Learning by Lobbying

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Abstract

How do interest groups learn about and influence politicians over time? We develop a game-theoretic model where an interest group can lobby a politician while learning about their ideological alignment. Our analysis reveals a fundamental tradeoff: interest groups must balance gathering information against exerting immediate influence, while politicians strategically manage their reputations to shape future interactions. These strategic forces generate systematic dynamics: policies and transfers shift in tandem, with early-career politicians showing greater policy variance and extracting larger rents through reputation management than veterans. Uncertainty about alignment increases policy volatility as groups experiment with offers, while institutional features like committee power and revolving-door incentives systematically alter both learning incentives and influence strategies. Our results shed new light on how interest group influence evolves across political careers and varies with institutional context.

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1 Introduction

The art of effective lobbying rests on a fundamental principle: *understanding what politicians want*. As one seasoned lobbyist emphasizes, "it is not about what you want; it is about what the other person needs" (Levine 2008, p. 163). While this knowledge clearly helps interest groups tailor their lobbying strategies, a crucial question remains: how do interest groups acquire this knowledge in the first place? Some groups attempt to purchase information directly through background research or revolving-door hiring. However, many groups could also learn through another powerful mechanism: the act of lobbying itself.

Through repeated interactions, interest groups gain valuable insights into politicians' preferences, which in turn shapes their future advocacy efforts. As one congressman notes, effective lobbyists must "be prepared to change your argument or strategy based on what you learn" (Levine 2008, p. 147).[1] This dynamic interplay between learning about and influencing politicians presents a fundamental challenge for understanding how outside interests shape policy. The process involves three interconnected forces: (i) lobbying efforts affect how much groups learn about policymakers' preferences, (ii) newly acquired information shapes future influence attempts, and (iii) politicians' concern for how interest groups perceive them creates reputation and signaling incentives (Egerod and Tran 2023).

We examine two central questions about this learning-lobbying nexus: How do interest groups approach lobbying while simultaneously trying to learn about politicians? And how does this dual dynamic shape the evolution of lobbying relationships over time?

Despite substantial evidence that experience in lobbying specific policymakers is valuable (Kerr, Lincoln and Mishra 2014; Drutman 2015), we lack a comprehensive theoretical understanding of how strategic forces shape these relationships over time. Existing theories typically analyze settings with complete information or one-shot interactions, limiting their

[1] Echoing this point, a veteran lobbyist explains that "[w]e build relationships with the staffers and [...] we also get to know what their political inclinations are and where their policy interests lie, and we use that as well" (Leech 2014, p. 180).

ability to capture the interplay between learning and influence in sustained lobbying relationships. This analytical challenge is particularly significant given the substantial initial investments that interest groups must make to create and maintain these relationships (Snyder 1992; Rosenthal 2008).

Uncertainty about alignment creates distinct complexities in how interest groups exert influence—groups must calibrate their lobbying attempts to optimize influence across different possible preferences, creating spillover effects. Securing optimal terms from aligned politicians requires carefully considered approaches to misaligned ones, and vice versa.

While interest groups can pursue various strategies for learning about politicians' preferences—including hiring revolving-door lobbyists with insider knowledge (Godwin, Ainsworth and Godwin 2012; Strickland 2020, 2023)[2]—they always have the option to learn through direct interaction. Indeed, this universally available learning channel shapes how groups approach their broader information-gathering efforts: their willingness to invest in alternative sources of information depends crucially on the costs and benefits of learning through lobbying itself.

To analyze these complexities, we develop a formal model of evolving relationships between interest groups and politicians. Our two-period framework captures the essential dynamic: an interest group lobbies a politician whose preferences—either aligned or divergent—are initially unknown. The group offers menus pairing policy proposals with potential benefits and learns about the politician through first-period choices.[3] This approach allows us to parse how learning and influence interact in ongoing lobbying relationships.

Our model reveals several interconnected insights about the dynamics of lobbying relation-

[2]Indeed, a primary motive for hiring lobbyists is "buying advice on who is likely to be sympathetic to them on a particular issue, how best to win the support of particular members" (Drutman 2015, p. 163).

[3]A 'menu' in our game theoretic model can be interpreted as equivalent to the outcome of a bargaining process between an interest group and politician. That is, an interest group may have certain policy demands and may make a quid-pro-quo promise in return to the politician.

ships. At its core, there is a fundamental connection between lobbying and learning about politicians' ideological alignment. While interest groups gain valuable information by observing politicians' responses to lobbying, politicians strategically manage these perceptions to secure better future terms. This creates a dynamic strategic tension: interest groups must calibrate their lobbying strategies to both extract information and account for politicians' reputational incentives. This tension produces distinctive patterns in lobbying relationships. Politicians who appear closely aligned early in their careers may appear less aligned over time—not due to preference changes, but because interest groups refine their strategies. We may even observe politicians choosing policies that appear less favorable to interest groups than no lobbying at all, a pattern that emerges from the strategic revelation of preferences. The analysis reveals particularly rich dynamics around newly elected politicians. While standard approaches suggest that lobbying should consistently push policies toward interest group preferences, we find that politicians may receive notably favorable deals or enact seemingly unfavorable policies as part of the learning process. Even without active lobbying, politicians may moderate their positions to shape future lobbying terms.

The Strategic Dynamics of Learning and Influence. Our analysis reveals how interest groups balance learning against influence over time, while politicians navigate between immediate policy gains and long-term reputation. These competing priorities create interconnected trade-offs that shape behavior in both periods. First-period lobbying and policymaking reflect both immediate incentives and long-term considerations. The interest group weighs using current knowledge to tailor offers against the potential for learning that enables more targeted future lobbying. Meanwhile, the politician balances selecting immediately favorable options versus cultivating a reputation that will elicit more favorable future terms.

The informational aspects of these dynamics stem from the strategic value of learning about or concealing politicians' preferences. This information directly impacts how effectively each side can pursue its objectives: both prefer policy closer to their ideal points, while the interest group aims to minimize transfers and the politician seeks to maximize them.

Equilibrium Patterns and Strategic Behavior. The interest group's optimal strategy depends critically on its prior belief about alignment and both actors' valuation of the future. When alignment appears highly unlikely, the interest group prioritizes securing favorable terms with misaligned politicians through carefully calibrated offers, while extracting additional policy concessions from aligned politicians relative to complete information benchmarks. Under moderate uncertainty about alignment, the group adjusts offers meant for misaligned politicians, with distortions increasing in the probability of alignment. When alignment appears likely, the group combines efficient offers to aligned types with strategically conservative misaligned offers, sometimes accepting policies further from its ideal than under no lobbying to maintain screening effectiveness. However, when the interest group is sufficiently impatient and alignment appears unlikely, it forgoes learning in favor of a unified approach treating all politicians as misaligned, sacrificing valuable information for immediate gains.

Lobbying Dynamics and Empirical Implications. Our equilibrium analysis reveals systematic patterns in how policies and transfers evolve over time. Policy and transfer adjustments move in tandem, either both shifting towards or away from the interest group's preferred position. These shifts more frequently favor the interest group when it assigns lower probability to alignment with the politician. Policy volatility varies systematically with uncertainty—in contexts of greater uncertainty, such as with political newcomers, we observe more variable policy choices as groups balance learning against influence. Politicians with longer time horizons can secure greater early-career benefits but also face more intensive screening from interest groups, suggesting distinct patterns of influence across career stages.

These theoretical insights help interpret empirical patterns in lobbying relationships. When interest groups successfully screen politicians early in their careers, aligned politicians' policy choices will decrease or remain constant over time, while misaligned politicians' choices become increasingly favorable to interest groups. This convergence in late-career behavior—

where aligned and misaligned politicians exhibit more similar choices than at the start—reflects the diminishing role of screening as uncertainty resolves.

Extensions and Broader Implications. Our analysis extends in four key directions that further illuminate the learning-lobbying nexus. Regarding the value of early-career information, we analyze when knowledge about politicians' preferences creates value—a question central to understanding the prevalence of revolving-door lobbyists (Blanes i Vidal, Draca and Fons-Rosen 2012; McCrain 2018; Strickland 2020). This value increases with politicians' patience, as more forward-looking politicians have stronger incentives to strategically manage their reputations. These findings suggest why firms often engage specialized lobbying intermediaries who can distribute learning costs across multiple clients, particularly when dealing with young politicians whose long career horizons make direct screening more costly.

Regarding early access, our analysis reveals conditions under which interest groups may benefit from temporarily deferring active lobbying of new politicians, allowing them to learn from others' interactions while avoiding immediate screening costs. This finding contributes to our understanding of when and how interest groups cultivate relationships with politicians across their careers.

On institutional constraints, we analyze how variations in policymaking constraints—such as committee leadership or party discipline—affect interest groups' learning and influence incentives. More institutionally powerful politicians generate both stronger learning incentives and expanded learning opportunities, helping explain systematic differences in lobbying patterns between committee chairs, party leaders, and rank-and-file members.

Finally, our investigation of revolving door incentives between politics and lobbying reveals two countervailing effects: these opportunities can reduce interest groups' screening costs while simultaneously altering politicians' incentives to manage their reputations. This provides new perspectives on how career prospects influence political decision-making even before any transitions occur.

Theoretical and Empirical Contributions. Our framework enriches our understanding of interest group influence in three interconnected ways. It illuminates how the interplay between learning and influence shapes lobbying relationships over time, highlighting dynamics that emerge from the interaction of uncertainty and repeated engagement. The analysis provides theoretical foundations for interpreting empirical patterns in lobbying behavior and policy choices across politicians' careers. Additionally, it generates new insights about how institutional features—from committee power to revolving door opportunities—systematically affect both learning and influence strategies.

These contributions advance scholarship on interest group politics in two key directions. Theoretically, we demonstrate how standard approaches to lobbying can be enriched by incorporating uncertainty and dynamic learning, revealing mechanisms that help explain observed patterns of political influence. Empirically, our framework provides new tools for analyzing how lobbying relationships evolve, generating testable predictions about when and how interest groups adjust their strategies as they learn about politicians' preferences.

Beyond interest group politics, our analysis speaks to fundamental questions about democratic representation and special interest influence. In an era of increasing political complexity and the rise of "political amateurs" (Porter and Treul 2024), the ability to understand and anticipate policymakers' preferences becomes increasingly crucial for shaping legislation. By illuminating the complex interplay between learning and influence in lobbying relationships, we provide a framework for understanding how these dynamics shape both outside influence and policy outcomes.

2 Relationship to Existing Literature

We shed new light on the dynamics of lobbying relationships and political influence.[4] Our analysis contributes to the understanding of interest groups and lobbying by providing the

[4]For empirical work on the dynamics of lobbying, see e.g., Kerr, Lincoln and Mishra (2014). For broader overviews of theoretical and empirical work on lobbying, see Grossman and Helpman (2001), Bombardini and Trebbi (2020), and Schnakenberg and Turner (2023).

first theoretical framework that captures how lobbying relationships evolve over time as groups learn about politicians' preferences through repeated interactions. The key innovation lies in modeling the bidirectional strategic dynamic: interest groups must balance learning against influence, while politicians manage their reputations to shape future interactions. This contrasts with existing work that typically assumes either that interest groups lobby once (Bernheim and Whinston 1986; Martimort and Semenov 2008; Minaudier 2022), know their target's preferences (Iaryczower and Oliveros 2017, 2023; Chen and Zápal 2022; Bils, Duggan and Judd 2021), or both (Hall and Wayman 1990; Austen-Smith and Wright 1994; Besley and Coate 2001; Schnakenberg 2017).

By parsing the interest group's learning process and the politician's reputational incentives in lobbying relationships, our model fills a crucial gap in the existing literature. This aligns with the observation that beliefs about legislators' preferences are updated through the lobbying process itself (Austen-Smith and Wright 1992). Our focus contrasts with existing studies of learning in ongoing lobbying relationships, where politicians learn about interest groups' characteristics such as their preferences or truthfulness (Groll and Ellis 2014, 2017; Ellis and Groll 2024). This emphasis on learning through repeated interactions provides a novel lens for understanding the evolution of lobbying relationships and their impact on policy over time, generating new predictions about how lobbying strategies and policy outcomes may evolve as relationships develop (Kerr, Lincoln and Mishra 2014).

We model lobbying as *exchange* (Grossman and Helpman 2001), where interest groups can influence policy via quid-pro-quo. Although lobbying can be modeled in various other ways (e.g., legislative subsidies (Hall and Deardorff 2006) or information transmission (Schnakenberg 2017; Awad 2020)), a common theme is that interest groups aim to influence politicians' behavior, and their efforts to do so will depend on the preferences of both sides.

To model the dynamic aspects of learning and lobbying influence, we build on established models of dynamic contracts in economics (Hart and Tirole 1988; Laffont and Tirole 1990;

Salanié 2005). These models typically feature a ratchet effect (Freixas, Guesnerie and Tirole 1985), where informed players have incentives to conceal their true characteristics to avoid future exploitation of this information. However, our setting introduces a novel technical challenge: the spatial nature of policy preferences creates complex spillover effects when offering menus and screening different types. Due to this departure from standard single-crossing conditions,[5] equilibrium conditions from menu pricing models do not apply. Thus, we make a modest technical contribution by providing new analytical techniques that may prove useful for studying other forms of political influence.

Our extensions shed light on various related interest group tactics and considerations including access, the revolving door, and policymaking constraints. These extensions relate to existing work on access (Judd 2023), the informational value of lobbyists (Hirsch et al. 2023), and the effects of hiring from special interests (Hübert, Rezaee and Colner 2023). Additionally, we consider how political constraints affect the dynamic aspects of lobbying (Bils, Duggan and Judd 2021).

3 The Model

We model a two-period interaction between an interest group and a politician. In each period, the interest group lobbies by offering combinations of policies and transfers, while the politician can accept one combination or set policy independently.[6] The interest group is uncertain about the politician's ideology, which may be either aligned or misaligned with its interests. By observing the politician's responses to lobbying efforts, the interest group updates its beliefs about their alignment. Simultaneously, the politician strategically manages its reputation, aware of this learning process. Our model thus captures the iterative nature

[5]For example, in a seller-buyer or firm-worker environment, this single-crossing condition is typically implicitly assumed, as in e.g., Beccuti and Möller (2018), Gerardi and Maestri (2020) and Breig (2022).

[6]Note that this is another departure from related models in economics as discussed in the literature review—in standard models, either the agent cannot do anything after rejection, or the game simply ends.

of lobbying and the dynamic process of learning about politicians' preferences through their responses to lobbying and subsequent policy decisions.

Players. There are two players: an interest group, G, and a politician, P.

Timing. There are two periods of policymaking. In each period $t \in \{1, 2\}$, P will enact a policy and G can lobby by offering P a menu M_t of policy-transfer pairs, with G choosing both (i) how many pairs to include and (ii) for each pair (x, T), the exact policy x and transfer T. Next, P observes M_t and then either selects one pair or rejects all of them. If P selects a pair (x, T) from M_t, then the enacted policy is $x_t = x$ and G transfers T to P. Otherwise, if P rejects the menu, then P chooses x_t freely. Since P's selection determines the realized policy and transfer—that is, we abstract from short-term commitment issues—we follow the literature on menu auctions and refer to each pair in a menu as a *contract*. Accordingly, we define an arbitrary contract as $c = (x, T)$.

Payoffs. In each period, G's utility function is $\Pi(x, T) := -(x - 1)^2 - T$, where x is the implemented policy, 1 is G's ideal point, and T is the accepted transfer. Similarly, P's utility function is $U(x, T) := -(x - \theta)^2 + T$, where θ denotes P's ideal point. Notably, P's ideal point can be either $\underline{\theta}$ or $\overline{\theta}$, so we also refer to θ as P's *type*. We assume $\underline{\theta} < \overline{\theta} < 1$, so that $\underline{\theta}$ can be interpreted as the *misaligned* type and $\overline{\theta}$ as the *aligned* type, with the latter being closer to G.[7]

Each player's cumulative payoff is the sum of their utility across both periods. Each player discounts second-period utility with (potentially different) discount factors: $\delta_P, \delta_G \in [0, 1]$.

Information. The interest group G does not know P's ideal point, $\theta \in \{\underline{\theta}, \overline{\theta}\}$. All other features of the game are common knowledge. At the beginning of the game, G's prior belief puts probability $\mu_0 \in [0, 1]$ on $\theta = \overline{\theta}$. Thus, a key aspect of the interaction is that G will

[7]Alternatively, the misaligned type could be viewed as G's *adversary* and the aligned type as its *ally*.

update its belief about P after observing her first-period behavior.

Strategies and Equilibrium Concept. We study Perfect Bayesian Equilibria (PBE) in pure strategies.[8] Thus, strategies are sequentially rational at every information set and Bayes' rule is applied wherever possible. Since off-path information sets arise in an equilibrium, we apply the Never a Weak Best Response (NWBR) refinement.[9] This refinement ensures that off-equilibrium-path beliefs assign higher probability to types that are more inclined to deviate.

Our results in the main text focus on equilibrium strategies of the interest group and politician. Formal statements of beliefs and all proofs are provided in the Appendix.

Model Discussion. Our model focuses on the interplay between learning and influence in lobbying relationships. Our baseline setting maintains several assumptions that highlight the core strategic dynamics and each serve a specific analytical purpose.

First, we model the interest group's uncertainty about the politician's preferences using a binary type space—the politician is either aligned or misaligned with the interest group. This parsimonious approach captures a fundamental distinction: some politicians are more inclined to support the interest group's preferred policies, even absent lobbying pressure.

Second, our two-period framework reflects the dynamic nature of lobbying relationships while remaining tractable. The first period captures initial interactions under uncertainty, while the second period shows how behavior evolves with learning. This focus on early learning

[8]We focus on pure strategies primarily for analytical tractability. Although mixed strategies could potentially yield different equilibrium outcomes (see Bester and Strausz 2001, for more details), the pure strategy analysis captures important strategic considerations in lobbying relationships. Moreover, our approach follows the precedent set by canonical models with menu auctions (e.g., Bernheim and Whinston 1986), which provides a justification for restricting the set of menu options.

[9]In particular, the politician may choose to reject an on-path offer, leading to an off-path information set. Alternatively, the interest group may make an off-path offer, leading to an off-path information set as well. In both such cases, applying the NWBR refinement helps specify what the interest group's beliefs are off the path of play. For an overview of refinements in signaling games, see Fudenberg and Tirole (1991) and Manelli (1997).

is well-motivated: as Kerr, Lincoln and Mishra (2014, p. 344) note, "[f]irms may gain from learning about policymakers' private dispositions, which may not be fully reflected in their public positions," and crucially, "the costs of learning and establishing relationships with policymakers are likely to be the highest in a firm's first several years of lobbying." We assume neither player can commit to future behavior, creating tension between immediate gains and long-term considerations. This reflects real-world constraints on political promises and allows us to study how both sides navigate important strategic trade-offs in how they learn and influence.

Third, we model lobbying as an *exchange* of policy for transfers (see, e.g., Grossman and Helpman 1994, 2001). The "transfers" can represent various forms of political support, including campaign contributions, charitable donations, or other legislative assistance, making the framework applicable across diverse lobbying scenarios. The *exchange* approach captures the granular nature of real-world influence.[10] As one lobbyist notes, "[w]hat matters is getting stuff put in the bill, a line here, a line there... What you're looking to do is put a line in a law, get something tweaked. You're looking to change this line in subsection B. You just need one person to make that change" (Drutman 2015, p. 31). Indeed, Rosenthal (2008, p. 218) observes that lobbyists are "among a handful of people who control the details. And the details are usually quite important." While lobbying can be modeled in other ways, our framework is particularly suited for analyzing how interest groups adjust their strategies as they learn about politicians' preferences. It clearly delineates the mechanisms of influence and learning while maintaining analytical tractability. We implement this exchange through a menu-auction framework (Bernheim and Whinston 1986), where the interest group periodically offers menus of policy-transfer pairs. This structure allows for sophisticated lobbying strategies that simultaneously probe preferences and exert influence.

Finally, we abstract from certain real-world complexities such as electoral pressures and

[10]Moreover, the exchange model is particularly relevant for studying prominent normative concerns about quid pro quo arrangements in politics.

competing interest groups, which have been studied elsewhere (Austen-Smith and Wright 1994; Bils, Duggan and Judd 2021). While important, incorporating these elements would complicate the analysis without fundamentally changing our core insights about learning and influence. Our extensions exploring policymaking constraints and early-career access touch on some of these considerations, demonstrating how they interact with our core insights on lobbying dynamics.

4 Analysis

Our analysis proceeds in several stages. We begin by characterizing behavior under complete information in two benchmark cases: with and without lobbying. We then begin our main analysis, studying lobbying in a static setting with incomplete information, which corresponds to the second period of our model. Working backward, we study the first-period interaction, where lobbying occurs in a dynamic setting with incomplete information. To conclude our main analysis, we flesh out the connections between equilibrium behavior across both periods to parse the dynamics of policies and transfers. Finally, we extend the baseline model to study how early-career information, access, revolving-door incentives, and policymaking constraints affect our main insights.

Two benchmarks

To set the stage for our main analysis, we characterize behavior in two benchmark settings: (i) no lobbying and (ii) lobbying with complete information.

Benchmark 1 (No lobbying)**.** *If the interest group cannot lobby, then the politician will set policy at her ideal point, θ, in both periods.*

Without lobbying, P obtains a payoff of zero in both periods, regardless of her type. However, P's type does impact G's payoff, $-(1+\delta_G)(1-\theta)^2$. Specifically, G's payoff is lower if P is misaligned, i.e., $\theta = \underline{\theta}$ than if P is aligned with $\theta = \overline{\theta}$.

Benchmark 2 (Lobbying with complete information)**.** *If G can lobby and has complete information about P, then in each period P enacts policy $x_\theta := \frac{1}{2}(1+\theta)$ and receives transfer*

$t_{\theta} := \frac{1}{4}\,(1-\theta)^2$.

If G knows P's ideal point, θ, then it can perfectly calibrate its lobbying. Notably, P's ideal point, θ, acts as a reservation policy since she will enact it if she rejects G's menu. Thus, θ determines both (i) the smallest transfer that is required for P to deviate from her ideal policy and (ii) G's willingness to provide that minimal compensation. Specifically, to induce P to enact any other policy x, G must offer her a transfer that is at least as large as $T_{\theta}(x) := (\theta - x)^2$.

In each period, G optimally balances its marginal benefit of more favorable policy against its marginal cost of increasing the transfer. If G and P are aligned, then G induces a mild policy shift at a moderate cost. Otherwise, G has a larger marginal gain from shifting P's policy and therefore induces a larger shift at a higher cost.

The politician never receives any surplus, since she is indifferent between accepting and rejecting the offered contract in both periods. Thus, we refer to $c_{\underline{\theta}} := (x_{\underline{\theta}}, t_{\underline{\theta}})$ as the $\underline{\theta}$-*efficient contract*, and define $c_{\overline{\theta}}$ analogously.[11] Accordingly, we define $\pi_{\underline{\theta}} := -(x_{\underline{\theta}} - 1)^2 - t_{\underline{\theta}}$ as G's $\underline{\theta}$-*efficient payoff* and define $\pi_{\overline{\theta}}$ as G's efficient payoff given the aligned type $\overline{\theta}$.

Lobbying with incomplete information

We now begin our main analysis, in which the interest group does not know the politician's type. We study how the interest group's influence is shaped as a function of *uncertainty* about the legislator's alignment and the effect of strategic *dynamic* considerations. Since we focus on pure strategies, the Revelation Principle implies that it is without loss of generality to focus on menus with at most three options: two 'type-specific' contracts and an 'empty' contract. The type-specific contracts are distinct: a 'low contract' $\underline{c} = (\underline{x}, \underline{T})$ intended for type $\underline{\theta}$ and a 'high contract' $\overline{c} = (\overline{x}, \overline{T})$ intended for type $\overline{\theta}$. We say a menu is *separating* if $\underline{c} \neq \overline{c}$ and *pooling* if $\underline{c} = \overline{c}$.

[11] That is, $c_{\underline{\theta}}$ and $c_{\overline{\theta}}$ are the two possible full-information contracts.

Second-period lobbying and policymaking

In the second period, G's sole focus is influence. Although this focus parallels Benchmark 2, a key distinction is that G may not know P's ideal point, θ. Based on first-period equilibrium strategies, G has belief $\mu_1 \in [0, 1]$ about θ, where $\mu_1 := \mu(\theta = \overline{\theta}|h)$ represents the posterior probability that P and G are aligned given history h.

The interest group wants to induce more favorable policy without overpaying or underlobbying. However, that is impossible unless G knows θ. That is, G's equilibrium menu cannot include both efficient contracts $c_{\underline{\theta}}$ and $c_{\overline{\theta}}$. If such a menu were offered, then a $\overline{\theta}$-type politician would choose $c_{\underline{\theta}}$, so G would infer they are overpaying a $\overline{\theta}$-type politician to potentially enact *worse* policy, $\underline{x}_2$ instead of $\overline{x}_2 > \underline{x}_2$. More broadly, G's menu must ensure *incentive compatibility* for P to ensure that each type selects the intended contract.

Since G cannot prevent P from sabotaging G's lobbying attempts by misrepresenting its preference, it will proactively distort lobbying itself through the offered menu. Specifically, G's second-period menu will not include both $c_{\underline{\theta}}$ and $c_{\overline{\theta}}$. This distortion could in principle arise as either a *separating menu* with two options or a *pooling menu* with only one option. A separating menu may in principle require too many distortions, such that it is better to provide only a single contract. However, in the second period of any equilibrium, we show that the absence of future considerations will induce G to offer a separating menu.

Thus, in G's optimal separating menu, at least one contract will be distorted to induce different behaviors from the types. To deter a $\overline{\theta}$-type politician from selecting the $\underline{\theta}$-contract, G offers a menu that either (i) overpays $\overline{\theta}$ to ensure type $\overline{\theta}$ chooses its 'efficient' policy $x_{\overline{\theta}}$ or (ii) makes an underaggressive $\underline{\theta}$-offer (i.e., $x_2 < x_{\underline{\theta}}$ with $t_2 = T_{\underline{\theta}}(x_2)$), extracting fewer policy concessions from type $\underline{\theta}$. The particular distortion G chooses depends on μ_1, G's updated belief about P.

If G believes P is probably aligned (i.e., μ_1 is sufficiently high), then G offers a separating menu with only one distortion: an underaggressive $\underline{\theta}$-offer. This menu pairs (i) a $\overline{\theta}$-efficient contract and (ii) an overly conservative $\underline{\theta}$-contract. Essentially, G makes the misaligned contract less appealing to the aligned $\overline{\theta}$-type so that she would choose the $\overline{\theta}$-efficient contract. Although this sacrifices efficiency in the $\underline{\theta}$-contract, it is relatively unlikely to materialize. To minimize the distortion, the contracts are calibrated so that (i) a $\overline{\theta}$-type is indifferent between them (and rejecting) and (ii) a $\underline{\theta}$-type is indifferent between accepting the misaligned contract and rejecting the interest group's offer.

Otherwise, if μ_1 is lower, G focuses more on getting a good deal when lobbying type $\underline{\theta}$ and offers a separating menu with two distortions: an excessive $\overline{\theta}$-transfer *and* an underaggressive $\underline{\theta}$-offer. Specifically, this menu pairs (i) a $\overline{\theta}$-contract that overpays type $\overline{\theta}$ for policy $x_{\overline{\theta}}$ with (ii) a $\underline{\theta}$-contract that is again overly conservative but to a lesser degree. The size of these distortions varies inversely with μ_1. As it decreases, making $\underline{\theta}$ more likely, G increasingly prioritizes efficiency in the $\underline{\theta}$-contract while offering increasingly excessive (but less likely) $\overline{\theta}$-payment to ensure incentive compatibility.

Lemma 1 precisely characterizes second-period policy and transfers. A key factor is whether G's belief μ_1 is relatively high or low. Let $\tilde{\mu}_1 := \frac{1-\overline{\theta}}{1-\underline{\theta}} \in (0,1)$, which defines a cutpoint distinguishing the two qualitatively distinct equilibrium behaviors.

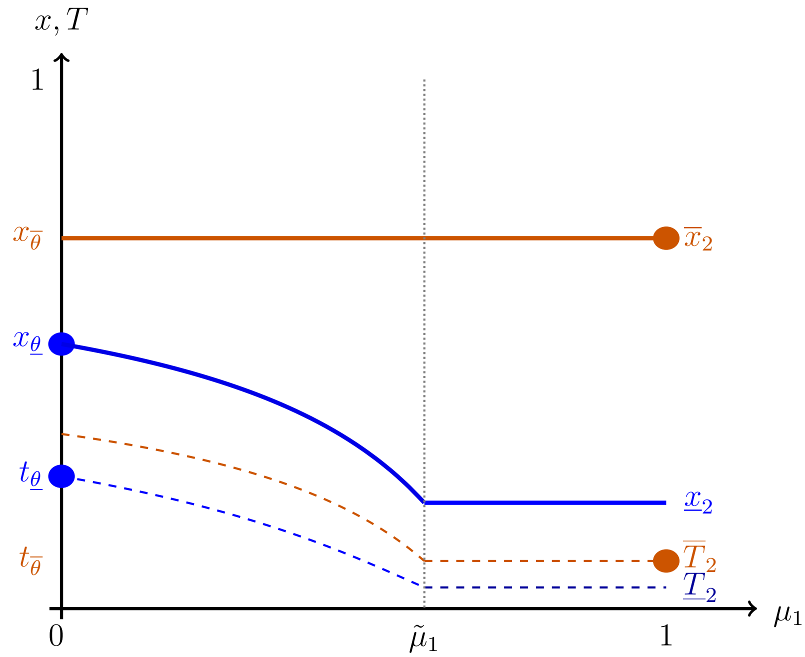
Lemma 1. *In any second-period history and for every belief μ_1, the interest group's optimal menu in equilibrium is separating.*

1. *If $\mu_1 \leq \tilde{\mu}_1$, then (i) a $\overline{\theta}$-type politician enacts $\overline{x}_2 = x_{\overline{\theta}}$ and receives $\overline{t}_2 = T_{\overline{\theta}}(\overline{x}_2) + \frac{(\overline{\theta}-\underline{\theta})(1-\overline{\theta}-\mu_1(1-\underline{\theta}))}{1-\mu_1}$, whereas (ii) a $\underline{\theta}$-type politician enacts $\underline{x}_2 = x_{\underline{\theta}} - \frac{\mu_1}{1-\mu_1}\frac{\overline{\theta}-\underline{\theta}}{2}$ and receives $\underline{t}_2 = T_{\underline{\theta}}(\underline{x}_2)$.*

2. *If $\mu_1 > \tilde{\mu}_1$, then (i) a $\overline{\theta}$-type politician enacts policy $\overline{x}_2 = x_{\overline{\theta}}$ and receives transfer $\overline{t}_2 = t_{\overline{\theta}}$, whereas (ii) a $\underline{\theta}$-type politician enacts $\underline{x}_2 = x_{\underline{\theta}} - \frac{1-\overline{\theta}}{2}$ and receives $\underline{t}_2 = T_{\underline{\theta}}(\underline{x}_2)$.*

Figure 1 illustrates Lemma 1 by displaying equilibrium policies (solid lines) and transfers (dashed lines) as functions of G's updated belief μ_1, for both aligned (orange) and misaligned (blue) types. The blue and orange dotted contracts illustrate the efficient contracts $c_{\underline{\theta}}$ and

Figure 1: Second-Period Policies and Transfers



Note: The figure illustrates equilibrium policies and transfers in Lemma 1. Blue lines represent the optimal contract for the misaligned type ($\underline{\theta} = 0$), while orange lines to the optimal contract for the aligned type ($\overline{\theta} = \frac{2}{5}$). The dots indicate the efficient contracts $c_{\underline{\theta}}$ given to the misaligned type if $\mu_1 = 0$ and $c_{\overline{\theta}}$ to the aligned type if $\mu_1 = 1$.

$c_{\overline{\theta}}$ that G would pick given complete information ($\mu_1 = 0$ and $\mu_1 = 1$). Any difference from these efficient contracts is an uncertainty-driven distortion of lobbying.

Figure 1 illustrates that policies and transfers are not sensitive to μ_1 above the cutpoint $\tilde{\mu}_1$, but they are below. The aligned type always receives an identical policy offer, but their transfer either decreases with μ_1 or remains constant. In contrast, the misaligned type's policy- and transfer-offer are always sensitive to G's belief. This implies that it is less likely to observe heterogeneity in terms of the policy that aligned politicians choose compared to those chosen by misaligned politicians. The interest group is more likely to hold back on its influence attempts over misaligned politicians than aligned ones.

First-period lobbying and policymaking

We now analyze first-period lobbying and policymaking, unpacking how the prospect of future lobbying opportunities shapes these activities.

Both the politician and interest group balance first-period incentives against forward-looking considerations about second-period consequences. G wants to influence policy favorably today while also learning about P to facilitate future lobbying. Meanwhile, P wants to receive favorable terms today while also managing her reputation to ensure favorable outcomes later. Each player's static incentive to obtain favorable terms is analogous to the second period. However, their forward-looking incentives—learning and reputation management, respectively—introduce new forces. Moreover, these forces are interdependent: G's learning is affected by P's approach to reputation management, which is in turn affected by P's anticipation of G's inferences.

In equilibrium, both P and G anticipate how their first-period interaction will influence second-period play. A key factor is the impact of first-period behavior on μ_1, G's updated belief about P, which shapes second-period policymaking—as characterized in Lemma 1—and, consequently, each player's continuation value following the first period. These continuation values are the channel for feedback effects through which second-period equilibrium behavior influences first-period incentives.

By learning about the politician, G can lobby more effectively in the second period, thereby altering its continuation value. Remark 1 characterizes how G's continuation value varies with its updated belief, μ_1.

Remark 1. *The interest group's continuation value is as follows,*

$$V_G(\mu_1) = \begin{cases} (1-\mu_1)(\pi_{\underline{\theta}}) + \mu_1 \left[\frac{(\overline{\theta}-\underline{\theta})^2}{1-\mu_1} + \pi_{\underline{\theta}}\right] & \text{if } \mu_1 \leq \tilde{\mu}_1 \\ (1-\mu_1)\left(\pi_{\underline{\theta}} + \pi_{\overline{\theta}}\right) + \mu_1\left(\pi_{\overline{\theta}}\right) & \text{if } \mu_1 > \tilde{\mu}_1. \end{cases}$$

A key implication of Remark 1 is that G benefits from more information. With more precise beliefs, G can more confidently tailor its menu of offers towards the more probable type of politician, anticipating that this increasingly efficient offer will be chosen. In the limiting case where G has beliefs $\mu_1 = 0$ and $\mu_1 = 1$, G's ex ante equilibrium payoff converges to $(1-\mu_0)V_G(0) + \mu_0 V_G(1) = (1-\mu_0)(\pi_{\underline{\theta}}) + \mu_0(\pi_{\overline{\theta}})$ in the second period, thereby minimizing distortions.

In equilibrium, G's learning is binary: it either learns everything or nothing. Specifically, it will either offer a separating menu, resulting in full learning ($\mu_1 \in \{0, 1\}$); or offer a pooling menu that both types would accept, resulting in no learning ($\mu_1 = \mu_0$). Thus, G's incentive to learn about P is based on the forecasted change in its continuation value relative to lobbying at μ_0 in the second period.

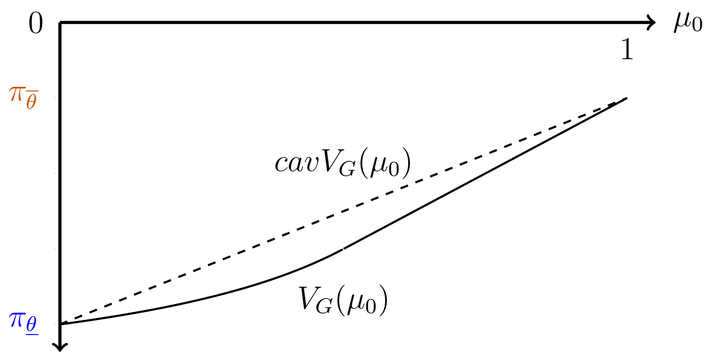
Definition 1. *In equilibrium, G's value of screening is* $W(\mu_0) := (1-\mu_0)\, V_G(0) + \mu_0\, V_G(1) - V_G(\mu_0)$.

The intensity of G's learning incentive varies with its prior belief, μ_0. Specifically, G is more inclined to screen P when it is more uncertain about P's type. Conversely, as μ_0 approaches 0 or 1, so G is quite certain about P, the benefits of screening weaken. Figure 2 depicts this relationship.

The information that G learns in the first period influences its second-period lobbying, so P has an incentive to strategically manage her reputation. Specifically, P's continuation value depends on G's updated beliefs. Remark 2 characterizes P's continuation value and clarifies its key properties.

Remark 2. *In equilibrium, (i) a* $\underline{\theta}$*-type politician's continuation value is zero independent of* μ_1*, whereas (ii) a* $\overline{\theta}$*-type politician's continuation value is strictly decreasing over* $\mu_1 \leq \tilde{\mu}_1$

Figure 2: The Interest Group's Dynamic Benefit of Screening



Note: The solid line represents G's continuation value as a function of its prior belief $\mu_0 \in [0, 1]$*, assuming that* $\overline{\theta} = \frac{1}{2}$ *and* $\underline{\theta} = 0$*. The dashed line depicts the concavification of* $V_G(\mu_0)$*, which is the upper bound of G's expected continuation value (resulting from full information about* θ*). The difference between* $cavV_G(\mu_0)$ *and* $V_G(\mu_0)$ *is the value of information for G.*

and constant over $\mu_1 > \tilde{\mu}_1$*. Specifically,* $V_P(\underline{\theta}, \mu_1) = 0$ *and*

$$V_P(\overline{\theta}, \mu_1) = \begin{cases} \frac{(\overline{\theta}-\underline{\theta})(1-\overline{\theta}-\mu_1(1-\underline{\theta}))}{1-\mu_1} & \text{if } \mu_1 \leq \tilde{\mu}_1 \\ 0 & \text{if } \mu_1 > \tilde{\mu}_1. \end{cases}$$

The politician can benefit from a reputation for seeming misaligned. Notably, since their continuation value decreases in μ_1, an aligned politician has a reputational incentive to appear misaligned, finding it optimal to ensure G believes that $\mu_1 = 0$.[12] By doing so, they would receive excess transfers in *both* periods.

These potential reputational incentives for P make learning costly for G. The prospect of future lobbying complicates G's efforts to induce P to reveal her type in the first period, relative to the second. Specifically, fixing a particular belief—i.e., $\mu_0 = \mu_1$—a separating menu in the first period must be more distorted than it would be in the second period.

The interplay between learning and reputational incentives that shape first-period lobbying

[12] In contrast, a misaligned politician's continuation value is constant in μ_1, so they do not have a reputational incentive to misrepresent their preferences.

is shaped by how much each player values the future. A more patient interest group is more willing to concede favorable first-period terms to facilitate learning about the politician and obtain more favorable second-period terms. Conversely, a more patient politician is more inclined to forego favorable first-period terms to misrepresent its preferences and receive more favorable second-period terms. Thus, increasing δ_G raises G's willingness to screen P's type, while increasing δ_P raises G's costs for doing so.

If P is likely to be aligned, then G always offers a separating menu in the first period. In this case, G prioritizes $\overline{\theta}$-efficiency and supports it with an overly-conservative $\underline{\theta}$-contract. Despite the low value of screening, it is cheap in expectation since the distortion is unlikely to materialize. Moreover, screening is cheap enough that it is always worthwhile and G offers a separating menu for all $\delta_G \in [0, 1]$.

Conversely, if P is unlikely to be aligned, then G's first-period offer will be a pooling menu under some conditions. Specifically, if δ_G is sufficiently low, then G offers a pooling menu containing only the $\underline{\theta}$-efficient contract, preventing any learning.[13] Otherwise, G is more inclined to learn, offering a separating menu that overpays $\overline{\theta}$ and is overly conservative towards $\underline{\theta}$. Although these distortions are qualitatively similar to second-period lobbying (at equivalent beliefs), their magnitude is amplified due to P's reputation incentive and increases with P's patience, δ_P.

Under certain conditions, first-period policy is always distorted, unlike the second period. If δ_P is sufficiently high and $\overline{\theta}$ is relatively unlikely, then G will distort $\overline{\theta}$-policy upward. Thus, $\overline{\theta}$-offers become overly aggressive in both transfers *and* policy. This additional distortion occurs because the substantial $\overline{\theta}$-transfer makes the $\overline{\theta}$-offer appealing to the $\underline{\theta}$ type. In particular, the incentive compatibility constraints bind for both types, so G adjusts the $\overline{\theta}$-policy to deter the $\underline{\theta}$-type from accepting the offer intended for $\overline{\theta}$.

[13]In the appendix, we formally define the threshold on δ_G (namely $\delta_G^{\dagger}$) that determines whether pooling can occur.

Lemma 2 characterizes first-period strategies in pure strategy PBE. Before stating the result, we define and highlight several thresholds that distinguish qualitatively different equilibria. In particular, these thresholds give rise to two or three regions (in terms of μ_0) for which different types of separating menus are offered. Let $\hat{\mu}_0 := \max\left\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\right\}$ be the first relevant threshold and $\tilde{\mu}_0 := \frac{(1+\delta_P)(1-\overline{\theta})}{1-\underline{\theta}+\delta_P(1-\overline{\theta})}$ be the second relevant threshold to determine equilibrium behavior.[14] Figure 3 illustrates the equilibrium categories depending on μ_0 and G, while Figure 4 illustrates the separating contracts as a function of G's belief μ_0 when δ_G is sufficiently high.

Lemma 2. *In any equilibrium, first-period behavior is as follows. If $\mu_0 < \tilde{\mu}_0$ and $\delta_G < \delta_G^\dagger$, then G offers a pooling menu and P enacts policy $x_1 = x_{\underline{\theta}}$ to receive transfer $t_1 = T_{\underline{\theta}}(x_1)$. Otherwise, G offers a separating menu in which:*

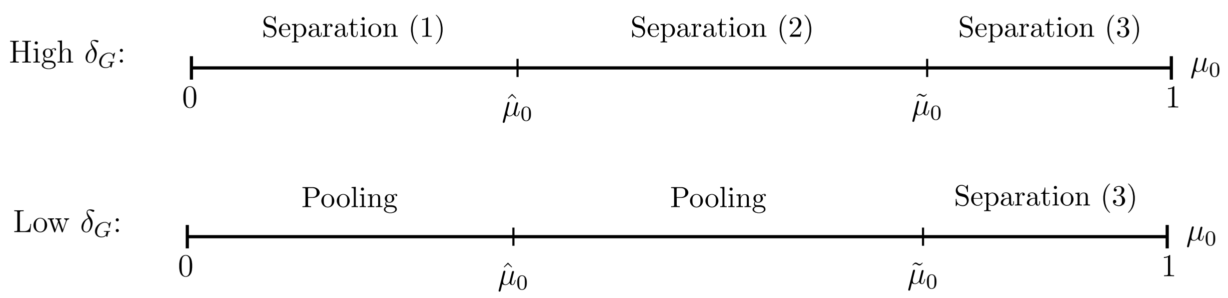
1. *if $\mu_0 \leq \hat{\mu}_0$, then (i) a $\underline{\theta}$-type politician will enact $\underline{x}_1 = x_{\underline{\theta}} - \delta_P\mu_0\frac{1-\overline{\theta}}{2}$ to receive $\underline{t}_1 = T_{\underline{\theta}}(\underline{x}_1)$, and (ii) a $\overline{\theta}$-type will enact $\overline{x}_1 = x_{\overline{\theta}} + \delta_P(1-\mu_0)\frac{1-\overline{\theta}}{2} - \frac{\overline{\theta}-\underline{\theta}}{2}$ to receive $\overline{t}_1 = T_{\overline{\theta}}(\overline{x}_1) + (\overline{\theta}-\underline{\theta})\bigg((1-\overline{\theta})(1+\delta_P(1-\mu_0))\bigg)$;*
2. *if $\mu_0 \in (\hat{\mu}_0, \tilde{\mu}_0]$, then (i) a $\underline{\theta}$-type politician will enact $\underline{x}_1 = x_{\underline{\theta}} - \frac{\mu_0}{1-\mu_0}\frac{\overline{\theta}-\underline{\theta}}{2}$ to receive $\underline{t}_1 = T_{\underline{\theta}}(\underline{x}_1)$, and (ii) a $\overline{\theta}$-type will enact $\overline{x}_1 = x_{\overline{\theta}}$ to receive $\overline{t}_1 = T_{\overline{\theta}}(\overline{x}_1) + \frac{\overline{\theta}-\underline{\theta}}{1-\mu_0}\bigg((1-\overline{\theta})(1+\delta_P(1-\mu_0)) - (1-\underline{\theta})\mu_0\bigg)$; and*
3. *if $\mu_0 > \tilde{\mu}_0$, then (i) a $\underline{\theta}$-type politician will enact $\underline{x}_1 = x_{\underline{\theta}} - (1+\delta_P)\frac{1-\overline{\theta}}{2}$ to receive $\underline{t}_1 = T_{\underline{\theta}}(\underline{x}_1)$, and (ii) a $\overline{\theta}$-type will enact $\overline{x}_1 = x_{\overline{\theta}}$ to receive $\overline{t}_1 = T_{\overline{\theta}}(x_{\overline{\theta}})$.*

Lemma 2 clarifies how learning and reputational considerations shape first-period lobbying and policymaking. Without reputational concerns ($\delta_P = 0$), the characterization mirrors that of Lemma 1. Otherwise, G alters its menu by either adjusting the terms it offers to screen P or by forgoing screening in favor of a pooling menu.

The politician's reputational incentives have several consequences. First, δ_P affects the

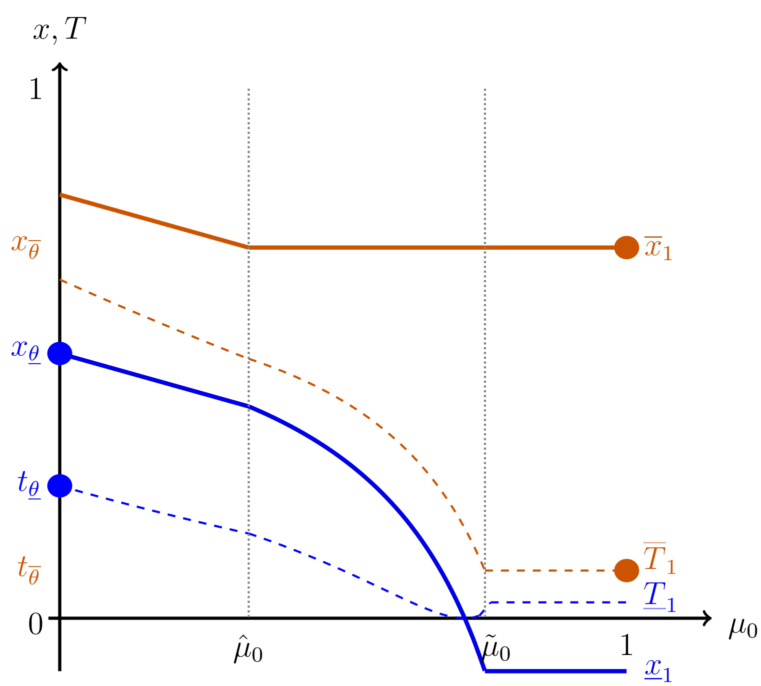
[14] Note that $\hat{\mu}_0 < \tilde{\mu}_0$ and $\tilde{\mu}_1 \in (0, \tilde{\mu}_0)$ always hold. Moreover, $\hat{\mu}_0 > 0$ if and only if $\delta_P > \frac{\overline{\theta}-\underline{\theta}}{1-\overline{\theta}}$.

Figure 3: Informativeness of First-period Lobbying



Note: The figure shows how the interest group's equilibrium strategy varies with its prior belief μ_0 and patience δ_G, as characterized in Lemma 2.

Figure 4: First-period Policies and Transfers



Note: The figure depicts equilibrium policies and transfers from Lemma 2, assuming $\delta_P = 1$. Blue lines represent the optimal contract for $\underline{\theta} = 0$ and orange lines for $\overline{\theta} = \frac{2}{5}$. Solid lines indicate policy-offers and dashed lines indicate transfer-offers.

conditions under which G offers a separating menu. As δ_P increases, P increasingly values the second-period lobbying gains from appearing misaligned, thereby forcing G to make an increasingly generous $\overline{\theta}$-offer to screen the $\overline{\theta}$-type. Therefore learning becomes more costly, requiring more patience from G. Thus, the interest group is less inclined to learn about the politician and will do so under fewer conditions.

Second, δ_P affects the particular contracts that G will include in its separating menu. Broadly, as δ_P increases, G's offer becomes more distorted. Moreover, depending on μ_0, G will adjust different aspects of the menu: (i) for low μ_0, it increases the $\overline{\theta}$-offer while decreasing the $\underline{\theta}$-offer; (ii) for intermediate μ_0, it only increases the $\overline{\theta}$-transfer; and (iii) for high μ_0, it only decreases the $\underline{\theta}$-offer. Proposition 1 makes these observations precise.

Proposition 1. *Suppose the politician's patience increases from δ_P to δ'_P and fix $\delta_G > \delta^{\dagger}_G$. In the first period:*

1. *if $\mu_0 < \hat{\mu}'_0$, then the $\underline{\theta}$-offer $(\underline{x}_1, \underline{t}_1)$ decreases and the $\overline{\theta}$-offer $(\overline{x}_1, \overline{t}_1)$ increases;*
2. *if $\mu_0 \in (\hat{\mu}'_0, \tilde{\mu}_0)$, then the $\overline{\theta}$-transfer increases while the $\overline{\theta}$-policy and $\underline{\theta}$-offer are both constant; and*
3. *if $\mu_0 > \tilde{\mu}_0$, then the $\overline{\theta}$-offer is constant and the $\underline{\theta}$-offer decreases.*

The politician's discount factor influences the dynamics of policymaking. Therefore, the policy choices of politicians with long time horizons (e.g., with secure seats) are likely to develop differently than those with shorter time horizons (e.g., those facing close elections). As δ_P increases, P increasingly emphasizes her second-period payoffs, thereby strengthening her reputational incentive to misrepresent her type. Thus, G must incur higher costs to screen P effectively. The particular way that G distorts its menu depends on its prior beliefs about P. Since some distortion is required, G prefers to do so in ways that minimize the associated policy or monetary costs. Notably, the cost of distorting a θ-contract is inversely related to the probability of that type.

How are equilibrium behaviors affected by G's beliefs about the pool of politicians?[15] First-period lobbying varies with G prior belief about P, μ_0, in several ways. Broadly, as μ_0 increases, G will decrease the policies and transfers that it offers. Yet, G adjusts fewer aspects of its offers as μ_0 increases: over low μ_0, G decreases policies and transfers for both types; over intermediate μ_0, G does not adjust the $\overline{\theta}$-policy; and over high μ_0, G's offer is constant. Proposition 2 formally states these observations.

Proposition 2. *Suppose μ_0 increases and fix $\delta_G > \delta_G^{\dagger}$.*

(i) If $\mu_0 < \hat{\mu}_0$, then the $\overline{\theta}$-offer $(\overline{x}_1, \overline{t}_1)$ and the $\underline{\theta}$-offer $(\underline{x}_1, \underline{t}_1)$ will decrease.

(ii) If $\mu_0 \in (\hat{\mu}_0, \tilde{\mu}_0)$ then the $\underline{\theta}$-offer will decrease or increase, the $\overline{\theta}$-transfer will decrease while the $\overline{\theta}$-policy is constant.

(iii) If $\mu_0 > \tilde{\mu}_0$, then the $\overline{\theta}$-offer and $\underline{\theta}$-offer are constant.

As μ_0 increases, G more likely faces an aligned type. In the first case, this leads G to distort the aligned contract less by shifting it towards the first-best contract, while it simultaneously decreases its influence over the misaligned type to maintain effective screening. In the second case, G places more emphasis on providing an efficient offer to the aligned type, and it becomes less costly to ensure that the aligned type does not mimic the misaligned type. Thus, the aligned type receives a lower transfer, which reduces the inefficiency needed to maintain incentive compatibility. Finally, in the third region, equilibrium offers are constant because the aligned type already receives an efficient offer, and no alterations are needed to maintain incentive compatibility.

5 Dynamics of Policymaking and Lobbying

We now trace the dynamics of policies and transfers across periods. These dynamics are pinned down by G's prior belief, μ_0, and the discount factors of each player, δ_P and δ_G.

[15]Essentially, μ_0 can be interpreted as the composition of the pool of politicians that G faces before lobbying.

Proposition 3 characterizes the trajectories of policies and transfers. We show that they both shift in the same direction.

Proposition 3. *In equilibrium, policies and transfers move in the same direction over time, i.e.,* $x_2 \geq x_1$ *if and only if* $t_2 \geq t_1$*. Furthermore, if* P *is sufficiently likely to be aligned with* $\mu_0 > \tilde{\mu}_0$*, then* $x_2 \leq x_1$ *and* $t_2 \leq t_1$*. Otherwise,* $x_2 \geq x_1$ *and* $t_2 \geq t_1$ *is also possible.*

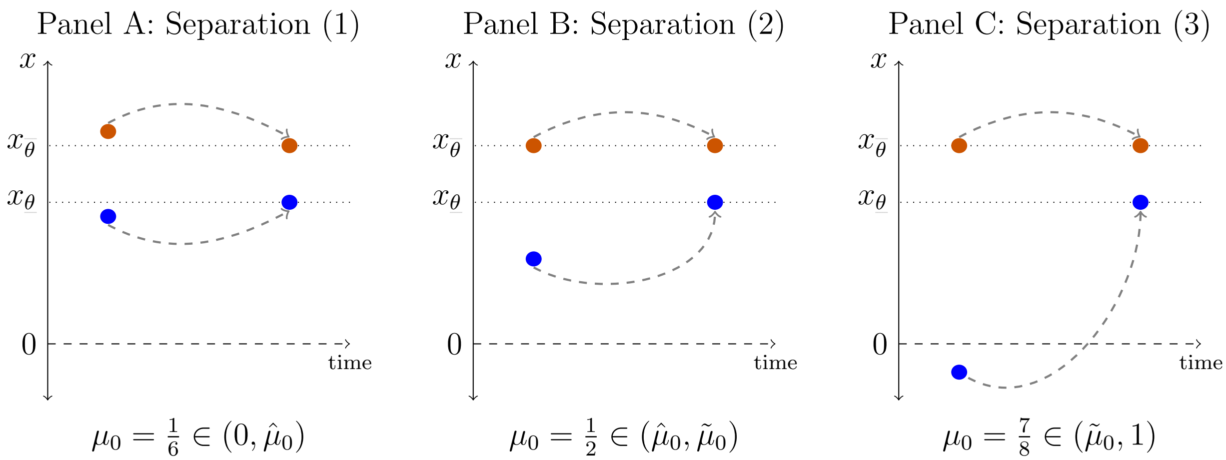
Yet, under broad conditions, the trajectories are ambiguous, with the potential to be increasing or decreasing. That is, it is unclear whether or not implemented policies always become more in line with what interest groups prefer. The only clear case is when P is likely to be aligned with G, when both policy and transfer will either remain constant or increase.

The trajectory of observed policies and transfers is primarily driven by G's learning. If G offers a separating menu to screen P in the first period, it learns information about P that facilitates an efficient second-period offer. Thus, if G is relatively patient, the observed policy and transfer will either (i) start low and then increase if P is a $\underline{\theta}$-type, or (ii) start high and then decrease or stay constant if P is a $\overline{\theta}$-type. However, the $\overline{\theta}$-offer will shift if and only if G is insufficiently certain that P is aligned ($\mu_0 < \tilde{\mu}_0$). Otherwise, G will simply repeat the efficient $\overline{\theta}$-offer. Figure 5 displays the three qualitatively different type of possibilities.

If G offers a pooling menu in the first period, then learning is delayed, and G will instead make a separating offer in the second period. Under these conditions, the observed policies and transfers will initially be low before shifting upward or downward depending on P's alignment. Figure 6 displays the two possible scenarios in which G offers only a single menu option in the first period, and two in the second period. In this case, second-period policies do not converge to the efficient ones, and the offer made to the misaligned type is distorted downwards.

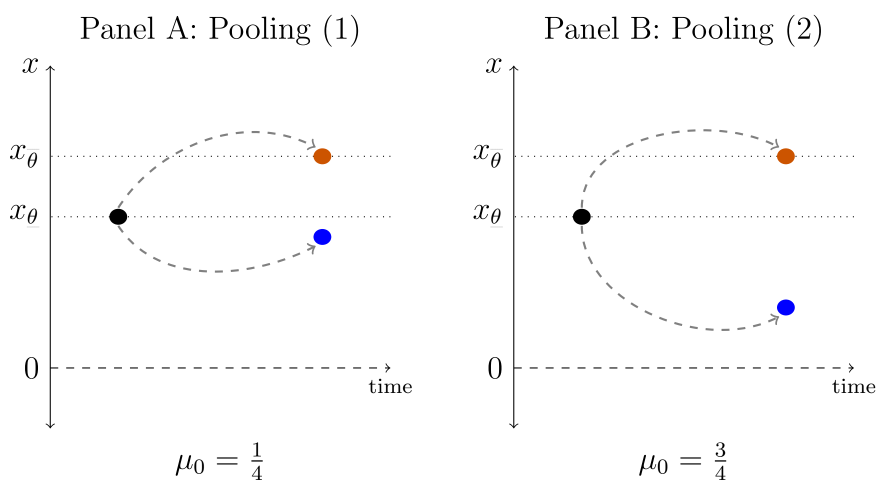
The distance with which observed policies and transfers shift over time depends on G prior belief (μ_0) and P's discount factor (δ_P). A higher δ_P induces larger shifts in both policy and transfers, regardless of P's type. This relationship emerges because G must offer more

Figure 5: Three Separating Equilibrium Paths



Note: In each panel, the dots on the left denote first-period policies while the dots on the right denote second-period policies. In every scenario, policies converge to the efficient policies $x_{\underline{\theta}}$ *and* $x_{\overline{\theta}}$*. Each depicts the case with* $\underline{\theta} = 0$, $\overline{\theta} = \frac{2}{5}$, $\delta_P = 1$ *and* δ_G *sufficiently large.*

Figure 6: Two Pooling Equilibrium Paths



Note: In each panel, the dot on the left denotes first-period policies while the dots on the right denote second-period policies. The black dot is the pooled policy, orange the policy for $\overline{\theta}$ *and blue for* $\underline{\theta}$*. Each depicts the case with* $\underline{\theta} = 0$, $\overline{\theta} = \frac{2}{5}$, $\delta_P = 1$ *and* δ_G *sufficiently small.*

distorted contracts to effectively screen patient politicians. In contrast, the impact of μ_0 varies with P's type: for higher μ_0, the $\underline{\theta}$-type offer shifts less, while the $\overline{\theta}$-type offer shifts more. For the limiting case where $\delta_P = 0$, the politician's reputational incentive disappears, and therefore its incentives to misrepresent are constant across periods. Thus, G can offer contracts closer to the efficient aligned and misaligned contracts, while still accounting for the inherent screening obstacles that arise even without reputational considerations. Proposition 4 formally characterizes these comparative statics.

Proposition 4. *Suppose that the interest group is sufficiently patient, such that $\delta_G > \delta_G^\dagger$.*

1. *If δ_P increases, then $|x_1 - x_2|$ and $|t_1 - t_2|$ will increase.*
2. *If μ_0 increases, then: (i) $|\overline{x}_1 - \overline{x}_2|$ and $|\overline{t}_1 - \overline{t}_2|$ will decrease, but (ii) $|\underline{x}_1 - \underline{x}_2|$ and $|\underline{t}_1 - \underline{t}_2|$ will increase.*

Since policies and transfers can shift in either direction depending on the politician's type, ex-ante forecasts about the dynamics of policymaking and lobbying will depend on μ_0. There is a direct effect given that the weight of either the aligned or misaligned type becomes larger when μ_0 changes. There is also an indirect effect, however, since policies and transfers can depend on μ_0. Our focus below is on the expected policy and transfer, analyzing a weighted average of these equilibrium objects for the misaligned and aligned politician.

First, we focus on how transfers evolve over time through lobbying. If it is highly likely that the politician is misaligned, then the expected transfer to the politician decreases over time. This is because learning is relatively costly, requiring large transfers to screen the politician. Otherwise, if it is sufficiently likely that the politician is aligned, then the politician receives a higher expected transfer in the second period than in the first one. Another effect dominates here—as the politician is more likely to be aligned, the interest group is very passive when making an offer meant for the misaligned type. As a result, once G learns that P is misaligned ($\underline{\theta}$), it can make an offer that gets more out of the misaligned politician, without having to worry about the incentive constraint of the aligned type.

Second, when focusing on policies, we observe that—on average—interest groups become more influential over time if they decide to screen in the first period. In expectation, the policies chosen by the politician (the weighted average of the aligned and misaligned type) move closer to the interest group's preferred policy. The reason is that G must slow down its influence in the first period to successfully learn P's alignment. This is clearly visible in Figure 5 as well in Panel B and C, and even in Panel A—although $\overline{\theta}$'s policy decreases over time—the expected policy increases.

Alternatively, when G does not aim to screen P, it simply treats P as if P were misaligned. In this case, in the second period, it is more aggressive when lobbying $\overline{\theta}$ and more conservative when lobbying $\underline{\theta}$. When assessing the expected value of x_1 and x_2, it can either be the case that G is equally influential in both periods ($\mu_0 < \tilde{\mu}_1$) or becomes more influential over time ($\mu_0 > \tilde{\mu}_1$).[16]

Finally, when assessing policy variance over time, it is clear that this simply depends on whether G provides a separating or pooling offer to P. As clearly displayed in Figure 5, first-period separating policy offers are more removed from each other than second-period policy offers. On the other hand, however, given pooling, Figure 6 shows that the lack of variance in the first period is followed by more variable policy predictions in the second, conditional on the pool of politicians, μ_0.

6 Extensions

We study four extensions to study the impact of other political features such as hiring lobbyists, making campaign contributions, veto players, voting rules, and revolving-door hiring. Specifically, we analyze how our main analysis is impacted by: (i) early-career information, (ii) early-career access, (iii) policymaking constraints, and (iv) revolving-door incentives. For each, we highlight the impact on policies and transfers, their dynamics, as

[16] The stated condition here focuses on second-period equilibrium policies, while implicitly assuming that G still finds it optimal to give a pooling offer to P.

well as the degree of G's policy influence.

The value of early-career information

Interest groups often have access to various tools for learning about politicians' motivations and interests before engaging in lobbying activities. These tools may include conducting interviews with staff members, researching politicians' histories and preferences, or hiring lobbyists with established connections. Given these avenues for acquiring information prior to lobbying, we address two key questions. First, what is the value of obtaining such information for interest groups compared to "learning by lobbying"? And, second, which factors influence the value of this information? By examining these questions, we aim to provide insights into the strategic value of early-career information in political lobbying and its implications for interest group behavior.

We analyze the value of early-career information by comparing G's payoff under full information to G's payoff in the main model, where G begins with a prior belief μ_0 about P's alignment, as established in Lemma 1 and Lemma 2. This extension clarifies the importance of initial information about politicians' preferences and motivations. Using this comparison, we quantify the value of early-career information and explore the factors that determine its importance in the lobbying process.

Under full information, as established at the outset of our model, G earns $\pi_{\underline{\theta}}$ and $\pi_{\overline{\theta}}$, depending on the politician's type. From G's first-period perspective, its expected payoff is:

$$V_G^{informed} = (1 + \delta_G)\left((1 - \mu_0)\pi_{\underline{\theta}} + \mu_0 \pi_{\overline{\theta}}\right).$$

There are two main cases. In the first, G fully screens P in the first period. Here, information is only valuable in the first period, since G will be fully informed in the second period regardless. The value of information is proportionate to the distortions G induces in making policy- and transfer-offers.

In the second case, G makes a pooling offer in the first period, and then a screening offer in the second. Here, information is valuable in both periods, since it allows for efficient contracting compared to the lack of efficient contracts absent information. In the first period, the misaligned type receives an efficient contract, while the aligned type's contract is distorted. Thus, the first-period value of information equals $\mu_0(\pi_{\underline{\theta}} - \pi_{\overline{\theta}})$. In the second period, the value of information depends on the prior, μ_0. If $\mu_0 \leq \tilde{\mu}_1$, then G's second-period value of information is $\mu_0(\pi_{\overline{\theta}} - \frac{(\overline{\theta}-\underline{\theta})^2}{1-\mu_0} - \pi_{\underline{\theta}})$. Otherwise, G's second-period value of information is $(1-\mu_0)(-\pi_{\overline{\theta}})$.[17] Proposition 5 summarizes these findings.

Proposition 5. *The value of early-career information is positive. Furthermore, it is (i) weakly increasing in the politician's patience, δ_P, but (ii) weakly decreasing in the interest group's patience, δ_G.*

Naturally, it is always valuable to know more about the politician's preferences. The interest group's willingness to pay for this information, however, depends on the political context. When politicians are more forward-looking, screening becomes more expensive, making G willing to pay more to avoid this screening cost. When G is more patient, it is more willing to screen P, making early-career information relatively less beneficial.

The value of early-career access

In the realm of political influence, interest groups face uncertainty in securing access to politicians. This access often requires strategic investments to establish relationships, such as hiring revolving-door lobbyists or providing campaign contributions. To quantify the importance of early-career access, we compare two settings: (i) *full access*, where G can lobby in both periods (as in our main model), and (ii) *late-career access*, where G can only lobby in the second period.

We define the *early-career value of access* as the difference between G's equilibrium payoff with full access versus its payoff with only late-career access. This quantity measures the

[17] Recall that $\pi_{\overline{\theta}}$ is negative, implying a positive value of information.

strategic importance of early engagement with politicians.

We show that politicians may have incentives to misrepresent their positions even when G is absent in the first period and does not lobby. These incentives stem from the anticipation of lobbying efforts in the second period. Similar to our main model, aligned politicians may be motivated to feign disagreement to secure more favorable terms from interest groups in subsequent interactions. The strength of these incentives can be substantial. In some cases, aligned politicians might moderate their chosen policies without receiving any immediate transfer, solely based on the prospect of future lobbying. This finding underscores the complex interplay between politicians' strategic behavior and the temporal dynamics of interest group influence.

Two equilibrium categories may arise. First, both the aligned and misaligned politician may choose their preferred policies, θ, ensuring that the interest group successfully learns the politician's preferences even without screening. Second, the aligned type may mimic the misaligned politician and choose policy $x_1 = \underline{\theta}$, which implies that G does not learn any new information about P.[18]

The value of early-career access depends on several factors, especially P's patience as well as G's belief about P's alignment. Depending on those factors, G will behave in two distinct ways. First, G may influence P directly by actively lobbying, paying extra costs to successfully screen and learn P's preferences. Alternatively, G may opt to not buy access, potentially learning about P for free. Proposition 6 characterizes the value of access.

Proposition 6. *Suppose $\delta_G > \delta_G^{\dagger}$. If $\delta_P \leq \frac{\overline{\theta}-\underline{\theta}}{1-\overline{\theta}}$, then G's value of early-career access is positive and constant in δ_G but decreasing in δ_P. Otherwise, G's value of early-career access may be positive or negative, and is increasing in δ_G but decreasing in δ_P.*

Interestingly, there are cases in which G may opt to forego access. At the cost of not being

[18] Mixed strategy equilibria also exist, causing G to only partially learn what P prefers. For the sake of presentation, we omit such equilibria given our focus on pure strategy equilibria.

able to influence P in the first period, G may be better off learning about P's alignment for free. This is especially the case when the politician has a long time horizon, which would significantly increase the interest group's cost of screening. As a result, in dynamic contexts with uncertainty, interest groups may not always want to invest in access, as politician's reputation management incentives may thwart efficient learning.

Lobbying with policymaking constraints

In complex political systems, politicians have different degrees of influence. While some are bound by party lines, others play active roles in developing and drafting proposals. Even the most powerful politicians often face constraints when drafting proposals, making amendments, or casting votes. These limitations raise an important question: how do such constraints affect the value of access to, and influence over, politicians?

To explore this question, we extend our model to incorporate restrictions on the set of policies that P can enact. Formally, we introduce a maximum policy, denoted $\bar{y}$, which P's policy cannot exceed. This extension yields two key strategic implications. First, the constraint raises G's cost of influencing P through transfers, since G can only offer P a more limited set of policies. Second, the constraint also lowers G's value of information obtained by screening P in the first period, since there is less freedom to use this information when influencing P in the second period. By parsing these implications, we provide new insights into how political constraints can impact the dynamics of lobbying relationships and the value of political influence.

Qualitatively, equilibrium strategies remain relatively similar to the main model, with the exception that the aligned type will enact policies $\bar{x}_1 = \bar{x}_2 = \bar{y}$. This difference has several effects. First, G can offer a lower transfer to encourage the aligned type to accept the contract rather than reject it. Second, the aligned contract becomes less attractive, so there are stronger incentives to switch to the misaligned contract. Third, to maintain incentive compatibility, G must either make the aligned contract more attractive (by increasing the transfer) or make the misaligned contract less attractive (by decreasing the transfer or policy). Fourth, G must ensure that the misaligned type is willing to accept the offer relative to rejecting, implying that a decrease in transfer must coincide with a decrease in policy. Overall, these effects imply that G, when lobbying a more constrained P, reduces its influence (both directly and indirectly). Also, G is more likely to moderate its first-period influence given that the cost of screening increases while the benefit of screening decreases. Proposition 7 formalizes these observations.

Proposition 7. *The value of information increases in $\bar{y}$, i.e., as policymaking constraints loosen, subsequently leading to more screening. Furthermore, that effect is stronger if the interest group is more patient, i.e., as δ_G increases.*

Interest groups do not just have more to gain from influencing powerful politicians, but it is also more valuable to know their alignment. This highlights that committee leaders may not just attract more campaign contributions because of their power, but in dynamic contexts with uncertainty, the value of information may attract even greater contributions.

Revolving-door incentives

In our final extension, we analyze the implications of revolving-door incentives for lobbying and policymaking dynamics. We explore how the potential for politicians to be hired by interest groups after their tenure affects strategic incentives and informational considerations in contexts with learning by lobbying.

We introduce a new parameter, $R > 0$, representing the additional payoff P receives if hired by G after the second period. We assume that G is willing to hire P if only if it is sufficiently certain about their alignment. That is, we assume that the belief about P's type must be sufficiently high before hiring.

The benefit of appearing aligned with the interest group due to the revolving door has two important strategic implications. First, the politician's signaling incentives become more complex. Previously, aligned politicians were motivated to signal misalignment solely to secure better lobbying deals in the second period. Now, this incentive is counterbalanced by the desire to become a revolving-door lobbyist. Second, the interest group faces lower costs for effectively screening the politician. Specifically, it can offer lower transfers when screening the politician in the first period. Moreover, these screening costs decrease further as the value of becoming a revolving-door lobbyist increases. These strategic implications lead to the main empirical implication of this extension.

Proposition 8. *Increasing the value of the revolving door expands the set of parameters under which G successfully screens P.*

Interestingly, the interest group benefits from situations in which the politician strongly values becoming a revolving-door lobbyist. Especially larger firms, which can promise and offer higher future wages and better careers to incumbent politicians, may also learn more quickly whether they are dealing with an aligned or misaligned politician. This effect on G's expected payoff is amplified by P's patience, δ_P, and G's belief that it faces an aligned politician, μ_0.

Proposition 9. *The interest group's equilibrium payoff increases in the politician's value of the revolving door, which is itself increasing in δ_P and μ_0.*

This result suggests that powerful interest groups—such as Big Tech firms—do not just benefit from their ability to contribute more to politicians. They also benefit from the potential to attract those politicians, who then have more incentives to appear aligned to these interest groups. This alignment, in turn, benefits these interest groups in settings with "learning by lobbying."

7 Discussion and Conclusion

The dynamics of lobbying and policymaking involve complex relationships between influence, learning, and strategic behavior. Our analysis makes three key theoretical contributions. First, we show how learning and influence are fundamentally intertwined in lobbying relationships. Interest groups must balance gathering information about politicians' preferences against exerting immediate influence, while politicians strategically manage their reputations to shape future interactions. Second, we demonstrate how this learning process shapes the temporal evolution of both policies and transfers, generating testable predictions about when and how lobbying relationships stabilize. Third, we identify how key characteristics—like politicians' time horizons and interest groups' issue breadth—systematically affect these dynamics.

These theoretical insights generate several empirical implications for studying lobbying. Our model predicts systematic differences in how early versus late-career politicians engage with interest groups. Early-career politicians with unknown preferences should face higher "screening costs" from interest groups but may also extract greater rents due to their stronger reputational incentives. The theory also suggests that broad-based interest groups focused on long-term relationships should exhibit different lobbying patterns than single-issue groups seeking immediate policy changes. In particular, we expect more gradual evolution of policies and transfers when interest groups have longer time horizons and broader policy interests.

Our framework provides new predictions about variation in lobbying practices and policy outcomes. For instance, the model suggests that uncertainty about preferences should increase policy volatility but decrease transfer volatility. Politicians with secure positions should display stronger reputational incentives, leading to larger shifts in both policies and transfers over time. These predictions could be tested using data on campaign contributions, lobbying expenditures, and policy positions across politicians' careers.

The analysis also yields insights about institutional design. Our results suggest that transparency regulations and revolving door restrictions affect not just the flow of money in politics, but also how interest groups learn about policymakers' preferences. This learning channel may be as important as financial constraints in determining which groups successfully influence policy. Moreover, our extension analyzing revolving-door incentives reveals how post-political career opportunities create countervailing effects on both screening costs and lobbying influence.

Our findings contribute to fundamental questions about democratic representation and special interest influence. In an era of increasing political complexity and the proliferation of “political amateurs”—individuals entering politics without extensive public-service track records (Porter and Treul 2024)—the ability to understand and anticipate a policymaker’s preferences can be crucial for shaping legislation. By untangling the complex interplay between learning and influence in lobbying relationships, we provide a framework for understanding how these dynamics shape outside influence and policy outcomes.

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Supplementary Information for

"Learning by Lobbying"

Part I

Online Appendix

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A Appendix: Proofs of Main Results

A.1 Proof of Lemma 1

In the second period, the interest group has the following constrained maximization problem given belief $\mu := \mu_1 \in (0,1)$,

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-\left(\overline{x}-1\right)^2-\overline{T}\right)+(1-\mu)\left(-\left(\underline{x}-1\right)^2-\underline{T}\right)$$
$$s.t. \quad (IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}).$$

The four restrictions include two incentive compatibility constraints and two participation constraints:

$$-\left(\overline{x}-\overline{\theta}\right)^2+\overline{T} \geq -\left(\underline{x}-\overline{\theta}\right)^2+\underline{T}, \tag{$IC_{\overline{\theta}}$}$$
$$-\left(\underline{x}-\underline{\theta}\right)^2+\underline{T} \geq -\left(\overline{x}-\underline{\theta}\right)^2+\overline{T}, \tag{$IC_{\underline{\theta}}$}$$
$$-\left(\overline{x}-\overline{\theta}\right)^2+\overline{T} \geq 0, \tag{$P_{\overline{\theta}}$}$$
$$-\left(\underline{x}-\underline{\theta}\right)^2+\underline{T} \geq 0. \tag{$P_{\underline{\theta}}$}$$

We study the following relaxed problem and then verify that the solution satisfies the constraint $(IC_{\underline{\theta}})$:

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-\left(\overline{x}-1\right)^2-\overline{T}\right)+(1-\mu)\left(-\left(\underline{x}-1\right)^2-\underline{T}\right)$$
$$s.t. \quad (IC_{\overline{\theta}}), (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}).$$

We begin our analysis by setting up the Lagrangian for the relaxed problem:

$$\begin{aligned}\mathcal{L} = &\ \mu\left(-\left(\overline{x}-1\right)^2-\overline{T}\right)+(1-\mu)\left(-\left(\underline{x}-1\right)^2-\underline{T}\right)\\ &+\lambda_1\left(-\left(\overline{x}-\overline{\theta}\right)^2+\overline{T}+\left(\underline{x}-\overline{\theta}\right)^2-\underline{T}\right)\\ &+\lambda_2\left(-\left(\overline{x}-\overline{\theta}\right)^2+\overline{T}\right)\\ &+\lambda_3\left(-\left(\underline{x}-\underline{\theta}\right)^2+\underline{T}\right),\end{aligned}$$

where λ_1, λ_2 and λ_3 are the multipliers for $(IC_{\overline{\theta}})$, $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ respectively. Using this notation, the Kuhn-Tucker first-order necessary conditions are given as follows:

The first order conditions with respect to the tuple $(\underline{x}, \overline{x}, \underline{T}, \overline{T})$ are:

$$\frac{\partial \mathcal{L}}{\partial \underline{x}} = -2(1-\mu)(\underline{x}-1) + 2\lambda_1(\underline{x}-\overline{\theta}) - 2\lambda_3(\underline{x}-\underline{\theta}) = 0, \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial \overline{x}} = -2\mu(\overline{x}-1) - 2\lambda_1(\overline{x}-\overline{\theta}) - 2\lambda_2(\overline{x}-\overline{\theta}) = 0, \tag{2}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{T}} = -(1-\mu) - \lambda_1 + \lambda_3 = 0, \tag{3}$$

$$\frac{\partial \mathcal{L}}{\partial \overline{T}} = -\mu + \lambda_1 + \lambda_2 = 0. \tag{4}$$

The complementary slackness conditions for $(IC_{\overline{\theta}}), (P_{\overline{\theta}})$, and $(P_{\underline{\theta}})$ respectively are:

$$\lambda_1 \left(-\left(\overline{x}-\overline{\theta}\right)^2 + \overline{T} + \left(\underline{x}-\overline{\theta}\right)^2 - \underline{T} \right) = 0, \tag{5}$$

$$\lambda_2 \left(-\left(\overline{x}-\overline{\theta}\right)^2 + \overline{T} \right) = 0, \tag{6}$$

$$\lambda_3 \left(-\left(\underline{x}-\underline{\theta}\right)^2 + \underline{T} \right) = 0. \tag{7}$$

The non-negative Lagrangian multipliers and the constraints are:

$$\lambda_1, \lambda_2, \lambda_3 \geq 0, \tag{8}$$

$$(IC_{\overline{\theta}}), (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}). \tag{9}$$

From (3), we deduce that $\lambda_3 = \lambda_1 + (1-\mu) > 0$. Condition (7) then implies $\left(-\left(\underline{x}-\underline{\theta}\right)^2 + \underline{T}\right) = 0$. From (4), we deduce that $\lambda_1 + \lambda_2 = \mu$. We have three possible cases: (i) $\lambda_1 > 0$ and $\lambda_2 > 0$, (ii) $\lambda_1 = \mu$, $\lambda_2 = 0$, and (iii) $\lambda_1 = 0$, $\lambda_2 = \mu$.

Case (i). The fact that $\lambda_1 > 0$ and $\lambda_2 > 0$ implies that constraints $(IC_{\overline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are all binding. These binding constraints plus the first-order conditions create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}^* = \frac{\underline{\theta}+\overline{\theta}}{2},\ \overline{x}^* = \frac{1+\overline{\theta}}{2},\ \underline{T}^* = \frac{1}{4}\left(\overline{\theta}-\underline{\theta}\right)^2, \overline{T}^* = \frac{1}{4}\left(1-\overline{\theta}\right)^2,$$
$$\lambda_1^* = \frac{(1-\mu)(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_2^* = \frac{\mu(1-\underline{\theta})-(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3^* = \frac{(1-\mu)(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

Note that $\lambda_1^*, \lambda_3^* > 0$. Also, $\lambda_2^* > 0$ if $\mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}$. Thus, if $\mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}$, the vector $\left(\underline{x}^*, \overline{x}^*, \underline{T}^*, \overline{T}^*, \lambda_1^*, \lambda_2^*, \lambda_3^*\right)$ satisfies the Kuhn-Tucker first-order necessary conditions of the relaxed problem.

Case (ii). Now $(IC_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and—together with the first-order conditions—

create a system of 6 equations and 6 unknowns with the following solution:

$$\underline{x}' = \frac{1+\underline{\theta}-\mu(1+\overline{\theta})}{2(1-\mu)},\ \overline{x}' = \frac{1+\overline{\theta}}{2},$$
$$\underline{T}' = \frac{1}{4}\frac{(1-\underline{\theta}-\mu(1+\overline{\theta}-2\underline{\theta}))^2}{(1-\mu)^2},\ \overline{T}' = \frac{1}{4}\frac{(1-\overline{\theta})(1+3\overline{\theta}-4\underline{\theta})-\mu(1+\overline{\theta}-2\underline{\theta})^2}{(1-\mu)},$$
$$\lambda_1' = \mu,\ \lambda_2' = 0,\ \lambda_3' = 1.$$

Replacing these values on the constraint $(P_{\overline{\theta}})$, we obtain that it must be that $\frac{(\overline{\theta}-\underline{\theta})(1-\overline{\theta}-\mu(1-\underline{\theta}))}{(1-\mu)} \geq 0$ which is satisfied if $\mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}$. Thus, if $\mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}$, the vector $\left(\underline{x}', \overline{x}', \underline{T}', \overline{T}', \lambda_1', \lambda_2', \lambda_3'\right)$ satisfies the Kuhn-Tucker first-order necessary conditions of the relaxed problem.

Case (iii). Now $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 6 equations and 6 unknowns with the following solution:

$$\underline{x}'' = \frac{\underline{\theta}+1}{2},\ \overline{x}'' = \frac{1+\overline{\theta}}{2},\ \underline{T}'' = \frac{1}{4}\left(1-\underline{\theta}\right)^2,\ \overline{T}'' = \frac{1}{4}\left(1-\overline{\theta}\right)^2,$$
$$\lambda_1'' = 0,\ \lambda_2'' = \mu,\ \lambda_3'' = (1-\mu).$$

Replacing these values on the constraint $(IC_{\overline{\theta}})$ we obtain that it must be that $-(1-\overline{\theta})(\overline{\theta}-\underline{\theta}) \geq 0$ which is never satisfied. Thus, there are no values that satisfy the Kuhn-Tucker first-order necessary conditions in case (iii).

Note that $\frac{\partial^2 \mathcal{L}}{\partial \underline{x}^2} = -2(1-\mu) + 2(\lambda_1 - \lambda_3)$. From (3) we have that $(\lambda_1 - \lambda_3) = -(1-\mu)$, and then $\frac{\partial^2 \mathcal{L}}{\partial \underline{x}^2} < 0$. Also, $\frac{\partial^2 \mathcal{L}}{\partial \overline{x}^2} = -2\mu - 2\lambda_1 - 2\lambda_2 < 0$. Thus, the Lagrangian function is strictly concave and the Kuhn-Tucker first-order conditions are also sufficient. In sum, the solution of the relaxed problem is the following:

$$(\underline{x}, \overline{x}, \underline{T}, \overline{T}) = \begin{cases} \left(\frac{\underline{\theta}+\overline{\theta}}{2}, \frac{1+\overline{\theta}}{2}, \frac{1}{4}\left(\overline{\theta}-\underline{\theta}\right)^2, \frac{1}{4}\left(1-\overline{\theta}\right)^2\right) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \left(\frac{1+\underline{\theta}-\mu(1+\overline{\theta})}{2(1-\mu)}, \frac{1+\overline{\theta}}{2}, \frac{1}{4}\frac{(1-\underline{\theta}-\mu(1+\overline{\theta}-2\underline{\theta}))^2}{(1-\mu)^2}, \frac{1}{4}\frac{(1-\overline{\theta})(1+3\overline{\theta}-4\underline{\theta})-\mu(1+\overline{\theta}-2\underline{\theta})^2}{(1-\mu)}\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

After some calculations, it is direct to see that the solution of the relaxed problem strictly satisfies $(IC_{\underline{\theta}})$. Thus, it is a solution of the original problem.

The interest group's expected payoff in the second period if it offers two contracts can be simplified to

$$V_2 = \begin{cases} (1-\mu)\left(-\frac{(1-\overline{\theta})^2}{2} - \frac{(1-\underline{\theta})^2}{2}\right) + \mu\left(-\frac{(1-\overline{\theta})^2}{2}\right) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \frac{1}{2(1-\mu)}\left((1-\mu)(-(1-\underline{\theta})^2) + \mu(\overline{\theta}-\underline{\theta})^2\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

We now consider the interest group's possibility to offer a single contract. In that case, incentive compatibility constraints are trivially satisfied, and only the participation constraints are relevant. Consider the following alternative set of contracts: (i) no type accepts, (ii) only the misaligned type accepts, (iii) only the aligned type accepts, or (iv) both types accept. These sets cover all possible single-contract offers. In each of the cases where only one type accepts, the participation constraint is binding since if that is not the case, the interest group can always decrease T by a small amount and strictly benefit from it. In case (i), since no type accepts, the interest group directly obtains:

$$V_0 = -\mu\left(\overline{\theta}-1\right)^2 - (1-\mu)\left(\underline{\theta}-1\right)^2.$$

In case (ii), the interest group offers a contract that is only accepted by $\underline{\theta}$. Using an analogue approach to the two-different contracts case, we find that the solution is

$$(x,T) = \left(\frac{\underline{\theta}+\overline{\theta}}{2}, \left(\frac{\underline{\theta}+\overline{\theta}}{2}-\underline{\theta}\right)^2\right).$$

The interest group's expected payoff in this case is

$$V_1^{\underline{\theta}} = (\overline{\theta}-1)^2(-\mu) - \frac{1}{4}(\mu-1)(\overline{\theta}+\underline{\theta}-2)^2.$$

In case (iii), the solution is the following:

$$(x,T) = \left(\frac{1+\overline{\theta}}{2}, \left(\frac{1+\overline{\theta}}{2}-\overline{\theta}\right)^2\right).$$

The interest group's expected payoff in this case is

$$V_1^{\overline{\theta}} = (1-\mu)(-(\underline{\theta}-1)^2) + \mu\left(-\frac{1}{2}(1-\overline{\theta})^2\right).$$

Case (iv) is included in the feasible set of the original problem where the interest group offers two contracts. Thus, in general, the expected payoff of the interest group is $V = \max\{V_2, V_0, V_1^{\underline{\theta}}, V_1^{\overline{\theta}}\}$. After some algebra, it is straightforward to verify that $V = V_2$ using the explicit expressions for the interest group's expected payoff in each of the cases.

A.2 Proofs of Remarks 1 and 2

By Lemma 1, the expected payoffs for the politician and the interest group in equilibrium as a function of the belief μ are the following:

$$V_P(\theta,\mu) = \begin{cases} \frac{(\overline{\theta}-\underline{\theta})(1-\mu-\overline{\theta}+\mu\underline{\theta})}{1-\mu} & \text{if } \theta=\overline{\theta} \text{ and } \mu \le \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ 0 & \text{if } \theta=\overline{\theta} \text{ and } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ 0 & \text{if } \theta=\underline{\theta}. \end{cases}$$

$$V_G(\mu) = \begin{cases} \frac{1}{2(1-\mu)}\left((1-\mu)(-(1-\underline{\theta})^2) + \mu(\overline{\theta}-\underline{\theta})^2\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ (1-\mu)\left(-\frac{(1-\overline{\theta})^2}{2} - \frac{(1-\underline{\theta})^2}{2}\right) + \mu\left(-\frac{(1-\overline{\theta})^2}{2}\right) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

Note that both functions are continuous in μ given that they are equal at the threshold of $\frac{1-\overline{\theta}}{1-\underline{\theta}}$ and continuous everywhere else.

A.3 Proof of Lemma 2

Suppose $\mu \in (0,1)$. We proceed in two steps. Step 1 focuses on separating contracts. Step 2 focuses on a pooling contract.

Step 1. Separation. If the interest group offers separating contracts implies that continuation values are $V_P(\overline{\theta},1) = V_P(\underline{\theta},0) = 0$ in equilibrium. By deviating, type $\underline{\theta}$ would earn $V_P(\underline{\theta},\mu) = 0$ for all $\mu \in [0,1]$. By mimicking $\underline{\theta}$, type $\overline{\theta}$ would earn $V_P(\overline{\theta},0) = (\overline{\theta}-\underline{\theta})(1-\overline{\theta})$.

Conditional on separation, the interest group's optimization problem is:

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G(1)\right) + (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G(0)\right)$$
$$s.t.\ (IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}),$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P(\overline{\theta},1) \geq -(\underline{x}-\overline{\theta})^2 + \underline{T} + \delta_P V_P(\overline{\theta},0), \tag{$IC_{\overline{\theta}}$}$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P(\underline{\theta},0) \geq -(\overline{x}-\underline{\theta})^2 + \overline{T} + \delta_P V_P(\underline{\theta},1), \tag{$IC_{\underline{\theta}}$}$$
$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P(\overline{\theta},1) \geq 0, \tag{$P_{\overline{\theta}}$}$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P(\underline{\theta},0) \geq 0. \tag{$P_{\underline{\theta}}$}$$

We consider the Never a Weak Best Response (NWBR) equilibrium refinement (Fudenberg and Tirole 1991). This refinement implies that any politician type obtains zero payoff from an off-path deviation.[19] We begin our analysis by setting up the Lagrangian

$$\begin{aligned}\mathcal{L} =& \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G(1)\right) + (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G(0)\right) \\ &+\lambda_1\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P(\overline{\theta},1) + (\underline{x}-\overline{\theta})^2 - \underline{T} - \delta_P V_P(\overline{\theta},0)\right) \\ &+\lambda_2\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P(\underline{\theta},0) + (\overline{x}-\underline{\theta})^2 - \overline{T} - \delta_P V_P(\underline{\theta},1)\right) \\ &+\lambda_3\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P(\overline{\theta},1)\right) \\ &+\lambda_4\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P(\underline{\theta},0)\right).\end{aligned}$$

[19] There are two cases. In some cases a profitable rejection must come from a type $\overline{\theta}$. In this case, the refinement requires $\mu = 1$. In the other case, NWBR does not apply, and we directly impose that $\mu = 1$.

The first-order conditions with respect to $\underline{x}, \overline{x}, \underline{T}, \overline{T}$ are

$$\frac{\partial \mathcal{L}}{\partial \underline{x}} = -2(1-\mu)(\underline{x}-1) + 2\lambda_1(\underline{x}-\overline{\theta}) - 2\lambda_2(\underline{x}-\underline{\theta}) - 2\lambda_4(\underline{x}-\underline{\theta}) = 0, \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial \overline{x}} = -2\mu(\overline{x}-1) - 2\lambda_1(\overline{x}-\overline{\theta}) + 2\lambda_2(\overline{x}-\underline{\theta}) - 2\lambda_3(\overline{x}-\overline{\theta}) = 0, \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{T}} = -(1-\mu) - \lambda_1 + \lambda_2 + \lambda_4 = 0, \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial \overline{T}} = -\mu + \lambda_1 - \lambda_2 + \lambda_3 = 0. \tag{13}$$

The complementary slackness conditions are:

$$\lambda_1\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P(\overline{\theta},1) + (\underline{x}-\overline{\theta})^2 - \underline{T} - \delta_P V_P(\overline{\theta},0)\right) = 0, \tag{14}$$

$$\lambda_2\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P(\underline{\theta},0) + (\overline{x}-\underline{\theta})^2 - \overline{T} - \delta_P V_P(\underline{\theta},1)\right) = 0, \tag{15}$$

$$\lambda_3\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P(\overline{\theta},1)\right) = 0, \tag{16}$$

$$\lambda_4\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P(\underline{\theta},0)\right) = 0. \tag{17}$$

Thus,

$$\lambda_1\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + (\underline{x}-\overline{\theta})^2 - \underline{T} - \delta_P(\overline{\theta}-\underline{\theta})(1-\overline{\theta})\right) = 0, \tag{18}$$

$$\lambda_2\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + (\overline{x}-\underline{\theta})^2 - \overline{T}\right) = 0, \tag{19}$$

$$\lambda_3\left(-(\overline{x}-\overline{\theta})^2 + \overline{T}\right) = 0, \tag{20}$$

$$\lambda_4\left(-(\underline{x}-\underline{\theta})^2 + \underline{T}\right) = 0. \tag{21}$$

Suppose first that $\lambda_2 = 0$. By (12) we know that $\lambda_4 = \lambda_1 + (1-\mu) > 0$. This implies that $(-(\underline{x}-\underline{\theta})^2 + \underline{T}) = 0 \iff \underline{T} = (\underline{x}-\underline{\theta})^2$. Then by (13) we know that $\mu_0 = \lambda_1 + \lambda_3$. Given that λ_1, λ_3 are non-negative, we know that there are three cases: (i) both $\lambda_1 > 0$ and $\lambda_3 > 0$, (ii) $\lambda_1 = \mu$ and $\lambda_3 = 0$, and (iii) $\lambda_1 = 0$ and $\lambda_3 = \mu$.

Case (i). The fact that $\lambda_1 > 0$ and $\lambda_3 > 0$ implies that constraints $(IC_{\overline{\theta}})$, $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are all binding. These binding constraints plus the first-order conditions create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}^* = \frac{1}{2}(\overline{\theta} + \underline{\theta} - \delta_P(1-\overline{\theta})),\ \overline{x}^* = \frac{1+\overline{\theta}}{2},\ \underline{T}^* = \frac{1}{4}(\overline{\theta} - \underline{\theta} - \delta_P(1-\overline{\theta}))^2,\ \overline{T}^* = \frac{(1-\overline{\theta})^2}{4},$$

$$\lambda_1^* = \frac{(1+\delta_P)(1-\mu)(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3^* = \frac{\delta_P(\mu(1-\overline{\theta}) + \overline{\theta} - 1) + \mu(-\underline{\theta}) + \mu + \overline{\theta} - 1}{\overline{\theta}-\underline{\theta}},$$

$$\lambda_4^* = \frac{(1-\mu)(\delta_P(1-\overline{\theta}) + 1 - \underline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

Under the condition $\mu > \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, the multiplier $\lambda_3^* > 0$ and $(IC_{\underline{\theta}})$ is satisfied, which implies that $\left(\underline{x}^*, \overline{x}^*, \underline{T}^*, \overline{T}^*, \lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*\right)$ with $\lambda_2^* = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.

Case (ii). Now $(IC_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 6 equations and 6 unknowns with the following solution:

$$\begin{aligned}
\underline{x}' &= \frac{1+\underline{\theta}-\mu(1+\overline{\theta})}{2(1-\mu)},\ \overline{x}' = \frac{1+\overline{\theta}}{2},\\
\underline{T}' &= \frac{(1-\underline{\theta}-\mu(1+\overline{\theta}-2\underline{\theta}))^2}{4(1-\mu)^2},\\
\overline{T}' &= \frac{\mu\left((1-4\delta_P)\overline{\theta}^2+\overline{\theta}(4\delta_P(\underline{\theta}+1)-4\underline{\theta}+2)-4(\delta_P+1)\underline{\theta}+4\underline{\theta}^2+1\right)}{4(\mu-1)}\\
&\quad+\frac{(\overline{\theta}-1)((4\delta_P+3)\overline{\theta}-4(\delta_P+1)\underline{\theta}+1)}{4(\mu-1)},\\
\lambda_1' &= \mu,\ \lambda_3'=0,\ \lambda_4'=1.
\end{aligned}$$

Replacing these values on the constraint $(P_{\overline{\theta}})$ we obtain that it is satisfied if $\mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$. Also, the constraint $(IC_{\underline{\theta}})$ is satisfied if $\mu > \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}$. Note that $\frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})} < \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, and $\frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}$ is positive if and only if $\delta_P > \frac{(\overline{\theta}-\underline{\theta})}{(1-\overline{\theta})}$. Thus, if $\max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\} < \mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, the vector $\left(\underline{x}', \overline{x}', \underline{T}', \overline{T}', \lambda_1', \lambda_2', \lambda_3', \lambda_4'\right)$ with $\lambda_2'=0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.

Case (iii). Now $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 6 equations and 6 unknowns with the following solution:

$$\begin{aligned}
&\underline{x}'' = \frac{1+\underline{\theta}}{2},\ \overline{x}'' = \frac{1+\overline{\theta}}{2},\ \underline{T}'' = \frac{1}{4}(1-\underline{\theta})^2,\ \overline{T}'' = \frac{1}{4}(1-\overline{\theta})^2,\\
&\lambda_1''=0,\ \lambda_3''=\mu,\ \lambda_4''=(1-\mu).
\end{aligned}$$

Replacing these values on the constraint $(IC_{\overline{\theta}})$ yields $-(1+\delta_P)(1-\overline{\theta})(\overline{\theta}-\underline{\theta}) \geq 0$, which is false. Thus, there are no solutions that satisfy the conditions in case (iii).

Now, suppose $\lambda_2 > 0$. Suppose also that $\lambda_4 > 0$. We have three other cases: (iv) both $\lambda_1 > 0$ and $\lambda_3 > 0$, (v) $\lambda_1 > 0$ and $\lambda_3 = 0$, and (vi) $\lambda_1 = 0$ and $\lambda_3 > 0$.

Case (iv). The fact that $\lambda_1 > 0$, $\lambda_2 > 0$ and $\lambda_3 > 0$ implies that all the constraints $(IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding. These binding constraints plus the first-order conditions create a system of 8 equations and 8 unknowns with the following solution:

$$\underline{x}^* = \frac{1}{2}(\overline{\theta} + \underline{\theta} - \delta_P(1-\overline{\theta})),\ \overline{x}^* = \frac{\overline{\theta}+\underline{\theta}}{2}, \underline{T}^* = \frac{1}{4}(\overline{\theta} - \underline{\theta} - \delta_P(1-\overline{\theta}))^2,\ \overline{T}^* = \frac{1}{4}(\overline{\theta}-\underline{\theta})^2,$$
$$\lambda_1^* = \frac{(1+\delta_P)(1-\overline{\theta})(1-\mu)}{\overline{\theta}-\underline{\theta}},\ \lambda_2^* = -\frac{\mu(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3^* = \frac{-\delta_P(1-\mu)(1-\overline{\theta})-(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},$$
$$\lambda_4^* = \frac{\delta_P(1-\mu)(1-\overline{\theta})+(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

In this case we have that $\lambda_2^* < 0$. Thus, there are no solutions that satisfy the conditions in case (iv).

Case (v). Now $(IC_{\overline{\theta}}), (IC_{\underline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\hat{\underline{x}} = \frac{1}{2}(1+\underline{\theta} - \delta_P\mu(1-\overline{\theta})),\ \hat{\overline{x}} = \frac{1}{2}(1+\underline{\theta}+\delta_P(1-\mu)(1-\overline{\theta})),$$
$$\hat{\underline{T}} = \frac{1}{4}(\delta_P\mu(1-\overline{\theta})-(1-\underline{\theta}))^2,$$
$$\hat{\overline{T}} = \frac{1}{4}(\delta_P(1-\mu)(1-\overline{\theta})+(1-\underline{\theta}))^2,$$
$$\hat{\lambda}_1 = \frac{\delta_P(1-\overline{\theta})\mu\,(1-\mu)}{\overline{\theta}-\underline{\theta}},\ \hat{\lambda}_2 = \mu\frac{(\delta_P(1-\mu)(1-\overline{\theta})-(\overline{\theta}-\underline{\theta}))}{\overline{\theta}-\underline{\theta}},\ \hat{\lambda}_4 = 1.$$

We have that $\hat{\lambda}_1 \geq 0$, but $\hat{\lambda}_2 \geq 0$ if and only if $\mu \leq \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}$. Replacing the solution on the constraint $(P_{\overline{\theta}})$ we obtain that it is satisfied. Thus, when $\frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})} > 0$, if $\mu \leq \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}$, the vector $\left(\hat{\underline{x}}, \hat{\overline{x}}, \hat{\underline{T}}, \hat{\overline{T}}, \hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3, \hat{\lambda}_4\right)$ with $\hat{\lambda}_3 = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.

Case (vi). Now $(IC_{\underline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}'' = \frac{1+\underline{\theta}}{2},\ \overline{x}'' = \frac{\overline{\theta}+\underline{\theta}}{2},\ \underline{T}'' = \frac{1}{4}(1-\underline{\theta})^2,\ \overline{T}'' = \frac{(\overline{\theta}-\underline{\theta})^2}{4},$$
$$\lambda_2'' = -\frac{\mu(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3'' = -\frac{(1-\overline{\theta})\mu}{\overline{\theta}-\underline{\theta}},\ \lambda_4'' = \frac{(\overline{\theta}-\underline{\theta})+\mu(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

We have that $\lambda_2'' < 0$. Thus, there are no solutions that satisfy the conditions in case (vi). Using an analogous procedure, we can discard the cases where $\lambda_2 > 0$ and $\lambda_4 = 0$.

Note that $\frac{\partial^2\mathcal{L}}{\partial\underline{x}^2} = -2(1-\mu)+2(\lambda_1-\lambda_2-\lambda_4)$. From (12) we have that $(\lambda_1-\lambda_2-\lambda_4) = -(1-\mu)$, and then $\frac{\partial^2\mathcal{L}}{\partial\underline{x}^2} < 0$. Also, $\frac{\partial^2\mathcal{L}}{\partial\overline{x}^2} = -2\mu - 2\lambda_1 + 2\lambda_2 - 2\lambda_3 < 0$ when $\lambda_2 = 0$. In the case $\lambda_2 > 0$, we have that $-2\mu - 2\lambda_1 + 2\lambda_2 - 2\lambda_3 = -2\mu < 0$.

Thus, the Lagrangian function is strictly concave and the Kuhn-Tucker first-order conditions are also sufficient. In sum, the solution of the relaxed problem is the following:

$$(\underline{x}, \overline{x}, \underline{T}, \overline{T}) = \begin{cases} \left(\underline{x}^*, \overline{x}^*, \underline{T}^*, \overline{T}^*\right) & \text{if } \mu > \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}, \\ \left(\underline{x}', \overline{x}', \underline{T}', \overline{T}'\right) & \text{if } \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\} < \mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}, \\ \left(\hat{\underline{x}}, \hat{\overline{x}}, \hat{\underline{T}}, \hat{\overline{T}}\right) & \text{if } 0 \leq \mu \leq \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\}. \end{cases} \tag{22}$$

The interest group's expected payoff from separation equals

$$V^{SEP} = (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G(0)\right) + \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G(1)\right),$$

where $(\underline{x}, \overline{x}, \underline{T}, \overline{T})$ is as in equation (22) and $V_G(0)$ and $V_G(1)$ are the interest group's continuation values as in Remark 1.

Step 2. Pooling. Next, we assume that the interest group chooses a pooling offer. Conditional on pooling, the interest group's constrained maximization problem is

$$\max_{x,T\in\mathbb{R}} \mu\left(-(x-1)^2 - T + \delta_G V_G(\mu)\right) + (1-\mu)\left(-(x-1)^2 - T + \delta_G V_G(\mu)\right)$$
$$s.t.\ (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}),$$

where the constraints are

$$-(x-\overline{\theta})^2 + T + \delta_P V_P(\overline{\theta}, \mu) \geq 0, \tag{$P_{\overline{\theta}}$}$$
$$-(x-\underline{\theta})^2 + T + \delta_P V_P(\underline{\theta}, \mu) \geq 0. \tag{$P_{\underline{\theta}}$}$$

The left-hand side denotes the payoff of accepting and the right-hand side each type's payoff of rejecting. The solution to this problem is to offer $(x, T) = \left(\frac{1+\underline{\theta}}{2}, \frac{(1-\underline{\theta})^2}{4}\right)$. The interest group's expected payoff of the pooling strategy is equal to

$$V^{POOL} = -\left(\frac{1+\underline{\theta}}{2} - 1\right)^2 - \frac{(1-\underline{\theta})^2}{4} + \delta_G V_P(\mu).$$

Finally, a direct comparison of V^{SEP} and V^{POOL} shows that there exists a threshold $\delta_G^\dagger$ such that $V^{SEP} \geq V^{POOL}$ if and only if $\delta_G \geq \delta_G^\dagger$. The cut-off is the following:

$$\delta_G^\dagger = \begin{cases} \frac{\delta_P^2(1-\mu)^2(1-\overline{\theta})^2}{(\overline{\theta}-\underline{\theta})(\mu(\overline{\theta}+\underline{\theta}-2)-2\overline{\theta}+2)} & \text{if } 0 \leq \mu \leq \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\} \\ \frac{2\delta_P(1-\mu)(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{2+\mu(\overline{\theta}+\underline{\theta}-2)-2\overline{\theta}} & \text{if } \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\} < \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}} \\ \frac{\mu(\overline{\theta}-\underline{\theta})\left(2\delta_P(1-\mu)(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})\right)}{(1-\mu)^2(1-\overline{\theta})^2} & \text{if } \frac{1-\overline{\theta}}{1-\underline{\theta}} < \mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})} \\ \delta_P^2 + 2\delta_P - \frac{\mu(1-\underline{\theta})^2}{(1-\mu)(1-\overline{\theta})^2} + \frac{1}{1-\mu} & \text{if } \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})} < \mu. \end{cases}$$

The cut-off $\delta_G^\dagger$ is continuous in μ. Also, if $\mu > \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}$, then $\delta_G^\dagger < 0$ so in this region screening is always optimal. Also $\delta_G^\dagger$ is increasing in δ_P and if $\Delta = \overline{\theta} - \underline{\theta}$, $\delta_G^\dagger$ is decreasing in Δ.

B Appendix: Proofs of Extensions

B.1 Proof of Proposition 5

The result follows from taking the difference between the interest group expected payoff under full information (Benchmark 2) and in the main model (Lemma 1 and 2).

B.2 Proof of Proposition 6

We first characterize the equilibrium behavior when there is a probability α_t of having the chance to lobby in the period t. The result follows by comparing the interest group's expected payoff from our main model with the equilibrium payoff when $\alpha_1 = 0$ and $\alpha_2 = 1$.

The analysis follows in two steps. In the first step, we analyze equilibrium behavior in the second period. In the second step, we analyze equilibrium behavior in the first period.

Step 1. In case the interest group is active, the equilibrium behavior in the second period follows from Lemma 1. If the interest group is not active, then each politician type chooses his ideal policy. Hence, from the perspective of the first period, continuation values for the politician and the interest group are as listed in Remark 2, but taking into consideration the probability of being active in the second period:

$$V_P^A(\theta,\mu) = \begin{cases} 0 & \text{if } \theta = \underline{\theta}, \\ 0 & \text{if } \theta = \overline{\theta} \text{ and } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \alpha_2 \frac{(\overline{\theta}-\underline{\theta})(1-\mu-\overline{\theta}+\mu\underline{\theta})}{1-\mu} & \text{if } \theta = \overline{\theta} \text{ and } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \end{cases}$$

$$V_G^A(\mu) = \begin{cases} \frac{\alpha_2}{2(1-\mu)}\left((1-\mu)(-(1-\underline{\theta})^2) + \mu(\overline{\theta}-\underline{\theta})^2\right) & \\ +(1-\alpha_2)\left(-(1-\mu)(1-\underline{\theta})^2 - \mu(1-\overline{\theta})^2\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \alpha_2\left((1-\mu)\left(-\frac{(1-\overline{\theta})^2}{2} - \frac{(1-\underline{\theta})^2}{2}\right) + \mu\left(-\frac{(1-\overline{\theta})^2}{2}\right)\right) & \\ +(1-\alpha_2)\left(-(1-\mu)(1-\underline{\theta})^2 - \mu(1-\overline{\theta})^2\right) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

Step 2. We now look at the first period, and divide the analysis in two cases: (1) the interest group is active, and (2) the interest group is not active.

Case (1). In this case, the interest group is active in the first period. We again compare separating and pooling equilibria. Conditional on separation, the interest group's maximization problem is

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G^A(1)\right) + (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G^A(0)\right)$$
$$s.t.\ (IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}),$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P V_P^A(\overline{\theta},1) \geq -(\underline{x}-\overline{\theta})^2+\underline{T}+\delta_P V_P^A(\overline{\theta},0), \qquad (IC_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2+\underline{T}+\delta_P V_P^A(\underline{\theta},0) \geq -(\overline{x}-\underline{\theta})^2+\overline{T}+\delta_P V_P^A(\underline{\theta},1), \qquad (IC_{\underline{\theta}})$$
$$-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P V_P^A(\overline{\theta},1) \geq 0, \qquad (P_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2+\underline{T}+\delta_P V_P^A(\underline{\theta},0) \geq 0. \qquad (P_{\underline{\theta}})$$

The problem can be rewritten as follows:

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-(\overline{x}-1)^2-\overline{T}+\alpha_2\delta_G V_G^A(1)\right)+(1-\mu)\left(-(\underline{x}-1)^2-\underline{T}+\alpha_2\delta_G V_G^A(0)\right)$$
$$+(1-\alpha_2)\left(-(1-\mu)(1-\underline{\theta})^2-\mu(1-\overline{\theta})^2\right)$$
$$s.t.\ (IC_{\overline{\theta}}),(IC_{\underline{\theta}}),(P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}),$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2+\overline{T}+\alpha_2\delta_P V_P(\overline{\theta},1) \geq -(\underline{x}-\overline{\theta})^2+\underline{T}+\alpha_2\delta_P V_P(\overline{\theta},0), \qquad (IC_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2+\underline{T}+\alpha_2\delta_P V_P(\underline{\theta},0) \geq -(\overline{x}-\underline{\theta})^2+\overline{T}+\alpha_2\delta_P V_P(\underline{\theta},1), \qquad (IC_{\underline{\theta}})$$
$$-(\overline{x}-\overline{\theta})^2+\overline{T}+\alpha_2\delta_P V_P(\overline{\theta},1) \geq 0, \qquad (P_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2+\underline{T}+\alpha_2\delta_P V_P(\underline{\theta},0) \geq 0. \qquad (P_{\underline{\theta}})$$

Note that the term $(1-\alpha_2)\left(-(1-\mu)(1-\underline{\theta})^2-\mu(1-\overline{\theta})^2\right)$ is constant. If we let $\delta_P'=\alpha_2\delta_P$ and $\delta_G'=\alpha_2\delta_G$, the maximization problem is mathematically equivalent to the one in Lemma 2 in case of a separating contracts. Thus, the solution is the same than Lemma 2 replacing δ_P and δ_G by $\alpha_2\delta_P$ and $\alpha_2\delta_G$ respectively. In case of a pooling offer the same argument applies. Thus, the optimal offer follows.

Case 2. Suppose the interest group is not active in the first period. Still the politician chooses a policy x. Note that $V_P^A(\underline{\theta},\mu)=0$ for every $\mu\in[0,1]$. Thus, independent of the interest group's belief, politician type $\underline{\theta}$ obtains a continuation value of 0. Thus, this type will choose his ideal policy $\underline{\theta}$.

We now turn our attention to the politician type $\overline{\theta}$. Our first step is to rule out in equilibrium any policy different than $\{\underline{\theta},\overline{\theta}\}$. By contradiction, suppose that type $\overline{\theta}$ chooses $x\notin\{\underline{\theta},\overline{\theta}\}$. Since in every equilibrium type $\underline{\theta}$ chooses $\underline{\theta}$, then by Bayesian consistency it must be that beliefs jump to $\mu=1$. By Remark 2, $V_P^A(\overline{\theta},1)=0$. Thus, the politician's expected utility for $x\notin\{\underline{\theta},\overline{\theta}\}$ is $-(x-\overline{\theta})^2+\delta_P\hat{V}_P(\overline{\theta},1)=-(x-\overline{\theta})^2$. Instead, by choosing $\overline{\theta}$, this type can secure a payoff of 0 which is strictly higher than $-(x-\overline{\theta})^2$. Thus, in equilibrium, type $\overline{\theta}$ chooses between policies $\{\underline{\theta},\overline{\theta}\}$.

We first analyze separating equilibria (case 2.1) and then pooling equilibria (case 2.2).

Case 2.1. By previous arguments, in a separating equilibria it must be that type $\overline{\theta}$ chooses policy $\overline{\theta}$ and obtains payoff 0 because in the second period $V_P^A(\overline{\theta},1)=0$. Consider a deviation to policy $\underline{\theta}$. In this case, the second-period payoff would be $V_P^A(\overline{\theta},0)=\alpha_2(\overline{\theta}-\underline{\theta})(1-\overline{\theta})$. For a separating equilibrium to exist we must ensure there is no profitable deviation, which translates to the following condition:

$$0 \geq -(\underline{\theta} - \overline{\theta})^2 + \delta_P \alpha_2 (\overline{\theta} - \underline{\theta})(1 - \overline{\theta}).$$

This condition is equivalent to $\delta_P \leq \frac{(\overline{\theta}-\underline{\theta})}{(1-\overline{\theta})}\frac{1}{\alpha_2}$. Some sufficient conditions for the requirement to hold are (i) $\overline{\theta} \geq \frac{1+\underline{\theta}}{2}$, or (ii) $\overline{\theta} < \frac{1+\underline{\theta}}{2}$ and $\delta_P \leq \frac{\overline{\theta}-\underline{\theta}}{1-\underline{\theta}}$, or (iii) $\overline{\theta} < \frac{1+\underline{\theta}}{2}$ and $\delta_P > \frac{\overline{\theta}-\underline{\theta}}{1-\underline{\theta}}$ and $\alpha_2 \leq \frac{\overline{\theta}-\underline{\theta}}{\delta_P(1-\overline{\theta})}$.

Finally, consider an off-path policy $x \notin \{\underline{\theta}, \overline{\theta}\}$, where Bayes' rule does not apply. From previous arguments, politician type $\underline{\theta}$ does not have a profitable deviation independent of the interest group's belief $\mu \in [0,1]$. Thus, for any $x \notin \{\underline{\theta}, \overline{\theta}\}$ such that there is a belief for which the deviation is profitable, then $\mu = 1$. A deviation gives $-(x - \overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, \mu)$, which is maximized at $\mu = 0$. Thus, for any x that satisfies the following condition it must be that $\mu = 1$

$$-(x - \overline{\theta})^2 + \alpha_2 \delta_P (\overline{\theta} - \underline{\theta})(1 - \overline{\theta}) > 0$$
$$\iff x \in \left(\overline{\theta} - \sqrt{\alpha_2 \delta_P (\overline{\theta} - \underline{\theta})(1 - \overline{\theta})}, \overline{\theta} + \sqrt{\alpha_2 \delta_P (\overline{\theta} - \underline{\theta})(1 - \overline{\theta})}\right).$$

A direct computation shows that type $\overline{\theta}$ does not have incentives to deviate for this set of policies since $0 > -(x - \overline{\theta})^2 + 0$ for any $x \neq \overline{\theta}$.

Finally, for all $x \notin \left(\overline{\theta} - \sqrt{\alpha_2 \delta_P (\overline{\theta} - \underline{\theta})(1 - \overline{\theta})}, \overline{\theta} + \sqrt{\alpha_2 \delta_P (\overline{\theta} - \underline{\theta})(1 - \overline{\theta})}\right) \cup \{\underline{\theta}\}$, NWBR does not restrict the beliefs and we directly impose $\mu = \mu_0$. The set of policies is characterized by $-(x - \overline{\theta})^2 + \alpha_2 \delta_P (\overline{\theta} - \underline{\theta})(1 - \overline{\theta}) < 0$. If the type $\overline{\theta}$ deviates to such a policy, it obtains $-(x - \overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, \mu_0)$. Since $V_P^A(\overline{\theta}, \mu_0) < V_P^A(\overline{\theta}, 1)$, type $\overline{\theta}$ does not have incentives to deviate for this set of policies either. Note that our results are independent of the belief we consider for this set of policies.

Case 2.2. In a pooling equilibrium type $\overline{\theta}$ chooses $\underline{\theta}$ and obtains payoff $-(\underline{\theta} - \overline{\theta})^2$ in the first period, and $V_P^A(\overline{\theta}, \mu_0)$ in the second period. Note that $V_P^A(\overline{\theta}, \mu_0) = 0 \iff \mu_0 \geq \frac{1-\overline{\theta}}{1-\underline{\theta}}$. Thus, if $\mu_0 \geq \frac{1-\overline{\theta}}{1-\underline{\theta}}$ there is no pooling equilibrium since type $\overline{\theta}$ secures a payoff of at least 0 choosing $\overline{\theta}$. Assume that $\mu_0 < \frac{1-\overline{\theta}}{1-\underline{\theta}}$. We now apply NWBR. Now all policies $x \neq \underline{\theta}$ are off the equilibrium path. Similar to before, if politician type $\overline{\theta}$ strictly prefers to deviate for some off-path belief, then $\mu = 1$. Thus, for any x that satisfies the following condition it must be that $\mu = 1$

$$-(x - \overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, 0) > -(\underline{\theta} - \overline{\theta})^2 + \delta_P V_P^A(\overline{\theta}, \mu_0) \iff$$
$$x \in \left(\overline{\theta} - \sqrt{(\underline{\theta} - \overline{\theta})^2 + \delta_P \left(V_P^A(\overline{\theta}, 0) - V_P^A(\overline{\theta}, \mu_0)\right)}, \overline{\theta} + \sqrt{(\underline{\theta} - \overline{\theta})^2 + \delta_P \left(V_P^A(\overline{\theta}, 0) - V_P^A(\overline{\theta}, \mu_0)\right)}\right),$$

In this case $V_P^A(\overline{\theta}, \mu_0) = \alpha_2 \frac{(\overline{\theta}-\underline{\theta})(1-\mu_0-\overline{\theta}+\mu_0\underline{\theta})}{1-\mu_0}$ and $V_P^A(\overline{\theta}, 0) = \alpha_2(\overline{\theta} - \underline{\theta})(1 - \overline{\theta})$. For policies in this set, type $\overline{\theta}$ has no profitable deviation if the following inequality is satisfied

$$\begin{aligned} & -(\underline{\theta}-\overline{\theta})^2+\delta_P V_P^A(\overline{\theta},\mu_0) \geq -(x-\overline{\theta})^2+\delta_P V_P^A(\overline{\theta},1) \\ \iff & -(\underline{\theta}-\overline{\theta})^2+\delta_P V_P^A(\overline{\theta},\mu_0) \geq -(x-\overline{\theta})^2. \end{aligned}$$

An equivalent condition for the previous inequality is that $\delta_P \geq \frac{(\overline{\theta}-\underline{\theta})}{(1-\mu_0-\overline{\theta}+\mu_0\underline{\theta})}\frac{1-\mu_0}{\alpha_2}$. Some sufficient conditions for the condition to hold are $\overline{\theta} < \frac{1+\underline{\theta}}{2}$ and $\delta_P \geq \frac{(\overline{\theta}-\underline{\theta})(1-\mu_0)}{(1-\mu_0)-\overline{\theta}+\mu_0\underline{\theta}}$ and $\alpha_2 \geq \frac{(\overline{\theta}-\underline{\theta})(1-\mu_0)}{\delta_P(1-\mu_0-\overline{\theta}+\mu_0\underline{\theta})}$ and $\mu_0 < \frac{1-2\overline{\theta}+\underline{\theta}}{1-\overline{\theta}}$. Note that $\frac{(\overline{\theta}-\underline{\theta})}{(1-\overline{\theta})}\frac{1}{\alpha_2} < \frac{(\overline{\theta}-\underline{\theta})}{(1-\mu_0-\overline{\theta}+\mu_0\underline{\theta})}\frac{1-\mu_0}{\alpha_2}$.

For all other off-path x not in this set, by assumption $\mu = \mu_0$. For policies in this set, type $\overline{\theta}$ has no profitable deviation if the following inequality is satisfied

$$-(\underline{\theta}-\overline{\theta})^2+\delta_P V_P^A(\overline{\theta},\mu_0) \geq -(x-\overline{\theta})^2+\delta_P V_P^A(\overline{\theta},\mu_0).$$

Since $V_P^A(\overline{\theta},0) > V_P^A(\overline{\theta},\mu_0)$, then $-(\underline{\theta}-\overline{\theta})^2+\delta_P V_P^A(\overline{\theta},\mu_0) > -(x-\overline{\theta})^2+\delta_P V_P^A(\overline{\theta},0) > -(x-\overline{\theta})^2+\delta_P V_P^A(\overline{\theta},\mu_0)$. Note that our results are independent of the belief we consider for this set of policies.

B.3 Proof of Proposition 7

We first characterize the equilibrium behavior when policy x is constrained to the interval $[\underline{y},\overline{y}]$. Similar than Lemma 2, we found a cut-off $\delta_G^{\dagger,\overline{y}}$ such that the interest group offers separating contracts if and only if $\delta_G \geq \delta_G^{\dagger,\overline{y}}$. We then calculate the derivative of the interest group's expected payoff in equilibrium with respect to the upper limit $\overline{y}$.

For our analysis we assume that $\underline{y} \leq \underline{\theta}$ and $\frac{1+\underline{\theta}}{2} \leq \overline{y} \leq \frac{1+\overline{\theta}}{2}$. That is, the interval restricts the policy only at the right side. We first study the second-period equilibrium behavior. Lemma 1 implies that if $\overline{y} \leq \frac{1+\overline{\theta}}{2}$, the marginal benefit of an increase in $\overline{x}$ when $\overline{x}=\overline{y}$ is strictly positive. Then, it must be that $\overline{x}=\overline{y}$. The interest group maximization problem is the follows:

$$\max_{\underline{y}\leq\underline{x}\leq\overline{y},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-\left(\overline{y}-1\right)^2-\overline{T}\right)+(1-\mu)\left(-\left(\underline{x}-1\right)^2-\underline{T}\right)$$

$$s.t.$$

$$\begin{aligned} -\left(\overline{y}-\overline{\theta}\right)^2+\overline{T} &\geq -\left(\underline{x}-\overline{\theta}\right)^2+\underline{T}, && (IC_{\overline{\theta}}) \\ -\left(\underline{x}-\underline{\theta}\right)^2+\underline{T} &\geq -\left(\overline{y}-\underline{\theta}\right)^2+\overline{T}, && (IC_{\underline{\theta}}) \\ -\left(\overline{y}-\overline{\theta}\right)^2+\overline{T} &\geq 0, && (P_{\overline{\theta}}) \\ -\left(\underline{x}-\underline{\theta}\right)^2+\underline{T} &\geq 0. && (P_{\underline{\theta}}) \end{aligned}$$

The solution is as follows.

$$(\underline{x}, \overline{x}, \underline{T}, \overline{T}) = \begin{cases} \left(\frac{\overline{\theta}+\underline{\theta}}{2}, \overline{y}, \left(\left(\frac{\overline{\theta}+\underline{\theta}}{2}\right) - \underline{\theta}\right)^2, \left(\overline{y} - \overline{\theta}\right)^2\right) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \left(\frac{1+\underline{\theta}-\mu(\overline{\theta}+1)}{2(1-\mu)}, \overline{y}, \left(\left(\frac{1+\underline{\theta}-\mu(\overline{\theta}+1)}{2(1-\mu)}\right) - \underline{\theta}\right)^2, \frac{(\overline{\theta}-\underline{\theta})(1-\mu-\overline{\theta}+\mu\underline{\theta})}{1-\mu} + \left(\overline{y} - \overline{\theta}\right)^2\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

Using these results, the continuation value for the politician and interest group are the following, respectively:

$$V_P^{\overline{y}}(\theta, \mu) = \begin{cases} 0 & \text{if } \theta = \underline{\theta}, \\ 0 & \text{if } \theta = \overline{\theta} \text{ and } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \frac{(\overline{\theta}-\underline{\theta})(1-\mu-\overline{\theta}+\mu\underline{\theta})}{1-\mu} & \text{if } \theta = \overline{\theta} \text{ and } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \end{cases}$$

$$V_G^{\overline{y}}(\mu) = \begin{cases} \frac{1}{2}\left(-(1+\mu)\overline{\theta}^2 + \overline{\theta}(\mu(4\overline{y}-2)+2) + (1-\mu)(2-\underline{\theta})\underline{\theta} - 4\mu\overline{y}^2 + 4\mu\overline{y} - 2\right) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \frac{1}{2(1-\mu)}\left(\mu^2(1+\overline{\theta}-2\overline{y})^2 - 2\mu\left(\overline{\theta}\underline{\theta} - 2(1+\overline{\theta})\overline{y} + \underline{\theta} + \overline{\theta} - \underline{\theta}^2 + 2\overline{y}^2\right) - (1-\underline{\theta})^2\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

Now we focus on the first period. Conditional on separation, the interest group's maximization problem is

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G^{\overline{y}}(1)\right) + (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G^{\overline{y}}(0)\right)$$
$$s.t.\ (IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}}), (P_{\underline{\theta}}) \text{ and } (LP),$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^{\overline{y}}(\overline{\theta}, 1) \geq -(\underline{x}-\overline{\theta})^2 + \underline{T} + \delta_P V_P^{\overline{y}}(\overline{\theta}, 0), \qquad (IC_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^{\overline{y}}(\underline{\theta}, 0) \geq -(\overline{x}-\underline{\theta})^2 + \overline{T} + \delta_P V_P^{\overline{y}}(\underline{\theta}, 1), \qquad (IC_{\underline{\theta}})$$
$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^{\overline{y}}(\overline{\theta}, 1) \geq 0, \qquad (P_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^{\overline{y}}(\underline{\theta}, 0) \geq 0, \qquad (P_{\underline{\theta}})$$
$$(\overline{y}-\overline{x}) \geq 0. \qquad (LP)$$

We begin our analysis by setting up the Lagrangian

$$\begin{aligned}\mathcal{L} =& \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G^{\overline{y}}(1)\right) + (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G^{\overline{y}}(0)\right) \\ &+\lambda_1\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^{\overline{y}}(\overline{\theta}, 1) + (\underline{x}_1-\overline{\theta})^2 - \underline{T} - \delta_P V_P^{\overline{y}}(\overline{\theta}, 0)\right) \\ &+\lambda_2\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^{\overline{y}}(\underline{\theta}, 0) + (\overline{x}-\underline{\theta})^2 - \overline{T} - \delta_P V_P^{\overline{y}}(\underline{\theta}, 1)\right) \\ &+\lambda_3\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^{\overline{y}}(\overline{\theta}, 1)\right) \\ &+\lambda_4\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^{\overline{y}}(\underline{\theta}, 0)\right) \\ &+\lambda_5\left(\overline{y}-\overline{x}\right).\end{aligned}$$

The first-order conditions with respect to $\underline{x}, \overline{x}, \underline{T}, \overline{T}$ are

$$\frac{\partial \mathcal{L}}{\partial \underline{x}} = -2(1-\mu)(\underline{x}-1) + 2\lambda_1(\underline{x}-\overline{\theta}) - 2\lambda_2(\underline{x}-\underline{\theta}) - 2\lambda_4(\underline{x}-\underline{\theta}) = 0, \tag{23}$$

$$\frac{\partial \mathcal{L}}{\partial \overline{x}} = -2\mu(\overline{x}-1) - 2\lambda_1(\overline{x}-\overline{\theta}) + 2\lambda_2(\overline{x}-\underline{\theta}) - 2\lambda_3(\overline{x}-\overline{\theta}) - \lambda_5 = 0, \tag{24}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{T}} = -(1-\mu) - \lambda_1 + \lambda_2 + \lambda_4 = 0, \tag{25}$$

$$\frac{\partial \mathcal{L}}{\partial \overline{T}} = -\mu + \lambda_1 - \lambda_2 + \lambda_3 = 0. \tag{26}$$

The complementary slackness conditions are:

$$\lambda_1\left(-(\overline{x}-\overline{\theta})^2 + \overline{T} + (\underline{x}-\overline{\theta})^2 - \underline{T} - \delta_P(\overline{\theta}-\underline{\theta})(1-\overline{\theta})\right) = 0, \tag{27}$$

$$\lambda_2\left(-(\underline{x}-\underline{\theta})^2 + \underline{T} + (\overline{x}-\underline{\theta})^2 - \overline{T}\right) = 0, \tag{28}$$

$$\lambda_3\left(-(\overline{x}-\overline{\theta})^2 + \overline{T}\right) = 0, \tag{29}$$

$$\lambda_4\left(-(\underline{x}-\underline{\theta})^2 + \underline{T}\right) = 0, \tag{30}$$

$$\lambda_5\left(\overline{y}-\overline{x}\right) = 0. \tag{31}$$

Suppose first that $\lambda_2 = 0$. By (25) we know that $\lambda_4 = \lambda_1 + (1-\mu) > 0$. This implies that $(-(\underline{x}-\underline{\theta})^2 + \underline{T}) = 0 \iff \underline{T} = (\underline{x}-\underline{\theta})^2$. Then by (26) we know that $\mu_0 = \lambda_1 + \lambda_3$. Given that λ_1, λ_3 are non-negative, we know that there are three cases: (i) both $\lambda_1 > 0$ and $\lambda_3 > 0$, (ii) $\lambda_1 = \mu$ and $\lambda_3 = 0$, and (iii) $\lambda_1 = 0$ and $\lambda_3 = \mu$.

Case (i). The fact that $\lambda_1 > 0$ and $\lambda_3 > 0$ implies that constraints $(IC_{\overline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are all binding. If $\lambda_5 = 0$ the solution violates constraint (LP). If $\lambda_5 > 0$, then $\overline{x} = \overline{y}$. The binding constraints plus the first-order conditions create a system of 8 equations and 8 unknowns with the following solution:

$$\underline{x}^* = \frac{1}{2}(\overline{\theta} + \underline{\theta} - \delta_P(1-\overline{\theta})),\ \overline{x}^* = \overline{y},\ \underline{T}^* = \frac{1}{4}(\overline{\theta} - \underline{\theta} - \delta_P(1-\overline{\theta}))^2,\ \overline{T}^* = (\overline{\theta}-\overline{y})^2,$$

$$\lambda_1^* = \frac{(1+\delta_P)(1-\mu)(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3^* = \frac{\delta_P(\mu(1-\overline{\theta}) + \overline{\theta} - 1) + \mu(-\underline{\theta}) + \mu + \overline{\theta} - 1}{\overline{\theta}-\underline{\theta}},$$

$$\lambda_4^* = \frac{(1-\mu)(\delta_P(1-\overline{\theta}) + 1 - \underline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_5^* = 2\mu(1+\overline{\theta}-2\overline{y}).$$

We have that $\lambda_5^* > 0$. Under the condition $\mu > \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, the multiplier $\lambda_3^* > 0$ and $(IC_{\underline{\theta}})$ is satisfied, which imply that $\left(\underline{x}^*, \overline{x}^*, \underline{T}^*, \overline{T}^*, \lambda_1^*, \lambda_2^*, \lambda_3^*, \lambda_4^*, \lambda_5^*\right)$ with $\lambda_2^* = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.

Case (ii). Now $(IC_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding. If $\lambda_5 = 0$ the solution violates constraint (LP). If $\lambda_5 > 0$, then $\overline{x} = \overline{y}$. The binding constraints together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}' = \frac{1+\underline{\theta}-\mu(1+\overline{\theta})}{2(1-\mu)}, \overline{x}' = \overline{y},$$
$$\underline{T}' = \frac{(1-\underline{\theta}-\mu(1+\overline{\theta}-2\underline{\theta}))^2}{4(1-\mu)^2},$$
$$\overline{T}' = \frac{\mu\left(-(\delta_P-1)\overline{\theta}^2+\overline{\theta}(\delta_P\underline{\theta}+\delta_P-\underline{\theta}-2\overline{y}+1)-(\delta_P+1)\underline{\theta}+\underline{\theta}^2+\overline{y}^2\right)}{(\mu-1)}$$
$$+\frac{\delta_P\overline{\theta}^2-\overline{\theta}(\delta_P\underline{\theta}+\delta_P+\underline{\theta}-2\overline{y}+1)+\delta_P\underline{\theta}+\underline{\theta}-\overline{y}^2}{(\mu-1)},$$
$$\lambda_1' = \mu,\ \lambda_3' = 0,\ \lambda_4' = 1,\ \lambda_5' = 2\mu(1+\overline{\theta}-2\overline{y}).$$

We have that $\lambda_5' > 0$. Replacing these values on the constraint $(P_{\overline{\theta}})$ we obtain that it is satisfied if $\mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$. Also, the constraint $(IC_{\underline{\theta}})$ is satisfied if $\mu > \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}$. Note that $\frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}} < \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, and $\frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}$ is positive if and only if $\delta_P > \frac{2\overline{y}-1-\underline{\theta}}{(1-\overline{\theta})}$. Thus, if $\max\{0, \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}\} < \mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, the vector $\left(\underline{x}', \overline{x}', \underline{T}', \overline{T}', \lambda_1', \lambda_2', \lambda_3', \lambda_4'\right)$ with $\lambda_2' = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.

Case (iii). Now $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding. If $\lambda_5 = 0$ the solution violates constraint (LP). If $\lambda_5 > 0$, then $\overline{x} = \overline{y}$. The binding constraints together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}'' = \frac{1+\underline{\theta}}{2},\ \overline{x}'' = \overline{y},\ \underline{T}'' = \frac{1}{4}(1-\underline{\theta})^2,\ \overline{T}'' = (\overline{\theta}-y)^2,$$
$$\lambda_1'' = 0,\ \lambda_3'' = \mu,\ \lambda_4'' = (1-\mu),\ \lambda_5'' = 2\mu(1+\overline{\theta}-2\overline{y}).$$

We have that $\lambda_5'' > 0$. Replacing these values on the constraint $(IC_{\overline{\theta}})$ we obtain that $-(1+\delta_P)(1-\overline{\theta})(\overline{\theta}-\underline{\theta}) \geq 0$, which is false. Thus, there are no solutions that satisfy the conditions in case (iii).

Now, suppose $\lambda_2 > 0$. Suppose also that $\lambda_4 > 0$. We have three other cases: (iv) both $\lambda_1 > 0$ and $\lambda_3 > 0$, (v) $\lambda_1 > 0$ and $\lambda_3 = 0$, and (vi) $\lambda_1 = 0$ and $\lambda_3 > 0$.

Case (iv). The fact that $\lambda_1 > 0$, $\lambda_2 > 0$ and $\lambda_3 > 0$ implies that constraints $(IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are all binding. If $\lambda_5 = 0$ the solution does not satisfy the KKT conditions since it implies that $\lambda_2 < 0$. If $\lambda_5 > 0$, then $\overline{x} = \overline{y}$. These binding constraints plus the first-order conditions create a system of 9 equations and 9 unknowns which has empty solution. Thus, there are no solutions that satisfy the conditions in case (iv).

Case (v). Now $(IC_{\overline{\theta}}), (IC_{\underline{\theta}})$ and $(P_{\underline{\theta}})$ are binding. When $\lambda_5 > 0$, together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\hat{\underline{x}} = \frac{1}{2}(2\overline{y} - \delta_P(1-\overline{\theta})),\ \hat{\overline{x}} = \overline{y},$$
$$\hat{\underline{T}} = \frac{1}{4}(\delta_P\overline{\theta} - \delta_P - 2\underline{\theta} + 2\overline{y})^2,\ \hat{\overline{T}} = (\underline{\theta} - \overline{y})^2,$$
$$\hat{\lambda}_1 = \frac{(\mu-1)(\delta_P\overline{\theta} - \delta_P - \underline{\theta} + 2\overline{y} - 1)}{\overline{\theta} - \underline{\theta}},$$
$$\hat{\lambda}_2 = \frac{\delta_P\overline{\theta}\mu + \delta_P(-\overline{\theta}) - \delta_P\mu + \delta_P - \overline{\theta}\mu + \underline{\theta} + 2\mu\overline{y} - \mu - 2\overline{y} + 1}{\overline{\theta} - \underline{\theta}},$$
$$\hat{\lambda}_4 = 1,\ \hat{\lambda}_5 = 2(\delta_P\overline{\theta}\mu + \delta_P(-\overline{\theta}) - \delta_P\mu + \delta_P + \underline{\theta} - 2\overline{y} + 1).$$

We have that $\hat{\lambda}_1 \geq 0$ if and only if $\delta_P > \frac{2\overline{y}-1-\underline{\theta}}{1-\overline{\theta}}$. Also, $\hat{\lambda}_2 \geq 0$ if and only if $\mu \leq \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}$. Also $\hat{\lambda}_5 \geq 0$ if and only if $\mu < \frac{\delta_P\overline{\theta}-\delta_P-\underline{\theta}+2\overline{y}-1}{\delta_P(\overline{\theta}-1)}$. Replacing the solution on the constraint $(P_{\overline{\theta}})$ we obtain that it is satisfied. Note that when $\delta_P > \frac{2\overline{y}-1-\underline{\theta}}{1-\overline{\theta}}$, we have that $0 < \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}} < \frac{\delta_P\overline{\theta}-\delta_P-\underline{\theta}+2\overline{y}-1}{\delta_P(\overline{\theta}-1)}$. Thus, when $\delta_P > \frac{2\overline{y}-1-\underline{\theta}}{1-\overline{\theta}}$, if $\mu \leq \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}$, the vector $\left(\hat{\underline{x}}, \hat{\overline{x}}, \hat{\underline{T}}, \hat{\overline{T}}, \hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3, \hat{\lambda}_4\right)$ with $\hat{\lambda}_3 = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.
If $\lambda_5 = 0$, then the solution is

$$\hat{\underline{x}} = \frac{1}{2}(1 + \underline{\theta} - \delta_P\mu(1-\overline{\theta})), \hat{\overline{x}} = \frac{1}{2}(1 + \underline{\theta} + \delta_P(1-\mu)(1-\overline{\theta})),$$
$$\hat{\underline{T}} = \frac{1}{4}(\delta_P\mu(1-\overline{\theta}) - (1-\underline{\theta}))^2,$$
$$\hat{\overline{T}} = \frac{1}{4}(\delta_P(1-\mu)(1-\overline{\theta}) + (1-\underline{\theta}))^2,$$
$$\hat{\lambda}_1 = \frac{\delta_P(1-\overline{\theta})\mu(1-\mu)}{\overline{\theta}-\underline{\theta}}, \hat{\lambda}_2 = \mu\frac{(\delta_P(1-\mu)(1-\overline{\theta}) - (\overline{\theta}-\underline{\theta}))}{\overline{\theta}-\underline{\theta}}, \hat{\lambda}_4 = 1.$$

For the set of parameters where $\hat{\lambda}_1 \geq 0$, we have that $\hat{\overline{x}} > y$, which violates constraint (LP). Thus, there are no solutions that satisfy the conditions in case (v) when $\lambda_5 = 0$.

Case (vi). Now $(IC_{\underline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding. If $\lambda_5 = 0$ the solution does not satisfies the KKT conditions since $\lambda_2^{*\prime\prime} < 0$. If $\lambda_5 > 0$, then $\overline{x} = \overline{y}$. and, together with the first-order conditions, create a system of 8 equations and 8 unknowns which has empty solution. Thus, there are no solutions that satisfy the conditions in case (vi).
Using an analog procedure, we can discard the cases where $\lambda_2 > 0$ and $\lambda_4 = 0$. In sum, the solution of the maximization problem is the following:

$$(\underline{x}, \overline{x}, \underline{T}, \overline{T}) = \begin{cases} \left(\underline{x}^*, \overline{x}^*, \underline{T}^*, \overline{T}^*\right) & \text{if } \mu > \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}, \\ \left(\underline{x}', \overline{x}', \underline{T}', \overline{T}'\right) & \text{if } \max\{0, \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}\} < \mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}, \\ \left(\hat{\underline{x}}, \hat{\overline{x}}, \hat{\underline{T}}, \hat{\overline{T}}\right) & \text{if } 0 \leq \mu \leq \max\{0, \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}\}. \end{cases} \quad (32)$$

The interest group's expected payoff from separation equals

$$V^{SEP} = (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_2^{\overline{y}}(0)\right) + \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_2^{\overline{y}}(1)\right),$$

where $(\underline{x}, \overline{x}, \underline{T}, \overline{T})$ is as in equation (32) and $V_G^{\overline{y}}(0)$ and $V_G^{\overline{y}}(1)$ are the interest group's continuation values.

In case of a pooling offer, the interest group solves:

$$\max_{\underline{y}\le x\le\overline{y}, T\in\mathbb{R}} \mu\left(-(x-1)^2 - T + \delta_G V_G^{\overline{y}}(\mu)\right) + (1-\mu)\left(-(x-1)^2 - T + \delta_G V_G^{\overline{y}}(\mu)\right)$$
$$s.t.\ (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}),$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^{\overline{y}}(\overline{\theta},\mu) \ge 0, \qquad (P_{\overline{\theta}})$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^{\overline{y}}(\underline{\theta},\mu) \ge 0. \qquad (P_{\underline{\theta}})$$

The solution to this problem is to offer $(x,T) = \left(\frac{1+\underline{\theta}}{2}, \frac{(1-\underline{\theta})^2}{4}\right)$. The interest group's expected payoff of the pooling strategy is equal to

$$V^{POOL} = -\left(\frac{1+\underline{\theta}}{2} - 1\right)^2 - \frac{(1-\underline{\theta})^2}{4} + \delta_G V_G^{\overline{y}}(\mu).$$

In Lemma 2, we found that there is a cut-off $\delta_G^{\dagger}$ such that $V^{SEP} \ge V^{POOL}$ if and only if $\delta_G \ge \delta_G^{\dagger}$. Here, we obtain a similar result: there is a cut-off $\delta_G^{\dagger,\overline{y}}$ such that $V^{SEP} \ge V^{POOL}$ if and only if $\delta_G \ge \delta_G^{\dagger,\overline{y}}$, where $\delta_G^{\dagger,\overline{y}}$ equals to

$$\begin{cases}
\left[\frac{(\mu-1)\left(\delta_P^2(\mu-1)(\overline{\theta}-1)^2 - 2\delta_P(\mu-1)(\overline{\theta}-1)(\underline{\theta}-2\overline{y}+1) - (\underline{\theta}-2\overline{y}+1)^2\right)}{\mu(\overline{\theta}-\underline{\theta})(\mu(\overline{\theta}+\underline{\theta}-2)-2\overline{\theta}+2)}\right] \\
\text{if } 0 \le \mu \le \max\{0, \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}\} \\
\left[\frac{\mu\left((2\delta_P-1)\overline{\theta}^2 - 2\overline{\theta}(\delta_P\underline{\theta}+\delta_P-2\overline{y}+1)+2\delta_P\underline{\theta}-4\overline{y}^2+4\overline{y}-1\right) - 2\delta_P\overline{\theta}^2 + 2\overline{\theta}(\delta_P\underline{\theta}+\delta_P+\underline{\theta}-2\overline{y}+1) - 2\delta_P\underline{\theta} - \underline{\theta}^2 + 4\overline{y}^2 - 4\overline{y}+1}{(\overline{\theta}-\underline{\theta})(\mu(\overline{\theta}+\underline{\theta}-2)-2\overline{\theta}+2)}\right] \\
\text{if } \max\{0, \frac{\delta_P(1-\overline{\theta})+1+\underline{\theta}-2\overline{y}}{\delta_P(1-\overline{\theta})+1+\overline{\theta}-2\overline{y}}\} < \mu \le \frac{1-\overline{\theta}}{1-\underline{\theta}} \\
\left[\frac{\mu\left(\mu\left((2\delta_P-1)\overline{\theta}^2 - 2\overline{\theta}(\delta_P\underline{\theta}+\delta_P-2\overline{y}+1)+2\delta_P\underline{\theta}-4\overline{y}^2+4\overline{y}-1\right) - 2\delta_P\overline{\theta}^2 + 2\overline{\theta}(\delta_P\underline{\theta}+\delta_P+\underline{\theta}-2\overline{y}+1) - 2\delta_P\underline{\theta} - \underline{\theta}^2 + 4\overline{y}^2 - 4\overline{y}+1\right)}{(1-\mu)^2(1-\overline{\theta})^2}\right] \\
\text{if } \frac{1-\overline{\theta}}{1-\underline{\theta}} < \mu \le \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})} \\
\left[\frac{\delta_P^2(\mu-1)(\overline{\theta}-1)^2 + 2\delta_P(\mu-1)(\overline{\theta}-1)^2 - (\mu+1)\overline{\theta}^2 + \overline{\theta}(4\mu\overline{y}-2\mu+2) + \mu\underline{\theta}^2 - 2\mu\underline{\theta} - 4\mu\overline{y}^2 + 4\mu\overline{y}-1}{(\mu-1)(1-\overline{\theta})^2}\right] \\
\text{if } \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})} < \mu.
\end{cases}$$

A direct comparison of $\delta_G^{\dagger,\overline{y}}$ and $\delta_G^{\dagger}$ shows that $\delta_G^{\dagger,\overline{y}} > \delta_G^{\dagger}$. Thus, a constraint in the feasible policies decreases the incentives for screening relative to pooling. Also, after some calculations, we obtain that $\delta_G^{\dagger,\overline{y}}$ is decreasing in $\overline{y}$ and that the interest group expected payoff in equilibrium increases in $\overline{y}$.

B.4 Proofs of Propositions 8 and 9

We first characterize the equilibrium behavior when there is an extra benefit $R > 0$ for the politician in the second period if it is sufficiently likely that the politician is aligned. Proposition 8 follows from calculating the cut-off $\delta_G^{\dagger,R}$ for which the interest group is indifferent between offering a separating offer and a polling offer in equilibrium. Proposition 9 follows from taking the derivative of the interest group's expected payoff in equilibrium with respect to R.

For simplicity, assume that the politician obtains the benefit R in the second period only if the interest group's belief is sufficiently high $\mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}$ after the first period. The politician and interest group continuation values are, respectively

$$V_P^R(\theta,\mu) = \begin{cases} R & \text{if } \theta = \underline{\theta} \text{ and } \mu \geq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ 0 & \text{if } \theta = \underline{\theta} \text{ and } \mu < \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ R & \text{if } \theta = \overline{\theta} \text{ and } \mu \geq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ \frac{(\overline{\theta}-\underline{\theta})(1-\mu-\overline{\theta}+\mu\underline{\theta})}{1-\mu}, & \text{if } \theta = \overline{\theta} \text{ and } \mu < \frac{1-\overline{\theta}}{1-\underline{\theta}}, \end{cases}$$

$$V_G^R(\mu) = \begin{cases} \frac{1}{2(1-\mu)}\left((1-\mu)(-(1-\underline{\theta})^2) + \mu(\overline{\theta}-\underline{\theta})^2\right) & \text{if } \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}}, \\ (1-\mu)\left(-\frac{(1-\overline{\theta})^2}{2} - \frac{(1-\underline{\theta})^2}{2}\right) + \mu\left(-\frac{(1-\overline{\theta})^2}{2}\right) & \text{if } \mu > \frac{1-\overline{\theta}}{1-\underline{\theta}}. \end{cases}$$

Conditional on separation, the interest group maximization problem is

$$\max_{\underline{x},\overline{x},\underline{T},\overline{T}\in\mathbb{R}} \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G^R(1)\right) + (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G^R(0)\right)$$
$$s.t.\ (IC_{\overline{\theta}}), (IC_{\underline{\theta}}), (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}),$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^R(\overline{\theta},1) \geq -(\underline{x}-\overline{\theta})^2 + \underline{T} + \delta_P V_P^R(\overline{\theta},0), \tag{$IC_{\overline{\theta}}$}$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^R(\underline{\theta},0) \geq -(\overline{x}-\underline{\theta})^2 + \overline{T} + \delta_P V_P^R(\underline{\theta},1), \tag{$IC_{\underline{\theta}}$}$$
$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P^R(\overline{\theta},1) \geq 0, \tag{$P_{\overline{\theta}}$}$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P^R(\underline{\theta},0) \geq 0. \tag{$P_{\underline{\theta}}$}$$

We begin our analysis by setting up the Lagrangian

$$\begin{aligned}\mathcal{L} =&\mu\left(-(\overline{x}-1)^2-\overline{T}+\delta_G V_G^R(1)\right)+(1-\mu)\left(-(\underline{x}-1)^2-\underline{T}+\delta_G V_G^R(0)\right)\\ &+\lambda_1\left(-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P V_P^R(\overline{\theta},1)+(\underline{x}_1-\overline{\theta})^2-\underline{T}-\delta_P V_P^R(\overline{\theta},0)\right)\\ &+\lambda_2\left(-(\underline{x}-\underline{\theta})^2+\underline{T}+\delta_P V_P^R(\underline{\theta},0)+(\overline{x}-\underline{\theta})^2-\overline{T}-\delta_P V_P^R(\underline{\theta},1)\right)\\ &+\lambda_3\left(-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P V_P^R(\overline{\theta},1)\right)\\ &+\lambda_4\left(-(\underline{x}-\underline{\theta})^2+\underline{T}+\delta_P V_P^R(\underline{\theta},0)\right).\end{aligned}$$

The first-order conditions with respect to $\underline{x},\overline{x},\underline{T},\overline{T}$ are

$$\frac{\partial\mathcal{L}}{\partial\underline{x}}=-2(1-\mu)(\underline{x}-1)+2\lambda_1(\underline{x}-\overline{\theta})-2\lambda_2(\underline{x}-\underline{\theta})-2\lambda_4(\underline{x}-\underline{\theta})=0, \tag{33}$$

$$\frac{\partial\mathcal{L}}{\partial\overline{x}}=-2\mu(\overline{x}-1)-2\lambda_1(\overline{x}-\overline{\theta})+2\lambda_2(\overline{x}-\underline{\theta})-2\lambda_3(\overline{x}-\overline{\theta})=0, \tag{34}$$

$$\frac{\partial\mathcal{L}}{\partial\underline{T}}=-(1-\mu)-\lambda_1+\lambda_2+\lambda_4=0, \tag{35}$$

$$\frac{\partial\mathcal{L}}{\partial\overline{T}}=-\mu+\lambda_1-\lambda_2+\lambda_3=0. \tag{36}$$

The complementary slackness conditions are:

$$\lambda_1\left(-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P R+(\underline{x}-\overline{\theta})^2-\underline{T}-\delta_P(\overline{\theta}-\underline{\theta})(1-\overline{\theta})\right)=0, \tag{37}$$

$$\lambda_2\left(-(\underline{x}-\underline{\theta})^2+\underline{T}+(\overline{x}-\underline{\theta})^2-\overline{T}-\delta_P R\right)=0, \tag{38}$$

$$\lambda_3\left(-(\overline{x}-\overline{\theta})^2+\overline{T}+\delta_P R\right)=0, \tag{39}$$

$$\lambda_4\left(-(\underline{x}-\underline{\theta})^2+\underline{T}\right)=0. \tag{40}$$

Suppose first that $\lambda_2=0$. By (35) we know that $\lambda_4=\lambda_1+(1-\mu)>0$. This implies that $(-(\underline{x}-\underline{\theta})^2+\underline{T})=0 \iff \underline{T}=(\underline{x}-\underline{\theta})^2$. Then by (36) we know that $\mu=\lambda_1+\lambda_3$. Given that λ_1,λ_3 are non-negative, we know that there are three cases: (i) both $\lambda_1>0$ and $\lambda_3>0$, (ii) $\lambda_1=\mu$ and $\lambda_3=0$, and (iii) $\lambda_1=0$ and $\lambda_3=\mu$.

Case (i). The fact that $\lambda_1>0$ and $\lambda_3>0$ implies that constraints $(IC_{\overline{\theta}}),(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are all binding. These binding constraints plus the first-order conditions create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}^*=\frac{1}{2}(\overline{\theta}+\underline{\theta}-\delta_P(1-\overline{\theta})),\ \overline{x}^*=\frac{1+\overline{\theta}}{2},\ \underline{T}^*=\frac{1}{4}(\overline{\theta}-\underline{\theta}-\delta_P(1-\overline{\theta}))^2,\ \overline{T}^*=\frac{(1-\overline{\theta})^2}{4}-\delta_P R,$$

$$\lambda_1^*=\frac{(1+\delta_P)(1-\mu)(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3^*=\frac{\delta_P(\mu(1-\overline{\theta})+\overline{\theta}-1)+\mu(-\underline{\theta})+\mu+\overline{\theta}-1}{\overline{\theta}-\underline{\theta}},$$

$$\lambda_4^*=\frac{(1-\mu)(\delta_P(1-\overline{\theta})+1-\underline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

It is direct to check that $(IC_{\underline{\theta}})$ is satisfied. Under the condition $\mu>\frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$, the multiplier

$\lambda_3^* > 0$ implies that $\left(\underline{x}^*, \overline{x}^*, \underline{T}^*, \overline{T}^*, \lambda_1^*, \lambda_3^*, \lambda_4^*\right)$ satisfies the Kuhn-Tucker first-order necessary conditions of the problem.

Case (ii). Now $(IC_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 6 equations and 6 unknowns with the following solution:

$$\underline{x}' = \frac{1+\underline{\theta}-\mu(1+\overline{\theta})}{2(1-\mu)},\ \overline{x}' = \frac{1+\overline{\theta}}{2},$$
$$\underline{T}' = \frac{(1-\underline{\theta}-\mu(1+\overline{\theta}-2\underline{\theta}))^2}{4(1-\mu)^2},$$
$$\overline{T}' = \frac{\mu\left((1-4\delta_P)\overline{\theta}^2+\overline{\theta}(4\delta_P(\underline{\theta}+1)-4\underline{\theta}+2)-4(\delta_P+1)\underline{\theta}+4\underline{\theta}^2+1\right)}{4(\mu-1)} - \delta_P R$$
$$+\frac{(\overline{\theta}-1)((4\delta_P+3)\overline{\theta}-4(\delta_P+1)\underline{\theta}+1)}{4(\mu-1)},$$
$$\lambda_1' = \mu,\ \lambda_3' = 0,\ \lambda_4' = 1.$$

Replacing these values on the constraint $(P_{\overline{\theta}})$ we obtain that it is satisfied if $\mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$. If we replace on $(IC_{\underline{\theta}})$ we obtain $\mu \geq \frac{\delta_P\overline{\theta}-\delta_P+\overline{\theta}-\underline{\theta}}{\delta_P(\overline{\theta}-1)}$. Note that $\frac{\delta_P\overline{\theta}-\delta_P+\overline{\theta}-\underline{\theta}}{\delta_P(\overline{\theta}-1)} < \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$. Thus, if $\frac{\delta_P\overline{\theta}-\delta_P+\overline{\theta}-\underline{\theta}}{\delta_P(\overline{\theta}-1)} \leq \mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{1+\delta_P-\delta_P\overline{\theta}-\underline{\theta}}$ the vector $\left(\underline{x}', \overline{x}', \underline{T}', \overline{T}', \lambda_1', \lambda_3', \lambda_4'\right)$ satisfies the Kuhn-Tucker first-order necessary conditions of the problem.

Case (iii). Now $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 6 equations and 6 unknowns with the following solution:

$$\underline{x}'' = \frac{1+\underline{\theta}}{2},\ \overline{x}'' = \frac{1+\overline{\theta}}{2},\ \underline{T}'' = \frac{1}{4}(1-\underline{\theta})^2,\ \overline{T}'' = \frac{1}{4}(1-\overline{\theta})^2 - \delta_P R,$$
$$\lambda_1'' = 0,\ \lambda_3'' = \mu,\ \lambda_4'' = (1-\mu).$$

Replacing these values on the constraint $(IC_{\overline{\theta}})$ we obtain that $-(1+\delta_P)(1-\overline{\theta})(\overline{\theta}-\underline{\theta}) \geq 0$, which is false. Thus, there are no solutions that satisfy the conditions in case (iii).

Now, suppose $\lambda_2 > 0$. We have three other cases: (iv) both $\lambda_1 > 0$ and $\lambda_3 > 0$, (v) $\lambda_1 > 0$ and $\lambda_3 = 0$, and (vi) $\lambda_1 = 0$ and $\lambda_3 > 0$.

Case (iv). The fact that $\lambda_1 > 0$, $\lambda_2 > 0$ and $\lambda_3 > 0$ implies that constraints $(IC_{\overline{\theta}})$, $(IC_{\underline{\theta}})$, $(P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are all binding. These binding constraints plus the first-order conditions create a system of

8 equations and 8 unknowns with the following solution:

$$\underline{x}^* = \frac{1}{2}(\overline{\theta} + \underline{\theta} - \delta_P(1-\overline{\theta})),\ \overline{x}^* = \frac{\overline{\theta}+\underline{\theta}}{2},\ \underline{T}^* = \frac{1}{4}(\overline{\theta} - \underline{\theta} - \delta_P(1-\overline{\theta}))^2,\ \overline{T}^* = \frac{1}{4}(\overline{\theta}-\underline{\theta})^2 - \delta_P R,$$
$$\lambda_1^* = \frac{(1+\delta_P)(1-\overline{\theta})(1-\mu)}{\overline{\theta}-\underline{\theta}},\ \lambda_2^* = -\frac{\mu(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3^* = \frac{-\delta_P(1-\mu)(1-\overline{\theta})-(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}},$$
$$\lambda_4^* = \frac{\delta_P(1-\mu)(1-\overline{\theta})+(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

In this case $\lambda_2^* < 0$. Thus, there are no solutions that satisfy the conditions in case (iv).

Case (v). Now $(IC_{\overline{\theta}}), (IC_{\underline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\hat{\underline{x}} = \frac{1}{2}(1+\underline{\theta} - \delta_P\mu(1-\overline{\theta})),\ \hat{\overline{x}} = \frac{1}{2}(1+\underline{\theta}+\delta_P(1-\mu)(1-\overline{\theta})),$$
$$\hat{\underline{T}} = \frac{1}{4}(\delta_P\mu(1-\overline{\theta}) - (1-\underline{\theta}))^2,$$
$$\hat{\overline{T}} = \frac{1}{4}(\delta_P(1-\mu)(1-\overline{\theta})+(1-\underline{\theta}))^2 - \delta_P R,$$
$$\hat{\lambda}_1 = \frac{\delta_P(1-\overline{\theta})\mu\,(1-\mu)}{\overline{\theta}-\underline{\theta}},\ \hat{\lambda}_2 = \mu\frac{(\delta_P(1-\mu)(1-\overline{\theta})-(\overline{\theta}-\underline{\theta}))}{\overline{\theta}-\underline{\theta}},\ \hat{\lambda}_4 = 1.$$

We have that $\hat{\lambda}_1 \geq 0$, but $\hat{\lambda}_2 \geq 0$ if and only if $\mu \leq \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}$. Replacing the solution on the constraint $(P_{\overline{\theta}})$ we obtain that it is satisfied. Thus, when $\frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})} > 0$, if $\mu \leq \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}$, the vector $\left(\hat{\underline{x}}, \hat{\overline{x}}, \hat{\underline{T}}, \hat{\overline{T}}, \hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3, \hat{\lambda}_4\right)$ with $\hat{\lambda}_3 = 0$ satisfies the Kuhn-Tucker first-order necessary conditions of the maximization problem.

Case (vi). Now $(IC_{\underline{\theta}}), (P_{\overline{\theta}})$ and $(P_{\underline{\theta}})$ are binding and, together with the first-order conditions, create a system of 7 equations and 7 unknowns with the following solution:

$$\underline{x}'' = \frac{1+\underline{\theta}}{2},\ \overline{x}'' = \frac{\overline{\theta}+\underline{\theta}}{2},\ \underline{T}'' = \frac{1}{4}(1-\underline{\theta})^2,\ \overline{T}'' = \frac{(\overline{\theta}-\underline{\theta})^2}{4} - \delta_P R,$$
$$\lambda_2'' = -\frac{\mu(1-\underline{\theta})}{\overline{\theta}-\underline{\theta}},\ \lambda_3'' = -\frac{(1-\overline{\theta})\mu}{\overline{\theta}-\underline{\theta}},\ \lambda_4'' = \frac{(\overline{\theta}-\underline{\theta})+\mu(1-\overline{\theta})}{\overline{\theta}-\underline{\theta}}.$$

We have that $\lambda_2'' < 0$. Thus, there are no solutions that satisfy the conditions in case (vi).

Note that $\frac{\partial^2\mathcal{L}}{\partial \underline{x}^2} = -2(1-\mu)+2(\lambda_1-\lambda_2-\lambda_4)$. From (12) we have that $(\lambda_1-\lambda_2-\lambda_4) = -(1-\mu)$, and then $\frac{\partial^2\mathcal{L}}{\partial \underline{x}^2} < 0$. Also, $\frac{\partial^2\mathcal{L}}{\partial \overline{x}^2} = -2\mu - 2\lambda_1 + 2\lambda_2 - 2\lambda_3 < 0$ when $\lambda_2 = 0$. In the case $\lambda_2 > 0$, we have that $-2\mu - 2\lambda_1 + 2\lambda_2 - 2\lambda_3 = -2\mu < 0$. Similar to Lemma 2, the Lagrangian function is strictly concave and the Kuhn-Tucker first-order conditions are also sufficient. In sum, the solution of the maximization problem is the following:

$$(\underline{x},\overline{x},\underline{T},\overline{T}) = \begin{cases} \left(\underline{x}^*,\overline{x}^*,\underline{T}^*,\overline{T}^*\right) & \text{if } \mu > \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}, \\ \left(\underline{x}',\overline{x}',\underline{T}',\overline{T}'\right) & \text{if } \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\} \leq \mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})}, \\ \left(\hat{\underline{x}},\hat{\overline{x}},\hat{\underline{T}},\hat{\overline{T}}\right) & \text{if } 0 \leq \mu < \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\}. \end{cases} \tag{41}$$

The interest group's expected payoff from separation equals

$$V^{SEP} = (1-\mu)\left(-(\underline{x}-1)^2 - \underline{T} + \delta_G V_G^R(0)\right) + \mu\left(-(\overline{x}-1)^2 - \overline{T} + \delta_G V_G^R(1)\right),$$

where $(\underline{x},\overline{x},\underline{T},\overline{T})$ is as in equation (41) and $V_G^R(0)$ and $V_G^R(1)$ are the interest group's continuation values.

Conditional on pooling, the interest group's maximization problem is

$$\max_{x,T\in\mathbb{R}} \mu\left(-(x-1)^2 - T + \delta_G V_G^R(\mu)\right) + (1-\mu)\left(-(x-1)^2 - T + \delta_G V_G^R(\mu)\right)$$
$$s.t.\ (P_{\overline{\theta}}) \text{ and } (P_{\underline{\theta}}),$$

where the constraints are

$$-(\overline{x}-\overline{\theta})^2 + \overline{T} + \delta_P V_P(\overline{\theta},\mu_0) \geq 0, \tag{$P_{\overline{\theta}}$}$$
$$-(\underline{x}-\underline{\theta})^2 + \underline{T} + \delta_P V_P(\underline{\theta},\mu_0) \geq 0. \tag{$P_{\underline{\theta}}$}$$

The solution to this problem is to offer $(x,T) = \left(\frac{1+\underline{\theta}}{2}, \frac{(1-\underline{\theta})^2}{4}\right)$. The interest group's expected payoff of the pooling strategy is equal to

$$V^{POOL} = -\left(\frac{1+\underline{\theta}}{2} - 1\right)^2 - \frac{(1-\underline{\theta})^2}{4} + \delta_G V_G^R(\mu).$$

In Lemma 2, we found that there is a cut-off $\delta_G^\dagger$ such that $V^{SEP} \geq V^{POOL}$ if and only if $\delta_G \geq \delta_G^\dagger$. Here, we obtain a similar result: there is a cut-off $\delta_G^{\dagger,R}$ such that $V^{SEP} \geq V^{POOL}$ if and only if $\delta_G \geq \delta_G^{\dagger,R}$, where

$$\delta_G^{\dagger,R} = \begin{cases} \frac{\delta_P^2(1-\mu)^2(1-\overline{\theta})^2}{(\overline{\theta}-\underline{\theta})(\mu(\overline{\theta}+\underline{\theta}-2)-2\overline{\theta}+2)} - \frac{2\delta_P(1-\mu)R}{(\overline{\theta}-\underline{\theta})(\mu(\overline{\theta}+\underline{\theta}-2)+2(1-\overline{\theta}))} & \text{if } \mu \leq \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\} \\ \frac{2\delta_P(1-\mu)(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{2+\mu(\overline{\theta}+\underline{\theta}-2)-2\overline{\theta}} - \frac{2\delta_P(1-\mu)R}{(\overline{\theta}-\underline{\theta})(\mu(\overline{\theta}+\underline{\theta}-2)+2(1-\overline{\theta}))} & \text{if } \max\{0, \frac{\delta_P(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})}{\delta_P(1-\overline{\theta})}\} < \mu \leq \frac{1-\overline{\theta}}{1-\underline{\theta}} \\ \frac{\mu(\overline{\theta}-\underline{\theta})\left(2\delta_P(1-\mu)(1-\overline{\theta})-(\overline{\theta}-\underline{\theta})\right)}{(1-\mu)^2(1-\overline{\theta})^2} - \frac{2\delta_P\mu R}{(1-\mu)(1-\overline{\theta})^2} & \text{if } \frac{1-\overline{\theta}}{1-\underline{\theta}} < \mu \leq \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})} \\ \delta_P^2 + 2\delta_P - \frac{\mu(1-\underline{\theta})^2}{(1-\mu)(1-\overline{\theta})^2} + \frac{1}{1-\mu} - \frac{2\delta_P\mu R}{(1-\mu)(1-\overline{\theta})^2} & \text{if } \frac{(1+\delta_P)(1-\overline{\theta})}{(1+\delta_P-\delta_P\overline{\theta}-\underline{\theta})} < \mu. \end{cases}$$

A direct comparison of $\delta_G^{\dagger,R}$ and $\delta_G^\dagger$ shows that $\delta_G^{\dagger,R} < \delta_G^\dagger$. Thus, revolving door incentives increases the incentives for screening relative pooling. Also, we obtain that $\hat{\delta}_G^r$ is decreasing in R, which proves the result.