta}})$ *for* $\mu_1 = 1$*. We assume that* $\underline{\theta} = 0$ *and* $\overline{\theta} = \frac{2}{5}$*.*

ceive favorable terms today while also managing her reputation to ensure favorable lobbying later. Each player's static incentive to obtain favorable terms is analogous to the second period. That is, all the incentives that are present in the second period (given the same beliefs about $P$'s alignment) are also present in the first period. In the first period, however, their forward-looking incentives—learning and reputation—introduce new forces above and beyond static incentives. Moreover, these forces are interdependent. On the one hand, $G$'s learning is affected by $P$'s reputational incentives. On the other, $P$'s reputational incentives are affected by $P$'s anticipation of $G$'s learning.

A key factor is how first-period behavior impacts G's updated belief about $P$, which shapes second-period lobbying and policymaking. How $G$ and $P$ interact in the second period then affects each player's expected future payoffs (i.e., their continuation value following the first period).[13] By learning $P$'s preferences in the first period, $G$ can lobby more effectively in the second. Remark 1 characterizes how $G$'s continuation value varies with its updated belief, $\mu_1$.

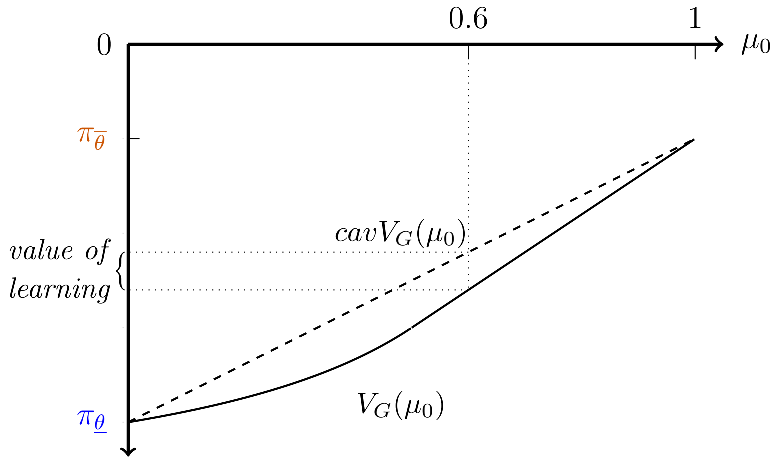
**Remark 1.** *The interest group's continuation value is:*

$$V_G(\mu_1) = \begin{cases} (1-\mu_1)(\pi_{\underline{\theta}}) + \mu_1 \left[\pi_{\underline{\theta}} + \frac{(\overline{\theta}-\underline{\theta})^2}{1-\mu_1}\right] & \textit{if } \mu_1 \leq \tilde{\mu}_1, \\ (1-\mu_1)\left(\pi_{\underline{\theta}} + \pi_{\overline{\theta}}\right) + \mu_1 \left(\pi_{\overline{\theta}}\right) & \textit{if } \mu_1 > \tilde{\mu}_1. \end{cases}$$

Remark 1 implies that $G$ benefits from more precise information. Then, $G$ can more confidently tailor its menu of offers towards the politician's type (adversary or ally), anticipating that this increasingly efficient offer will be chosen. In the limiting case where $G$ has degenerate beliefs, $\mu_1 = 0$ or $\mu_1 = 1$, $G$'s ex ante equilibrium payoff converges to $(1-\mu_0)V_G(0) + \mu_0 V_G(1) = (1-\mu_0)(\pi_{\underline{\theta}}) + \mu_0(\pi_{\overline{\theta}})$ in the second period. More precise information facilitates the provision of efficient lobbying offers and minimizes distortions.

[13]Formally, these continuation values are the channel for feedback effects through which second-period equilibrium behavior affects first-period incentives.

Figure 2: The Interest Group's Value of Learning




*Note: The solid line represents G's continuation value as a function of its prior belief* $\mu_0 \in [0, 1]$*. The dashed line depicts the concavification of* $V_G(\mu_0)$*, which is the upper bound of G's expected continuation value (resulting from having full information about* $\theta$*). G's value of information is* $cavV_G(\mu_0) - V_G(\mu_0)$*. For an example belief of* $\mu_0 = 0.6$*, we illustrate the value of information by the bracketed part on the y-axis. We assume* $\overline{\theta} = \frac{1}{2}$ *and* $\underline{\theta} = 0$*.*

In equilibrium, $G$'s learning is binary: it either learns everything or nothing. It will either offer a separating menu that distinguishes $P$'s preferences ($\mu_1 \in \{0, 1\}$); or a pooling menu that both types accept ($\mu_1 = \mu_0$). Thus, $G$'s *value of learning* $P$'s alignment is based on the comparison of (i) $G$ learning $P$'s alignment, and (ii) $G$ lobbying $P$ at the same belief. Definition 1 formalizes this comparison and the value of learning.

**Definition 1.** *In equilibrium, G's* ***value of learning*** *is* $W(\mu_0) := (1-\mu_0)\, V_G(0) + \mu_0\, V_G(1) - V_G(\mu_0)$*.*

$G$ is more inclined to learn $P$'s preferences when $G$ is less certain about $P$. The value of learning disappears when $G$ is almost certain whether $P$ is an ally or adversary. Figure 2 depicts this relationship.

As $G$'s first-period learning affects second-period lobbying, $P$ has incentives to strategically manage her reputation. Specifically, $P$'s continuation value depends on $G$'s updated belief. Remark 2 characterizes $P$'s continuation value and clarifies its key properties.

**Remark 2.** *In equilibrium, (i) an adversary politician's continuation value is always zero,* *whereas (ii) an ally politician's continuation value is strictly decreasing over* $\mu_1 \leq \tilde{\mu}_1$ *and constant over* $\mu_1 > \tilde{\mu}_1$. *Specifically,* $V_P(\underline{\theta}, \mu_1) = 0$ *and*

$$V_P(\overline{\theta}, \mu_1) = \begin{cases} \frac{(\overline{\theta}-\underline{\theta})(1-\overline{\theta}-\mu_1(1-\underline{\theta}))}{1-\mu_1} & \text{if } \mu_1 \leq \tilde{\mu}_1, \\ 0 & \text{if } \mu_1 > \tilde{\mu}_1. \end{cases}$$

Notably, since an ally's continuation value decreases in $\mu_1$, an ally has incentives to appear adversarial and induce $G$'s belief to be $\mu_1 = 0$.[14] By doing so, they would receive a better lobbying deal in the second period, receiving more than needed to accept.

$P$'s potential reputational incentives make learning costly for $G$. The prospect of future lobbying complicates $G$'s efforts to induce $P$ to reveal her preferences in the first period.[15] The interplay between learning and reputational incentives that shape first-period lobbying is determined by how much each player values the future. A more patient *interest group* is more willing to concede favorable first-period terms to learn about $P$ and obtain more favorable second-period terms. Conversely, a more patient *politician* is more inclined to forego favorable first-period terms to misrepresent its preferences and receive more favorable second-period terms. Thus, a higher $\delta_G$ results in $G$ having a higher willingness to learn $P$'s preferences, while a higher $\delta_P$ results in $G$ facing greater costs of learning about $P$'s alignment.[16]

If $P$ is likely to be an ally, then $G$'s first-period menu is separating. In this case, $G$ prioritizes ally-efficiency and supports it with an overly-conservative adversary-contract. Despite the

[14]In contrast, an adversary's continuation value is constant in $\mu_1$, removing any incentive to misrepresent their preferences.

[15]That is, inducing $P$ to reveal her preferences in the second period is already costly (even without dynamic considerations), but this is even more the case in the first. Fixing a particular belief—i.e., $\mu_0 = \mu_1$—a separating menu in the first period must be more distorted than it would be in the second period.

[16]An alternative interpretation is to compare a politician $P$ with a low $\delta_P$ and a high $\delta_P$. The same applies to other parameters and players.

low value of learning, $G$ chooses to learn through offering separating contracts as it is cheap in expectation.[17] Moreover, learning is cheap enough that it is always worthwhile and $G$ offers a separating menu for all $\delta_G \in [0, 1]$.

Conversely, if $P$ is unlikely to be an ally, then $G$'s first-period offer can be a pooling menu. Specifically, if $G$ is sufficiently impatient, then $G$ offers a pooling menu containing only the adversary-efficient contract, preventing any learning.[18] Otherwise, if $G$ is more patient, $G$ is more inclined to learn, offering a separating menu that overpays an ally politician and is overly conservative towards an adversary politician. Although these distortions are qualitatively similar to second-period lobbying (at equivalent beliefs), their magnitude is amplified due to $P$'s reputation incentive. In turn, the magnitude of distortions increases with $P$'s patience.

First-period distortions may be shaped somewhat differently than in the second period. If $G$ is sufficiently patient and believes $P$ is relatively unlikely to be an ally, then $G$ will distort the ally-policy upward. Thus, the ally-contract become overly aggressive in both transfer *and* policy. This additional distortion occurs because the substantial ally-transfer makes the ally-contract appealing to an adversary. The incentive compatibility constraints bind for both types, so $G$ adjusts the ally-policy to deter an adversary from accepting the ally-contract.

Lemma 2 characterizes $G$ and $P$'s first-period equilibrium behavior. To state the result, we define several belief thresholds. Let $\hat{\mu}_0 := \max\left\{0, \frac{\delta_P(1-\bar{\theta})-(\bar{\theta}-\underline{\theta})}{\delta_P(1-\bar{\theta})}\right\}$ and $\tilde{\mu}_0 := \frac{(1+\delta_P)(1-\bar{\theta})}{1-\underline{\theta}+\delta_P(1-\bar{\theta})}$.[19]

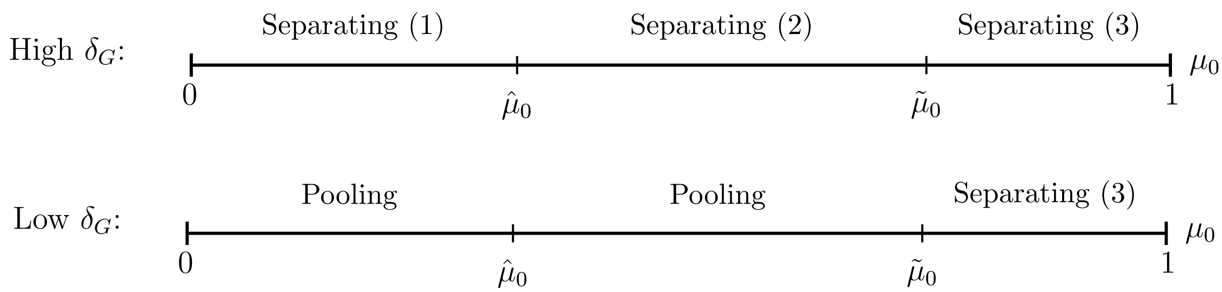
**Lemma 2.** *In any equilibrium, first-period behavior is as follows. If $\mu_0 < \tilde{\mu}_0$ and $\delta_G < \delta_G^{\dagger}$, then $G$ offers a pooling menu and $P$ enacts policy $x_1 = x_{\underline{\theta}}$ to receive transfer $t_1 = T_{\underline{\theta}}(x_1)$.*

[17] This is because distortions are unlikely to materialize in this case, namely only in the unlikely event that $P$ is an adversary.

[18] In the appendix, we formally define the threshold on $\delta_G$ (namely $\delta_G^{\dagger}$) that determines whether pooling can occur.

[19] Note that $\hat{\mu}_0 < \tilde{\mu}_0$ and $\tilde{\mu}_1 \in (0, \tilde{\mu}_0)$ always hold. Moreover, $\hat{\mu}_0 > 0$ if and only if $\delta_P > \frac{\bar{\theta}-\underline{\theta}}{1-\bar{\theta}}$.

Figure 3: Informativeness of First-period Lobbying




*Note: The figure illustrates the qualitatively different cases of Lemma 2 (i.e., two cases of $\delta_G$) to show how $G$'s equilibrium strategy varies with its prior belief $\mu_0$. The figure presents the three separating cases and the pooling case as listed in Lemma 2.*

*Otherwise, $G$ offers a separating menu in which:*

1. *if $\mu_0 \leq \hat{\mu}_0$, then (i) an ally will enact $\overline{x}_1 = x_{\overline{\theta}} + \delta_P(1-\mu_0)\frac{1-\overline{\theta}}{2} - \frac{\overline{\theta}-\underline{\theta}}{2}$ to receive $\overline{t}_1 = T_{\overline{\theta}}(\overline{x}_1) + (\overline{\theta}-\underline{\theta})\Big((1-\overline{\theta})(1+\delta_P(1-\mu_0))\Big)$, and (ii) an adversary will enact $\underline{x}_1 = x_{\underline{\theta}} - \delta_P\mu_0\frac{1-\overline{\theta}}{2}$ to receive $\underline{t}_1 = T_{\underline{\theta}}(\underline{x}_1)$;*

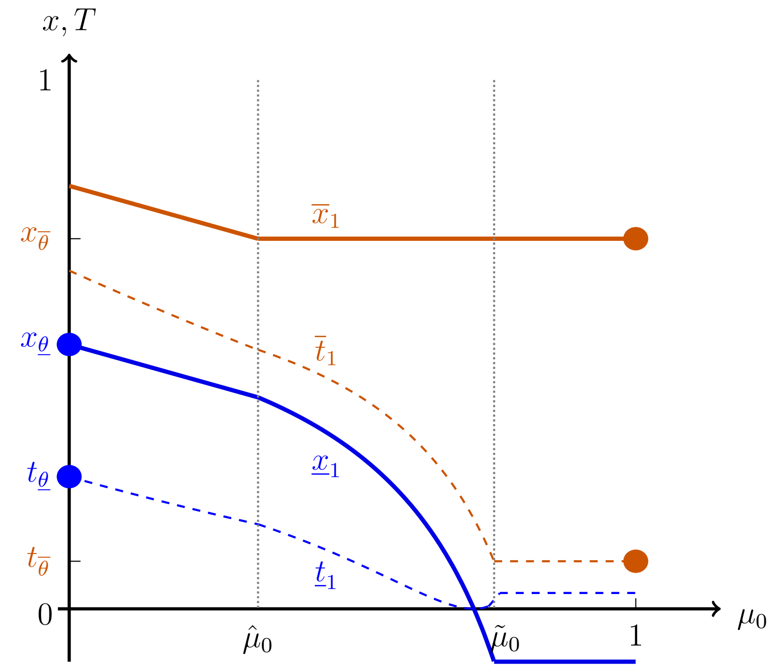
2. *if $\mu_0 \in (\hat{\mu}_0, \tilde{\mu}_0]$, then (i) an ally will enact $\overline{x}_1 = x_{\overline{\theta}}$ to receive $\overline{t}_1 = T_{\overline{\theta}}(\overline{x}_1) + \frac{\overline{\theta}-\underline{\theta}}{1-\mu_0}\Big((1-\overline{\theta})(1+\delta_P(1-\mu_0)) - (1-\underline{\theta})\mu_0\Big)$, and (ii) an adversary will enact $\underline{x}_1 = x_{\underline{\theta}} - \frac{\mu_0}{1-\mu_0}\frac{\overline{\theta}-\underline{\theta}}{2}$ to receive $\underline{t}_1 = T_{\underline{\theta}}(\underline{x}_1)$; and*

3. *if $\mu_0 > \tilde{\mu}_0$, then (i) an ally will enact $\overline{x}_1 = x_{\overline{\theta}}$ to receive $\overline{t}_1 = T_{\overline{\theta}}(x_{\overline{\theta}})$, and (ii) an adversary will enact $\underline{x}_1 = x_{\underline{\theta}} - (1+\delta_P)\frac{1-\overline{\theta}}{2}$ to receive $\underline{t}_1 = T_{\underline{\theta}}(\underline{x}_1)$.*

Lemma 2 reveals the impact of learning and reputation considerations. Figure 3 illustrates different equilibrium behaviors depending on $G$'s belief $\mu_0$ and patience $\delta_G$, while Figure 4 illustrates the separating contracts as a function of $G$'s belief $\mu_0$ when $G$ is sufficiently patient. Without reputation concerns ($\delta_P = 0$), the characterization mirrors the one presented in Lemma 1. Otherwise, $G$ alters its menu by either adjusting the terms it offers to learn $P$'s preferences or by forgoing learning in favor of a pooling menu.

The politician's reputation incentives have several consequences. First, $P$'s patience affects

Figure 4: First-period Policies and Transfers




*Note: The figure depicts equilibrium policies and transfers from Lemma 2. Blue lines represent the equilibrium adversary-contract (where solid lines illustrate the adversary-policy and dashed lines the adversary-transfer) and orange lines represent the equilibrium ally-contract (where solid lines illustrate the ally-policy and dashed lines the ally-transfer). The dots indicate the efficient contracts:* $c_{\underline{\theta}} = (x_{\underline{\theta}}, t_{\underline{\theta}})$ *for* $\mu_0 = 0$ *and* $c_{\overline{\theta}} = (x_{\overline{\theta}}, t_{\overline{\theta}})$ *for* $\mu_0 = 1$*. We assume that* $\delta_P = 1$*,* $\underline{\theta} = 0$*, and* $\overline{\theta} = \frac{2}{5}$*.*

the conditions under which $G$ offers a separating menu. As $P$'s patience increases, $P$ (if she is an ally) increasingly values the second-period gains from appearing adversarial, thereby forcing $G$ to make an increasingly generous ally-contract to screen $P$. Therefore learning becomes more costly, requiring more patience from $G$. Thus, $G$ is less inclined to learn about $P$'s preferences and will do so under fewer circumstances.

Second, $P$'s patience also affects the equilibrium contracts that $G$ offers in its separating menu. Broadly, as $P$'s patience increases, $G$'s offers become more distorted. Moreover, depending on the belief $\mu_0$, $G$ will adjust different aspects of the menu: (i) for low $\mu_0$, it increases the ally-contract while decreasing the adversary-contract; (ii) for intermediate $\mu_0$, it only increases the ally-transfer; and (iii) for high $\mu_0$, it only decreases the adversary-offer. Proposition 1 makes these observations precise.

**Proposition 1.** *Suppose $P$'s patience increases from $\delta_P$ to $\delta'_P$ and fix $\delta_G > \delta^{\dagger}_G$. In the first period:*

1. *if $\mu_0 < \hat{\mu}'_0$, then the adversary-contract $(\underline{x}_1, \underline{t}_1)$ decreases and the ally-contract $(\overline{x}_1, \overline{t}_1)$ increases;*
2. *if $\mu_0 \in (\hat{\mu}'_0, \tilde{\mu}_0)$, then the ally-transfer increases while the ally-policy and adversary-contract are both constant; and*
3. *if $\mu_0 > \tilde{\mu}_0$, then the ally-contract is constant and the adversary-contract decreases.*

A more patient politician increasingly emphasizes her second-period payoffs, thereby strengthening her reputational incentive to misrepresent her preference. Thus, $G$ must incur higher costs to learn $P$'s preference effectively. The particular way that $G$ distorts its menu depends on its prior beliefs about $P$'s preferences. Since some distortion is required, $G$ prefers to do so in ways that minimize the associated policy or monetary costs. Notably, the cost of distorting a $\theta$-contract is inversely related to the probability of that type.

How are equilibrium behaviors affected by $G$'s beliefs about $P$'s preferences? First-period lobbying varies with $G$'s belief about $P$'s preference, in several ways. Broadly, as $P$ is more likely to be an ally, $G$ will reduce offered transfers and request fewer policy concessions. Yet, $G$ adjusts fewer aspects of its offers as $\mu_0$ increases: over low $\mu_0$, $G$ decreases policies and transfers for both types; over intermediate $\mu_0$, $G$ does not adjust the ally-policy; and over high $\mu_0$, $G$'s offer is constant. Proposition 2 formally states these observations.

**Proposition 2.** *Suppose $\mu_0$ increases and fix $\delta_G > \delta_G^\dagger$.*

*(i) If $\mu_0 < \hat{\mu}_0$, then the ally-contract $(\overline{x}_1, \bar{t}_1)$ and the adversary-contract $(\underline{x}_1, \underline{t}_1)$ will decrease.*

*(ii) If $\mu_0 \in (\hat{\mu}_0, \tilde{\mu}_0)$ then the adversary-contract will decrease or increase, the ally-transfer will decrease while the ally-policy is constant.*

*(iii) If $\mu_0 > \tilde{\mu}_0$, then the ally-contract and adversary-contract are constant.*

As $\mu_0$ increases, $G$ is more likely to face an ally. In the first case, this leads $G$ to distort the ally-contract less by shifting it towards the ally-efficient-contract, while it simultaneously decreases its influence over an adversary to ensure that $G$ can successfully learn $P$'s preferences. In the second case, $G$ places more emphasis on providing an efficient ally-contract, and it becomes less costly to ensure that an ally would not mimic an adversary. Thus, an ally receives a lower transfer.[20] Finally, in the third region, $G$ finds it highly likely that $P$ is an ally. In this case, equilibrium offers are constant because the ally-contract is efficient, and no adjustments are needed to maintain incentive compatibility.

# Dynamics of Policymaking and Lobbying

We now trace the dynamics of policies and transfers over time. These dynamics depend on $G$'s prior belief about $P$'s preferences, as well as each player's discount factor.

Proposition 3 characterizes the trajectories of policies and transfers. As the policy moves

[20]To maintain incentive compatibility, $G$ further distorts the adversary-contract to ensure that an ally picks the ally-contract.

closer to $G$'s ideal point, $G$'s transfer to $P$ increases; and as the policy moves further away, $G$'s transfer decreases.

**Proposition 3.** *Equilibrium policies and transfers move in the same direction over time, i.e., $x_2 \geq x_1$ if and only if $t_2 \geq t_1$. Furthermore, if $P$ is sufficiently likely to be an ally ($\mu_0 > \tilde{\mu}_0$), then $x_1 \leq x_2$ and $t_1 \leq t_2$. Otherwise, $x_1 \geq x_2$ and $t_1 \geq t_2$ is also possible.*

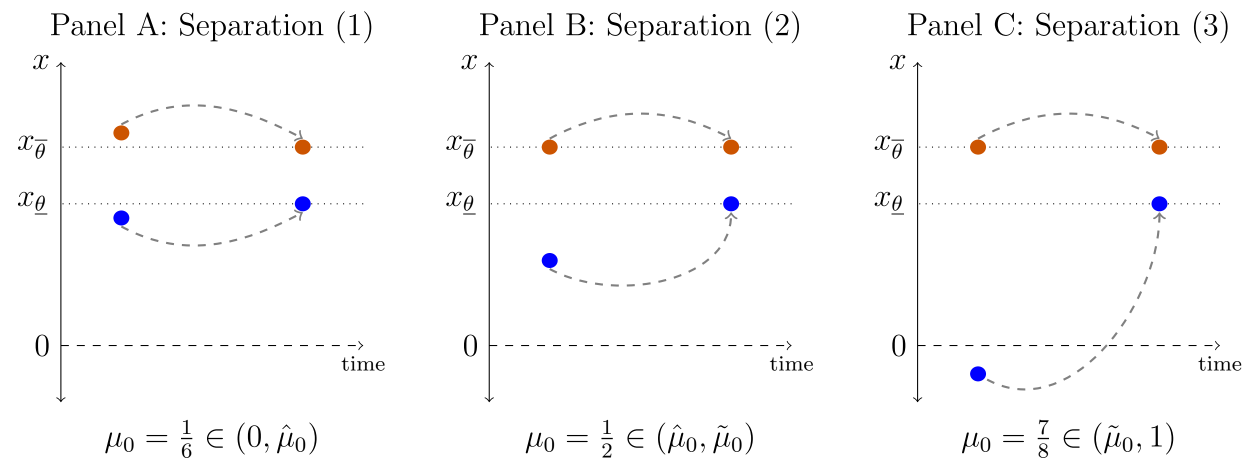
Yet, under broad conditions, the trajectories are ambiguous, with the potential to be increasing or decreasing. One would expect intuitively that $G$ becomes more influential over time. However, the only clear case of this happening is when $P$ is likely an ally, when the implemented policy either remains constant or increases.

The trajectory of observed policies and transfers is primarily driven by $G$'s learning. If $G$ offers a separating menu to learn $P$'s preference in the first period, that information facilitates an efficient second-period offer. Thus, if $G$ is relatively patient, the observed policy and transfer will either (i) start low and then increase if $P$ is an adversary, or (ii) start high and then decrease or stay constant if $P$ is an ally. Figure 5 displays the three qualitatively different possibilities.

If $G$'s first-period menu is pooling, then learning is delayed and $G$ will instead make a separating offer in the second period. Under these conditions, the observed policies will initially be low before shifting upward or downward depending on $P$'s alignment. Figure 6 displays the two possible scenarios in which $G$ offers only a single menu option in the first period, and two in the second period. In this case, second-period policies do not converge to the efficient ones, and the adversary offer is distorted downwards.

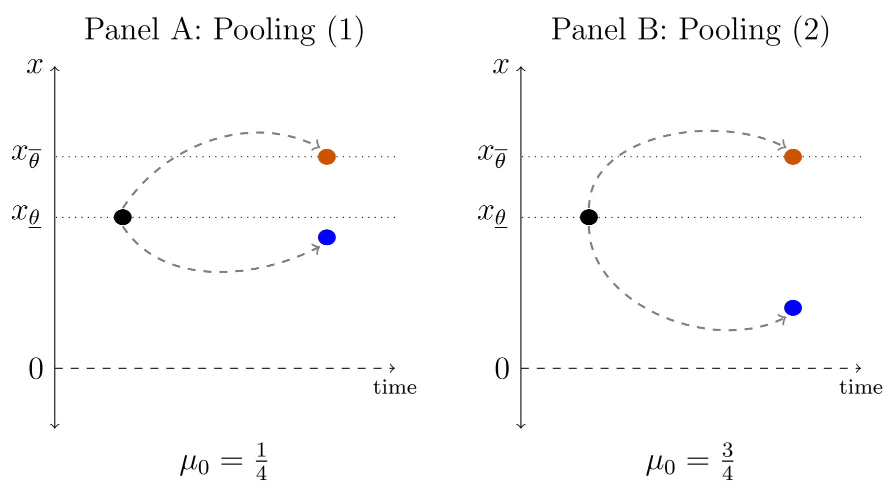
Several factors impact the magnitude of changes in observed policies and transfers over time. As $P$ becomes more patient, the shifts in both policy and transfers become larger, regardless of $P$'s type. This relationship emerges because $G$ must offer more distorted contracts to effectively learn about a patient $P$'s preferences. In contrast, the impact of $G$'s beliefs depends on $P$'s preference: for higher $\mu_0$, the adversary-contract shifts less, while the ally-

Figure 5: Three Separating Equilibrium Paths




*Note: In each panel, the dots on the left denote first-period policies while the dots on the right denote second-period policies. The blue dots indicate adversary-policies and the orange dots indicate ally-policies in equilibrium. In every scenario, equilibrium policies converge to the efficient policies* $x_{\underline{\theta}}$ *and* $x_{\overline{\theta}}$. *Each panel depicts the case with* $\underline{\theta} = 0$, $\overline{\theta} = \frac{2}{5}$, $\delta_P = 1$ *and* $\delta_G$ *sufficiently large.*

Figure 6: Two Pooling Equilibrium Paths




*Note: In each panel, the dot on the left denotes first-period policies while the dots on the right denote second-period policies. The black dot is the pooled policy for both the adversary and ally, the orange dot is the ally-policy and the blue dot is the adversary-policy. Each panel depicts the case with* $\underline{\theta} = 0$, $\overline{\theta} = \frac{2}{5}$, $\delta_P = 1$ *and* $\delta_G$ *sufficiently small.*

contract shifts more. In the limiting case where $P$ is entirely short-sighted $\delta_P = 0$, $P$'s reputational incentive disappears, and therefore its incentives to misrepresent are constant across periods. Thus, $G$ can offer contracts closer to the efficient ally and adversary contracts, while still accounting for the inherent screening obstacles that arise even without reputational considerations. Proposition 4 formally characterizes these comparative statics.

**Proposition 4.** *Suppose that the interest group is sufficiently patient, such that $\delta_G > \delta_G^{\dagger}$.*

1. *If $\delta_P$ increases, then $|x_1 - x_2|$ and $|t_1 - t_2|$ will increase.*
2. *If $\mu_0$ increases, then: (i) $|\overline{x}_1 - \overline{x}_2|$ and $|\overline{t}_1 - \overline{t}_2|$ will decrease, but (ii) $|\underline{x}_1 - \underline{x}_2|$ and $|\underline{t}_1 - \underline{t}_2|$ will increase.*

Since policies and transfers can shift in either direction depending on $P$'s type, forecasts about the dynamics of policymaking and lobbying will depend on the probability of facing an ally politician. Our focus below is on the expected policy and transfer, analyzing a weighted average of these equilibrium objects for the adversary and ally types.[21]

First, we focus on how transfers evolve over time through lobbying. When the probability of an adversary politician is high, the expected transfer to $P$ decreases over time. This is because learning is relatively costly as it requires large transfers. Otherwise, if an ally is sufficiently likely, then $P$ receives a higher expected transfer in the second period than the first period. Another effect dominates here—as an ally is more likely, $G$ is underaggressive in its adversary-contract. As a result, once $G$ learns that $P$ is an adversary, it can make a more aggressive offer, without having to worry about an ally's incentive constraint.

Second, when focusing on policies, we observe that—on average—interest groups become more influential over time if they decide to learn politicians' preferences in the first period. In expectation, $P$'s policy choice shifts towards $G$'s ideal point. The reason is that $G$ must reduce its influence in the first period to successfully learn $P$'s alignment. This is visible in

[21] There is a direct effect determined by the weights of these objects that vary with $\mu_0$. There is also an indirect effect, however, since policies and transfers can depend on $\mu_0$.

Figure 5: although the ally-policy decreases or stays constant over time, the expected policy increases.

Alternatively, when $G$ does not aim to learn politicians' preferences, it simply treats $P$ like an adversary. In this case, in the second period, $G$ is more aggressive when lobbying an ally and more conservative when lobbying an adversary. When assessing the expected value of $x_1$ and $x_2$, it can either be the case that $G$ is equally influential in both periods ($\mu_0 < \tilde{\mu}_1$) or becomes more influential over time ($\mu_0 > \tilde{\mu}_1$).[22]

Finally, policy variance simply depends on whether $G$ provides a separating or pooling offer to $P$. As displayed in Figure 5, first-period separating policy offers are more distinct than second-period policy offers. On the other hand, however, given first-period pooling, Figure 6 shows that the lack of variance in the first period is followed by more uncertain policy predictions in the second, where the level of uncertainty depends on $G$'s belief, $\mu_0$.

# Extensions

We study four extensions capturing key aspects of other political features such as hiring lobbyists, making campaign contributions, veto players, voting rules, or revolving-door hiring. We extend our main analysis to analyze: (i) early-career information, (ii) early-career access, (iii) policymaking constraints, and (iv) revolving-door incentives. For each, we highlight the impact on policies and transfers, their dynamics, as well as the degree of $G$'s policy influence.

## The value of early-career information

Interest groups often have access to various tools for learning about politicians' motivations and interests before lobbying: conducting interviews with staff members, researching

[22] The stated condition here focuses on second-period equilibrium policies, while implicitly assuming that $G$ still finds it optimal to give a pooling offer to $P$.

politicians' lawmaking and voting records, or hiring lobbyists with established connections.[23] Given these avenues for acquiring information, we address two key questions. First, what is the value of obtaining such information for interest groups compared to *learning-by-lobbying*? And, second, which factors influence the value of this information?

We analyze the value of early-career information by comparing $G$'s payoff under full information to $G$'s payoff in the main model.[24]

Under full information, $G$ receives $\pi_{\underline{\theta}}$ and $\pi_{\overline{\theta}}$, depending on $P$'s type. From $G$'s first-period perspective, its expected payoff is:

$$V_G^{informed} = (1 + \delta_G)\left((1 - \mu_0)\pi_{\underline{\theta}} + \mu_0 \pi_{\overline{\theta}}\right).$$

There are two main cases. In the first, $G$ offers a separating menu and learns $P$'s preferences in the first period. Here, information is only valuable in the first period, since $G$ will be fully informed in the second period regardless of having access to early-career information. The value of information is proportionate to the distortions $G$ induces when lobbying.

In the second case, $G$ makes a pooling offer in the first period, and then a separating offer in the second. Here, information is valuable in both periods, since it allows for efficient lobbying compared to the lack of efficient contracts absent information. In the first period, the adversary-contract is efficient, while the ally-contract is distorted. Thus, the first-period value of information equals $\mu_0(\pi_{\underline{\theta}} - \pi_{\overline{\theta}})$. In the second period, the value of information depends on the prior, $\mu_0$. If $\mu_0 \leq \tilde{\mu}_1$, then $G$'s second-period value of information is $\mu_0(\pi_{\overline{\theta}} - \frac{(\overline{\theta}-\underline{\theta})^2}{1-\mu_0} - \pi_{\underline{\theta}})$. Otherwise, it is $(1-\mu_0)(-\pi_{\overline{\theta}})$.[25] Proposition 5 summarizes these findings.

[23]For instance, a primary motive for revolving-door hiring is "buying advice on who is likely to be sympathetic to them on a particular issue, how best to win the support of particular members" (Drutman 2015, p. 163).

[24]See Lemma 1 and Lemma 2.

[25]Recall that $\pi_{\overline{\theta}}$ is negative, implying a positive value of information.

**Proposition 5.** *The value of early-career information is positive. Furthermore, it is (i) weakly increasing in the politician's patience, $\delta_P$, but (ii) weakly decreasing in the interest group's patience, $\delta_G$.*

It is always valuable to know more about $P$'s preferences but $G$'s willingness to pay for this information depends on the political context. When politicians are more forward-looking, screening becomes more expensive, making $G$ willing to pay more to avoid this screening cost. When $G$ is more patient, it is more willing to learn $P$'s preferences in the first place, making early-career information relatively less beneficial.

## The value of early-career access

To lobby particular politicians, interest groups typically need access (Hansen 1991). This access often requires strategic investments to establish relationships, such as hiring lobbyists (Blanes i Vidal, Draca and Fons-Rosen 2012; Bertrand, Bombardini and Trebbi 2014) or providing campaign contributions (Fouirnaies 2018; Kim, Stuckatz and Wolters 2025). To quantify the importance of early-career access, we compare two settings: (i) *full access*, where $G$ can lobby in both periods (as in our main model), and (ii) *late-career access*, where $G$ can only lobby in the second period.

We define the *early-career value of access* as the difference between $G$'s equilibrium payoff with full access versus its payoff with only late-career access. This quantity measures the strategic importance of early engagement with politicians.

We show that politicians may have incentives to misrepresent their positions even when $G$ is absent in the first period and does not lobby. These incentives stem from the anticipation of lobbying efforts in the second period. Similar to our main model, ally politicians may be motivated to feign disagreement to secure more favorable terms from interest groups in subsequent interactions. The strength of these incentives can be substantial. In some cases, ally politicians might moderate their chosen policies without receiving any immediate transfer, solely based on the prospect of future lobbying.

Two equilibrium categories may arise. First, both politician types may choose their preferred policies, $\theta$, ensuring that $G$ successfully learns $P$'s preferences even without screening. Second, an ally may mimic an adversary and choose policy $x_1 = \underline{\theta}$, which implies that $G$ does not learn any new information about $P$.[26]

The value of early-career access depends on several factors, especially $P$'s patience and $G$'s belief about $P$'s alignment. Proposition 6 characterizes the value of access.

**Proposition 6.** *Suppose $\delta_G > \delta_G^{\dagger}$. If $\delta_P \leq \frac{\overline{\theta}-\theta}{1-\overline{\overline{\theta}}}$, then $G$'s value of early-career access is positive and constant in $\delta_G$ but decreasing in $\delta_P$. Otherwise, $G$'s value of early-career access may be positive or negative, and is increasing in $\delta_G$ but decreasing in $\delta_P$.*

Interestingly, there are cases in which $G$ may opt to forego access. At the cost of not being able to influence $P$ in the first period, $G$ may be better off learning about $P$'s alignment for free. Especially if $P$ is more patient, which raises $G$'s costs of learning-by-lobbying.

This extension highlights how politicians' job security, career stage, or familiarity can affect interest groups' desire to cultivate relationships with them. Our learning-by-lobbying mechanism sheds new light on why the value of access varies across politicians based on factors like their procedural rights (Fouirnaies and Hall 2018; Berry and Fowler 2018).

## Lobbying with policymaking constraints

Politicians have different degrees of policy influence, with some playing more active roles in developing and drafting proposals on particular issues than others. Yet, even the most powerful politicians often face constraints when crafting proposals due to, e.g., voting rules or veto players. How do such policymaking constraints affect lobbying, influence, and the value of access?

To study this question, we extend our model by restricting the set of policies that $P$ can

[26] Mixed strategy equilibria also exist, causing $G$ to only partially learn what $P$ prefers. For the sake of presentation, we omit such equilibria given our focus on pure strategy equilibria.

enact.[27] Formally, we introduce a maximum policy, denoted $\bar{y}$, which $P$'s policy cannot exceed. This extension yields two key strategic implications. First, the constraint raises $G$'s cost of influencing $P$ through transfers, since $G$ can only offer $P$ a more limited set of policies. Second, the constraint also lowers $G$'s value of information obtained by screening $P$ in the first period, since there is less freedom to use this information when influencing $P$ in the second period.

Qualitatively, equilibrium strategies remain relatively similar to the main model, with the exception that an ally will enact policies $\bar{x}_1 = \bar{x}_2 = \bar{y}$. This difference has several effects. First, $G$ can offer a lower transfer to encourage an ally to accept the contract rather than reject it. Second, the ally contract becomes less attractive, so there are stronger incentives to choose the adversary-contract. Third, to maintain incentive compatibility, $G$ must either make a more attractive ally-contract (by increasing its associated transfer) or make a less attractive adversary-contract (by decreasing its associated transfer or policy). Fourth, $G$ must ensure that an adversary is willing to accept the offer relative to rejecting, implying that a decrease in transfer must coincide with a decrease in policy. Overall, these effects imply that $G$, when lobbying a more constrained $P$, reduces its influence. Also, $G$ is more likely to moderate its first-period influence given that the costs of learning increase while the benefits of learning decrease. Proposition 7 formalizes these observations.

**Proposition 7.** *The value of information increases in $\bar{y}$ (i.e., policymaking constraints loosen) subsequently leading to more learning. Furthermore, that effect is stronger if the interest group is more patient, i.e., as $\delta_G$ increases.*

Interest groups do not just have more to gain from influencing powerful politicians, but it is also more valuable to know their alignment. This highlights that committee leaders may not just attract more campaign contributions because of their power, but in dynamic contexts

[27] Alternatively, suppose moving policy from a status quo is costlier for some politicians than others. Since lobbying politicians who can shift policy more easily is more rewarding, learning about them is more valuable. Thus, qualitatively similar results should arise. We thank a reviewer for this point.

with uncertainty, the value of information may attract even greater contributions.

## Revolving-door incentives

In our final extension, we study how politicians' revolving-door incentives can affect lobbying and policymaking dynamics. The revolving door is a lucrative post-government option (Blanes i Vidal, Draca and Fons-Rosen 2012; Bertrand, Bombardini and Trebbi 2014; McCrain 2018) that can encourage politicians to signal their alignment with potential future employers while still in office (Cornaggia, Cornaggia and Xia 2016; Tabakovic and Wollmann 2018). We analyze how this incentive interacts with our core learning-by-lobbying forces to shape lobbying and politicians' observed in-office behavior.

We introduce a new parameter, $R > 0$, representing the additional payoff $P$ receives if hired by $G$ after the second period. We assume that $G$ is willing to hire $P$ if only if it is sufficiently certain about their alignment. That is, we assume that the belief about $P$'s type must be sufficiently high before hiring.

The benefit of appearing an ally has two key strategic implications. First, $P$'s signaling incentives become more complex. Previously, an ally was motivated to signal misalignment solely to secure better second-period lobbying. Now, this incentive is counterbalanced by the desire to become a revolving-door lobbyist. Second, $G$ faces lower costs for learning $P$'s preferences. It can offer lower transfers when providing $P$ a separating first-period contract. These learning costs decrease with the value of becoming a revolving-door lobbyist. These strategic implications lead to the main empirical implication of this extension.

**Proposition 8.** *Increasing the value of the revolving door expands the set of parameters under which $G$ successfully learns $P$'s preferences.*

Interestingly, $G$ benefits from situations in which $P$ strongly values becoming a revolving-door lobbyist. Especially larger firms, which can promise and offer higher future wages and better careers to incumbent politicians, may also learn more quickly whether they are dealing with an ally or adversary. This effect on $G$'s expected payoff is amplified by $P$'s patience,

$\delta_P$, and $G$'s belief that it faces an ally politician, $\mu_0$.

**Proposition 9.** *The interest group's equilibrium payoff increases in the politician's value of the revolving door, which is itself increasing in $\delta_P$ and $\mu_0$.*

This result suggests that powerful and wealthy interest groups do not just benefit from their ability to contribute more to politicians. They also benefit from the potential to attract those politicians, who then have more incentives to appear as allies. This alignment, in turn, benefits these interest groups in settings with *learning-by-lobbying*.

# Discussion and Conclusion

We study how lobbying relationships evolve when interest groups may learn about politicians by lobbying them. Our game-theoretic analysis clarifies how *learning-by-lobbying* can impact patterns of lobbying behavior throughout political careers and across institutional contexts. This fundamental mechanism complements previous insights about information transmission, legislative subsidies, or financial incentives. In doing so, we sharpen theoretical understanding of information, influence, and policy outcomes.

Our main analysis has implications for empirical patterns in lobbying. Early-career politicians with secure positions can attract intense lobbying partly due to their stronger reputational considerations. Additionally, our learning-by-lobbying mechanism may contribute to observed patterns of generous treatment toward allies, especially early in politicians' careers—a pattern that standard exchange models struggle to explain. Our model also predicts systematic variation in policy uncertainty across career stages, with more dispersed policy choices by newcomers and a convergence in late-career lobbying as learning and reputation forces diminish. Politicians receiving aggressive lobbying early might face milder approaches later, and vice versa—not because their preferences changed, but because interest groups learned about them.

Our extensions offer additional empirical insights into broader interest group activities. Our early-career information extension suggests interest groups are especially inclined to solicit information about patient politicians with secure positions. In contrast, our early-career access extension suggests groups are less inclined to seek access to those same politicians. Our policymaking constraint extension suggests they are less inclined to solicit information about politicians with limited discretion. Notably, some interest group activities—like hiring former staffers—serve both information and access functions, complicating empirical expectations. Finally, our revolving-door extension suggests learning-by-lobbying forces can encourage wealthy industries with lucrative post-career opportunities to favor access cultivation over background research and enjoy lower overall lobbying costs.

Our work has broader implications for democratic representation and special interest influence. Since new politicians increasingly lack extensive public-service track records (Porter and Treul 2025), the ability to understand and anticipate policymakers' preferences is increasingly valuable for shaping legislation. We set aside factors such as competing interest groups, multiple politicians, and electoral considerations that have been emphasized elsewhere (Austen-Smith and Wright 1994; Bils, Duggan and Judd 2021; Groseclose and Snyder 1996). Although relevant in certain contexts, such factors are not central to our core insights about learning and influence. They are promising avenues for future work.

## References

Austen-Smith, David. 1995. "Campaign Contributions and Access." *American Political Science Review* 89(3):566–581.

Austen-Smith, David and John R. Wright. 1992. "Competitive Lobbying for a Legislator's Vote." *Social Choice and Welfare* 9(3):229–257.

Austen-Smith, David and John R. Wright. 1994. "Counteractive Lobbying." *American Journal of Political Science* 38(1):25–44.

Awad, Emiel. 2020. "Persuasive Lobbying with Allied Legislators." *American Journal of Political Science* 64(4):938–951.

Bar-Isaac, Heski and Joel Shapiro. 2011. "Credit Ratings Accuracy and Analyst Incentives." *American Economic Review: Papers and Proceedings* 101(3):120–124.

Beccuti, Juan and Marc Möller. 2018. "Dynamic Adverse Selection with a Patient Seller." *Journal of Economic Theory* 173:95–117.

Bernheim, B. Douglas and Michael D. Whinston. 1986. "Menu Auctions, Resource Allocation, and Economic Influence." *Quarterly Journal of Economics* 101(1):1–31.

Berry, Christopher R. and Anthony Fowler. 2018. "Congressional Committees, Legislative Influence, and the Hegemony of Chairs." *Journal of Public Economics* 158:1–11.

Bertrand, Marianne, Matilde Bombardini and Francesco Trebbi. 2014. "Is It Whom You Know or What You Know? An Empirical Assessment of the Lobbying Process." *American Economic Review* 104(12):3885–3920.

Besley, Timothy and Stephen Coate. 2001. "Lobbying and Welfare in a Representative Democracy." *The Review of Economic Studies* 68(1):67–82.

Bester, Helmut and Roland Strausz. 2001. "Contracting with Imperfect Commitment and the Revelation Principle: The Single Agent Case." *Econometrica* 69(4):1077–1098.

Bils, Peter, John Duggan and Gleason Judd. 2021. "Lobbying and Policy Extremism in Repeated Elections." *Journal of Economic Theory* 193:105223.

Blanes i Vidal, Jordi, Mirko Draca and Christian Fons-Rosen. 2012. "Revolving Door Lobbyists." *American Economic Review* 102(7):3731–3748.

Bombardini, Matilde and Francesco Trebbi. 2020. "Empirical Models of Lobbying." *Annual Review of Economics* 12:391–413.

Breig, Zachary. 2022. "Repeated Contracting without Commitment." *Journal of Economic Theory* 204:105514.

Che, Yeon-Koo. 1995. "Revolving Doors and the Optimal Tolerance for Agency Collusion." *RAND Journal of Economics* 26(3):378–397.

Chen, Ying and Jan Zápal. 2022. "Sequential Vote Buying." *Journal of Economic Theory* 205:105529.

Cornaggia, Jess, Kimberly J. Cornaggia and Han Xia. 2016. "Revolving Doors on Wall Street." *Journal of Financial Economics* 120(2):400–419.

Drutman, Lee. 2015. *The Business of America is Lobbying*. Oxford University Press New York.

Egerod, Benjamin C.K. and Hai Tran. 2023. "Extremists Not on Board: Labor Market Costs to Radical Behavior in Elected Office." *Journal of Politics* 85(3):1161–1165.

Ellis, Christopher J. and Thomas Groll. 2024. "Who Lobbies Whom? Special Interests and Hired Guns." Unpublished manuscript.

Fouirnaies, Alexander. 2018. "When Are Agenda Setters Valuable?" *American Journal of Political Science* 62(1):176–191.

Fouirnaies, Alexander and Andrew B. Hall. 2018. "How Do Interest Groups Seek Access to Committees?" *American Journal of Political Science* 62(1):132–147.

Freixas, Xavier, Roger Guesnerie and Jean Tirole. 1985. "Planning under Incomplete Information and the Ratchet Effect." *The Review of Economic Studies* 52(2):173–191.

Fudenberg, Drew and Jean Tirole. 1991. *Game Theory*. MIT Press.

Gerardi, Dino and Lucas Maestri. 2020. "Dynamic Contracting with Limited Commitment and the Ratchet Effect." *Theoretical Economics* 15(2):583–623.

Groll, Thomas and Christopher J. Ellis. 2014. "A Simple Model of the Commercial Lobbying Industry." *European Economic Review* 70:299–316.

Groll, Thomas and Christopher J. Ellis. 2017. "Repeated Lobbying by Commercial Lobbyists and Special Interests." *Economic Inquiry* 55(4):1868–1897.

Groseclose, Tim and James M. Snyder. 1996. "Buying Supermajorities." *The American Political Science Review* 90(2):303–315.

Grossman, Gene M. and Elhanan Helpman. 1994. "Protection for Sale." *American Economic Review* 84(4):833–850.

Grossman, Gene M. and Elhanan Helpman. 2001. *Special Interest Politics*. MIT Press.

Hall, Richard L. and Alan V. Deardorff. 2006. "Lobbying as Legislative Subsidy." *American Political Science Review* 100(1):69–84.

Hall, Richard L and Frank W Wayman. 1990. "Buying Time: Moneyed Interests and the Mobilization of Bias in Congressional Committees." *American Political Science Review* 84(3):797–820.

Hansen, John Mark. 1991. *Gaining Access: Congress and the Farm Lobby, 1919-1981*. University of Chicago Press.

Hart, Oliver D and Jean Tirole. 1988. "Contract Renegotiation and Coasian Dynamics." *The Review of Economic Studies* 55(4):509–540.

Hirsch, Alexander V., Karam Kang, B. Pablo Montagnes and Hye Young You. 2023. "Lobbyists as Gatekeepers: Theory and Evidence." *The Journal of Politics* 85(2):731–748.

Hübert, Ryan, Janna King Rezaee and Jonathan Colner. 2023. "Going into Government: How Hiring from Special Interests Reduces Their Influence." *American Journal of Political Science* 67(2):485–498.

Iaryczower, Matías and Santiago Oliveros. 2017. "Competing for Loyalty: The Dynamics of Rallying Support." *American Economic Review* 107(10):2990–3005.

Judd, Gleason. 2023. "Access to Proposers and Influence in Collective Policy Making." *The Journal of Politics* 85(4):1430–1443.

Kerr, William R., William F. Lincoln and Prachi Mishra. 2014. "The Dynamics of Firm Lobbying." *American Economic Journal: Economic Policy* 6(4):343–379.

Kim, In Song, Jan Stuckatz and Lukas Wolters. 2025. "Strategic and Sequential Links Between Campaign: Donations and Lobbying." *Journal of Politics* forthcoming.

Kolotilin, Anton, Tymofiy Mylovanov, Andriy Zapechelnyuk and Ming Li. 2017. "Persuasion of a Privately Informed Receiver." *Econometrica* 85(6):1949–1964.

Laffont, Jean-Jacques and Jean Tirole. 1990. "Adverse Selection and Renegotiation in Procurement." *The Review of Economic Studies* 57(4):597–625.

LaPira, Timothy and Herschel F. Thomas III. 2017. *Revolving Door Lobbying: Public Service, Private Influence, and the Unequal Representation of Interests*. University Press of Kansas.

Levine, Bertram J. 2008. *The Art of Lobbying: Building Trust and Selling Policy*. CQ Press.

Manelli, Alejandro M. 1997. "The Never-a-weak-best-response Test in Infinite Signaling Games." *Journal of Economic Theory* 74(1):152–173.

Martimort, David and Aggey Semenov. 2008. "Ideological Uncertainty and Lobbying Competition." *Journal of Public Economics* 92(3-4):456–481.

McChesney, Fred S. 1997. *Money for Nothing: Politicians, Rent Extraction, and Political Extortion*. Harvard University Press.

McCrain, Joshua. 2018. "Revolving Door Lobbyists and the Value of Congressional Staff Connections." *The Journal of Politics* 80(4):1369–1383.

Minaudier, Clement. 2022. "The Value of Confidential Policy Information: Persuasion, Transparency, and Influence." *The Journal of Law, Economics, and Organization* 38(2):570–612.

Porter, Rachel and Sarah A. Treul. 2025. "Evaluating (in)experience in Congressional Elections." *American Journal of Political Science* 69(1):284–298.

Salanié, Bernard. 2005. *The Economics of Contracts: A Primer*. MIT Press.

Schnakenberg, Keith E. 2017. "Informational Lobbying and Legislative Voting." *American Journal of Political Science* 61(1):129–145.

Schnakenberg, Keith E. and Ian R. Turner. 2023. "Formal Theories of Special Interest Influence." *Annual Review of Political Science* 27:401–421.

Tabakovic, Haris and Thomas G. Wollmann. 2018. From Revolving Doors to Regulatory Capture? Evidence from Patent Examiners. Technical report National Bureau of Economic Research.

Supplementary Information for

"Learning by Lobbying"

# Part I

# Online Appendix

## Table of Contents

# A Appendix: Proofs of Main Results

## A.1 Proof of Lemma 1

In the second period, the interest group has the following constrained maximization problem given belief $\mu := \mu_1 \in (0,1)$,

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-\left(\overline{x}-1\right)^2-\overline{T}\right)+(1-\mu)\left(-\left(\underline{x}-1\right)^2-\underline{T}\right)$$
$$s.t. \quad (IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}).$$

The four restrictions include two incentive compatibility constraints and two participation constraints:

$$-\left(\overline{x}-\overline{\theta}\right)^2+\overline{T} \geq -\left(\underline{x}-\overline{\theta}\right)^2+\underline{T}, \tag{$IC_{\overline{\theta}}$}$$
$$-\left(\underline{x}-\underline{\theta}\right)^2+\underline{T} \geq -\left(\overline{x}-\underline{\theta}\right)^2+\overline{T}, \tag{$IC_{\underline{\theta}}$}$$
$$-\left(\overline{x}-\overline{\theta}\right)^2+\overline{T} \geq 0, \tag{$P_{\overline{\theta}}$}$$
$$-\left(\underline{x}-\underline{\theta}\right)^2+\underline{T} \geq 0. \tag{$P_{\underline{\theta}}$}$$

We study the following relaxed problem and then verify that the solution satisfies the constraint $(IC_{\underline{\theta}})$:

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-\left(\overline{x}-1\right)^2-\overline{T}\right)+(1-\mu)\left(-\left(\underline{x}-1\right)^2-\underline{T}\right)$$
$$s.t. \quad (IC_{\overline{\theta}}), (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}).$$

We begin our analysis by setting up the Lagrangian for the relaxed problem:

$$\begin{aligned}\mathcal{L} = &\ \mu\left(-\left(\overline{x}-1\right)^2-\overline{T}\right)+(1-\mu)\left(-\left(\underline{x}-1\right)^2-\underline{T}\right)\\ &+\lambda_1\left(-\left(\overline{x}-\overline{\theta}\right)^2+\overline{T}+\left(\underline{x}-\overline{\theta}\right)^2-\underline{T}\right)\\ &+\lambda_2\left(-\left(\overline{x}-\overline{\theta}\right)^2+\overline{T}\right)\\ &+\lambda_3\left(-\left(\underline{x}-\underline{\theta}\right)^2+\underline{T}\right),\end{aligned}$$

where $\lambda_1$, $\lambda_2$ and $\lambda_3$ are the multipliers for $(IC_{\overline{\theta}})$, $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ respectively. Using this notation, the Kuhn-Tucker first-order necessary conditions are given as follows:

The first order conditions with respect to the tuple $(\underline{x}, \overline{x}, \underline{T}, \overline{T})$ are:

$$\frac{\partial \mathcal{L}}{\partial \underline{x}} = -2(1-\mu)(\underline{x}-1) + 2\lambda_1(\underline{x}-\overline{\theta}) - 2\lambda_3(\underline{x}-\underline{\theta}) = 0, \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial \overline{x}} = -2\mu(\overline{x}-1) - 2\lambda_1(\overline{x}-\overline{\theta}) - 2\lambda_2(\overline{x}-\overline{\theta}) = 0, \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{T}} = -(1-\mu) - \lambda_1 + \lambda_3 = 0, \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial \overline{T}} = -\mu + \lambda_1 + \lambda_2 = 0. \tag{4}$$

The complementary slackness conditions for $(IC_{\overline{\theta}}), (P_{\overline{\theta}})$, and $(P_{\underline{\theta}})$ respectively are:

$$\lambda_1 \left(-\left(\overline{x}-\overline{\theta}\right)^2 + \overline{T} + \left(\underline{x}-\overline{\theta}\right)^2 - \underline{T}\right) = 0, \tag{5}$$

$$\lambda_2 \left(-\left(\overline{x}-\overline{\theta}\right)^2 + \overline{T}\right) = 0, \tag{6}$$

$$\lambda_3 \left(-\left(\underline{x}-\underline{\theta}\right)^2 + \underline{T}\right) = 0. \tag{7}$$

The non-negative Lagrangian multipliers and the constraints are:

$$\lambda_1, \lambda_2, \lambda_3 \geq 0, \tag{8}$$

$$(IC_{\overline{\theta}}), (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}). \tag{9}$$

From (3), we deduce that $\lambda_3 = \lambda_1 + (1-\mu) > 0$. Condition (7) then implies $\left(-\left(\underline{x}-\underline{\theta}\right)^2 + \underline{T}\right) = 0$. From (4), we deduce that $\lambda_1 + \lambda_2 = \mu$. We have three possible cases: (i) $\lambda_1 > 0$ and $\lambda_2 > 0$, (ii) $\lambda_1 = \mu$, $\lambda_2 = 0$, and (iii) $\lambda_1 = 0$, $\lambda_2 = \mu$.

*Case (i).* The fact that $\lambda_1 > 0$ and $\lambda_2 > 0$ implies that constraints $(IC_{\overline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are all binding. These binding constraints plus the first-order conditions create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}^* = \frac{\underline{\theta}+\overline{\theta}}{2},\ \overline{x}^* = \frac{1+\overline{\theta}}{2},\ \underline{T}^* = \frac{1}{4}\left(\overline{\theta}-\underline{\theta}\right)^2, \overline{T}^* = \frac{1}{4}\left(1-\overline{\theta}\right)^2,$$
$$\lambda_1^* = \frac{(1-\mu)(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_2^* = \frac{\mu(1-\underline{\theta})-(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3^* = \frac{(1-\mu)(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

Note that $\lambda_1^*, \lambda_3^* > 0$. Also, $\lambda_2^* > 0$ if $\mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}$. Thus, if $\mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}$, the vector $\left(\underline{x}^*, \overline{x}^*, \underline{T}^*, \overline{T}^*, \lambda_1^*, \lambda_2^*, \lambda_3^*\right)$ satisfies the Kuhn-Tucker first-order necessary conditions of the relaxed problem.

*Case (ii).* Now $(IC_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and—together with the first-order conditions—

create a system of 6 equations and 6 unknowns with the following solution:

$$\underline{x}' = \frac{1+\underline{\theta}-\mu(1+\overline{\theta})}{2(1-\mu)},\ \overline{x}' = \frac{1+\overline{\theta}}{2},$$
$$\underline{T}' = \frac{1}{4}\frac{(1-\underline{\theta}-\mu(1+\overline{\theta}-2\underline{\theta}))^2}{(1-\mu)^2},\ \overline{T}' = \frac{1}{4}\frac{(1-\overline{\theta})(1+3\overline{\theta}-4\underline{\theta})-\mu(1+\overline{\theta}-2\underline{\theta})^2}{(1-\mu)},$$
$$\lambda_1' = \mu,\ \lambda_2' = 0,\ \lambda_3' = 1.$$

Replacing these values on the constraint $(P_{\overline{\theta}})$, we obtain that it must be that $\frac{(\overline{\theta}-\underline{\theta})(1-\overline{\theta}-\mu(1-\underline{\theta}))}{(1-\mu)} \geq 0$ which is satisfied if $\mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}$. Thus, if $\mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}$, the vector $\left(\underline{x}', \overline{x}', \underline{T}', \overline{T}', \lambda_1', \lambda_2', \lambda_3'\right)$ satisfies the Kuhn-Tucker first-order necessary conditions of the relaxed problem.

*Case (iii).* Now $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 6 equations and 6 unknowns with the following solution:

$$\underline{x}'' = \frac{\underline{\theta}+1}{2},\ \overline{x}'' = \frac{1+\overline{\theta}}{2},\ \underline{T}'' = \frac{1}{4}(1-\underline{\theta})^2,\ \overline{T}'' = \frac{1}{4}\left(1-\overline{\theta}\right)^2,$$
$$\lambda_1'' = 0,\ \lambda_2'' = \mu,\ \lambda_3'' = (1-\mu).$$

Replacing these values on the constraint $(IC_{\overline{\theta}})$ we obtain that it must be that $-(1-\overline{\theta})(\overline{\theta}-\underline{\theta}) \geq 0$ which is never satisfied. Thus, there are no values that satisfy the Kuhn-Tucker first-order necessary conditions in case (iii).

Note that $\frac{\partial^2 \mathcal{L}}{\partial \underline{x}^2} = -2(1-\mu) + 2(\lambda_1 - \lambda_3)$. From (3) we have that $(\lambda_1 - \lambda_3) = -(1-\mu)$, and then $\frac{\partial^2 \mathcal{L}}{\partial \underline{x}^2} < 0$. Also, $\frac{\partial^2 \mathcal{L}}{\partial \overline{x}^2} = -2\mu - 2\lambda_1 - 2\lambda_2 < 0$. Thus, the Lagrangian function is strictly concave and the Kuhn-Tucker first-order conditions are also sufficient. In sum, the solution of the relaxed problem is the following:

$$(\underline{x}, \overline{x}, \underline{T}, \overline{T}) = \begin{cases} \left(\frac{\underline{\theta}+\overline{\theta}}{2}, \frac{1+\overline{\theta}}{2}, \frac{1}{4}\left(\overline{\theta}-\underline{\theta}\right)^2, \frac{1}{4}\left(1-\overline{\theta}\right)^2\right) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \left(\frac{1+\underline{\theta}-\mu(1+\overline{\theta})}{2(1-\mu)}, \frac{1+\overline{\theta}}{2}, \frac{1}{4}\frac{(1-\underline{\theta}-\mu(1+\overline{\theta}-2\underline{\theta}))^2}{(1-\mu)^2}, \frac{1}{4}\frac{(1-\overline{\theta})(1+3\overline{\theta}-4\underline{\theta})-\mu(1+\overline{\theta}-2\underline{\theta})^2}{(1-\mu)}\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

After some calculations, it is direct to see that the solution of the relaxed problem strictly satisfies $(IC_{\underline{\theta}})$. Thus, it is a solution of the original problem.

The interest group's expected payoff in the second period if it offers two contracts can be simplified to

$$V_2 = \begin{cases} (1-\mu)\left(-\frac{(1-\overline{\theta})^2}{2} - \frac{(1-\underline{\theta})^2}{2}\right) + \mu\left(-\frac{(1-\overline{\theta})^2}{2}\right) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \frac{1}{2(1-\mu)}\left((1-\mu)(-(1-\underline{\theta})^2) + \mu(\overline{\theta}-\underline{\theta})^2\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

We now consider $G$'s possibility to offer a single contract. In that case, incentive compatibility constraints are trivially satisfied, and only the participation constraints are relevant. Consider the following alternative set of contracts: (i) no type accepts, (ii) only an adversary accepts, (iii) only an ally accepts, or (iv) both types accept. These sets cover all possible single-contract offers. In each of the cases where only one type accepts, the participation constraint is binding since if that is not the case, $G$ can always decrease $T$ by a small amount and strictly benefit from it. In case (i), since no type accepts, $G$ directly obtains:

$$V_0 = -\mu \left(\overline{\theta} - 1\right)^2 - (1-\mu)\left(\underline{\theta} - 1\right)^2.$$

In case (ii), $G$ offers a contract that is only accepted by $\underline{\theta}$. Using an analogue approach to the two-different contracts case, we find that the solution is

$$(x, T) = \left(\frac{\underline{\theta} + \overline{\theta}}{2}, \left(\frac{\underline{\theta} + \overline{\theta}}{2} - \underline{\theta}\right)^2\right).$$

The interest group's expected payoff in this case is

$$V_1^{\underline{\theta}} = (\overline{\theta} - 1)^2(-\mu) - \frac{1}{4}(\mu - 1)(\overline{\theta} + \underline{\theta} - 2)^2.$$

In case (iii), the solution is the following:

$$(x, T) = \left(\frac{1 + \overline{\theta}}{2}, \left(\frac{1 + \overline{\theta}}{2} - \overline{\theta}\right)^2\right).$$

The interest group's expected payoff in this case is

$$V_1^{\overline{\theta}} = (1-\mu)(-(\underline{\theta} - 1)^2) + \mu\left(-\frac{1}{2}(1 - \overline{\theta})^2\right).$$

Case (iv) is included in the feasible set of the original problem where $G$ offers two contracts. Thus, in general, the expected payoff of $G$ is $V = \max\{V_2, V_0, V_1^{\underline{\theta}}, V_1^{\overline{\theta}}\}$. After some algebra, it is straightforward to verify that $V = V_2$ using the explicit expressions for $G$'s expected payoff in each of the cases.

## A.2 Proofs of Remarks 1 and 2

By Lemma 1, the expected payoffs for $P$ and $G$ in equilibrium as a function of the belief $\mu$ are the following:

$$V_P(\theta, \mu) = \begin{cases} \frac{(\overline{\theta} - \underline{\theta})(1 - \mu - \overline{\theta} + \mu\underline{\theta})}{1-\mu} & \text{if } \theta = \overline{\theta} \text{ and } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ 0 & \text{if } \theta = \overline{\theta} \text{ and } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ 0 & \text{if } \theta = \underline{\theta}. \end{cases}$$

$$V_G(\mu)=\begin{cases}\frac{1}{2(1-\mu)}\left((1-\mu)(-(1-\underline{\theta})^2)+\mu(\overline{\theta}-\underline{\theta})^2\right) & \text{if } \mu\leq\frac{1-\overline{\theta}}{1-\underline{\theta}},\\ (1-\mu)\left(-\frac{(1-\overline{\theta})^2}{2}-\frac{(1-\underline{\theta})^2}{2}\right)+\mu\left(-\frac{(1-\overline{\theta})^2}{2}\right) & \text{if } \mu>\frac{1-\overline{\theta}}{1-\underline{\theta}}.\end{cases}$$

Note that both functions are continuous in $\mu$ given that they are equal at the threshold of $\frac{1-\overline{\theta}}{1-\underline{\theta}}$ and continuous everywhere else.

## A.3 Proof of Lemma 2

Suppose $\mu\in(0,1)$. We proceed in two steps. Step 1 focuses on separating contracts. Step 2 focuses on a pooling contract.

*Step 1. Separation.* If $G$ offers separating contracts implies that continuation values are $V_P(\overline{\theta},1)=V_P(\underline{\theta},0)=0$ in equilibrium. By deviating, type $\underline{\theta}$ would earn $V_P(\underline{\theta},\mu)=0$ for all $\mu\in[0,1]$. By mimicking $\underline{\theta}$, type $\overline{\theta}$ would earn $V_P(\overline{\theta},0)=(\overline{\theta}-\underline{\theta})(1-\overline{\theta})$.

Conditional on separation, $G$'s optimization problem is:

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}}\mu\left(-(\overline{x}-1)^2-\overline{T}+\delta_G V_G(1)\right)+(1-\mu)\left(-(\underline{x}-1)^2-\underline{T}+\delta_G V_G(0)\right)$$
$$s.t.\ (IC_{\overline{\theta}}),(IC_{\underline{\theta}}),(P_{\overline{\theta}})\text{ and }(P_{\underline{\theta}}),$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P V_P(\overline{\theta},1)\geq-(\underline{x}-\overline{\theta})^2+\underline{T}+\delta_P V_P(\overline{\theta},0), \qquad (IC_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2+\underline{T}+\delta_P V_P(\underline{\theta},0)\geq-(\overline{x}-\underline{\theta})^2+\overline{T}+\delta_P V_P(\underline{\theta},1), \qquad (IC_{\underline{\theta}})$$
$$-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P V_P(\overline{\theta},1)\geq 0, \qquad (P_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2+\underline{T}+\delta_P V_P(\underline{\theta},0)\geq 0. \qquad (P_{\underline{\theta}})$$

We consider the Never a Weak Best Response (NWBR) equilibrium refinement (Fudenberg and Tirole 1991). This refinement implies that any politician type obtains zero payoff from an off-path deviation.[28] We begin our analysis by setting up the Lagrangian

$$\begin{aligned}\mathcal{L}=&\mu\left(-(\overline{x}-1)^2-\overline{T}+\delta_G V_G(1)\right)+(1-\mu)\left(-(\underline{x}-1)^2-\underline{T}+\delta_G V_G(0)\right)\\&+\lambda_1\left(-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P V_P(\overline{\theta},1)+(\underline{x}-\overline{\theta})^2-\underline{T}-\delta_P V_P(\overline{\theta},0)\right)\\&+\lambda_2\left(-(\underline{x}-\underline{\theta})^2+\underline{T}+\delta_P V_P(\underline{\theta},0)+(\overline{x}-\underline{\theta})^2-\overline{T}-\delta_P V_P(\underline{\theta},1)\right)\\&+\lambda_3\left(-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P V_P(\overline{\theta},1)\right)\\&+\lambda_4\left(-(\underline{x}-\underline{\theta})^2+\underline{T}+\delta_P V_P(\underline{\theta},0)\right).\end{aligned}$$

[28] There are two cases. In some cases a profitable rejection must come from a type $\overline{\theta}$. In this case, the refinement requires $\mu=1$. In the other case, NWBR does not apply, and we directly impose that $\mu=1$.

The first-order conditions with respect to $\underline{x}, \overline{x}, \underline{T}, \overline{T}$ are

$$\frac{\partial \mathcal{L}}{\partial \underline{x}} = -2(1-\mu)(\underline{x}-1) + 2\lambda_1(\underline{x}-\overline{\theta}) - 2\lambda_2(\underline{x}-\underline{\theta}) - 2\lambda_4(\underline{x}-\underline{\theta}) = 0, \tag{10}$$
$$\frac{\partial \mathcal{L}}{\partial \overline{x}} = -2\mu(\overline{x}-1) - 2\lambda_1(\overline{x}-\overline{\theta}) + 2\lambda_2(\overline{x}-\underline{\theta}) - 2\lambda_3(\overline{x}-\overline{\theta}) = 0, \tag{11}$$
$$\frac{\partial \mathcal{L}}{\partial \underline{T}} = -(1-\mu) - \lambda_1 + \lambda_2 + \lambda_4 = 0, \tag{12}$$
$$\frac{\partial \mathcal{L}}{\partial \overline{T}} = -\mu + \lambda_1 - \lambda_2 + \lambda_3 = 0. \tag{13}$$

The complementary slackness conditions are:

$$\lambda_1\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P(\overline{\theta},1) + (\underline{x}-\overline{\theta})^2 - \underline{T} - \delta_P V_P(\overline{\theta},0)\right) = 0, \tag{14}$$
$$\lambda_2\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P(\underline{\theta},0) + (\overline{x}-\underline{\theta})^2 - \overline{T} - \delta_P V_P(\underline{\theta},1)\right) = 0, \tag{15}$$
$$\lambda_3\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P(\overline{\theta},1)\right) = 0, \tag{16}$$
$$\lambda_4\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P(\underline{\theta},0)\right) = 0. \tag{17}$$

Thus,

$$\lambda_1\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + (\underline{x}-\overline{\theta})^2 - \underline{T} - \delta_P(\overline{\theta}-\underline{\theta})(1-\overline{\theta})\right) = 0, \tag{18}$$
$$\lambda_2\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + (\overline{x}-\underline{\theta})^2 - \overline{T}\right) = 0, \tag{19}$$
$$\lambda_3\left(-(\overline{x}-\overline{\theta})^2 + \overline{T}\right) = 0, \tag{20}$$
$$\lambda_4\left(-(\underline{x}-\underline{\theta})^2 + \underline{T}\right) = 0. \tag{21}$$

Suppose first that $\lambda_2 = 0$. By (12) we know that $\lambda_4 = \lambda_1 + (1-\mu) > 0$. This implies that $(-(\underline{x}-\underline{\theta})^2 + \underline{T}) = 0 \iff \underline{T} = (\underline{x}-\underline{\theta})^2$. Then by (13) we know that $\mu_0 = \lambda_1 + \lambda_3$. Given that $\lambda_1, \lambda_3$ are non-negative, we know that there are three cases: (i) both $\lambda_1 > 0$ and $\lambda_3 > 0$, (ii) $\lambda_1 = \mu$ and $\lambda_3 = 0$, and (iii) $\lambda_1 = 0$ and $\lambda_3 = \mu$.

*Case (i).* The fact that $\lambda_1 > 0$ and $\lambda_3 > 0$ implies that constraints $(IC_{\overline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are all binding. These binding constraints plus the first-order conditions create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}^* = \frac{1}{2}(\overline{\theta} + \underline{\theta} - \delta_P(1-\overline{\theta})),\ \overline{x}^* = \frac{1+\overline{\theta}}{2},\ \underline{T}^* = \frac{1}{4}(\overline{\theta} - \underline{\theta} - \delta_P(1-\overline{\theta}))^2,\ \overline{T}^* = \frac{(1-\overline{\theta})^2}{4},$$
$$\lambda_1^* = \frac{(1+\delta_P)(1-\mu)(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3^* = \frac{\delta_P(\mu(1-\overline{\theta}) + \overline{\theta} - 1) + \mu(-\underline{\theta}) + \mu + \overline{\theta} - 1}{\overline{\theta}-\underline{\theta}},$$
$$\lambda_4^* = \frac{(1-\mu)(\delta_P(1-\overline{\theta}) + 1 - \underline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

Under the condition $\mu > \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, the multiplier $\lambda_3^* > 0$ and $(IC_{\underline{\theta}})$ is satisfied, which implies that $\left(\underline{x}^*, \overline{x}^*, \underline{T}^*, \overline{T}^*, \lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*\right)$ with $\lambda_2^* = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.

*Case (ii).* Now $(IC_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 6 equations and 6 unknowns with the following solution:

$$\begin{aligned}
\underline{x}' &= \frac{1+\underline{\theta}-\mu(1+\overline{\theta})}{2(1-\mu)},\ \overline{x}' = \frac{1+\overline{\theta}}{2},\\
\underline{T}' &= \frac{(1-\underline{\theta}-\mu(1+\overline{\theta}-2\underline{\theta}))^2}{4(1-\mu)^2},\\
\overline{T}' &= \frac{\mu\left((1-4\delta_P)\overline{\theta}^2+\overline{\theta}(4\delta_P(\underline{\theta}+1)-4\underline{\theta}+2)-4(\delta_P+1)\underline{\theta}+4\underline{\theta}^2+1\right)}{4(\mu-1)}\\
&\quad+\frac{(\overline{\theta}-1)((4\delta_P+3)\overline{\theta}-4(\delta_P+1)\underline{\theta}+1)}{4(\mu-1)},\\
\lambda_1' &= \mu,\ \lambda_3'=0,\ \lambda_4'=1.
\end{aligned}$$

Replacing these values on the constraint $(P_{\overline{\theta}})$ we obtain that it is satisfied if $\mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$. Also, the constraint $(IC_{\underline{\theta}})$ is satisfied if $\mu > \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}$. Note that $\frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})} < \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, and $\frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}$ is positive if and only if $\delta_P > \frac{(\overline{\theta}-\underline{\theta})}{(1-\overline{\theta})}$. Thus, if $\max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\} < \mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, the vector $\left(\underline{x}', \overline{x}', \underline{T}', \overline{T}', \lambda_1', \lambda_2', \lambda_3', \lambda_4'\right)$ with $\lambda_2' = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.

*Case (iii).* Now $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 6 equations and 6 unknowns with the following solution:

$$\begin{aligned}
&\underline{x}'' = \frac{1+\underline{\theta}}{2},\ \overline{x}'' = \frac{1+\overline{\theta}}{2},\ \underline{T}'' = \frac{1}{4}(1-\underline{\theta})^2,\ \overline{T}'' = \frac{1}{4}(1-\overline{\theta})^2,\\
&\lambda_1''=0,\ \lambda_3''=\mu,\ \lambda_4''=(1-\mu).
\end{aligned}$$

Replacing these values on the constraint $(IC_{\overline{\theta}})$ yields $-(1+\delta_P)(1-\overline{\theta})(\overline{\theta}-\underline{\theta}) \geq 0$, which is false. Thus, there are no solutions that satisfy the conditions in case (iii).

Now, suppose $\lambda_2 > 0$. Suppose also that $\lambda_4 > 0$. We have three other cases: (iv) both $\lambda_1 > 0$ and $\lambda_3 > 0$, (v) $\lambda_1 > 0$ and $\lambda_3 = 0$, and (vi) $\lambda_1 = 0$ and $\lambda_3 > 0$.

*Case (iv).* The fact that $\lambda_1 > 0$, $\lambda_2 > 0$ and $\lambda_3 > 0$ implies that all the constraints $(IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding. These binding constraints plus the first-order conditions create a system of 8 equations and 8 unknowns with the following solution:

$$\underline{x}^* = \frac{1}{2}(\overline{\theta} + \underline{\theta} - \delta_P(1-\overline{\theta})),\ \overline{x}^* = \frac{\overline{\theta}+\underline{\theta}}{2}, \underline{T}^* = \frac{1}{4}(\overline{\theta} - \underline{\theta} - \delta_P(1-\overline{\theta}))^2,\ \overline{T}^* = \frac{1}{4}(\overline{\theta}-\underline{\theta})^2,$$
$$\lambda_1^* = \frac{(1+\delta_P)(1-\overline{\theta})(1-\mu)}{\overline{\theta}-\underline{\theta}},\ \lambda_2^* = -\frac{\mu(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3^* = \frac{-\delta_P(1-\mu)(1-\overline{\theta})-(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},$$
$$\lambda_4^* = \frac{\delta_P(1-\mu)(1-\overline{\theta})+(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

In this case we have that $\lambda_2^* < 0$. Thus, there are no solutions that satisfy the conditions in case (iv).

*Case (v).* Now $(IC_{\overline{\theta}}), (IC_{\underline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\hat{\underline{x}} = \frac{1}{2}(1+\underline{\theta} - \delta_P\mu(1-\overline{\theta})),\ \hat{\overline{x}} = \frac{1}{2}(1+\underline{\theta}+\delta_P(1-\mu)(1-\overline{\theta})),$$
$$\hat{\underline{T}} = \frac{1}{4}(\delta_P\mu(1-\overline{\theta})-(1-\underline{\theta}))^2,$$
$$\hat{\overline{T}} = \frac{1}{4}(\delta_P(1-\mu)(1-\overline{\theta})+(1-\underline{\theta}))^2,$$
$$\hat{\lambda}_1 = \frac{\delta_P(1-\overline{\theta})\mu(1-\mu)}{\overline{\theta}-\underline{\theta}},\ \hat{\lambda}_2 = \mu\frac{(\delta_P(1-\mu)(1-\overline{\theta})-(\overline{\theta}-\underline{\theta}))}{\overline{\theta}-\underline{\theta}},\ \hat{\lambda}_4 = 1.$$

We have that $\hat{\lambda}_1 \geq 0$, but $\hat{\lambda}_2 \geq 0$ if and only if $\mu \leq \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}$. Replacing the solution on the constraint $(P_{\overline{\theta}})$ we obtain that it is satisfied. Thus, when $\frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})} > 0$, if $\mu \leq \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}$, the vector $\left(\hat{\underline{x}}, \hat{\overline{x}}, \hat{\underline{T}}, \hat{\overline{T}}, \hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3, \hat{\lambda}_4\right)$ with $\hat{\lambda}_3 = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.

*Case (vi).* Now $(IC_{\underline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}'' = \frac{1+\underline{\theta}}{2},\ \overline{x}'' = \frac{\overline{\theta}+\underline{\theta}}{2},\ \underline{T}'' = \frac{1}{4}(1-\underline{\theta})^2,\ \overline{T}'' = \frac{(\overline{\theta}-\underline{\theta})^2}{4},$$
$$\lambda_2'' = -\frac{\mu(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3'' = -\frac{(1-\overline{\theta})\mu}{\overline{\theta}-\underline{\theta}},\ \lambda_4'' = \frac{(\overline{\theta}-\underline{\theta})+\mu(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

We have that $\lambda_2'' < 0$. Thus, there are no solutions that satisfy the conditions in case (vi). Using an analogous procedure, we can discard the cases where $\lambda_2 > 0$ and $\lambda_4 = 0$.

Note that $\frac{\partial^2 \mathcal{L}}{\partial \underline{x}^2} = -2(1-\mu)+2(\lambda_1-\lambda_2-\lambda_4)$. From (12) we have that $(\lambda_1-\lambda_2-\lambda_4) = -(1-\mu)$, and then $\frac{\partial^2 \mathcal{L}}{\partial \underline{x}^2} < 0$. Also, $\frac{\partial^2 \mathcal{L}}{\partial \overline{x}^2} = -2\mu - 2\lambda_1 + 2\lambda_2 - 2\lambda_3 < 0$ when $\lambda_2 = 0$. In the case $\lambda_2 > 0$, we have that $-2\mu - 2\lambda_1 + 2\lambda_2 - 2\lambda_3 = -2\mu < 0$.

Thus, the Lagrangian function is strictly concave and the Kuhn-Tucker first-order conditions are also sufficient. In sum, the solution of the relaxed problem is the following:

$$(\underline{x}, \overline{x}, \underline{T}, \overline{T}) = \begin{cases} \left(\underline{x}^*, \overline{x}^*, \underline{T}^*, \overline{T}^*\right) & \text{if } \mu > \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}, \\ \left(\underline{x}', \overline{x}', \underline{T}', \overline{T}'\right) & \text{if } \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\} < \mu \le \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}, \\ \left(\hat{\underline{x}}, \hat{\overline{x}}, \hat{\underline{T}}, \hat{\overline{T}}\right) & \text{if } 0 \le \mu \le \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\}. \end{cases} \tag{22}$$

$G$'s expected payoff from separation equals

$$V^{SEP} = (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G(0)\right) + \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G(1)\right),$$

where $(\underline{x}, \overline{x}, \underline{T}, \overline{T})$ is as in equation (22) and $V_G(0)$ and $V_G(1)$ are $G$'s continuation values as in Remark 1.

*Step 2. Pooling.* Next, we assume that $G$ chooses a pooling offer. Conditional on pooling, $G$'s constrained maximization problem is

$$\max_{x,T\in\mathbb{R}} \mu\left(-(x-1)^2 - T + \delta_G V_G(\mu)\right) + (1-\mu)\left(-(x-1)^2 - T + \delta_G V_G(\mu)\right)$$
$$s.t.\ (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}),$$

where the constraints are

$$-(x-\overline{\theta})^2 + T + \delta_P V_P(\overline{\theta}, \mu) \ge 0, \tag{$P_{\overline{\theta}}$}$$
$$-(x-\underline{\theta})^2 + T + \delta_P V_P(\underline{\theta}, \mu) \ge 0. \tag{$P_{\underline{\theta}}$}$$

The left-hand side denotes the payoff of accepting and the right-hand side each type's payoff of rejecting. The solution to this problem is to offer $(x, T) = \left(\frac{1+\underline{\theta}}{2}, \frac{(1-\underline{\theta})^2}{4}\right)$. The interest group's expected payoff of the pooling strategy is equal to

$$V^{POOL} = -\left(\frac{1+\underline{\theta}}{2} - 1\right)^2 - \frac{(1-\underline{\theta})^2}{4} + \delta_G V_P(\mu).$$

Finally, a direct comparison of $V^{SEP}$ and $V^{POOL}$ shows that there exists a threshold $\delta_G^\dagger$ such that $V^{SEP} \ge V^{POOL}$ if and only if $\delta_G \ge \delta_G^\dagger$. The cut-off is the following:

$$\delta_G^\dagger = \begin{cases} \frac{\delta_P^2(1-\mu)^2(1-\overline{\theta})^2}{(\overline{\theta}-\underline{\theta})(\mu(\overline{\theta}+\underline{\theta}-2)-2\overline{\theta}+2)} & \text{if } 0 \le \mu \le \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\} \\ \frac{2\delta_P(1-\mu)(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{2+\mu(\overline{\theta}+\underline{\theta}-2)-2\overline{\theta}} & \text{if } \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\} < \mu \le \frac{1-\overline{\theta}}{1-\underline{\theta}} \\ \frac{\mu(\overline{\theta}-\underline{\theta})\left(2\delta_P(1-\mu)(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})\right)}{(1-\mu)^2(1-\overline{\theta})^2} & \text{if } \frac{1-\overline{\theta}}{1-\underline{\theta}} < \mu \le \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})} \\ \delta_P^2 + 2\delta_P - \frac{\mu(1-\underline{\theta})^2}{(1-\mu)(1-\overline{\theta})^2} + \frac{1}{1-\mu} & \text{if } \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})} < \mu. \end{cases}$$

The cut-off $\delta_G^\dagger$ is continuous in $\mu$. Also, if $\mu > \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}$, then $\delta_G^\dagger < 0$ so in this region screening is always optimal. Also $\delta_G^\dagger$ is increasing in $\delta_P$ and if $\Delta = \overline{\theta} - \underline{\theta}$, $\delta_G^\dagger$ is decreasing in $\Delta$.

# B Appendix: Proofs of Extensions

## B.1 Proof of Proposition 5

The result follows from taking the difference between the interest group expected payoff under full information (Benchmark 2) and in the main model (Lemma 1 and 2).

## B.2 Proof of Proposition 6

We first characterize the equilibrium behavior when there is a probability $\alpha_t$ of having the chance to lobby in the period $t$. The result follows by comparing $G$'s expected payoff from our main model with the equilibrium payoff when $\alpha_1 = 0$ and $\alpha_2 = 1$.

The analysis follows in two steps. In the first step, we analyze equilibrium behavior in the second period. In the second step, we analyze equilibrium behavior in the first period.

*Step 1.* In case $G$ is active, the equilibrium behavior in the second period follows from Lemma 1. If $G$ is not active, then each politician type chooses their ideal policy. Hence, from the perspective of the first period, continuation values for $P$ and $G$ are as listed in Remark 2, but taking into consideration the probability of being active in the second period:

$$V_P^A(\theta,\mu) = \begin{cases} 0 & \text{if } \theta = \underline{\theta}, \\ 0 & \text{if } \theta = \overline{\theta} \text{ and } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \alpha_2 \frac{(\overline{\theta}-\underline{\theta})(1-\mu-\overline{\theta}+\mu\underline{\theta})}{1-\mu} & \text{if } \theta = \overline{\theta} \text{ and } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \end{cases}$$

$$V_G^A(\mu) = \begin{cases} \frac{\alpha_2}{2(1-\mu)}\left((1-\mu)(-(1-\underline{\theta})^2) + \mu(\overline{\theta}-\underline{\theta})^2\right) & \\ +(1-\alpha_2)\left(-(1-\mu)(1-\underline{\theta})^2 - \mu(1-\overline{\theta})^2\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \alpha_2\left((1-\mu)\left(-\frac{(1-\overline{\theta})^2}{2} - \frac{(1-\underline{\theta})^2}{2}\right) + \mu\left(-\frac{(1-\overline{\theta})^2}{2}\right)\right) & \\ +(1-\alpha_2)\left(-(1-\mu)(1-\underline{\theta})^2 - \mu(1-\overline{\theta})^2\right) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

*Step 2.* We now look at the first period, and divide the analysis in two cases: (1) $G$ is active, and (2) $G$ is not active.

*Case (1).* In this case, $G$ is active in the first period. We again compare separating and pooling equilibria. Conditional on separation, $G$'s maximization problem is

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G^A(1)\right) + (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G^A(0)\right)$$
$$s.t.\ (IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}),$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^A(\overline{\theta},1) \geq -(\underline{x}-\overline{\theta})^2 + \underline{T} + \delta_P V_P^A(\overline{\theta},0), \quad (IC_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^A(\underline{\theta},0) \geq -(\overline{x}-\underline{\theta})^2 + \overline{T} + \delta_P V_P^A(\underline{\theta},1), \quad (IC_{\underline{\theta}})$$
$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^A(\overline{\theta},1) \geq 0, \quad (P_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^A(\underline{\theta},0) \geq 0. \quad (P_{\underline{\theta}})$$

The problem can be rewritten as follows:

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-(\overline{x}-1)^2-\overline{T}+\alpha_2\delta_G V_G^A(1)\right)+(1-\mu)\left(-(\underline{x}-1)^2-\underline{T}+\alpha_2\delta_G V_G^A(0)\right)$$
$$+(1-\alpha_2)\left(-(1-\mu)(1-\underline{\theta})^2-\mu(1-\overline{\theta})^2\right)$$
$$s.t.\ (IC_{\overline{\theta}}),(IC_{\underline{\theta}}),(P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}),$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2+\overline{T}+\alpha_2\delta_P V_P(\overline{\theta},1)\geq-(\underline{x}-\overline{\theta})^2+\underline{T}+\alpha_2\delta_P V_P(\overline{\theta},0), \tag{$IC_{\overline{\theta}}$}$$
$$-(\underline{x}-\underline{\theta})^2+\underline{T}+\alpha_2\delta_P V_P(\underline{\theta},0)\geq-(\overline{x}-\underline{\theta})^2+\overline{T}+\alpha_2\delta_P V_P(\underline{\theta},1), \tag{$IC_{\underline{\theta}}$}$$
$$-(\overline{x}-\overline{\theta})^2+\overline{T}+\alpha_2\delta_P V_P(\overline{\theta},1)\geq 0, \tag{$P_{\overline{\theta}}$}$$
$$-(\underline{x}-\underline{\theta})^2+\underline{T}+\alpha_2\delta_P V_P(\underline{\theta},0)\geq 0. \tag{$P_{\underline{\theta}}$}$$

Note that the term $(1-\alpha_2)\left(-(1-\mu)(1-\underline{\theta})^2-\mu(1-\overline{\theta})^2\right)$ is constant. If we let $\delta_P'=\alpha_2\delta_P$ and $\delta_G'=\alpha_2\delta_G$, the maximization problem is mathematically equivalent to the one in Lemma 2 in case of a separating contracts. Thus, the solution is the same than Lemma 2 replacing $\delta_P$ and $\delta_G$ by $\alpha_2\delta_P$ and $\alpha_2\delta_G$ respectively. In case of a pooling offer the same argument applies. Thus, the optimal offer follows.

*Case 2.* Suppose $G$ is not active in the first period. Still $P$ chooses a policy $x$. Note that $V_P^A(\underline{\theta},\mu)=0$ for every $\mu\in[0,1]$. Thus, independent of $G$'s belief, politician type $\underline{\theta}$ obtains a continuation value of 0. Thus, this type will choose her ideal policy $\underline{\theta}$.

We now turn our attention to the politician type $\overline{\theta}$. Our first step is to rule out in equilibrium any policy different than $\{\underline{\theta},\overline{\theta}\}$. By contradiction, suppose that type $\overline{\theta}$ chooses $x\notin\{\underline{\theta},\overline{\theta}\}$. Since in every equilibrium type $\underline{\theta}$ chooses $\underline{\theta}$, then by Bayesian consistency it must be that beliefs jump to $\mu=1$. By Remark 2, $V_P^A(\overline{\theta},1)=0$. Thus, $P$'s expected utility for $x\notin\{\underline{\theta},\overline{\theta}\}$ is $-(x-\overline{\theta})^2+\delta_P\hat{V}_P(\overline{\theta},1)=-(x-\overline{\theta})^2$. Instead, by choosing $\overline{\theta}$, this type can secure a payoff of 0 which is strictly higher than $-(x-\overline{\theta})^2$. Thus, in equilibrium, type $\overline{\theta}$ chooses between policies $\{\underline{\theta},\overline{\theta}\}$.

We first analyze separating equilibria (case 2.1) and then pooling equilibria (case 2.2).

*Case 2.1.* By previous arguments, in a separating equilibria it must be that type $\overline{\theta}$ chooses policy $\overline{\theta}$ and obtains payoff 0 because in the second period $V_P^A(\overline{\theta},1)=0$. Consider a deviation to policy $\underline{\theta}$. In this case, the second-period payoff would be $V_P^A(\overline{\theta},0)=\alpha_2(\overline{\theta}-\underline{\theta})(1-\overline{\theta})$. For a separating equilibrium to exist we must ensure there is no profitable deviation, which translates to the following condition:

$$0\geq-(\underline{\theta}-\overline{\theta})^2+\delta_P\alpha_2(\overline{\theta}-\underline{\theta})(1-\overline{\theta}).$$

This condition is equivalent to $\delta_P\leq\frac{(\overline{\theta}-\underline{\theta})}{(1-\overline{\theta})}\frac{1}{\alpha_2}$. Some sufficient conditions for the requirement to hold are (i) $\overline{\theta}\geq\frac{1+\underline{\theta}}{2}$, or (ii) $\overline{\theta}<\frac{1+\underline{\theta}}{2}$ and $\delta_P\leq\frac{\overline{\theta}-\underline{\theta}}{1-\underline{\theta}}$, or (iii) $\overline{\theta}<\frac{1+\underline{\theta}}{2}$ and $\delta_P>\frac{\overline{\theta}-\underline{\theta}}{1-\underline{\theta}}$ and $\alpha_2\leq\frac{\overline{\theta}-\underline{\theta}}{\delta_P(1-\overline{\theta})}$.

Finally, consider an off-path policy $x \notin \{\underline{\theta}, \overline{\theta}\}$, where Bayes' rule does not apply. From previous arguments, politician type $\underline{\theta}$ does not have a profitable deviation independent of $G$'s belief $\mu \in [0,1]$. Thus, for any $x \notin \{\underline{\theta}, \overline{\theta}\}$ such that there is a belief for which the deviation is profitable, then $\mu = 1$. A deviation gives $-(x-\overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, \mu)$, which is maximized at $\mu = 0$. Thus, for any $x$ that satisfies the following condition it must be that $\mu = 1$

$$\begin{aligned} &-(x-\overline{\theta})^2 + \alpha_2 \delta_P (\overline{\theta} - \underline{\theta})(1-\overline{\theta}) > 0 \\ \iff\ & x \in \left(\overline{\theta} - \sqrt{\alpha_2 \delta_P (\overline{\theta} - \underline{\theta})(1-\overline{\theta})}, \overline{\theta} + \sqrt{\alpha_2 \delta_P (\overline{\theta} - \underline{\theta})(1-\overline{\theta})}\right). \end{aligned}$$

A direct computation shows that type $\overline{\theta}$ does not have incentives to deviate for this set of policies since $0 > -(x-\overline{\theta})^2 + 0$ for any $x \neq \overline{\theta}$.

Finally, for all $x \notin \left(\overline{\theta} - \sqrt{\alpha_2 \delta_P (\overline{\theta} - \underline{\theta})(1-\overline{\theta})}, \overline{\theta} + \sqrt{\alpha_2 \delta_P (\overline{\theta} - \underline{\theta})(1-\overline{\theta})}\right) \cup \{\underline{\theta}\}$, NWBR does not restrict the beliefs and we directly impose $\mu = \mu_0$. The set of policies is characterized by $-(x-\overline{\theta})^2 + \alpha_2 \delta_P (\overline{\theta} - \underline{\theta})(1-\overline{\theta}) < 0$. If the type $\overline{\theta}$ deviates to such a policy, it obtains $-(x-\overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, \mu_0)$. Since $V_P^A(\overline{\theta}, \mu_0) < V_P^A(\overline{\theta}, 1)$, type $\overline{\theta}$ does not have incentives to deviate for this set of policies either. Note that our results are independent of the belief we consider for this set of policies.

*Case 2.2.* In a pooling equilibrium type $\overline{\theta}$ chooses $\underline{\theta}$ and obtains payoff $-(\underline{\theta} - \overline{\theta})^2$ in the first period, and $V_P^A(\overline{\theta}, \mu_0)$ in the second period. Note that $V_P^A(\overline{\theta}, \mu_0) = 0 \iff \mu_0 \geq \frac{1-\overline{\theta}}{1-\underline{\theta}}$. Thus, if $\mu_0 \geq \frac{1-\overline{\theta}}{1-\underline{\theta}}$ there is no pooling equilibrium since type $\overline{\theta}$ secures a payoff of at least 0 choosing $\overline{\theta}$. Assume that $\mu_0 < \frac{1-\overline{\theta}}{1-\underline{\theta}}$. We now apply NWBR. Now all policies $x \neq \underline{\theta}$ are off the equilibrium path. Similar to before, if politician type $\overline{\theta}$ strictly prefers to deviate for some off-path belief, then $\mu = 1$. Thus, for any $x$ that satisfies the following condition it must be that $\mu = 1$

$$\begin{aligned} &-(x-\overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, 0) > -(\underline{\theta} - \overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, \mu_0) \iff \\ &x \in \left(\overline{\theta} - \sqrt{(\underline{\theta} - \overline{\theta})^2 + \delta_P \left(V_P^A(\overline{\theta}, 0) - V_P^A(\overline{\theta}, \mu_0)\right)}, \overline{\theta} + \sqrt{(\underline{\theta} - \overline{\theta})^2 + \delta_P \left(V_P^A(\overline{\theta}, 0) - V_P^A(\overline{\theta}, \mu_0)\right)}\right), \end{aligned}$$

In this case $V_P^A(\overline{\theta}, \mu_0) = \alpha_2 \frac{(\overline{\theta} - \underline{\theta})(1-\mu_0-\overline{\theta}+\mu_0\underline{\theta})}{1-\mu_0}$ and $V_P^A(\overline{\theta}, 0) = \alpha_2(\overline{\theta} - \underline{\theta})(1-\overline{\theta})$. For policies in this set, type $\overline{\theta}$ has no profitable deviation if the following inequality is satisfied

$$\begin{aligned} &-(\underline{\theta} - \overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, \mu_0) \geq -(x-\overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, 1) \\ \iff\ &-(\underline{\theta} - \overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, \mu_0) \geq -(x-\overline{\theta})^2. \end{aligned}$$

An equivalent condition for the previous inequality is that $\delta_P \geq \frac{(\overline{\theta} - \underline{\theta})}{(1-\mu_0-\overline{\theta}+\mu_0\underline{\theta})} \frac{1-\mu_0}{\alpha_2}$. Some sufficient conditions for the condition to hold are $\overline{\theta} < \frac{1+\underline{\theta}}{2}$ and $\delta_P \geq \frac{(\overline{\theta} - \underline{\theta})(1-\mu_0)}{(1-\mu_0)-\overline{\theta}+\mu_0\underline{\theta}}$ and $\alpha_2 \geq \frac{(\overline{\theta} - \underline{\theta})(1-\mu_0)}{\delta_P(1-\mu_0-\overline{\theta}+\mu_0\underline{\theta})}$ and $\mu_0 < \frac{1-2\overline{\theta}+\underline{\theta}}{1-\overline{\theta}}$. Note that $\frac{(\overline{\theta} - \underline{\theta})}{(1-\overline{\theta})} \frac{1}{\alpha_2} < \frac{(\overline{\theta} - \underline{\theta})}{(1-\mu_0-\overline{\theta}+\mu_0\underline{\theta})} \frac{1-\mu_0}{\alpha_2}$.

For all other off-path $x$ not in this set, by assumption $\mu = \mu_0$. For policies in this set, type $\overline{\theta}$ has no profitable deviation if the following inequality is satisfied

$$-(\underline{\theta} - \overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, \mu_0) \geq -(x - \overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, \mu_0).$$

Since $V_P^A(\overline{\theta}, 0) > V_P^A(\overline{\theta}, \mu_0)$, then $-(\underline{\theta} - \overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, \mu_0) > -(x - \overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, 0) > -(x - \overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, \mu_0)$. Note that our results are independent of the belief we consider for this set of policies.

## B.3 Proof of Proposition 7

We first characterize the equilibrium behavior when policy $x$ is constrained to the interval $[\underline{y}, \overline{y}]$. Similar than Lemma 2, we found a cut-off $\delta_G^{\dagger,\overline{y}}$ such that $G$ offers separating contracts if and only if $\delta_G \geq \delta_G^{\dagger,\overline{y}}$. We then calculate the derivative of $G$'s expected payoff in equilibrium with respect to the upper limit $\overline{y}$.

For our analysis we assume that $\underline{y} \leq \underline{\theta}$ and $\frac{1+\underline{\theta}}{2} \leq \overline{y} \leq \frac{1+\overline{\theta}}{2}$. That is, the interval restricts the policy only at the right side. We first study the second-period equilibrium behavior. Lemma 1 implies that if $\overline{y} \leq \frac{1+\overline{\theta}}{2}$, the marginal benefit of an increase in $\overline{x}$ when $\overline{x} = \overline{y}$ is strictly positive. Then, it must be that $\overline{x} = \overline{y}$. The interest group maximization problem is the follows:

$$\max_{\underline{y} \leq \underline{x} \leq \overline{y}, \underline{T}, \overline{T} \in \mathbb{R}} \mu \left(-\left(\overline{y} - 1\right)^2 - \overline{T}\right) + (1 - \mu)\left(-\left(\underline{x} - 1\right)^2 - \underline{T}\right)$$
$$s.t.$$

$$\begin{aligned}
-\left(\overline{y} - \overline{\theta}\right)^2 + \overline{T} &\geq -\left(\underline{x} - \overline{\theta}\right)^2 + \underline{T}, && (IC_{\overline{\theta}})\\
-\left(\underline{x} - \underline{\theta}\right)^2 + \underline{T} &\geq -\left(\overline{y} - \underline{\theta}\right)^2 + \overline{T}, && (IC_{\underline{\theta}})\\
-\left(\overline{y} - \overline{\theta}\right)^2 + \overline{T} &\geq 0, && (P_{\overline{\theta}})\\
-\left(\underline{x} - \underline{\theta}\right)^2 + \underline{T} &\geq 0. && (P_{\underline{\theta}})
\end{aligned}$$

The solution is as follows.

$$(\underline{x}, \overline{x}, \underline{T}, \overline{T}) = \begin{cases} \left(\frac{\overline{\theta}+\underline{\theta}}{2}, \overline{y}, \left(\left(\frac{\overline{\theta}+\underline{\theta}}{2}\right) - \underline{\theta}\right)^2, \left(\overline{y} - \overline{\theta}\right)^2\right) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}},\\ \left(\frac{1+\underline{\theta}-\mu(\overline{\theta}+1)}{2(1-\mu)}, \overline{y}, \left(\left(\frac{1+\underline{\theta}-\mu(\overline{\theta}+1)}{2(1-\mu)}\right) - \underline{\theta}\right)^2, \frac{(\overline{\theta}-\underline{\theta})(1-\mu-\overline{\theta}+\mu\underline{\theta})}{1-\mu} + \left(\overline{y} - \overline{\theta}\right)^2\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

Using these results, the continuation value for $P$ and interest group are the following, respectively:

$$V_P^{\overline{y}}(\theta, \mu) = \begin{cases} 0 & \text{if } \theta = \underline{\theta},\\ 0 & \text{if } \theta = \overline{\theta} \text{ and } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}},\\ \frac{(\overline{\theta}-\underline{\theta})(1-\mu-\overline{\theta}+\mu\underline{\theta})}{1-\mu} & \text{if } \theta = \overline{\theta} \text{ and } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \end{cases}$$

$$V_G^{\overline{y}}(\mu) = \begin{cases} \frac{1}{2}\Big(-(1+\mu)\overline{\theta}^2 + \overline{\theta}(\mu(4\overline{y}-2)+2) + (1-\mu)(2-\underline{\theta})\underline{\theta} - 4\mu\overline{y}^2 + 4\mu\overline{y} - 2\Big) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \frac{1}{2(1-\mu)}\left(\mu^2(1+\overline{\theta}-2\overline{y})^2 - 2\mu\left(\overline{\theta}\underline{\theta} - 2(1+\overline{\theta})\overline{y} + \underline{\theta} + \overline{\theta} - \underline{\theta}^2 + 2\overline{y}^2\right) - (1-\underline{\theta})^2\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

Now we focus on the first period. Conditional on separation, $G$'s maximization problem is

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G^{\overline{y}}(1)\right) + (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G^{\overline{y}}(0)\right)$$
$$s.t.\ (IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}}), (P_{\underline{\theta}}) \text{ and } (LP),$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^{\overline{y}}(\overline{\theta},1) \geq -(\underline{x}-\overline{\theta})^2 + \underline{T} + \delta_P V_P^{\overline{y}}(\overline{\theta},0), \qquad (IC_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^{\overline{y}}(\underline{\theta},0) \geq -(\overline{x}-\underline{\theta})^2 + \overline{T} + \delta_P V_P^{\overline{y}}(\underline{\theta},1), \qquad (IC_{\underline{\theta}})$$
$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^{\overline{y}}(\overline{\theta},1) \geq 0, \qquad (P_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^{\overline{y}}(\underline{\theta},0) \geq 0, \qquad (P_{\underline{\theta}})$$
$$(\overline{y}-\overline{x}) \geq 0. \qquad (LP)$$

We begin our analysis by setting up the Lagrangian

$$\begin{aligned}\mathcal{L} =& \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G^{\overline{y}}(1)\right) + (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G^{\overline{y}}(0)\right) \\ &+\lambda_1\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^{\overline{y}}(\overline{\theta},1) + (\underline{x}_1-\overline{\theta})^2 - \underline{T} - \delta_P V_P^{\overline{y}}(\overline{\theta},0)\right) \\ &+\lambda_2\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^{\overline{y}}(\underline{\theta},0) + (\overline{x}-\underline{\theta})^2 - \overline{T} - \delta_P V_P^{\overline{y}}(\underline{\theta},1)\right) \\ &+\lambda_3\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^{\overline{y}}(\overline{\theta},1)\right) \\ &+\lambda_4\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^{\overline{y}}(\underline{\theta},0)\right) \\ &+\lambda_5\left(\overline{y}-\overline{x}\right).\end{aligned}$$

The first-order conditions with respect to $\underline{x}, \overline{x}, \underline{T}, \overline{T}$ are

$$\frac{\partial\mathcal{L}}{\partial\underline{x}} = -2(1-\mu)(\underline{x}-1) + 2\lambda_1(\underline{x}-\overline{\theta}) - 2\lambda_2(\underline{x}-\underline{\theta}) - 2\lambda_4(\underline{x}-\underline{\theta}) = 0, \qquad (23)$$
$$\frac{\partial\mathcal{L}}{\partial\overline{x}} = -2\mu(\overline{x}-1) - 2\lambda_1(\overline{x}-\overline{\theta}) + 2\lambda_2(\overline{x}-\underline{\theta}) - 2\lambda_3(\overline{x}-\overline{\theta}) - \lambda_5 = 0, \qquad (24)$$
$$\frac{\partial\mathcal{L}}{\partial\underline{T}} = -(1-\mu) - \lambda_1 + \lambda_2 + \lambda_4 = 0, \qquad (25)$$
$$\frac{\partial\mathcal{L}}{\partial\overline{T}} = -\mu + \lambda_1 - \lambda_2 + \lambda_3 = 0. \qquad (26)$$

The complementary slackness conditions are:

$$\lambda_1 \left(-(\overline{x}-\overline{\theta})^2+\overline{T}+(\underline{x}-\overline{\theta})^2-\underline{T}-\delta_P(\overline{\theta}-\underline{\theta})(1-\overline{\theta})\right)=0, \tag{27}$$

$$\lambda_2 \left(-(\underline{x}-\underline{\theta})^2+\underline{T}+(\overline{x}-\underline{\theta})^2-\overline{T}\right)=0, \tag{28}$$

$$\lambda_3 \left(-(\overline{x}-\overline{\theta})^2+\overline{T}\right)=0, \tag{29}$$

$$\lambda_4 \left(-(\underline{x}-\underline{\theta})^2+\underline{T}\right)=0, \tag{30}$$

$$\lambda_5 \left(\overline{y}-\overline{x}\right)=0. \tag{31}$$

Suppose first that $\lambda_2 = 0$. By (25) we know that $\lambda_4 = \lambda_1 + (1-\mu) > 0$. This implies that $(-(\underline{x}-\underline{\theta})^2+\underline{T}) = 0 \iff \underline{T} = (\underline{x}-\underline{\theta})^2$. Then by (26) we know that $\mu_0 = \lambda_1 + \lambda_3$. Given that $\lambda_1, \lambda_3$ are non-negative, we know that there are three cases: (i) both $\lambda_1 > 0$ and $\lambda_3 > 0$, (ii) $\lambda_1 = \mu$ and $\lambda_3 = 0$, and (iii) $\lambda_1 = 0$ and $\lambda_3 = \mu$.

*Case (i).* The fact that $\lambda_1 > 0$ and $\lambda_3 > 0$ implies that constraints $(IC_{\overline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are all binding. If $\lambda_5 = 0$ the solution violates constraint $(LP)$. If $\lambda_5 > 0$, then $\overline{x} = \overline{y}$. The binding constraints plus the first-order conditions create a system of 8 equations and 8 unknowns with the following solution:

$$\underline{x}^* = \frac{1}{2}(\overline{\theta}+\underline{\theta}-\delta_P(1-\overline{\theta})),\ \overline{x}^* = \overline{y},\ \underline{T}^* = \frac{1}{4}(\overline{\theta}-\underline{\theta}-\delta_P(1-\overline{\theta}))^2,\ \overline{T}^* = (\overline{\theta}-\overline{y})^2,$$

$$\lambda_1^* = \frac{(1+\delta_P)(1-\mu)(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3^* = \frac{\delta_P(\mu(1-\overline{\theta})+\overline{\theta}-1)+\mu(-\underline{\theta})+\mu+\overline{\theta}-1}{\overline{\theta}-\underline{\theta}},$$

$$\lambda_4^* = \frac{(1-\mu)(\delta_P(1-\overline{\theta})+1-\underline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_5^* = 2\mu(1+\overline{\theta}-2\overline{y}).$$

We have that $\lambda_5^* > 0$. Under the condition $\mu > \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, the multiplier $\lambda_3^* > 0$ and $(IC_{\underline{\theta}})$ is satisfied, which imply that $\left(\underline{x}^*, \overline{x}^*, \underline{T}^*, \overline{T}^*, \lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*, \lambda_5^*\right)$ with $\lambda_2^* = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.

*Case (ii).* Now $(IC_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding. If $\lambda_5 = 0$ the solution violates constraint $(LP)$. If $\lambda_5 > 0$, then $\overline{x} = \overline{y}$. The binding constraints together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}' = \frac{1+\underline{\theta}-\mu(1+\overline{\theta})}{2(1-\mu)}, \overline{x}' = \overline{y},$$

$$\underline{T}' = \frac{(1-\underline{\theta}-\mu(1+\overline{\theta}-2\underline{\theta}))^2}{4(1-\mu)^2},$$

$$\overline{T}' = \frac{\mu\left(-(\delta_P-1)\overline{\theta}^2+\overline{\theta}(\delta_P\underline{\theta}+\delta_P-\underline{\theta}-2\overline{y}+1)-(\delta_P+1)\underline{\theta}+\underline{\theta}^2+\overline{y}^2\right)}{(\mu-1)} + \frac{\delta_P\overline{\theta}^2-\overline{\theta}(\delta_P\underline{\theta}+\delta_P+\underline{\theta}-2\overline{y}+1)+\delta_P\underline{\theta}+\underline{\theta}-\overline{y}^2}{(\mu-1)},$$

$$\lambda_1' = \mu,\ \lambda_3' = 0,\ \lambda_4' = 1,\ \lambda_5' = 2\mu(1+\overline{\theta}-2\overline{y}).$$

We have that $\lambda'_5 > 0$. Replacing these values on the constraint $(P_{\overline{\theta}})$ we obtain that it is satisfied if $\mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$. Also, the constraint $(IC_{\underline{\theta}})$ is satisfied if $\mu > \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}$. Note that $\frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}} < \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, and $\frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}$ is positive if and only if $\delta_P > \frac{2\overline{y}-1-\underline{\theta}}{(1-\overline{\theta})}$. Thus, if $\max\{0, \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}\} < \mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, the vector $\left(\underline{x}', \overline{x}', \underline{T}', \overline{T}', \lambda'_1, \lambda'_2, \lambda'_3, \lambda'_4\right)$ with $\lambda'_2 = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.

*Case (iii).* Now $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding. If $\lambda_5 = 0$ the solution violates constraint $(LP)$. If $\lambda_5 > 0$, then $\overline{x} = \overline{y}$. The binding constraints together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}'' = \frac{1+\underline{\theta}}{2},\ \overline{x}'' = \overline{y},\ \underline{T}'' = \frac{1}{4}(1-\underline{\theta})^2,\ \overline{T}'' = (\overline{\theta} - y)^2,$$
$$\lambda''_1 = 0,\ \lambda''_3 = \mu,\ \lambda''_4 = (1-\mu),\ \lambda''_5 = 2\mu(1+\overline{\theta}-2\overline{y}).$$

We have that $\lambda''_5 > 0$. Replacing these values on the constraint $(IC_{\overline{\theta}})$ we obtain that $-(1+\delta_P)(1-\overline{\theta})(\overline{\theta}-\underline{\theta}) \geq 0$, which is false. Thus, there are no solutions that satisfy the conditions in case (iii).

Now, suppose $\lambda_2 > 0$. Suppose also that $\lambda_4 > 0$. We have three other cases: (iv) both $\lambda_1 > 0$ and $\lambda_3 > 0$, (v) $\lambda_1 > 0$ and $\lambda_3 = 0$, and (vi) $\lambda_1 = 0$ and $\lambda_3 > 0$.

*Case (iv).* The fact that $\lambda_1 > 0$, $\lambda_2 > 0$ and $\lambda_3 > 0$ implies that constraints $(IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are all binding. If $\lambda_5 = 0$ the solution does not satisfy the KKT conditions since it implies that $\lambda_2 < 0$. If $\lambda_5 > 0$, then $\overline{x} = \overline{y}$. These binding constraints plus the first-order conditions create a system of 9 equations and 9 unknowns which has empty solution. Thus, there are no solutions that satisfy the conditions in case (iv).

*Case (v).* Now $(IC_{\overline{\theta}}), (IC_{\underline{\theta}})$ and $(P_{\underline{\theta}})$ are binding. When $\lambda_5 > 0$, together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\hat{\underline{x}} = \frac{1}{2}(2\overline{y} - \delta_P(1-\overline{\theta})),\ \hat{\overline{x}} = \overline{y},$$
$$\hat{\underline{T}} = \frac{1}{4}(\delta_P\overline{\theta} - \delta_P - 2\underline{\theta} + 2\overline{y})^2,\ \hat{\overline{T}} = (\underline{\theta} - \overline{y})^2,$$
$$\hat{\lambda}_1 = \frac{(\mu-1)(\delta_P\overline{\theta} - \delta_P - \underline{\theta} + 2\overline{y} - 1)}{\overline{\theta} - \underline{\theta}},$$
$$\hat{\lambda}_2 = \frac{\delta_P\overline{\theta}\mu + \delta_P(-\overline{\theta}) - \delta_P\mu + \delta_P - \overline{\theta}\mu + \underline{\theta} + 2\mu\overline{y} - \mu - 2\overline{y} + 1}{\overline{\theta} - \underline{\theta}},$$
$$\hat{\lambda}_4 = 1,\ \hat{\lambda}_5 = 2(\delta_P\overline{\theta}\mu + \delta_P(-\overline{\theta}) - \delta_P\mu + \delta_P + \underline{\theta} - 2\overline{y} + 1).$$

We have that $\hat{\lambda}_1 \geq 0$ if and only if $\delta_P > \frac{2\overline{y}-1-\underline{\theta}}{1-\overline{\theta}}$. Also, $\hat{\lambda}_2 \geq 0$ if and only if $\mu \leq \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}$. Also $\hat{\lambda}_5 \geq 0$ if and only if $\mu < \frac{\delta_P\overline{\theta}-\delta_P-\underline{\theta}+2\overline{y}-1}{\delta_P(\overline{\theta}-1)}$. Replacing the solution on the constraint $(P_{\overline{\theta}})$ we obtain that it is satisfied. Note that when $\delta_P > \frac{2\overline{y}-1-\underline{\theta}}{1-\overline{\theta}}$, we have that

$0 < \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}} < \frac{\delta_P\overline{\theta}-\delta_P-\underline{\theta}+2\overline{y}-1}{\delta_P(\overline{\theta}-1)}$. Thus, when $\delta_P > \frac{2\overline{y}-1-\theta}{1-\overline{\theta}}$, if $\mu \leq \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}$, the vector $\left(\hat{\underline{x}}, \hat{\overline{x}}, \hat{\underline{T}}, \hat{\overline{T}}, \hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3, \hat{\lambda}_4\right)$ with $\hat{\lambda}_3 = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.
If $\lambda_5 = 0$, then the solution is

$$\begin{aligned}
&\hat{\underline{x}} = \frac{1}{2}(1+\underline{\theta}-\delta_P\mu(1-\overline{\theta})), \hat{\overline{x}} = \frac{1}{2}(1+\underline{\theta}+\delta_P(1-\mu)(1-\overline{\theta})),\\
&\hat{\underline{T}} = \frac{1}{4}(\delta_P\mu(1-\overline{\theta})-(1-\underline{\theta}))^2,\\
&\hat{\overline{T}} = \frac{1}{4}(\delta_P(1-\mu)(1-\overline{\theta})+(1-\underline{\theta}))^2,\\
&\hat{\lambda}_1 = \frac{\delta_P(1-\overline{\theta})\mu\,(1-\mu)}{\overline{\theta}-\underline{\theta}}, \hat{\lambda}_2 = \mu\frac{(\delta_P(1-\mu)(1-\overline{\theta})-(\overline{\theta}-\underline{\theta}))}{\overline{\theta}-\underline{\theta}}, \hat{\lambda}_4 = 1.
\end{aligned}$$

For the set of parameters where $\hat{\lambda}_1 \geq 0$, we have that $\hat{\overline{x}} > y$, which violates constraint $(LP)$. Thus, there are no solutions that satisfy the conditions in case (v) when $\lambda_5 = 0$.

*Case (vi).* Now $(IC_{\underline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding. If $\lambda_5 = 0$ the solution does not satisfies the KKT conditions since $\lambda_2^{*\prime\prime} < 0$. If $\lambda_5 > 0$, then $\overline{x} = \overline{y}$. and, together with the first-order conditions, create a system of 8 equations and 8 unknowns which has empty solution. Thus, there are no solutions that satisfy the conditions in case (vi).
Using an analog procedure, we can discard the cases where $\lambda_2 > 0$ and $\lambda_4 = 0$. In sum, the solution of the maximization problem is the following:

$$(\underline{x}, \overline{x}, \underline{T}, \overline{T}) = \begin{cases} \left(\underline{x}^*, \overline{x}^*, \underline{T}^*, \overline{T}^*\right) & \text{if } \mu > \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})},\\ \left(\underline{x}', \overline{x}', \underline{T}', \overline{T}'\right) & \text{if } \max\{0, \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}\} < \mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})},\\ \left(\hat{\underline{x}}, \hat{\overline{x}}, \hat{\underline{T}}, \hat{\overline{T}}\right) & \text{if } 0 \leq \mu \leq \max\{0, \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}\}. \end{cases} \tag{32}$$

The interest group's expected payoff from separation equals

$$V^{SEP} = (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_2^{\overline{y}}(0)\right) + \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_2^{\overline{y}}(1)\right),$$

where $(\underline{x}, \overline{x}, \underline{T}, \overline{T})$ is as in equation (32) and $V_G^{\overline{y}}(0)$ and $V_G^{\overline{y}}(1)$ are $G$'s continuation values.

In case of a pooling offer, $G$ solves:

$$\begin{aligned}
&\max_{\underline{y}\leq x\leq\overline{y}, T\in\mathbb{R}} \mu\left(-(x-1)^2 - T + \delta_G V_G^{\overline{y}}(\mu)\right) + (1-\mu)\left(-(x-1)^2 - T + \delta_G V_G^{\overline{y}}(\mu)\right)\\
&\qquad s.t.\ (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}),
\end{aligned}$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P V_P^{\overline{y}}(\overline{\theta},\mu)\geq 0, \qquad (P_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2+\underline{T}+\delta_P V_P^{\overline{y}}(\underline{\theta},\mu)\geq 0. \qquad (P_{\underline{\theta}})$$

The solution to this problem is to offer $(x,T)=\left(\frac{1+\underline{\theta}}{2},\frac{(1-\underline{\theta})^2}{4}\right)$. The interest group's expected payoff of the pooling strategy is equal to

$$V^{POOL}=-\left(\frac{1+\underline{\theta}}{2}-1\right)^2-\frac{(1-\underline{\theta})^2}{4}+\delta_G V_G^{\overline{y}}(\mu).$$

In Lemma 2, we found that there is a cut-off $\delta_G^{\dagger}$ such that $V^{SEP}\geq V^{POOL}$ if and only if $\delta_G\geq\delta_G^{\dagger}$. Here, we obtain a similar result: there is a cut-off $\delta_G^{\dagger,\overline{y}}$ such that $V^{SEP}\geq V^{POOL}$ if and only if $\delta_G\geq\delta_G^{\dagger,\overline{y}}$, where $\delta_G^{\dagger,\overline{y}}$ equals to

$$\begin{cases}
\left[\frac{(\mu-1)\left(\delta_P^2(\mu-1)(\overline{\theta}-1)^2-2\delta_P(\mu-1)(\overline{\theta}-1)(\underline{\theta}-2\overline{y}+1)-(\underline{\theta}-2\overline{y}+1)^2\right)}{\mu(\overline{\theta}-\underline{\theta})(\mu(\overline{\theta}+\underline{\theta}-2)-2\overline{\theta}+2)}\right] \\
\text{if } 0\leq\mu\leq\max\{0,\frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}\} \\
\left[\frac{\mu\left((2\delta_P-1)\overline{\theta}^2-2\overline{\theta}(\delta_P\underline{\theta}+\delta_P-2\overline{y}+1)+2\delta_P\underline{\theta}-4\overline{y}^2+4\overline{y}-1\right)-2\delta_P\overline{\theta}^2+2\overline{\theta}(\delta_P\underline{\theta}+\delta_P+\underline{\theta}-2\overline{y}+1)-2\delta_P\underline{\theta}-\underline{\theta}^2+4\overline{y}^2-4\overline{y}+1}{(\overline{\theta}-\underline{\theta})(\mu(\overline{\theta}+\underline{\theta}-2)-2\overline{\theta}+2)}\right] \\
\text{if } \max\{0,\frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}\}<\mu\leq\frac{1-\overline{\theta}}{1-\underline{\theta}} \\
\left[\frac{\mu\left(\mu\left((2\delta_P-1)\overline{\theta}^2-2\overline{\theta}(\delta_P\underline{\theta}+\delta_P-2\overline{y}+1)+2\delta_P\underline{\theta}-4\overline{y}^2+4\overline{y}-1\right)-2\delta_P\overline{\theta}^2+2\overline{\theta}(\delta_P\underline{\theta}+\delta_P+\underline{\theta}-2\overline{y}+1)-2\delta_P\underline{\theta}-\underline{\theta}^2+4\overline{y}^2-4\overline{y}+1\right)}{(1-\mu)^2(1-\overline{\theta})^2}\right] \\
\text{if } \frac{1-\overline{\theta}}{1-\underline{\theta}}<\mu\leq\frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})} \\
\left[\frac{\delta_P^2(\mu-1)(\overline{\theta}-1)^2+2\delta_P(\mu-1)(\overline{\theta}-1)^2-(\mu+1)\overline{\theta}^2+\overline{\theta}(4\mu\overline{y}-2\mu+2)+\mu\underline{\theta}^2-2\mu\underline{\theta}-4\mu\overline{y}^2+4\mu\overline{y}-1}{(\mu-1)(1-\overline{\theta})^2}\right] \\
\text{if } \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}<\mu.
\end{cases}$$

A direct comparison of $\delta_G^{\dagger,\overline{y}}$ and $\delta_G^{\dagger}$ shows that $\delta_G^{\dagger,\overline{y}}>\delta_G^{\dagger}$. Thus, a constraint in the feasible policies decreases the incentives for screening relative to pooling. Also, after some calculations, we obtain that $\delta_G^{\dagger,\overline{y}}$ is decreasing in $\overline{y}$ and that the interest group expected payoff in equilibrium increases in $\overline{y}$ .

## B.4 Proofs of Propositions 8 and 9

We first characterize the equilibrium behavior when there is an extra benefit $R>0$ for $P$ in the second period if an ally politician is sufficiently likely. Proposition 8 follows from calculating the cut-off $\delta_G^{\dagger,R}$ for which $G$ is indifferent between offering a separating offer and a polling offer in equilibrium. Proposition 9 follows from taking the derivative of $G$'s expected payoff in equilibrium with respect to $R$.

For simplicity, assume that $P$ obtains the benefit $R$ in the second period only if $G$'s belief is sufficiently high $\mu>\frac{1-\overline{\theta}}{1-\underline{\theta}}$ after the first period. $P$ and interest group continuation values are, respectively

$$V_P^R(\theta,\mu) = \begin{cases} R & \text{if } \theta = \underline{\theta} \text{ and } \mu \geq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ 0 & \text{if } \theta = \underline{\theta} \text{ and } \mu < \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ R & \text{if } \theta = \overline{\theta} \text{ and } \mu \geq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \frac{(\overline{\theta}-\underline{\theta})(1-\mu-\overline{\theta}+\mu\underline{\theta})}{1-\mu}, & \text{if } \theta = \overline{\theta} \text{ and } \mu < \frac{1-\overline{\theta}}{1-\underline{\theta}}, \end{cases}$$

$$V_G^R(\mu) = \begin{cases} \frac{1}{2(1-\mu)}\left((1-\mu)(-(1-\underline{\theta})^2) + \mu(\overline{\theta}-\underline{\theta})^2\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ (1-\mu)\left(-\frac{(1-\overline{\theta})^2}{2} - \frac{(1-\underline{\theta})^2}{2}\right) + \mu\left(-\frac{(1-\overline{\theta})^2}{2}\right) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

Conditional on separation, the interest group maximization problem is

$$\begin{aligned} &\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G^R(1)\right) + (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G^R(0)\right) \\ &\quad s.t.\ (IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}), \end{aligned}$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^R(\overline{\theta},1) \geq -(\underline{x}-\overline{\theta})^2 + \underline{T} + \delta_P V_P^R(\overline{\theta},0), \tag{$IC_{\overline{\theta}}$}$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^R(\underline{\theta},0) \geq -(\overline{x}-\underline{\theta})^2 + \overline{T} + \delta_P V_P^R(\underline{\theta},1), \tag{$IC_{\underline{\theta}}$}$$
$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^R(\overline{\theta},1) \geq 0, \tag{$P_{\overline{\theta}}$}$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^R(\underline{\theta},0) \geq 0. \tag{$P_{\underline{\theta}}$}$$

We begin our analysis by setting up the Lagrangian

$$\begin{aligned} \mathcal{L} =& \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G^R(1)\right) + (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G^R(0)\right) \\ &+\lambda_1\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^R(\overline{\theta},1) + (\underline{x}_1-\overline{\theta})^2 - \underline{T} - \delta_P V_P^R(\overline{\theta},0)\right) \\ &+\lambda_2\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^R(\underline{\theta},0) + (\overline{x}-\underline{\theta})^2 - \overline{T} - \delta_P V_P^R(\underline{\theta},1)\right) \\ &+\lambda_3\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^R(\overline{\theta},1)\right) \\ &+\lambda_4\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^R(\underline{\theta},0)\right). \end{aligned}$$

The first-order conditions with respect to $\underline{x},\overline{x},\underline{T},\overline{T}$ are

$$\frac{\partial\mathcal{L}}{\partial\underline{x}} = -2(1-\mu)(\underline{x}-1) + 2\lambda_1(\underline{x}-\overline{\theta}) - 2\lambda_2(\underline{x}-\underline{\theta}) - 2\lambda_4(\underline{x}-\underline{\theta}) = 0, \tag{33}$$
$$\frac{\partial\mathcal{L}}{\partial\overline{x}} = -2\mu(\overline{x}-1) - 2\lambda_1(\overline{x}-\overline{\theta}) + 2\lambda_2(\overline{x}-\underline{\theta}) - 2\lambda_3(\overline{x}-\overline{\theta}) = 0, \tag{34}$$
$$\frac{\partial\mathcal{L}}{\partial\underline{T}} = -(1-\mu) - \lambda_1 + \lambda_2 + \lambda_4 = 0, \tag{35}$$
$$\frac{\partial\mathcal{L}}{\partial\overline{T}} = -\mu + \lambda_1 - \lambda_2 + \lambda_3 = 0. \tag{36}$$

The complementary slackness conditions are:

$$\lambda_1\left(-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P R+(\underline{x}-\overline{\theta})^2-\underline{T}-\delta_P(\overline{\theta}-\underline{\theta})(1-\overline{\theta})\right)=0, \tag{37}$$
$$\lambda_2\left(-(\underline{x}-\underline{\theta})^2+\underline{T}+(\overline{x}-\underline{\theta})^2-\overline{T}-\delta_P R\right)=0, \tag{38}$$
$$\lambda_3\left(-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P R\right)=0, \tag{39}$$
$$\lambda_4\left(-(\underline{x}-\underline{\theta})^2+\underline{T}\right)=0. \tag{40}$$

Suppose first that $\lambda_2=0$. By (35) we know that $\lambda_4=\lambda_1+(1-\mu)>0$. This implies that $(-(\underline{x}-\underline{\theta})^2+\underline{T})=0 \iff \underline{T}=(\underline{x}-\underline{\theta})^2$. Then by (36) we know that $\mu=\lambda_1+\lambda_3$. Given that $\lambda_1,\lambda_3$ are non-negative, we know that there are three cases: (i) both $\lambda_1>0$ and $\lambda_3>0$, (ii) $\lambda_1=\mu$ and $\lambda_3=0$, and (iii) $\lambda_1=0$ and $\lambda_3=\mu$.

*Case (i).* The fact that $\lambda_1>0$ and $\lambda_3>0$ implies that constraints $(IC_{\overline{\theta}}),(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are all binding. These binding constraints plus the first-order conditions create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}^*=\frac{1}{2}(\overline{\theta}+\underline{\theta}-\delta_P(1-\overline{\theta})),\ \overline{x}^*=\frac{1+\overline{\theta}}{2},\ \underline{T}^*=\frac{1}{4}(\overline{\theta}-\underline{\theta}-\delta_P(1-\overline{\theta}))^2,\ \overline{T}^*=\frac{(1-\overline{\theta})^2}{4}-\delta_P R,$$
$$\lambda_1^*=\frac{(1+\delta_P)(1-\mu)(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3^*=\frac{\delta_P(\mu(1-\overline{\theta})+\overline{\theta}-1)+\mu(-\underline{\theta})+\mu+\overline{\theta}-1}{\overline{\theta}-\underline{\theta}},$$
$$\lambda_4^*=\frac{(1-\mu)(\delta_P(1-\overline{\theta})+1-\underline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

It is direct to check that $(IC_{\underline{\theta}})$ is satisfied. Under the condition $\mu>\frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, the multiplier $\lambda_3^*>0$ implies that $\left(\underline{x}^*,\overline{x}^*,\underline{T}^*,\overline{T}^*,\lambda_1^*,\lambda_3^*,\lambda_4^*\right)$ satisfies the Kuhn-Tucker first-order necessary conditions of the problem.

*Case (ii).* Now $(IC_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 6 equations and 6 unknowns with the following solution:

$$\underline{x}'=\frac{1+\underline{\theta}-\mu(1+\overline{\theta})}{2(1-\mu)},\ \overline{x}'=\frac{1+\overline{\theta}}{2},$$
$$\underline{T}'=\frac{(1-\underline{\theta}-\mu(1+\overline{\theta}-2\underline{\theta}))^2}{4(1-\mu)^2},$$
$$\overline{T}'=\frac{\mu\left((1-4\delta_P)\overline{\theta}^2+\overline{\theta}(4\delta_P(\underline{\theta}+1)-4\underline{\theta}+2)-4(\delta_P+1)\underline{\theta}+4\underline{\theta}^2+1\right)}{4(\mu-1)}-\delta_P R$$
$$+\frac{(\overline{\theta}-1)((4\delta_P+3)\overline{\theta}-4(\delta_P+1)\underline{\theta}+1)}{4(\mu-1)},$$
$$\lambda_1'=\mu,\ \lambda_3'=0,\ \lambda_4'=1.$$

Replacing these values on the constraint $(P_{\overline{\theta}})$ we obtain that it is satisfied if $\mu\le\frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$. If we replace on $(IC_{\underline{\theta}})$ we obtain $\mu\ge\frac{\delta_P\overline{\theta}-\delta_P+\overline{\theta}-\underline{\theta}}{\delta_P(\overline{\theta}-1)}$. Note that $\frac{\delta_P\overline{\theta}-\delta_P+\overline{\theta}-\underline{\theta}}{\delta_P(\overline{\theta}-1)}<\frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$. Thus, if $\frac{\delta_P\overline{\theta}-\delta_P+\overline{\theta}-\underline{\theta}}{\delta_P(\overline{\theta}-1)} \leq \mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$ the vector $\left(\underline{x}', \overline{x}', \underline{T}', \overline{T}', \lambda_1', \lambda_3', \lambda_4'\right)$ satisfies the Kuhn-Tucker first-order necessary conditions of the problem.

*Case (iii).* Now $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 6 equations and 6 unknowns with the following solution:

$$\underline{x}'' = \frac{1+\underline{\theta}}{2},\ \overline{x}'' = \frac{1+\overline{\theta}}{2},\ \underline{T}'' = \frac{1}{4}(1-\underline{\theta})^2,\ \overline{T}'' = \frac{1}{4}(1-\overline{\theta})^2 - \delta_P R,$$
$$\lambda_1'' = 0,\ \lambda_3'' = \mu,\ \lambda_4'' = (1-\mu).$$

Replacing these values on the constraint $(IC_{\overline{\theta}})$ we obtain that $-(1+\delta_P)(1-\overline{\theta})(\overline{\theta}-\underline{\theta}) \geq 0$, which is false. Thus, there are no solutions that satisfy the conditions in case (iii).

Now, suppose $\lambda_2 > 0$. We have three other cases: (iv) both $\lambda_1 > 0$ and $\lambda_3 > 0$, (v) $\lambda_1 > 0$ and $\lambda_3 = 0$, and (vi) $\lambda_1 = 0$ and $\lambda_3 > 0$.

*Case (iv).* The fact that $\lambda_1 > 0$, $\lambda_2 > 0$ and $\lambda_3 > 0$ implies that constraints $(IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are all binding. These binding constraints plus the first-order conditions create a system of 8 equations and 8 unknowns with the following solution:

$$\underline{x}^* = \frac{1}{2}(\overline{\theta}+\underline{\theta}-\delta_P(1-\overline{\theta})),\ \overline{x}^* = \frac{\overline{\theta}+\underline{\theta}}{2},\ \underline{T}^* = \frac{1}{4}(\overline{\theta}-\underline{\theta}-\delta_P(1-\overline{\theta}))^2,\ \overline{T}^* = \frac{1}{4}(\overline{\theta}-\underline{\theta})^2 - \delta_P R,$$
$$\lambda_1^* = \frac{(1+\delta_P)(1-\overline{\theta})(1-\mu)}{\overline{\theta}-\underline{\theta}},\ \lambda_2^* = -\frac{\mu(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3^* = \frac{-\delta_P(1-\mu)(1-\overline{\theta})-(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},$$
$$\lambda_4^* = \frac{\delta_P(1-\mu)(1-\overline{\theta})+(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

In this case $\lambda_2^* < 0$. Thus, there are no solutions that satisfy the conditions in case (iv).

*Case (v).* Now $(IC_{\overline{\theta}}), (IC_{\underline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\hat{\underline{x}} = \frac{1}{2}(1+\underline{\theta}-\delta_P\mu(1-\overline{\theta})),\ \hat{\overline{x}} = \frac{1}{2}(1+\underline{\theta}+\delta_P(1-\mu)(1-\overline{\theta})),$$
$$\hat{\underline{T}} = \frac{1}{4}(\delta_P\mu(1-\overline{\theta})-(1-\underline{\theta}))^2,$$
$$\hat{\overline{T}} = \frac{1}{4}(\delta_P(1-\mu)(1-\overline{\theta})+(1-\underline{\theta}))^2 - \delta_P R,$$
$$\hat{\lambda}_1 = \frac{\delta_P(1-\overline{\theta})\mu(1-\mu)}{\overline{\theta}-\underline{\theta}},\ \hat{\lambda}_2 = \mu\frac{(\delta_P(1-\mu)(1-\overline{\theta})-(\overline{\theta}-\underline{\theta}))}{\overline{\theta}-\underline{\theta}},\ \hat{\lambda}_4 = 1.$$

We have that $\hat{\lambda}_1 \geq 0$, but $\hat{\lambda}_2 \geq 0$ if and only if $\mu \leq \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}$. Replacing the solution on the constraint $(P_{\overline{\theta}})$ we obtain that it is satisfied. Thus, when $\frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})} > 0$, if $\mu \leq \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}$, the vector $\left(\hat{\underline{x}}, \hat{\overline{x}}, \hat{\underline{T}}, \hat{\overline{T}}, \hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3, \hat{\lambda}_4\right)$ with $\hat{\lambda}_3 = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.

*Case (vi)*. Now $(IC_{\underline{\theta}})$, $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}'' = \frac{1+\underline{\theta}}{2},\ \overline{x}'' = \frac{\overline{\theta}+\underline{\theta}}{2},\ \underline{T}'' = \frac{1}{4}(1-\underline{\theta})^2,\ \overline{T}'' = \frac{(\overline{\theta}-\underline{\theta})^2}{4} - \delta_P R,$$
$$\lambda_2'' = -\frac{\mu(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3'' = -\frac{(1-\overline{\theta})\mu}{\overline{\theta}-\underline{\theta}},\ \lambda_4'' = \frac{(\overline{\theta}-\underline{\theta})+\mu(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

We have that $\lambda_2'' < 0$. Thus, there are no solutions that satisfy the conditions in case (vi).

Note that $\frac{\partial^2 \mathcal{L}}{\partial \underline{x}^2} = -2(1-\mu)+2(\lambda_1-\lambda_2-\lambda_4)$. From (12) we have that $(\lambda_1-\lambda_2-\lambda_4) = -(1-\mu)$, and then $\frac{\partial^2 \mathcal{L}}{\partial \underline{x}^2} < 0$. Also, $\frac{\partial^2 \mathcal{L}}{\partial \overline{x}^2} = -2\mu - 2\lambda_1 + 2\lambda_2 - 2\lambda_3 < 0$ when $\lambda_2 = 0$. In the case $\lambda_2 > 0$, we have that $-2\mu - 2\lambda_1 + 2\lambda_2 - 2\lambda_3 = -2\mu < 0$. Similar to Lemma 2, the Lagrangian function is strictly concave and the Kuhn-Tucker first-order conditions are also sufficient. In sum, the solution of the maximization problem is the following:

$$(\underline{x}, \overline{x}, \underline{T}, \overline{T}) = \begin{cases} \left(\underline{x}^*, \overline{x}^*, \underline{T}^*, \overline{T}^*\right) & \text{if } \mu > \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}, \\ \left(\underline{x}', \overline{x}', \underline{T}', \overline{T}'\right) & \text{if } \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\} \le \mu \le \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}, \\ \left(\hat{\underline{x}}, \hat{\overline{x}}, \hat{\underline{T}}, \hat{\overline{T}}\right) & \text{if } 0 \le \mu < \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\}. \end{cases} \tag{41}$$

The interest group's expected payoff from separation equals

$$V^{SEP} = (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G^R(0)\right) + \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G^R(1)\right),$$

where $(\underline{x}, \overline{x}, \underline{T}, \overline{T})$ is as in equation (41) and $V_G^R(0)$ and $V_G^R(1)$ are $G$'s continuation values.

Conditional on pooling, $G$'s maximization problem is

$$\max_{x,T\in\mathbb{R}} \mu\left(-(x-1)^2 - T + \delta_G V_G^R(\mu)\right) + (1-\mu)\left(-(x-1)^2 - T + \delta_G V_G^R(\mu)\right)$$
$$s.t.\ (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}),$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P(\overline{\theta}, \mu_0) \ge 0, \tag{$P_{\overline{\theta}}$}$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P(\underline{\theta}, \mu_0) \ge 0. \tag{$P_{\underline{\theta}}$}$$

The solution to this problem is to offer $(x, T) = \left(\frac{1+\underline{\theta}}{2}, \frac{(1-\underline{\theta})^2}{4}\right)$. The interest group's expected payoff of the pooling strategy is equal to

$$V^{POOL} = -\left(\frac{1+\underline{\theta}}{2} - 1\right)^2 - \frac{(1-\underline{\theta})^2}{4} + \delta_G V_G^R(\mu).$$

In Lemma 2, we found that there is a cut-off $\delta_G^\dagger$ such that $V^{SEP} \geq V^{POOL}$ if and only if $\delta_G \geq \delta_G^\dagger$. Here, we obtain a similar result: there is a cut-off $\delta_G^{\dagger,R}$ such that $V^{SEP} \geq V^{POOL}$ if and only if $\delta_G \geq \delta_G^{\dagger,R}$, where

$$\delta_G^{\dagger,R} = \begin{cases} \frac{\delta_P^2(1-\mu)^2(1-\bar{\theta})^2}{(\bar{\theta}-\underline{\theta})(\mu(\bar{\theta}+\underline{\theta}-2)-2\bar{\theta}+2)} - \frac{2\delta_P(1-\mu)R}{(\bar{\theta}-\underline{\theta})(\mu(\bar{\theta}+\underline{\theta}-2)+2(1-\bar{\theta}))} & \text{if } \mu \leq \max\{0, \frac{\delta_P(1-\bar{\theta})-(\bar{\theta}-\underline{\theta})}{\delta_P(1-\bar{\theta})}\} \\ \frac{2\delta_P(1-\mu)(1-\bar{\theta})-(\bar{\theta}-\underline{\theta})}{2+\mu(\bar{\theta}+\underline{\theta}-2)-2\bar{\theta}} - \frac{2\delta_P(1-\mu)R}{(\bar{\theta}-\underline{\theta})(\mu(\bar{\theta}+\underline{\theta}-2)+2(1-\bar{\theta}))} & \text{if } \max\{0, \frac{\delta_P(1-\bar{\theta})-(\bar{\theta}-\underline{\theta})}{\delta_P(1-\bar{\theta})}\} < \mu \leq \frac{1-\bar{\theta}}{1-\underline{\theta}} \\ \frac{\mu(\bar{\theta}-\underline{\theta})\left(2\delta_P(1-\mu)(1-\bar{\theta})-(\bar{\theta}-\underline{\theta})\right)}{(1-\mu)^2(1-\bar{\theta})^2} - \frac{2\delta_P\mu R}{(1-\mu)(1-\bar{\theta})^2} & \text{if } \frac{1-\bar{\theta}}{1-\underline{\theta}} < \mu \leq \frac{(1+\delta_P)(1-\bar{\theta})}{(1+\delta_P-\delta_P\bar{\theta}-\underline{\theta})} \\ \delta_P^2 + 2\delta_P - \frac{\mu(1-\underline{\theta})^2}{(1-\mu)(1-\bar{\theta})^2} + \frac{1}{1-\mu} - \frac{2\delta_P\mu R}{(1-\mu)(1-\bar{\theta})^2} & \text{if } \frac{(1+\delta_P)(1-\bar{\theta})}{(1+\delta_P-\delta_P\bar{\theta}-\underline{\theta})} < \mu. \end{cases}$$

A direct comparison of $\delta_G^{\dagger,R}$ and $\delta_G^\dagger$ shows that $\delta_G^{\dagger,R} < \delta_G^\dagger$. Thus, revolving door incentives increases the incentives for screening relative pooling. Also, we obtain that $\hat{\delta}_G^r$ is decreasing in $R$, which proves the result.