Electoral Competition with Targeted Voting Costs*

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Abstract

How do voting laws impact elections? We highlight how laws targeting a specific group of citizens can have weak effects on turnout and vote shares but substantial effects on policy platforms, thereby influencing substantive representation even if there are no observable effects on participation. To parse these effects, we analyze a model of electoral competition with endogenous turnout and targeted voting costs. Each party anticipates the direct effect of raising one side's voting costs: discouraging targeted citizens from voting. Consequently, both platforms shift towards the untargeted group. These platform adjustments mobilize targeted citizens and demobilize the untargeted, muting the net impact on turnout and vote shares—consistent with empirical evidence for small electoral effects. Policy effects, however, hurt targeted citizens and their aligned party. The targeted group's size amplifies these effects. Our results address party competition, participation, representation, and normative and empirical evaluations of voting laws.

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Introduction

In recent years, election law watchdogs have documented a "tidal wave" of new restrictive voting laws in the United States (Brennan Center 2021). Since 2021, state legislatures have enacted numerous laws making it harder for citizens to register to vote, stay registered, or cast their ballots—ranging from strict voter identification requirements to laws that criminalize passing out water to people waiting at the polls (Brennan Center 2021, 2023). A common complaint is that these laws appear *partisan*, with right-leaning state legislatures changing rules in ways designed to make it harder for left-leaning citizens to vote. Voting rights advocates have issued dire warnings about these new laws, stating that "we need to be very, very serious about this moment" and that "[o]ur democracy is in peril" (Johnson 2021). Scholars have also joined the fray, using these laws as a basis to conclude that states have been sites for significant democratic backsliding (Grumbach 2023).

Despite these concerns, the empirical evidence on the electoral impact of restrictive voting laws has been muted. Across a range of policies, researchers studying the impact of these laws on turnout and party vote shares have found little evidence of effects large enough to actually swing elections (Cantoni, Pons and Schafer 2025). Summarizing the available evidence, Grimmer and Hersh (2024) conclude that "the laws have small effects on turnout and essentially no effect on partisan advantage." This suggests a mismatch between the uproar surrounding recent restrictive voting laws and their actual impact. Such seemingly incongruous public and scholarly debates over voting laws are not new either, as earlier expansionary reforms were similarly contested on partisan grounds despite muddled evidence of electoral consequences (Berinsky 2005). One central lesson from those debates is that the electoral consequences of voting laws can depend not only on their direct effects on turnout, but also on indirect effects that can offset or amplify them (Burden et al. 2014).

In this paper, we offer a new perspective on voting laws by studying how they can affect not only voter participation, but also the substantive *representation* of voters. Such representation depends on the policies that parties choose; yet debates about voting laws have largely overlooked their potential indirect effects on party competition over policy—a classic electoral force (Downs 1957). Despite recent evidence that voting laws have affected policy in various contexts (Cascio and Washington 2014; Fujiwara 2015; Bertocchi et al. 2020),[1] our limited theoretical understanding of the role of party competition continues to impede sharper empirical and normative insights into the policy consequences of voting laws. We analyze how voting laws targeting particular groups, whether they increase or decrease costs, shape both which citizens turn out to vote and which platforms parties adopt.

In short, our theory suggests that even when a voting law does not visibly change turnout, it can still change the policies that parties choose in a way that affects substantive representation. When a law makes it harder for a particular group to vote, parties expect that fewer people from that group will participate. In response, both parties shift their platforms away from the targeted group's preferences, as those voters now matter less in the competition for votes. Importantly, these strategic policy shifts can undo much of the turnout effect we might otherwise expect: the targeted group may become more motivated to vote because the new platforms are worse for them, while the untargeted group may become less motivated. The net result is that turnout may not change much—making the law *appear* to have little electoral impact—even though policy is now less representative of the targeted group's preferences.

To formalize these ideas, we study a game-theoretic model of electoral competition with endogenous turnout and voting costs that vary by citizen ideology. In our model, policy-motivated parties strategically choose their policy platforms based on citizens' policy preferences and potential voting costs. Voters decide whether to vote and for whom, based on the

[1] Fujiwara (2015) links the adoption of easier voting technology to higher spending on healthcare. Bertocchi et al. (2020) studies how pre-registration policies can increase education spending by increasing participation among young voters. A handful of studies link the Voting Rights Act to improved policy outcomes for Black citizens (Cascio and Washington 2014; Bernini, Facchini and Testa 2023).

parties' platforms and their own costs of voting. Crucially, by allowing voting costs to vary with citizens' ideological leanings, we can analyze targeted voting laws that differentially affect specific groups of citizens in a partisan way. Moreover, our setup accommodates uncertainty in forecasting exactly how voting laws will affect voting costs, which may shift, spread, or skew in various ways.

Our model highlights how changes to voting costs can affect not only turnout, but also policy platforms. The direct effect of a targeted increase in voting costs discourages turnout among the targeted group, giving the opposing party an electoral advantage. Both parties respond by shifting their platforms away from the targeted group, but for distinct reasons. The advantaged party, now more secure electorally, converts some of that advantage into policy gains by moving toward its ideal point. The disadvantaged party, facing a harder path to victory, moderates further toward the other side to remain competitive. Because both shift in the same direction, targeted voting restrictions move policy away from the targeted group regardless of who wins the election.

In contrast to these policy effects, we show that the effects on relative voter participation and party vote shares can be minimal. Although partisan voting restrictions have direct effects that discourage the targeted group's turnout, the indirect effects on policy platforms feed back into turnout for both groups of citizens, who vote or abstain based on the adjusted platforms. This feedback blunts the relative impact of the direct effects on turnout by mobilizing the targeted side and demobilizing the untargeted side. Under expressive voting, this occurs because moderate voters in the targeted group find the other side's platform more offensive and their own side's platform more appealing, while moderate voters in the untargeted group experience the opposite. But this core insight—that resulting platform shifts induce offsetting mobilization effects—is more general, holding under various other turnout motivations including affinity voting. The overall effects of partisan voting restrictions therefore include (i) a direct effect on the targeted side's turnout and (ii) indirect effects on policy platforms *and* voter turnout for *both* sides. Ultimately, these direct and indirect effects combine to mute the electoral impact.

The magnitude of these effects varies with the relative sizes of ideological groups in the electorate. For example, platforms shift farther if the targeted group is larger. Thus, we should not expect large effects when that group is small. Additionally, we show that untargeted voting restrictions impacting both groups equally can shift policy toward the smaller group by moving platforms away from the larger group. Thus, even seemingly neutral voting restrictions can bias policy by shifting platforms away from the preferences of the majority of citizens.

Our results have implications for empirical, normative, and theoretical analyses of restrictive voting laws. On the whole, we aim to continue a cycle between theory and evidence in which empirical findings inform theoretical development and models guide empirical inquiry (Canen and Ramsay 2024). Broadly, we *reinterpret* existing empirical patterns, *elaborate* the implications of an alternative theory that would explain these patterns, and *distinguish* implications of different theories (Ashworth, Berry and Bueno de Mesquita 2021).[2] By constructing a simple but flexible representation of asymmetric changes to voting costs that allows parties to strategically adjust to these changes, we parse understudied channels for empirical researchers to explore further. We isolate the party-competition mechanism transparently and parsimoniously with only enough detail to elucidate the core mechanism, rather than piling on details to approximate "real world" complexities (Paine and Tyson 2020). Major changes to our assumptions could yield different results,[3] but our key insights can be generalized.

On the empirical side, the observed impact of voting laws on turnout and vote shares is consistently muted. There are various potential reasons for this regularity. Recent voting laws in the United States may simply have had small direct effects due to weak targeting or limited

[2]See also Spirling and Stewart (2025), who emphasize the value of "offering new explanations" and "elaborating new implications of those explanations" (pg. 1595).

[3]For example, under affinity voting, extreme polarization in the electorate could dampen the incentive for the targeted party to moderate in response to an increase in voting costs due to fears about alienating the party's base.

enforcement (Grimmer and Hersh 2024), or because they have been enacted in states where their marginal impact is low (Berry, Cox and Haile 2025). Earlier expansive laws may have primarily retained already-engaged voters rather than stimulating new participants (Berinsky 2005). And, in general, direct effects on participation may be offset by indirect effects via non-party actors such as the media and interest groups (Burden et al. 2014).

We highlight a distinct party-driven rationale for muted electoral effects: strategic platform adjustment. Essentially, changes to voting costs may induce platform adjustments that preserve electoral competitiveness. Thus, muted electoral effects can coincide with clear and meaningful platform effects. This has a key implication for empirical analyses of voting laws: when parties strategically adjust platforms in response to voting laws, direct effects on turnout and indirect effects through platforms become intertwined. This strategic interdependence can complicate the identification and interpretation of electoral effects, but analyzing effects on policies and platforms can be more straightforward and fruitful. We discuss these considerations more thoroughly in our Empirical Guidance section and suggest several potentially productive paths forward. More broadly, since our mechanism could operate in various contexts, our analysis can help scholars analyze historical settings where restrictions varied more substantially, evaluate counterfactual policy ideas, and understand why parties may invest resources in enacting voting laws even when electoral benefits appear minimal.

On the normative side, we highlight that voting laws can have welfare consequences even if they appear to have minimal electoral impact. Although scholars have sometimes downplayed normative concerns about voting laws in light of observed muted effects on turnout and vote shares (e.g., Cantoni and Pons 2021), focusing on electoral outcomes can overlook critical policy effects and thus obscure welfare and substantive representation consequences. In particular, (i) partisan voting restrictions can meaningfully affect party and voter welfare by shifting platforms away from the preferences of the targeted group, and (ii) even seemingly neutral restrictions can affect welfare by shifting platforms away from the larger group.

Our results also speak to broader, longstanding concerns with participation and representation. Existing studies on restrictive voting laws have largely focused on how they affect participation. Consistent with research linking participation to policies (Peress 2011; Cascio and Washington 2014; Fujiwara 2015; Bertocchi et al. 2020; Lo Prete and Revelli 2021; Oprea, Martin and Brennan 2024), our analysis of the policy consequences of restrictive voting laws highlights representation—and specifically *substantive representation*, meaning the alignment between policy and the interests of the represented (Pitkin 1967)—as another important consideration. These policy consequences are also relevant to legal analyses of voting restrictions based on the *alignment approach* to election law, which emphasizes this alignment as a relevant factor in determining the constitutional validity of a law (Stephanopoulos 2024).

On the theoretical side, our analysis emphasizes the feedback between voting behavior and politician behavior by endogenizing both turnout and platforms. Models with costly voting typically fix candidates and focus on voters' turnout (Borgers 2004; Krishna and Morgan 2012; Myatt 2015; Herrera, Morelli and Nunnari 2016; Tyson 2016; Arzumanyan and Polborn 2017), while classic studies of electoral competition do not feature abstention (Downs 1957; Wittman 1983; Calvert 1985).[4] By integrating these behaviors, we contribute to understanding how turnout and platforms impact each other (Ledyard 1984; Adams and Merrill III 2003; Hortala-Vallve and Esteve-Volart 2011; Bierbrauer, Tsyvinski and Werquin 2022).

The key gap we fill is parsing competitive consequences of partisan-biased changes to voting costs. Specifically, we highlight how politicians and parties may shift their platforms if voting costs change for particular groups of citizens. In this vein, Bertocchi et al. (2020) also allow voting costs to differ between voting blocs. That is not their main focus, however, and we allow for more general differences in voting costs in order to analyze a broader range of partisan-biased changes.[5] Additionally, they study purely office-motivated candidates, whereas we study policy-motivated candidates. Due largely to this difference, they

[4]For overviews of theoretical models, see Feddersen (2004) on voting and see Dewan and Shepsle (2011) on electoral competition.

[5]Their appendix analyzes uniform voting costs, a special case of our log-concave costs.

include incumbency advantage to generate interesting equilibrium behavior. In contrast, we set incumbency aside because it is not central to our interest in studying targeted voting laws.

Model

We analyze a spatial model of an election with policy-motivated parties making binding campaign platforms. There are two groups of citizens, left-leaning citizens and right-leaning citizens. To vote, each citizen must bear a cost and those costs can differ between the two groups. Moreover, both parties are uncertain about the left group's voting costs—capturing, e.g., uncertainty about the impacts of recent voting restrictions targeting those citizens. Additionally, to reflect that voting blocs can differ in size, we allow the groups to have different shares of the population. In the Appendix, we establish our key results under weaker assumptions.

Players. There are two parties, L and R, as well as a unit mass of citizens. Citizens are split into two groups, with a share $\alpha \in (0,1)$ in G_L and the remaining $1-\alpha$ in G_R.

Timing. First, the parties L and R simultaneously choose platforms in the one-dimensional policy space $X = [-1, 1]$.[6] Second, each citizen chooses whether to vote and, if so, which party to vote for. We refer to citizens who turn out to vote as *voters*. Finally, the party with the greater vote share wins the election and enacts its platform.[7]

Preferences. Both parties are purely policy motivated and prefer closer winning platforms. Specifically, each party j has an associated ideal point $\hat{x}_j$ and its utility from elected platform x is $u_j(x) = -|x - \hat{x}_j|$. Throughout, we assume $\hat{x}_L = -1$ and $\hat{x}_R = 1$. Pure policy motivation and linear loss streamline presentation but are not necessary—our main insights are robust to allowing some office motivation and risk aversion.

Citizens have policy preferences and prefer closer winning platforms, but incur a cost

[6] These choices could be committing to platforms or choosing candidates' ideal points.

[7] In our setting, ties occur with probability zero generically. To streamline analysis under non-generic conditions, we assume R wins if there is a tie.

if they vote. Each citizen i has an ideal point, $\hat{x}_i$, a voting cost, c_i, and their utility from elected platform x is $u_i(x) = -|x - \hat{x}_i| - c_i \cdot \mathbb{I}\{i \text{ votes}\}$. Like parties, these particular voter preferences are not crucial—our results are robust to partisan attachments and risk aversion.

Crucially, citizens' ideal points and voting costs are related to group membership. In G_R, citizen ideal points are uniformly distributed on $[0, 1]$ and every citizen has voting cost $c_R \geq 0$. In G_L, citizen ideal points are uniform on $[-1, 0)$ and every citizen has voting cost $c_L \geq 0$. Uniformly distributed ideal points in each group ease presentation but are not required for our main results.

Information. All features are common knowledge except for c_L, the voting cost for citizens in G_L. In particular, citizens in G_L observe c_L before they decide whether to vote, whereas the parties do not know c_L when choosing policies but they share a common belief represented by a distribution function F that has support $[\underline{c}, \overline{c}]$, where $\underline{c} \geq 0$, and associated density f that is log-concave.[8]

We assume uncertainty about only one side's voting costs as a tractable way to capture the essence of targeted voting laws. The prominent criticism that targeted voting restrictions are partisan suggests that the resulting increase in voting costs will disproportionately impact voters on one side. Yet, the exact intensity of that impact is hard to forecast, so parties choose their platforms without knowing how strongly those targeted citizens will be affected. We capture that disparate but uncertain partisan impact in a stark way. Our distributional assumption is quite general, however, and allows small differences in uncertainty.

Strategies and Equilibrium Concept. For each party j, a pure strategy specifies a policy platform $x_j \in X$. For each citizen i, a strategy is a mapping from policy pairs (x_L, x_R) and their voting cost c_i to their voting decision of whether to vote for L, vote for R, or abstain. An *equilibrium* is a strategy profile in which: (i) for each pair of platforms, each citizen votes for the closer platform if the difference in policy utility exceeds their voting cost and otherwise

[8]We allow $\overline{c} = \infty$ to accommodate common distributions such as the exponential.

they abstain, and (ii) each party's platform maximizes its expected payoff given the other party's platform, citizens' voting strategies, and its beliefs about voting costs.

In our main analysis, citizens' voting strategies are strategically equivalent to *expressive voting*—i.e., citizens receive expressive utility from voting but incur their turnout cost (as in, e.g., Hortala-Vallve and Esteve-Volart 2011). This form of voting and abstention is not crucial for our main insights, as shown in the Appendix. It follows in the tradition of analyzing *sincere voting* strategies (Calvert 1985; Wittman 1983) and, since each citizen votes as if they are pivotal, has the spirit of eliminating undominated voting strategies.[9] It induces *abstention by indifference*—i.e., each citizen votes only if their utility difference between the candidates is large enough—which has empirical support (Jessee 2009, 2010; Shor and Rogowski 2018). Our main insights hold more broadly, including with *affinity voting* that induces *abstention by alienation*—i.e., both candidates are too far away from the voter (as in, e.g., Adams and Merrill III 2003; Llavador 2006; Callander and Wilson 2007; Callander and Carbajal 2022).

Additionally, we focus in the main text on conditions ensuring that no citizen will surely vote for the opposing side's candidate. Specifically, (i) no citizen in G_R will vote for L's candidate and (ii) no citizen in G_L will always—i.e., for all realizations of c_L—vote for R's candidate. This property is substantively plausible and fairly innocuous—as we do allow that some citizens in G_L may have positive probability of *crossover voting*.[10] Furthermore, we are not primarily interested in the occurrence of crossover voting, so this streamlines the presentation.

Analysis

In our analysis, we start with citizens' voting behavior by characterizing how party platforms and voting costs determine who votes and which party they vote for. We then consider the par-

[9]With a continuum of citizens, no citizen is ever pivotal, so no voter strategy is dominated.

[10]Specifically, if some centrist citizens in G_L are closer to R's candidate, they would vote for them if c_L is low enough but abstain otherwise.

ties and characterize the policy platforms they choose in anticipation of the subsequent voting behavior. Using this foundation, we then analyze how voting costs affect various equilibrium features, such as policies, turnout, who wins the election, and party welfare. In particular, we distinguish between *targeted* versus *untargeted* changes to voting costs. Throughout the analysis, we also track how the size of each group of citizens shapes behavior and mediates the effects of voting costs.

Individual Turnout & Vote Choice

Each citizen has a simple voting calculus: they prefer the closer policy, but will not vote unless it is sufficiently closer than the other policy to outweigh their voting cost. Thus, in our baseline setting any equilibrium abstainers will be relatively moderate. As characterized in Remark 1, citizen i abstains if $|u_i(x_L) - u_i(x_R)| \leq c_i$ and otherwise votes for the closer policy.

Remark 1. *In equilibrium, if $x_L < x_R$, then citizen i will: (i) vote for L's candidate if $\hat{x}_i \leq \frac{x_L + x_R - c_i}{2}$, (ii) vote for R's candidate if $\hat{x}_i \geq \frac{x_L + x_R + c_i}{2}$, and (iii) abstain otherwise.*

Voters, Turnout, & Election Outcomes

We can aggregate individual voting behavior to characterize turnout, vote shares, and electoral outcomes. To streamline presentation in the main text, we focus primarily on *partisan voting*—that is, voters in G_L support L, voters in G_R support R, and abstaining citizens are an interval around 0. It drives the key equilibrium properties and, although low realizations of c_L may induce some *crossover voting* from G_L for some platform pairs (x_L, x_R), in equilibrium that is always less likely than partisan voting.

Party L wins if and only if it receives weakly greater vote share than R. Let the share of citizens who vote for party j, given policies (x_L, x_R) and voting costs $c = (c_L, c_R)$, be denoted $\tau_j(x_L, x_R; c)$. Then, with partisan voting, L wins if and only if:

$$\tau_L(x_L, x_R; c) = \alpha \left| \left(\frac{x_R + x_L}{2} - \frac{c_L}{2} \right) - (-1) \right| > (1 - \alpha) \left| 1 - \left(\frac{x_R + x_L}{2} + \frac{c_R}{2} \right) \right| = \tau_R(x_L, x_R; c).$$

Equivalently, since L's turnout is decreasing in c_L, party L wins if and only if c_L is sufficiently low. Remark 2 characterizes the cutpoint on c_L distinguishing whether L wins in equilibrium and, moreover, each party's win probability (see Lemmas 1–3 in the Appendix).

Remark 2. *In equilibrium, party L wins if and only if*

$$c_L < \frac{1}{\alpha}\left(x_R + x_L + (1-\alpha)\,c_R + 2(2\alpha - 1)\right) \equiv \hat{c}(x_L, x_R), \tag{1}$$

so the probability that L wins is $F(\hat{c}(x_L, x_R))$ and R wins with complementary probability.

Remark 2 highlights that—keeping in mind that we have not yet considered the parties' equilibrium policy choices—party L's electoral prospects are affected by the platforms (x_L, x_R), voting costs for right-leaning citizens (c_R), and G_L's population share (α). First, if x_R shifts rightward, then marginal voters on both sides will behave less favorably toward R. Specifically, (i) marginal voters in G_L become more concerned about R winning, so they become more inclined to turn out and vote for L; and (ii) marginal voters in G_R become less excited about R winning, so they become less inclined to turn out and vote for R. Second, if x_L shifts rightward, then marginal voters on both sides grow more favorable towards L, so turnout increases on the left and decreases on the right. Third, if c_R increases, then marginal voters in G_R will be less inclined to vote and thus turnout decreases among right-leaning citizens. Fourth, if α increases, there are both direct and indirect effects on L's electoral prospects. Most straightforwardly, an increase in α implies a greater share of left-leaning citizens, which improves L's electoral prospects.

Furthermore, the magnitudes of those direct effects are shaped by interaction effects between policies, voting costs, and population shares. Most importantly, α mediates the other effects: as G_L grows, platform shifts have stronger effects on the mobilization of left-leaning citizens but weaker effects on the demobilization of right-leaning citizens. Conversely, platforms and voting costs mediate α's direct effect by altering which citizens turn out on each side.

Party Competition: Electoral Chances and Policy Platforms

We now analyze party platforms. In general, each party j's expected payoff from a platform pair (x_L, x_R) is

$$\mathbb{E}_{c_L}[u_j(x_L; x_R)] = -|x_L - \hat{x}_j| \cdot Pr(L \text{ wins} \mid x_L, x_R) - |x_R - \hat{x}_j| \cdot (1 - Pr(L \text{ wins} \mid x_L, x_R)).$$

In equilibrium, Remark 2 ensures that $Pr(L \mid x_L, x_R) = F(\hat{c}(x_L, x_R))$, so

$$\mathbb{E}_{c_L}[u_L(x_L; x_R)] = u_L(x_R) + (u_L(x_L) - u_L(x_R)) \cdot F(\hat{c}(x_L, x_R)), \text{ and} \tag{2}$$

$$\mathbb{E}_{c_L}[u_R(x_R; x_L)] = u_R(x_R) + (u_R(x_L) - u_R(x_R)) \cdot F(\hat{c}(x_L, x_R)). \tag{3}$$

When choosing its platform, each party balances two familiar competing incentives. On one hand, choosing a more moderate policy increases its probability of winning by attracting more voters from its own side and turning off voters from the other side. On the other hand, doing so also decreases its benefit from winning, since its policy is farther from its own ideal point. The second term in (2) reflects L's expected gain from winning, whereas the second term in (3) reflects R's expected loss from losing. These terms are symmetric about zero and increase with the policy divergence.[11]

Using (2) and (3), the parties' marginal utilities from moderating are

$$\frac{\partial \mathbb{E}[u_L(x_L; x_R)]}{\partial x_L} = -F(\hat{c}(x_L, x_R)) + \frac{x_R - x_L}{\alpha} f(\hat{c}(x_L, x_R)), \text{ and} \tag{4}$$

$$-\frac{\partial \mathbb{E}[u_R(x_L; x_R)]}{\partial x_R} = -(1 - F(\hat{c}(x_L, x_R))) + \frac{x_R - x_L}{\alpha} f(\hat{c}(x_L, x_R)). \tag{5}$$

In both (4) and (5), the first term reflects the party's downside of moderation—lower policy payoff conditional on winning—weighted by their probability of winning. Victory becomes

[11] We use *divergence* following the Downsian tradition, since *polarization* has multiple widespread meanings (Esteban and Ray 1994).

less sweet, a sting that becomes especially poignant as the party's chances of winning rise. Meanwhile, the second term reflects the upside—greater probability of winning—weighted by the magnitude of the policy gain. Victory becomes more likely, which is especially valuable when parties' platforms are farther apart.

The marginal incentives can differ between the parties only through differences in their downsides of moderation. Specifically, a favored party would have a larger downside because they would be more likely to win and realize the cost of their moderation, while the other party would face less of a downside because they would probably lose anyway. Consequently, a favored party would be less inclined to moderate than its opponent.

Since both parties face the same upside of converging, however, in equilibrium they will be equally likely to win. Formally, combining (4) and (5) yields $F(\hat{c}(x_L^*, x_R^*)) = \frac{1}{2}$, so we say the election is a *toss-up*. This toss-up property is not sensitive to group size or, crucially, voting costs. The key point here is *not* that the election is even ex ante, since straightforward alterations can tilt electoral prospects toward one side or the other without changing the main takeaways.[12] Instead, as established in Proposition 1, the key point is that targeted changes to voting costs have no impact on ex-ante electoral prospects.

Proposition 1. *In equilibrium, neither party's probability of winning varies with F, the distribution of G_L's voting costs.*

Proposition 1 starkly highlights a more general insight: targeted changes to voting costs can have weak effects on electoral prospects. The underlying mechanism here—meaningful policy effects—stems from a familiar tradeoff in models of electoral competition with policy-motivated agents: electoral success versus policy gains. Each party wants to win the election, which pushes platforms to converge, but they also want to maximize the policy benefits that

[12]For instance, our affinity voting extension in the Appendix generates similar electoral effects but the win probability can be unequal. A favored party can also result from parties with different risk aversion (Farber 1980).

come with victory, which pulls against convergence. In equilibrium, this tension from electoral competition can mute the electoral impact of targeted voting costs because of endogenous policy responses that rebalance each party's chance of winning. These strategic forces highlight that, although empirical work has understandably focused on electoral outcomes and vote shares because they are easier to measure, those observables may obscure important effects.

The toss-up property does not imply that the election outcome is close. Win probability is an *ex ante* feature, whereas the observed margin is *ex post* and depends on realized voting costs. High costs for the left-leaning group dissuade moderate left-leaning citizens, giving the right party an advantage; low costs do the opposite. But *ex ante*, neither party has a greater chance of winning. Furthermore, the ex ante probability of winning (e.g., $F(\hat{c}(x_L^*, x_R^*))$ for L) differs from ex ante expected vote share (e.g., $\mathbb{E}\left[\frac{\tau_L^*}{\tau_R^*+\tau_L^*}\right]$ for L), which we analyze below. Notably, our toss-up property does not imply that expected vote shares are equal.

Nor does the toss-up property imply that both parties are equally well off. Instead, one party is strictly better off unless equilibrium platforms are symmetric around 0. With any asymmetry in platforms, the electoral balance implies that one party's ideal point is closer to the expected policy—which is the midpoint between the platforms—and thus they have a higher expected payoff.[13] Intuitively, differences in voting costs or group size can leave the parties on unequal competitive footing, leading the disadvantaged party to give up more in policy benefits to remain electorally competitive. Meanwhile, the advantaged party can remain electorally competitive without moderating as far.

To analyze party welfare, we can characterize the expected equilibrium policy, $\frac{x_L^*+x_R^*}{2}$, using (i) the definition of $\hat{c}(x_L, x_R)$ in (1) and (ii) the toss-up election property, $F(\hat{c}(x_L^*, x_R^*)) = \frac{1}{2}$. Specifically, (ii) implies that $\hat{c}(x_L^*, x_R^*)$ equals the median of F, denoted $\tilde{c}$, and thus substi-

[13]Each party's expected payoff is equal to its utility from the expected policy since $\hat{x}_L < x_L^* < x_R^* < \hat{x}_R$ and both parties are purely policy motivated with absolute loss.

tuting $\tilde{c}$, x_R^*, and x_L^* into (i) and rearranging yields

$$\frac{x_R^* + x_L^*}{2} = \frac{1}{2}(\alpha\tilde{c} - (1-\alpha)c_R) + (1-2\alpha). \tag{6}$$

Unlike win probability, the expected policy depends on both groups' voting costs and their relative size. Thus, even though they do not affect parties' electoral chances, these factors *do* affect welfare. They do so through two distinct channels, as expressed in (6). First, group size has a direct effect, $(1-2\alpha)$, so that enlarging a group shifts platforms toward that group's aligned party. Second, voting costs have a direct effect that depends on group size: $\frac{1}{2}(\alpha\tilde{c} - (1-\alpha)c_R)$. That is, higher voting costs for the left-leaning group—more precisely, a higher median—shift policy rightward, and higher voting costs for the right-leaning group shift policy leftward, with the magnitude of each effect increasing in the group's size.

Additionally, (4) and (5) imply the divergence between equilibrium platforms is

$$x_R^* - x_L^* = \frac{\alpha}{2\, f(\tilde{c})}. \tag{7}$$

Divergence is driven by uncertainty about the voting costs of citizens in G_L. The key component is $f(\tilde{c})$, which reflects party-level uncertainty about G_L's voting cost on the electoral margin (i.e., around $\tilde{c}$). Greater uncertainty about these costs implies less density around $\tilde{c}$ and thus a lower value of $f(\tilde{c})$, which increases equilibrium platform divergence, and vice versa. Group size again plays a mediating role, as it affects the magnitude of this force and, in turn, scales platform divergence. The uncertainty over costs plays a similar role in inducing divergence as uncertainty over the median voter's ideal point in models such as Wittman (1983), Calvert (1985), and Groseclose (2001).

Together, (6) and (7) pin down equilibrium platforms, expressed in Proposition 2.

Proposition 2. *In equilibrium, the party platforms are:*

$$x_L^* = (1-2\alpha) + \frac{1}{2}(\alpha\tilde{c} - (1-\alpha)c_R) - \frac{\alpha}{4\,f(\tilde{c})} \quad \textit{and} \tag{8}$$

$$x_R^* = (1-2\alpha) + \frac{1}{2}(\alpha\tilde{c} - (1-\alpha)c_R) + \frac{\alpha}{4\,f(\tilde{c})}. \tag{9}$$

Equilibrium platforms are determined by the midpoint and divergence, which is symmetric about the midpoint. Thus, they depend on the same forces that affect those features. The midpoint depends on the levels of voting costs, c_R and $\tilde{c}$, as well as group size, α. Divergence also depends on α, but the key factor is uncertainty about voting costs in G_L. Our characterization highlights that we can distinguish how beliefs about partisan voting costs can affect both platforms in different ways.

Thus far, we have characterized equilibrium behavior by analyzing the unique solution of two necessary first-order conditions. In the Appendix, we provide technical details establishing sufficient conditions on parameters for that solution to be the unique equilibrium. Essentially, these conditions ensure that neither party has a profitable deviation. First, we impose a regularity condition requiring that c_R is not too high relative to the distribution of G_L's voting costs, and additional conditions on c_R and $\tilde{c}$ guarantee turnout behavior at (x_L^*, x_R^*) that permits Remark 2. Second, log-concavity ensures that R does not have a profitable deviation. Third, if the distribution of G_L's voting costs is not too left-skewed—specifically, mode $F \leq \tilde{c}$—then L does not have a profitable deviation. This condition is satisfied by symmetric and right-skewed distributions, which are natural for voting costs given that costs have a lower bound but potentially long right tails.[14] Additionally, under these conditions the equilibrium is unique, as the necessary conditions for crossover equilibria are incompatible.

[14]Within the log-concave class, this condition holds for many common distributions including normal, exponential, gamma, and many others.

Effects of Voting Costs

We now analyze how changes in voting costs impact equilibrium policies, turnout, vote shares, representativeness, and party welfare. We consider two types of changes in voting costs: (i) *targeted* changes, which affect only one group of citizens, and (ii) *untargeted* changes, which affect both groups. For targeted changes, we consider either a straightforward shift for c_R or a distributional shift for c_L. We allow distributional shifts that change the shape of F, e.g., by shifting the median ($\tilde{c}$) by a different amount than the mean ($\mathbb{E}[c_L]$). For untargeted changes, we uniformly shift both groups' voting costs by ε.[15]

Policies. We start by analyzing how equilibrium platforms vary with voting costs. Proposition 3 follows from Proposition 2 and states these effects.[16] First, increasing the median cost for one group of citizens—a targeted change—shifts policies away from that group. Second, an untargeted increase in voting costs shifts platforms away from the larger group and towards the smaller group.

Proposition 3. *In equilibrium, each party's platform: (i) increases by $\frac{\alpha}{2}$ as $\tilde{c}$ increases, (ii) decreases by $\frac{1-\alpha}{2}$ as c_R increases, and (iii) changes by $\alpha - \frac{1}{2}$ if all voting costs increase equally.*

The key insight in Proposition 3 is that targeted changes to voting costs will shift both parties' policy platforms away from the targeted side. Essentially, the direct effect of targeted voting costs will encourage rightward platform shifts, which have reinforcing equilibrium effects that push further rightward. The direct effect is that citizens in G_L become less inclined to turn out, so L is more concerned with mobilizing them and R is less concerned with mobilizing its own support. Thus, since in equilibrium each party's win probability increases if and only if they shift inward,[17] both parties are more inclined to shift rightward. Such platform

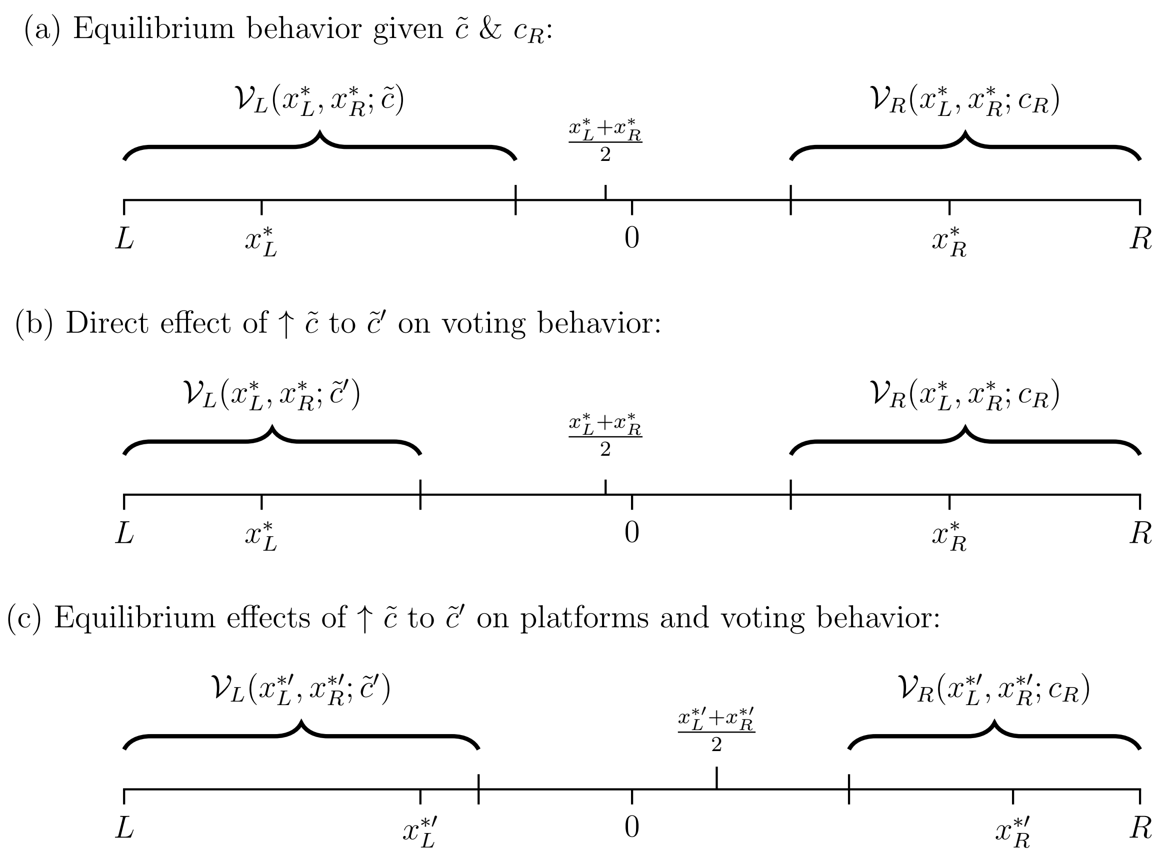
[15]For c_L, an original distribution F_0 with support $[\underline{c}, \overline{c}]$ is shifted to F_1 with support $[\underline{c} + \varepsilon, \overline{c} + \varepsilon]$, median $\tilde{c}_0 + \varepsilon \equiv \tilde{c}_1$, and $f_0(\tilde{c}) = f_1(\tilde{c}_1)$.

[16]Throughout, comparative statics are marginal effects derived by taking partial derivatives.

[17]Equilibrium platforms must balance higher win probability against lower policy utility.

adjustments also affect party incentives but, under broad conditions, that equilibrium effect reinforces itself: L shifting rightward makes R more inclined to shift rightward as well, and vice versa. Intuitively, L anticipates more downside from losing while R anticipates less, so L is even more inclined to mobilize voters while R is less. Figure 1 illustrates these effects.

Figure 1: Effects of changing G_L's voting costs
(by shifting $\tilde{c}$, the median of F)



Note: Figure 1 illustrates how increasing $\tilde{c} = \text{med } F$ affects party platforms and voter behavior. The set of citizens in group G_j who vote given platforms and (realized) cost c_j is denoted $\mathcal{V}_j(x_L, x_R; c_j)$. Figure 1(a) depicts a baseline case with voting cost c_R for G_R and median cost $\tilde{c}$ for G_L. The effects of increasing $\tilde{c}$ to $\tilde{c}'$ are depicted in Figure 1(b) and 1(c). First, given the platforms in 1(a), Figure 1(b) illustrates the direct effect on voting behavior: less turnout in G_L. Second, (c) illustrates the overall effects as platforms and voting behavior adjust in equilibrium.

Crucially, Proposition 3 sharpens our core insight from Proposition 1: targeted changes to voting costs can have weak effects on electoral prospects *that are muted due to meaningful* *policy effects that push away from the targeted side.* Thus, observed electoral effects are attenuated relative to the *ceteris paribus* direct effect and, furthermore, those weak effects can mask important policy consequences. Our baseline model makes this point starkly but this core insight is more general. It does not require linear policy utilities, uniform ideal point distributions, nor citizens' expressive motivation for turnout. In the Appendix, we study a smooth version of our model without imposing those assumptions. We show that if relative turnout across the groups is not overly sensitive to platforms, then the game is supermodular. Thus, increases in G_L's voting costs (under various stochastic orders) will shift both platforms rightward (Vives 2005) and thereby mute electoral effects.

To illustrate how these results might matter in practice, consider a few examples from the empirical literature on the effects of restrictive voting laws. One focus of the literature has been voter ID laws. Using detailed microdata from Texas, Fraga and Miller (2022) find that these laws disproportionately affect Black and Latino voters, who lean Democratic. Our theoretical results thus imply that these laws would shift party platforms rightward. Alternatively, consider same-day registration and pre-registration. Prior research suggests that these policies reduce voting costs for young citizens, who lean Democratic (Grumbach and Hill 2022; Bertocchi et al. 2020). Thus, our model implies that these policies would shift party platforms leftward. Indeed, consistent with this prediction, Bertocchi et al. (2020) find evidence that pre-registration policies raise education spending. Lastly, consider changes to polling place locations and long lines at the polls (e.g., Bagwe, Margitic and Stashko 2023; Pettigrew 2021). If these policies target specific groups of voters (e.g., Democratic-leaning urban voters), we would expect party platforms to shift away from the targeted group's preferences.

Proposition 3 also reveals that group size mediates the effects of voting costs, whether targeted or untargeted. As one group grows, both parties are more sensitive to changes in voting costs. For example, if G_L is larger than G_R—i.e., $\alpha > \frac{1}{2}$—then changing c_L will shift each party's platform further than if the change targeted G_R. Additionally, an untargeted cost increase when $\alpha > \frac{1}{2}$ will shift both platforms rightward. More broadly, we highlight that policy effects of voting cost changes can vary with groups' sizes and disparities, which suggests potentially important interaction between redistricting and voting laws.

Effects of group size. We now consider how group size affects equilibrium platforms. Proposition 4 characterizes how G_L's population share affects platforms.

Proposition 4. *In equilibrium, as G_L's population share grows (α increases): L shifts its platform by $\frac{\tilde{c}+c_R}{2} - 2 - \frac{1}{4\,f(\tilde{c})} < 0$, and R shifts its platform by $\frac{\tilde{c}+c_R}{2} - 2 + \frac{1}{4\,f(\tilde{c})}$.*

The total effect of α on equilibrium platforms combines two effects. First, there is a *direct effect* of group size, $\frac{\tilde{c}+c_R}{2} - 2$, that affects both platforms and shifts them leftward (since $\frac{\tilde{c}+c_R}{2} < 2$) as α increases, and vice versa. Essentially, as G_L grows, a larger proportion of citizens is willing to vote for a relatively extreme left platform, and conversely, a smaller share of citizens is willing to vote for a relatively extreme right platform. Thus, both platforms shift left. Moreover, this effect is magnified by larger voting costs, highlighting the complementarity between groups' sizes and voting costs.

Second, there is an *uncertainty effect* that shifts L's platform leftward by $\frac{1}{4\,f(\tilde{c})}$ and R's platform rightward by that same distance, fueling divergence. As α increases, aggregate turnout in G_L becomes more sensitive to c_L, creating greater uncertainty about the median voter's location and causing parties to diverge further.

In addition to studying how group size affects party platforms, we also analyze how it affects the distance between party platforms, or *divergence* $(x_R^* - x_L^*)$. Group size (α) does not generate divergence through its direct effect on platforms. Instead, divergence results from parties' policy motivation and electoral uncertainty. Yet, α does indirectly affect the *magnitude* of equilibrium divergence. By amplifying the uncertainty effect, it increases divergence between equilibrium platforms even as their midpoint shifts leftward. Corollary 1 establishes that, as usual in spatial electoral models, greater electoral uncertainty increases divergence.

Corollary 1. *In equilibrium, $\frac{\partial(x_R^* - x_L^*)}{\partial\alpha} = \frac{1}{2\,f(\tilde{c})}$, so platforms diverge more as the share of citizens in G_L grows.*

Turnout. We now focus on equilibrium turnout. Turnout is widely studied in empirical work on voting laws. It reflects the *ex post* side of the *ex ante* toss-up property that parties are equally likely to win in equilibrium. As previously noted, there may be *ex post* differences in turnout even if elections are *ex ante* toss-ups. Our results have implications for empirical studies of how restrictive voting laws affect turnout and vote shares, which we elaborate below in the Empirical Guidance section. To streamline discussion of our main insights, we focus on partisan voting.

Let τ_L^* denote realized equilibrium turnout for G_L and define τ_R^* analogously. Given a realized c_L, we have: $\tau_L^* = \alpha\left(2(1-\alpha) + \frac{1}{2}(\alpha\,\tilde{c} - (1-\alpha)c_R - c_L)\right)$. In equilibrium, L's expected turnout is $\mathbb{E}[\tau_L^*] = \alpha\left(2(1-\alpha) + \frac{1}{2}(\alpha\,\tilde{c} - (1-\alpha)c_R - \mathbb{E}[c_L])\right)$ and R's turnout is $\tau_R^* = \alpha\,(1-\alpha)\left(2 - \frac{1}{2}(\tilde{c} + c_R)\right)$.

Note that G_L's expected equilibrium turnout is equal to G_R's equilibrium turnout if G_L's expected voting cost equals G_L's median voting cost (i.e., $\mathbb{E}[c_L] = \tilde{c}$). One prominent class of log-concave distributions that always generate this equivalence are symmetric, single-peaked distributions (e.g., the normal distribution). If $\mathbb{E}[c_L] \neq \tilde{c}$, however, then the parties have different expected equilibrium turnout even though they are equally likely to win.

Proposition 5 characterizes how equilibrium turnout varies with different changes to voting costs. An increase in G_L's voting costs reduces each group's expected turnout by different amounts. Moreover, which effect is larger depends on whether the mean of F increases more than its median. In contrast, an increase in G_R's voting costs reduces expected turnout equally for both groups.

Proposition 5. *In equilibrium: (i) G_R's turnout decreases by $\frac{\alpha}{2}(1-\alpha)$ as either $\tilde{c}$ or c_R increases, whereas (ii) G_L's expected turnout decreases by $\frac{\alpha}{2}(1-\alpha)$ as c_R increases and changes by $-\frac{\alpha}{2}\left(\frac{\partial\mathbb{E}[c_L]}{\partial\tilde{c}} - \alpha\right)$ as $\tilde{c}$ increases.*

First, consider G_R's turnout. It decreases in response to increased voting costs for *either group.* That is, right-leaning citizens will stay at home in greater numbers not only when their own voting cost increases, but also when the *other* group's voting cost increases. This effect arises due to changes in equilibrium platforms. If platforms remained constant, changing $\tilde{c}$ would not affect G_R's turnout. But since a higher voting cost for G_L emboldens R to adopt a more extreme platform, turnout can decrease for both groups. Moreover, the magnitude of this effect is the same regardless of which group's costs increase. That is, G_R's turnout decreases by the same amount in response to an increase in its own voting costs as it does in response to an increase in the other group's voting costs. Finally, the effect is largest if the groups are evenly split in the electorate.

For G_L, voting costs have turnout effects that are similar but somewhat more complicated. Since c_L is a random variable, G_L's turnout is a random variable. Our focus on G_L's expected turnout reflects its prominence in the empirical literature (e.g., linear regression models involve conditional expectations). Proposition 5 implies that, as long as shifting F does not increase the median ($\tilde{c}$) too much faster than the mean ($\mathbb{E}[c_L]$),[18] an increase in G_L's voting costs will reduce expected turnout for G_L. This net reduction in turnout is muted by equilibrium effects, however, since L shifts its equilibrium platform rightward and counteracts the direct effects of G_L's higher voting costs. Additionally, increasing G_R's voting costs also reduces G_L's expected turnout. That is, like the other group, G_L's turnout can drop in response to an increase in either group's voting costs, not just its own.

Which group is likely to experience a larger drop in turnout? For increases in G_R's voting costs, the expected turnout effect is equal for both groups. For increases in G_L's voting costs, however, the effect depends on whether the median ($\tilde{c}$) increases by more or less than the mean ($\mathbb{E}[c_L]$).[19] If the mean increases faster than the median, then the effect is greater for G_L. Otherwise, the effect is greater for G_R. Hence, the effects on relative turnout depend critically on the type of shift in voting costs.

[18]Specifically, the condition is $\frac{\partial \mathbb{E}[c_L]}{\partial \tilde{c}} > \alpha$. Otherwise, an increase in G_L's voting costs would *increase* G_L's expected equilibrium turnout.

[19]As above, this assumes $\frac{\partial \mathbb{E}[c_L]}{\partial \tilde{c}} > \alpha$.

Finally, beyond targeted increases in voting costs, our model also provides insight into how *untargeted* cost increases might affect turnout. If G_L's expected voting cost equals its median voting cost, $\mathbb{E}[c_L] = \tilde{c}$, then an untargeted increase in voting costs preserving that equality would decrease each group's turnout by the same amount, $\frac{1}{2}\alpha \cdot (1 - \alpha)$. This effect is especially large when the party supporters are evenly split in the electorate, consistent with evidence that mail-in voting—which effectively decreases voting costs for all voters—seems to increase turnout across the board in evenly divided constituencies (Bonica et al. 2021).[20] But if $\mathbb{E}[c_L] \neq \tilde{c}$, then untargeted changes in voting costs may have asymmetric turnout effects.

Vote Shares and *Representativeness*. Many scholars are also interested in understanding the relationship between voting costs and two measures that are functions of turnout: (i) vote shares and (ii) the representativeness of voters. First, by *vote shares*, we mean the share of votes cast for one party out of the total votes cast (e.g., $\frac{\tau_L^*}{\tau_R^* + \tau_L^*}$). Scholars have often understood this measure as reflecting electoral competitiveness. Second, by *representativeness* of voters, we mean the similarity between the composition of voters who turn out versus the composition of eligible voters. In our context, representativeness can be measured by how close $\frac{\tau_L^*}{\tau_R^* + \tau_L^*}$ is to α.[21] Scholars have viewed this measure as an indicator of how well policy will align with public interests.

To illustrate how our analysis thus far can shed light on these measures, we fix ideas by focusing on average vote shares and average representativeness. Two widespread intuitions are that increasing voting costs will (i) lead to more imbalanced vote shares, thereby decreasing competitiveness, and (ii) reduce representativeness, thereby decreasing public alignment. Our

[20]Bonica et al. (2021) find an eight-percentage-point effect for all-mail voting in Colorado that does not meaningfully differ by party. This fairly large effect aligns with our model's predictions since Colorado is roughly evenly split (Colorado Secretary of State 2022).

[21]In our baseline with expressive voting, although extremists always vote and abstention occurs among centrists, it may be the case that one group has a disproportionate share of turnout. This latter aspect of representativeness is our focus.

analysis reveals two insights for these intuitions.

First, shifting the voting cost distribution does not necessarily change either measure—due to shifts in equilibrium platforms. To illustrate, suppose $\mathbb{E}[c_L] = \tilde{c}$ and let F denote the initial distribution of c_L. Then, if $\text{var}(F)$ is small enough, expected vote share is $\mathbb{E}\left[\frac{\tau_R^*}{\tau_R^*+\tau_L^*}\right] \approx \frac{\tau_R^*}{\tau_R^*+\mathbb{E}[\tau_L^*]} = \frac{1}{2}$ for both parties. Moreover, uniformly shifting F rightward has no effect on average vote share or average representativeness. Instead, to see changes in those quantities, any shift in F must change $|\mathbb{E}[c_L]-\tilde{c}|$. This observation is a stark illustration of a more general point: shifting the cost distribution is not sufficient to observe changes in these measures, as it is important to understand *how* that distribution changes.

Second, changes in average vote share or representativeness can occur without affecting expected policy payoffs. To illustrate, consider an initial cost distribution F that has $\mathbb{E}_F[c_L] = \tilde{c}$, and two different shifts in the cost distribution, F' and F'', that each increase the median cost to $\tilde{c}'$ but differ in that only F' preserves the equality of expected cost and median cost. Formally, $\tilde{c} < \tilde{c}' = \text{median}(F') = \text{median}(F'')$ and $\mathbb{E}_{F'}[c_L] = \tilde{c}' \neq \mathbb{E}_{F''}[c_L]$. Then, after either shift, expected payoffs change in the same way. In contrast, only the shift to F'' will change average vote share and average representativeness. Thus, changes in either of these measures, or the lack thereof, are not necessarily informative about welfare effects (as measured in our model by policy payoffs).

These observations have implications for interpreting empirical patterns in vote shares. For example, consider voter ID laws. As previously noted, prior research has found that these laws disproportionately affect Black and Latino voters, who lean Democratic (Fraga and Miller 2022). Based on this disparate impact, many scholars have hypothesized that voter ID laws will reduce Democratic turnout more than Republican turnout, thus decreasing Democratic vote shares. Our theoretical results imply that this hypothesis is not necessarily warranted, as an increase in voting costs targeting one group can reduce turnout for both groups, even if the targeting is perfect. We should therefore expect voter ID laws to reduce turnout among Democrats more than Republicans only under certain conditions about the shift in distribution, which could hold in some cases but not in others. This may partly explain the relatively small effects on vote share that have been observed empirically. Critically, however, this does *not* mean that voter ID laws are unimportant for electoral competition—the effects may simply be surfacing in the policies chosen rather than observed vote shares.[22]

Party Welfare. We now study how each party's *ex ante* equilibrium payoff changes with voting costs and group size. By doing so, we shed light on how strongly each party would want to change voting costs under different conditions.

Because the game is zero-sum for the parties, we analyze only one party. From the *ex ante* perspective, L's equilibrium welfare is

$$U_L^* = -1 - \frac{x_L^* + x_R^*}{2} = \frac{1}{2}((1-\alpha)\, c_R - \alpha\tilde{c}) - 2(1-\alpha).$$

Proposition 6 shows how party welfare varies with voting costs and group size.

Proposition 6. *In equilibrium, party L's welfare: (i) decreases by $\frac{\alpha}{2}$ as $\tilde{c}$ increases, (ii) increases by $\frac{1-\alpha}{2}$ as c_R increases, (iii) changes by $\frac{1}{2} - \alpha$ as all voting costs increase equally, and (iv) increases by $2 - \frac{1}{2}(c_R + \tilde{c})$ as α increases. Party R's welfare effects are symmetric.*

Unsurprisingly, each party benefits when their side's voting cost decreases or the other side's voting cost increases. The size of the affected group amplifies these gains and losses, though the magnitude of these effects depends on the functional form of the parties' preferences. An increase in group size also affects party welfare, by inducing the opposite party to shift its platform toward the growing group to capture enough of the vote to remain electorally competitive, while the aligned party shifts its platform toward its ideal point to convert some

[22]These implications for turnout and vote shares also apply to voting laws that facilitate voting, such as same-day registration and pre-registration, which prior research suggests reduce voting costs for young and thus Democratic-leaning citizens (Grumbach and Hill 2022; Bertocchi et al. 2020).

of these constituency gains into policy gains. But voting costs moderate this effect: with higher voting costs, citizens are more inclined to stay home, muting the effects of group size. Lastly, an *untargeted* increase in voting costs, expressed in the third comparative static, hurts the party aligned with the larger group and helps the party aligned with the smaller group.

Empirical Guidance

Empirical researchers studying restrictive voting laws have primarily focused on their effects on turnout and vote shares. Our model has implications for empirically analyzing these quantities. A key challenge is that parties may strategically adjust platforms in response to voting laws, so direct and indirect effects become entangled. We discuss what different empirical approaches can identify and suggest productive paths forward.

Consider a hypothetical experiment randomizing voter ID requirements across constituencies. Let $D_i \in \{0,1\}$ denote treatment assignment for constituency i, $X_i = (x_{L,i}, x_{R,i})$ denote party platforms, and Y_i denote turnout. The causal structure consists of a direct effect $(D_i \to Y_i)$ through voting costs and an indirect effect $(D_i \to X_i \to Y_i)$ through platform adjustments. Randomization ensures treatment independence: $D_i \perp\!\!\!\perp (Y_i(0), Y_i(1))$, which identifies the total average treatment effect $\text{ATE}_{\text{total}} = \mathbb{E}[Y_i(1) - Y_i(0)] = \mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0]$. This estimand captures the overall impact of voting laws on turnout, combining any direct effect through voting costs with any indirect effect operating through platform adjustments. Our model offers a prediction about this total effect: it may be small, even when the direct effect of voting costs on turnout is meaningful.

Researchers may seek to isolate the direct effect of voting costs on turnout, holding platforms fixed. For instance, researchers might target the *average controlled direct effect*: $\text{ACDE}(x') = \mathbb{E}[Y_i(1, x') - Y_i(0, x')]$, where x' is a fixed platform pair common to all constituencies. Identifying and interpreting this quantity has well-known considerations: *sequential unconfoundedness* must hold, and when treatment-mediator interactions are present, the controlled direct effect varies with the reference value at which the mediator is fixed (Acharya, Blackwell and Sen 2016). In our setting, the natural reference values—platforms under treatment versus control—differ precisely because parties optimally respond to voting costs, making the choice of reference value substantively loaded rather than merely technical.

A deeper issue concerns strategic interdependence of the platforms. The indirect effect, operating via platform repositioning, occurs because parties anticipate the direct effect on turnout. But the direct effect—how the law changes turnout holding platforms fixed—depends on the platforms, which result from strategic adjustment. Furthermore, platforms are themselves interdependent choices: each party's position responds to the other's. Standard mediation approaches require isolating direct and indirect effects, but here direct and indirect effects are determined jointly rather than sequentially, which is a thorny challenge for standard causal frameworks that wholly set aside strategic choices (Heckman and Pinto 2024).

Our analysis suggests several ways forward. First, analyze platform effects. Platform effects are key theoretical quantities and our analysis provides a clear prediction: targeted increases in voting costs shift both parties' platforms away from the targeted side. Standard estimands for the effect of voting laws on platforms, such as $\mathbb{E}[X_i|D_i = 1] - \mathbb{E}[X_i|D_i = 0]$, are identified by randomization without the complications discussed above. Observing platform shifts in the predicted direction, with heterogeneous magnitudes based on the targeted group's size, would provide evidence for the strategic mechanism we highlight. Recent work analyzing policy effects (Fujiwara 2015; Bertocchi et al. 2020) should encourage more work in that vein, which would complement and help interpret findings on turnout and vote shares.

Second, interpret total effects as equilibrium outcomes. A total effect like the $\text{ATE}_{\text{total}}$ bundles direct and indirect channels but is identified under standard assumptions and policy-relevant since it measures a net impact after voting laws change and parties respond. Proposition 1 shows that platform adjustments can offset direct turnout effects, so negligible total effects on turnout or vote shares, alongside meaningful platform shifts, would be consistent with our theory. Furthermore, they would suggest that focusing on electoral outcomes alone may obscure important policy consequences.

Third, exploit variation in the timing of voting law changes relative to election day. Unexpected legal changes close to elections, such as court decisions issued weeks before voting, limit parties' ability to adjust platforms. Comparing effects of changes announced months before elections (allowing full adjustment) versus weeks before (preventing adjustment) can reveal whether strategic responses attenuate direct effects in practice. Similar variation may arise from unexpected polling relocations due to natural disasters (Abramson and Thrower 2025).

Fourth, use empirical approaches that account for key aspects of voter and party behavior underlying electoral outcomes. Estimating a model of party and voter behavior defines these quantities precisely and disciplines how they are recovered from data (Canen and Ramsay 2024). For example, Berry, Cox and Haile (2025) study how voting laws affect U.S. House elections by estimating a model of turnout and vote choices that allows general indirect effects such as changes in party platforms or campaign spending. They find that higher voting costs disproportionately discourage Democratic-leaning citizens on the turnout margin, which is consistent with our interest in asymmetric changes to voting costs.

Concluding Discussion

Debates over restrictive voting laws typically emphasize their effects on turnout and vote shares. Our analysis suggests this focus may miss important policy effects: voting restrictions can reshape party platforms, altering policies and representation even when electoral effects are minimal. Using a formal model of electoral competition with endogenous turnout and targeted voting costs, we show that partisan voting restrictions induce parties to shift platforms away from targeted groups' preferences, creating policy consequences regardless of electoral outcomes. Our findings are relevant to both democratic theorists studying participation and representation, as well as empirical scholars analyzing specific laws.

Our model highlights that the key outcomes are not only *who wins* and *who votes*, but also *what policies result*. Thus, our results reframe the interpretation of common empirical patterns. Null or small effects on vote shares need not indicate voting laws lack consequence. Rather, they may reflect platform adjustments that preserve electoral competitiveness while shifting policy away from targeted groups.

We particularly urge empiricists to look at whether platforms and policies change in response to new restrictive voting laws. If restrictive voting laws discourage turnout, and turnout affects policy, then restrictive voting laws should also affect policy. For reasons outside our model, the policy effects of restrictive laws may be even more severe in practice. Our results rely on the assumption that parties can choose any available platform location. In practice, however, parties may sometimes nominate extreme candidates who are unwilling to choose moderate policies (Nielson and Visalvanich 2017; Hall 2019). In such cases, the candidate's inability to moderate could cause the other party to move its platform in an even more extreme direction.

Our analysis also yields insight into when parties have the sharpest incentives to enact restrictive voting laws. A key takeaway is that increasing a group's voting costs benefits the opposing party more when the targeted group is larger. It is thus unsurprising that as Texas becomes more purple, the Republican government instituted a sweeping restrictive voting law targeting Democratic-leaning citizens (Tulin and Sanchez 2023). Conversely, reducing voting costs for one party's supporters will benefit that party most when the opposing group is relatively small—a result that holds even when the reduction is untargeted and impacts all citizens. From this perspective, expanding voting access in states with large proportions of Democratic voters can benefit the Democratic Party even without being deliberately partisan.

We focus on voting costs, but these capture only one class of election laws that have drawn attention from scholars and advocates. Although the models would be different, a similarly broad focus on different outcomes—assessing the policy impact and not just turnout—may shed light on the effects of gerrymandering, voter purges, and other types of election laws. To analyze the impacts of these laws going forward, empirical researchers should consider a wider set of potential consequences.

References

Abramson, Scott and Sharece Thrower. 2025. "The Direct Cost to Voters of Polling Site Closures and Consolidation." *Political Science Research and Methods* pp. 1–22.

Acharya, Avidit, Matthew Blackwell and Maya Sen. 2016. "Explaining Causal Findings Without Bias: Detecting and Assessing Direct Effects." *American Political Science Review* 110(3):512–529.

Adams, James and Samuel Merrill III. 2003. "Voter Turnout and Candidate Strategies in American Elections." *Journal of Politics* 65(1):161–189.

Arzumanyan, Mariam and Mattias K. Polborn. 2017. "Costly Voting with Multiple Candidates Under Plurality Rule." *Games and Economic Behavior* 106:38–50.

Ashworth, Scott, Christopher R. Berry and Ethan Bueno de Mesquita. 2021. *Theory and Credibility: Integrating Theoretical and Empirical Social Science*. Princeton University Press.

Bagwe, Gaurav, Juan Margitic and Allison Stashko. 2023. "Polling Place Location and the Costs of Voting." *American Economic Journal: Economic Policy* forthcoming.

Berinsky, Adam J. 2005. "The Perverse Consequences of Electoral Reform in the United States." *American Politics Research* 33(4):471–491.

Bernini, Andrea, Giovanni Facchini and Cecilia Testa. 2023. "Race, Represenatation, and Local Governments in the US South: The Effect of the Voting Rights Act." *Journal of Political Economy* 131(4):994–1056.

Berry, Steven, Christian Cox and Philip A. Haile. 2025. "Selective Turnout, Voting Policy, and Partisan Bias: Evidence from Multi-Level Data." *NBER Working Paper* (w34149).

Bertocchi, Graziella, Arcangelo Dimico, Francesco Lancia and Alessia Russo. 2020. "Youth Enfranchisement, Political Responsiveness, and Education Expenditure: Evidence from the US." *American Economic Journal: Economic Policy* 12(3):76–106.

Bierbrauer, Felix, Aleh Tsyvinski and Nicolas Werquin. 2022. "Taxes and Turnout: When the Decisive Voter Stays at Home." *American Economic Review* 112(2):689–719.

Bonica, Adam, Jacob M. Grumbach, Charlotte Hill and Hakeem Jefferson. 2021. "All-mail Voting in Colorado Increases Turnout and Reduces Turnout Inequality." *Electoral Studies* 72:102363.

Borgers, Tilman. 2004. "Costly Voting." *American Economic Review* 94(1):57–66.

Brennan Center. 2021. "Voting Laws Roundup: December 2021.".
URL: *https://www.brennancenter.org/our-work/research-reports/voting-laws-roundup-december-2021*

Brennan Center. 2023. "Voting Laws Roundup: October 2023.".
URL: *https://www.brennancenter.org/our-work/research-reports/voting-laws-roundup-october-2023*

Burden, Barry C., David T. Canon, Kenneth R. Mayer and Donald P. Moynihan. 2014. "Election Laws, Mobilization, and Turnout: The Unanticipated Consequences of Election Reform." *American Journal of Political Science* 58(1):95–109.

Callander, Steven and Catherine H. Wilson. 2007. "Turnout, Polarization, and Duverger's Law." *Journal of Politics* 69(4):1047–1056.

Callander, Steven and Juan Carlos Carbajal. 2022. "Cause and Effect in Political Polarization: A Dynamic Analysis." *Journal of Political Economy* 130(4):825–880.

Calvert, Randall L. 1985. "Robustness of the Multidimensional Voting Model: Candidate Motivations, Uncertainty, and Convergence." *American Journal of Political Science* 29(1):69–95.

Canen, Nathan and Kristopher Ramsay. 2024. "Quantifying Theory in Politics: Identification, Interpretation, and the Role of Structural Methods." *Journal of Theoretical Politics* 36(4):301–327.

Cantoni, Enrico and Vincent Pons. 2021. "Strict ID Laws Don't Stop Voters: Evidence from a US Nationwide Panel, 2008–2018." *Quarterly Journal of Economics* 136(4):2615–2660.

Cantoni, Enrico, Vincent Pons and Jerome Schafer. 2025. "Voting Rules, Turnout, and Economic Policies." *Annual Review of Economics* 17:217–239.

Cascio, Elizabeth U. and Ebonya Washington. 2014. "Valuing the Vote: The Redistribution of Voting Rights and State Funds Following the Voting Rights Act of 1965." *Quarterly Journal of Economics* 129(1):379–433.

Dewan, Torun and Kenneth A. Shepsle. 2011. "Political Economy Models of Elections." *Annual Review of Political Science* 14:311–330.

Downs, Anthony. 1957. *An Economic Theory of Democracy.* Harper & Row.

Esteban, Joan-María and Debraj Ray. 1994. "On the Measurement of Polarization." *Econometrica* 62(4):819–851.

Farber, Henry S. 1980. "An Analysis of Final-offer Arbitration." *Journal of Conflict Resolution* 24(4):683–705.

Feddersen, Timothy J. 2004. "Rational Choice Theory and the Paradox Of Not Voting." *Journal of Economic Perspectives* 18(1):99–112.

Fraga, Bernard L. and Michael G. Miller. 2022. "Who do Voter ID Laws Keep from Voting?" *Journal of Politics* 84(2):1091–1105.

Fujiwara, Thomas. 2015. "Voting Technology, Political Responsiveness, and Infant Health: Evidence from Brazil." *Econometrica* 83(2):423–464.

Grimmer, Justin and Eitan Hersh. 2024. "How Election Rules Affect who Wins." *Journal of Legal Analysis* 16(1):1–25.

Groseclose, Tim. 2001. "A Model of Candidate Location When One Candidate Has a Valence Advantage." *American Journal of Political Science* 45(4):862–886.

Grumbach, Jacob M. 2023. "Laboratories of Democratic Backsliding." *American Political Science Review* 117(3):967–984.

Grumbach, Jacob M. and Charlotte Hill. 2022. "Rock the Registration: Same Day Registration Increases Turnout of Young Voters." *Journal of Politics* 84(1):405–417.

Hall, Andrew B. 2019. *Who Wants to Run?: How the Devaluing of Political Office Drives Polarization*. University of Chicago Press.

Heckman, James J. and Rodrigo Pinto. 2024. "Econometric Causality: The Central Role of Thought Experiments." *Journal of Econometrics* 243(1-2):105719.

Herrera, Helios, Massimo Morelli and Salvatore Nunnari. 2016. "Turnout Across Democracies." *American Journal of Political Science* 60(3):607–624.

Hortala-Vallve, Rafael and Berta Esteve-Volart. 2011. "Voter Turnout in a Multidimensional Policy Space." *Economics of Governance* 12(1):25–49.

Jessee, Stephen A. 2009. "Spatial Voting in the 2004 Presidential Election." *American Political Science Review* 103(1):59–81.

Jessee, Stephen A. 2010. "Partisan Bias, Political Information and Spatial Voting in the 2008 Presidential Election." *Journal of Politics* 72(2):327–340.

Johnson, Mary. 2021. "Advocates Sound Alarm as Restrictive Voting Laws Pile Up.".
URL: *https://thehill.com/homenews/campaign/552433-advocates-sound-alarm-as-restrictive-voting-laws-pile-up*

Krishna, Vijay and John Morgan. 2012. "Voluntary Voting: Costs and Benefits." *Journal of Economic Theory* 147(6):2083–2123.

Ledyard, John O. 1984. "The Pure Theory of Large Two-candidate Elections." *Public Choice* 44(1):7–41.

Llavador, Humberto. 2006. "Electoral Platforms, Implemented Policies, and Abstention." *Social Choice and Welfare* 27(1):55–81.

Lo Prete, Anna and Federico Revelli. 2021. "Voter Turnout and City Performance: Evidence from Italian Municipalities." *Journal of Law, Economics, and Organization* 37(1):168–197.

Myatt, David P. 2015. "A Theory of Voter Turnout." *Manuscript, London Bus. School* .

Nielson, Lindsay and Neil Visalvanich. 2017. "Primaries and Candidates: Examining the Influence of Primary Electorates on Candidate Ideology." *Political Science Research and Methods* 5(2):397–408.

of State, Colorado Secretary. 2022. "2022 Voter Registration Statistics.".
URL: *https://www.sos.state.co.us/pubs/elections/VoterRegNumbers/VoterRegNumbers.html*

Oprea, Alexandra, Lucy Martin and Geoffrey H. Brennan. 2024. "Moving Toward the Median: Compulsory Voting and Political Polarization." *American Political Science Review* 118(4):1951–1965.

Paine, Jack and Scott A. Tyson. 2020. Uses and Abuses of Formal Models in Political Science. In *SAGE Handbook of Political Science*. SAGE Publications Ltd pp. 188–202.

Peress, Michael. 2011. "Securing the Base: Electoral Competition Under Variable Turnout." *Public Choice* 148(1):87–104.

Pettigrew, Stephen. 2021. "The Downstream Consequences of Long Waits: How Lines at the Precinct Depress Future Turnout." *Electoral Studies* 71:1–17.

Pitkin, Hanna Fenichel. 1967. *The Concept of Representation*. University of California Press.

Shor, Boris and Jon C. Rogowski. 2018. "Ideology and the US Congressional Vote." *Political Science Research and Methods* 6(2):323–341.

Spirling, Arthur and Brandon M. Stewart. 2025. "What Good Is a Regression? Inference to the Best Explanation and the Practice of Political Science Research." *Journal of Politics* 87(4):1587–1599.

Stephanopoulos, Nicholas O. 2024. *Aligning Election Law*. Oxford University Press.

Tulin, Leah and Gabriella Sanchez. 2023. "Texas's Voter Suppression Law Is On Trial.". **URL:** *https://www.brennancenter.org/our-work/analysis-opinion/texas-voter-suppression-law-trial*

Tyson, Scott A. 2016. "Information Acquisition, Strategic Voting, and Improving the Quality of Democratic Choice." *Journal of Politics* 78(4):1016–1031.

Vives, Xavier. 2005. "Complementarities and Games: New Developments." *Journal of Economic Literature* 43(2):437–479.

Wittman, Donald. 1983. "Candidate Motivation: A Synthesis of Alternative Theories." *American Political Science Review* 77(1):142–157.

Online Appendix

Throughout the appendix, we maintain the following regularity condition: $c_R < \frac{1-F(c_R)}{f(c_R)}$. Since f is log-concave, the inverse hazard rate $\frac{1-F}{f}$ is strictly decreasing in c, so this condition defines an upper bound on c_R that depends on F. For example, if F is exponential with rate λ, the bound is $\frac{1}{\lambda} = \mathbb{E}[c_L]$. Substantively, this requires that c_R is mild relative to G_L's likely voting costs which is consistent with our interest in targeted restrictions that disproportionately affect G_L. Its role is to ensure positive turnout from G_R in equilibrium (Lemma 1) while allowing positive probability of crossover voting by some citizens in G_L.

We begin with three preliminary lemmas. Lemma 1 establishes that equilibrium platform divergence must exceed G_R's voting cost, which guarantees positive turnout from G_R. Lemma 2 shows that crossover voting from G_L (i.e., citizens in G_L voting for R) occurs with probability less than $\frac{1}{2}$. Lemma 3 combines these properties to characterize exactly when each party wins, facilitating the equilibrium characterization in Propositions 1 and 2.

Lemma 1. *In equilibrium,* $\Delta^* \equiv x_R^* - x_L^* \geq c_R$.

Proof. To begin, note that $x_L^* < x_R^*$ in equilibrium, since otherwise L could profitably deviate to some $x_L' < x_R^*$. Thus, we must have $\Delta^* > 0$.

To complete the proof, suppose for contradiction that $\Delta^* < c_R$. We proceed in three steps. First, we show that no G_R citizen votes when $\Delta^* < c_R$, so the election is determined entirely by G_L. Second, we characterize behavior in this no-G_R-turnout case and show that either R has a profitable deviation or L's best response is $x_L^* = -1$. Third, we show that if $x_L^* = -1$, then R has a profitable deviation to increase x_R. Thus, there is always one party with a profitable deviation, as desired.

Step 1: No turnout in G_R. For any citizen in G_R with ideal point $\hat{x}_i \in [0, 1]$, the utility difference between platforms is at most $x_R^* - x_L^* = \Delta^*$, with the maximum at $\hat{x}_i \geq x_R^*$. Since $\Delta^* < c_R$, no citizens in G_R will vote.

Step 2: With zero G_R turnout, must have $\frac{x_L^*+x_R^*}{2} > -\frac{1}{2}$ and $x_L^* = -1$. With no G_R turnout, the election is determined by G_L. Citizens in G_L with ideal points below the platform midpoint $\frac{x_L^*+x_R^*}{2}$ prefer L; those above $\frac{x_L^*+x_R^*}{2}$ prefer R.

If $\frac{x_L^*+x_R^*}{2} \leq -\frac{1}{2}$, the set of G_L citizens who vote for R is weakly larger than the set who vote for L. Therefore R wins whenever there is positive turnout, and wins the tie otherwise. Since R wins with probability 1, it has a profitable deviation to increase x_R.

If $\frac{x_L^*+x_R^*}{2} > -\frac{1}{2}$, then whenever there is positive turnout the set of G_L citizens who vote for L is always larger than the set who vote for R, so L wins. Therefore R wins iff $c_L \geq \Delta^*$, so no one votes. Thus L's payoff is $U_L(x_L^*) = -(1 + x_R^*) + F(\Delta^*) \cdot \Delta^*$, which has derivative $\frac{\partial U_L(x_L)}{\partial x_L} = -f(\Delta^*) \cdot \Delta^* - F(\Delta^*) < 0$. Thus, L has a profitable deviation unless $x_L^* = -1$.

Altogether, it follows that any equilibrium with zero G_R turnout must have $x_L^* = -1$.

Step 3: If $x_L^* = -1$, then R has a profitable deviation. Suppose $x_L^* = -1$. Then, $\Delta^* = x_R^* + 1$ and $\frac{x_L^*+x_R^*}{2} = \frac{x_R^*-1}{2}$. Furthermore, $\frac{x_L^*+x_R^*}{2} > -\frac{1}{2}$ and $\Delta^* < c_R$ together imply $x_R^* \in (0, c_R - 1)$.[23] Therefore R's payoff is $U_R(x_R^*) = -(1 - x_R^*) - F(x_R^* + 1) \cdot (x_R^* + 1)$, with derivative $\frac{dU_R(x_R)}{dx_R} = (1 - F(x_R + 1)) - (x_R + 1) \cdot f(x_R + 1)$. Since $x_R^* \in (0, c_R - 1)$, we have $x_R^* + 1 \in (1, c_R)$. Additionally, since f is log-concave, the inverse hazard rate $g = \frac{1-F}{f}$ is strictly decreasing. Then, due our maintained assumption that $c_R < g(c_R)$, we have that $x_R^* + 1 < c_R$ implies $x_R^* + 1 < c_R < g(c_R) < g(x_R^* + 1)$. Equivalently, $1 - F(x_R^* + 1) > (x_R^* + 1) \cdot f(x_R^* + 1)$, which implies $\frac{dU_R(x_R)}{dx_R} > 0$ for all $x_R \in (0, c_R - 1)$. Thus, R has a strictly profitable deviation to increase x_R.

Altogether, we have shown that (x_L^*, x_R^*) cannot be an equilibrium if $\Delta^* < c_R$, so $x_R^* - x_L^* \geq c_R$. ■

Let $c^\dagger(x_L, x_R) \equiv -(x_L + x_R)$ denote the highest c_L at which there are citizens in G_L who vote for R's candidate. Additionally, recall $\hat{c}(x_L, x_R) \equiv \frac{1}{\alpha}(x_R + x_L + (1-\alpha)c_R + 2(2\alpha - 1))$.

[23] If $c_R \leq 1$, this interval is empty, so any equilibrium with $x_L^* = -1$ and $\Delta^* < c_R$ must have $\frac{x_L^*+x_R^*}{2} \leq -\frac{1}{2}$, where R has a profitable deviation per Step 2.

Lemma 2. *In equilibrium,* $c^\dagger(x_L^*, x_R^*) < \hat{c}(x_L^*, x_R^*)$.

Proof. Suppose (x_L^*, x_R^*) is an equilibrium. Furthermore, suppose $c^\dagger(x_L^*, x_R^*) > \underline{c}$ since otherwise the result is trivial. First, we establish that the vote margin with G_L crossover voting is a constant. Specifically, suppressing dependence on the fixed platforms and c_R, we show that the vote differential $\tau_L(c_L) - \tau_R(c_L)$ is constant over $c_L < c^\dagger(x_L^*, x_R^*)$. To see this, observe that for $c_L < c^\dagger(x_L^*, x_R^*)$, each citizen $i \in G_L$ votes for L if $\hat{x}_i \in \left[-1, \frac{x_L^*+x_R^*-c_L}{2}\right]$ and votes for R if $\hat{x}_i \in \left[\frac{x_L^*+x_R^*+c_L}{2}, 0\right)$. Since citizens in G_R do not vote for L in this case, the vote differential is:

$$\tau_L(c_L) - \tau_R(c_L) = \alpha\left(\frac{x_L^* + x_R^* - c_L}{2} + 1\right) - \alpha\left(-\frac{x_L^* + x_R^* + c_L}{2}\right) - (1-\alpha)\left(1 - \frac{x_L^* + x_R^* + c_R}{2}\right),$$

which simplifies to $\tau_L(c_L) - \tau_R(c_L) = \alpha(x_L^* + x_R^* + 1) - (1-\alpha)\left(1 - \frac{x_L^*+x_R^*+c_R}{2}\right)$ and is constant in c_L. Furthermore, by continuity, this constant vote differential with crossover voting equals the vote differential at $c_L = c^\dagger(x_L^*, x_R^*)$ under partisan voting.

We complete the proof by showing a contradiction if $c^\dagger(x_L^*, x_R^*) \geq \hat{c}(x_L^*, x_R^*)$. Then, $\tau_L(c_L) \leq \tau_R(c_L)$ at $c_L = c^\dagger(x_L^*, x_R^*)$, since $\hat{c}(x_L^*, x_R^*)$ is defined as the value of c_L where $\tau_L(c_L) = \tau_R(c_L)$ under partisan voting and in that case $\tau_L(c_L) - \tau_R(c_L)$ is decreasing in c_L. Thus $\tau_L(c_L) \leq \tau_R(c_L)$ for all $c_L \leq c^\dagger(x_L^*, x_R^*)$. Moreover, for $c_L > c^\dagger(x_L^*, x_R^*) \geq \hat{c}(x_L^*, x_R^*)$, there is partisan voting and $\tau_L(c_L) < \tau_R(c_L)$. Therefore $\tau_L(c_L) \leq \tau_R(c_L)$ always holds, so $Pr(L \text{ wins} \mid x_L^*, x_R^*) = 0$. But then L would have a profitable deviation. Thus, we must have $c^\dagger(x_L^*, x_R^*) < \hat{c}(x_L^*, x_R^*)$. ■

Lemma 3. *In equilibrium, Party L wins if and only if* $c_L < \hat{c}(x_L^*, x_R^*)$.

Proof. Suppose (x_L^*, x_R^*) is an equilibrium. Lemma 1 implies that $\tau_R(x_L^*, x_R^*; c_R) > 0$ and Lemma 2 implies that $\tau_L(x_L^*, x_R^*; c_L) > \tau_R(x_L^*, x_R^*; c_R)$ for all $c_L < \hat{c}(x_L^*, x_R^*)$. Thus, L wins if and only if:

$$(1-\alpha)\left|1 - \left(\frac{x_R^* + x_L^*}{2} + \frac{c_R}{2}\right)\right| < \alpha\left|-1 - \left(\frac{x_R^* + x_L^*}{2} - \frac{c_L}{2}\right)\right|, \tag{10}$$

which reduces to $c_L < \frac{1}{\alpha}\bigg(x_R^* + x_L^* + (1-\alpha)\, c_R + 2(2\alpha - 1)\bigg) = \hat{c}(x_L^*, x_R^*)$, as desired. ■

Proof of Propositions 1 & 2. Suppose (x_L^*, x_R^*) is an equilibrium. By Lemma 3, party L wins if and only if $c_L < \hat{c}(x_L^*, x_R^*)$. To streamline notation, let $\hat{c}^* = \hat{c}(x_L^*, x_R^*)$. Then, $Pr(L \text{ wins} \mid x_L^*, x_R^*) = F(\hat{c}^*)$, so L's expected payoff is

$$\mathbb{E}[u_L(x_L^*; x_R^*)] = -|x_L^* - \hat{x}_L| \cdot Pr(L \mid x_L^*, x_R^*) - |x_R^* - \hat{x}_L| \cdot (1 - Pr(L \mid x_L^*, x_R^*)) \tag{11}$$

$$= -(1 + x_R^*) + (x_R^* - x_L^*) \cdot F(\hat{c}^*), \tag{12}$$

and R's expected payoff is

$$\mathbb{E}[u_R(x_R^*; x_L^*)] = -(1 - x_R^*) + (x_L^* - x_R^*) \cdot F(\hat{c}^*). \tag{13}$$

Therefore $\frac{\partial \mathbb{E}[u_L(x_L^*;x_R^*)]}{\partial x_L} = -F(\hat{c}^*) + \frac{x_R^* - x_L^*}{\alpha} f(\hat{c}^*)$ and $\frac{\partial \mathbb{E}[u_R(x_R^*;x_L^*)]}{\partial x_R} = (1 - F(\hat{c}^*)) - \frac{x_R^* - x_L^*}{\alpha} f(\hat{c}^*)$, which uses $\frac{\partial \hat{c}}{\partial x_L} = \frac{\partial \hat{c}}{\partial x_R} = \frac{1}{\alpha}$. Thus, the FOCs require

$$\frac{x_R^* - x_L^*}{\alpha} \frac{f(\hat{c}^*)}{F(\hat{c}^*)} = 1, \text{ and} \tag{14}$$

$$\frac{x_R^* - x_L^*}{\alpha} \frac{f(\hat{c}^*)}{1 - F(\hat{c}^*)} = 1. \tag{15}$$

Combining (14) and (15) yields $F(\hat{c}^*) = \frac{1}{2}$, so $\hat{c}^* = \tilde{c} = \text{median}(F)$.

Next, since $\tilde{c} = \hat{c}^* = \frac{1}{\alpha}(x_R^* + x_L^* + (1-\alpha)c_R + 2(2\alpha - 1))$, we rearrange to derive the midpoint of equilibrium platforms:

$$\frac{x_R^* + x_L^*}{2} = \frac{1}{2}(\alpha\tilde{c} - (1-\alpha)c_R) + (1 - 2\alpha). \tag{16}$$

Similarly, substituting $F(\hat{c}^*) = \frac{1}{2}$ into (14) and solving for $x_R^* - x_L^*$ yields the divergence between equilibrium platforms:

$$x_R^* - x_L^* = \frac{\alpha}{2\,f(\tilde{c})}. \tag{17}$$

Finally, combining (16) and (17) yields the equilibrium platforms:

$$x_L^* = \frac{1}{2}(\alpha\tilde{c} - (1-\alpha)c_R) + (1-2\alpha) - \frac{\alpha}{4\,f(\tilde{c})} \tag{18}$$

$$x_R^* = \frac{1}{2}(\alpha\tilde{c} - (1-\alpha)c_R) + (1-2\alpha) + \frac{\alpha}{4\,f(\tilde{c})}. \tag{19}$$

■

Proof of Proposition 3. Using (18) and (19), we have: (i) $\frac{\partial x_L^*}{\partial c_R} = \frac{\partial x_R^*}{\partial c_R} = -\frac{1-\alpha}{2} < 0$, (ii) $\frac{\partial x_L^*}{\partial \tilde{c}} = \frac{\partial x_R^*}{\partial \tilde{c}} = \frac{\alpha}{2} > 0$, and (iii) $\frac{\partial x_L^*}{\partial \varepsilon} = \frac{\partial x_R^*}{\partial \varepsilon} = \alpha - \frac{1}{2}$. ■

Proof of Proposition 4. Using (18) and (19), we have: (i) $\frac{\partial x_L^*}{\partial \alpha} = \frac{\tilde{c}+c_R}{2} - 2 - \frac{1}{4\,f(\tilde{c})} < 0$ and (ii) $\frac{\partial x_R^*}{\partial \alpha} = \frac{\tilde{c}+c_R}{2} - 2 + \frac{1}{4\,f(\tilde{c})}$. Additionally, since $\tilde{c} + c_R < 4$ is required for positive turnout, note that $\frac{\partial x_L^*}{\partial \alpha} < \frac{\partial x_R^*}{\partial \alpha}$ and $|\frac{\partial x_R^*}{\partial \alpha}| < |\frac{\partial x_L^*}{\partial \alpha}|$. ■

Proof of Corollary 1. Let $\Delta_x^* = x_R^* - x_L^* = \frac{\alpha}{2f(\tilde{c})}$. Then, we have $\frac{\partial \Delta_x^*}{\partial \alpha} = \frac{1}{2\,f(\tilde{c})} > 0$. ■

Proof of Proposition 5. Using (18) and (19), we have $x_L^* + x_R^* = \alpha\tilde{c} - (1-\alpha)c_R + 2(1-2\alpha)$.

For G_R, citizens vote if $\hat{x}_i \geq \frac{x_L^*+x_R^*+c_R}{2}$, so equilibrium turnout is

$$\tau_R^* = (1-\alpha)\left(1 - \frac{x_L^* + x_R^* + c_R}{2}\right) = \alpha(1-\alpha)\left(2 - \frac{\tilde{c} + c_R}{2}\right).$$

Therefore $\frac{\partial \tau_R^*}{\partial \tilde{c}} = \frac{\partial \tau_R^*}{\partial c_R} = -\frac{\alpha(1-\alpha)}{2}$.

For G_L, citizens vote if $\hat{x}_i \leq \frac{x_L^*+x_R^*-c_L}{2}$, so equilibrium turnout given realized c_L is

$$\tau_L^* = \alpha\left(\frac{x_L^* + x_R^* - c_L}{2} + 1\right) = \alpha\left(2(1-\alpha) + \frac{\alpha\tilde{c} - (1-\alpha)c_R - c_L}{2}\right),$$

with expectation $\mathbb{E}[\tau_L^*] = \alpha\left(2(1-\alpha) + \frac{\alpha\tilde{c}-(1-\alpha)c_R-\mathbb{E}[c_L]}{2}\right)$. Therefore $\frac{\partial\mathbb{E}[\tau_L^*]}{\partial c_R} = -\frac{\alpha(1-\alpha)}{2}$ and $\frac{\partial\mathbb{E}[\tau_L^*]}{\partial\tilde{c}} = \frac{\alpha}{2}\left(\alpha - \frac{\partial\mathbb{E}[c_L]}{\partial\tilde{c}}\right) = -\frac{\alpha}{2}\left(\frac{\partial\mathbb{E}[c_L]}{\partial\tilde{c}} - \alpha\right)$. ■

Proof of Proposition 6. Using (18) and (19) yields party L's equilibrium value: $U_L^* = -(1+x_R^*)+\frac{1}{2}(x_R^*-x_L^*) = \frac{1}{2}((1-\alpha)\,c_R-\alpha\tilde{c})-2(1-\alpha)$ and R's equilibrium value is $U_R^* = -2-U_L^*$. Therefore, $\frac{\partial U_L^*}{\partial\tilde{c}} = -\frac{\partial U_R^*}{\partial\tilde{c}} = -\frac{\alpha}{2}$, $\frac{\partial U_L^*}{\partial c_R} = -\frac{\partial U_R^*}{\partial c_R} = \frac{1-\alpha}{2}$, and $\frac{\partial U_L^*}{\partial\alpha} = -\frac{\partial U_R^*}{\partial\alpha} = 2 - \frac{1}{2}(c_R+\tilde{c})$. ■

Conditions for Existence

Let (x_L^*, x_R^*) denote a platform pair that solves (14) and (15), which we have already shown is necessary for an equilibrium. We now establish sufficient conditions for (x_L^*, x_R^*) to be an equilibrium. Moreover, later on (in Corollary 2) we will show it is unique.

Proposition 7. *Let (x_L^*, x_R^*) solve the first-order conditions (14) and (15). There exist bounds $\underline{c}_R, \overline{c}_R$ and functions $\underline{c}_L, \overline{c}_L$ such that $c_R \in (\underline{c}_R, \overline{c}_R)$ and $\tilde{c} \in (\underline{c}_L, \overline{c}_L)$ together imply (x_L^*, x_R^*) is an equilibrium.*

Proof of Proposition 7. Let (x_L^*, x_R^*) solve the first-order conditions (14) and (15). Define $q \equiv F^{-1}\left(\frac{2-\alpha}{2}\right)$, the bounds $\underline{c}_R \equiv \frac{\alpha\, q+2(1-2\alpha)}{2-\alpha}$ and $\overline{c}_R \equiv \min\left\{\frac{\alpha}{2f(\tilde{c})},\ \frac{1-F(c_R)}{f(c_R)}\right\}$, and the functions $\underline{c}_L \equiv \max\left\{\text{mode } F,\ \frac{(1-\alpha)c_R+2(2\alpha-1)}{1+\alpha}\right\}$ and $\overline{c}_L \equiv \min\left\{\frac{\alpha}{2f(\tilde{c})},\ \frac{(2-\alpha)c_R+2(2\alpha-1)}{\alpha}\right\}$.[24] Since voters are infinitesimal, it suffices to verify that neither party has a profitable deviation when (i) $c_R \in (\underline{c}_R, \overline{c}_R)$ and (ii) $\tilde{c} \in (\underline{c}_L, \overline{c}_L)$.

Step 1: Verify equilibrium properties. Conditions (i) and (ii) ensure three properties at (x_L^*, x_R^*). First, no citizen in G_R votes for L since the second term of $\overline{c}_L$ implies $x_L^* + x_R^* < c_R$. Second, crossover voting from G_L occurs with probability less than $\frac{1}{2}$ since the second term of $\underline{c}_L$ implies $x_L^* + x_R^* + \tilde{c} > 0$ and thus $c^\dagger(x_L^*, x_R^*) < \tilde{c}$. Third, both groups have positive turnout at (x_L^*, x_R^*) for all $c_L \leq \tilde{c}$, since $\max\{c_R, \tilde{c}\} < \frac{\alpha}{2f(\tilde{c})} = x_R^* - x_L^*$.

Step 2: Check party R deviations. Consider any deviation $x_R \neq x_R^*$. There are three cases.

[24]Under location shifts of F, the density at the median $f(\tilde{c})$ is constant, so these define proper intervals.

First, if x_R induces $\tau_R(x_L^*, x_R; c_R) = 0$, then R loses with probability 1. Since R has positive winning probability at $x_R^* > x_L^*$, this is not profitable.

Second, if $x_L^* + x_R \leq c_R$, then there is no crossover from G_R and thus there is partisan voting. The first-order condition $\frac{\partial \mathbb{E}[u_R(x_R; x_L^*)]}{\partial x_R} = 0$ is equivalent to $\frac{x_R - x_L^*}{\alpha} = \frac{1 - F(\hat{c}(x_L^*, x_R))}{f(\hat{c}(x_L^*, x_R))}$. The right-hand side is the inverse hazard rate evaluated at $\hat{c}(x_L^*, x_R)$, which is strictly decreasing in x_R since (i) the inverse hazard rate is strictly decreasing because f is log-concave and (ii) $\hat{c}$ is increasing in x_R. The left-hand side is strictly increasing in x_R. Therefore these expressions cross exactly once, at x_R^*, so x_R^* is uniquely optimal for R in this case.

Third, if $x_L^* + x_R > c_R$, then some citizens in G_R vote for L. Therefore:

$$\mathbb{E}[u_R(x_R; x_L^*)] \leq \mathbb{E}[u_R(x_R; x_L^*) \mid G_R \text{ partisan voting}] \tag{20}$$

$$\leq \mathbb{E}[u_R(x_R^*; x_L^*) \mid G_R \text{ partisan voting}] = \mathbb{E}[u_R(x_R^*; x_L^*)], \tag{21}$$

where (20) follows because R is weakly more likely to win if G_R-crossover voters are ignored, (21) follows because x_R^* solves R's maximization problem given x_L^* and partisan voting from G_R, and the equality holds because (x_L^*, x_R^*) does not induce any crossover voting by citizens in G_R.

Step 3: Check party L deviations. By arguments analogous to Step 2, L does not have a profitable deviation to any x_L such that $x_L + x_R^* \leq c_R$. Thus, the only potential profitable deviations are platforms that would induce G_R-crossover voting for L. Those are $x_L \in (x^\dagger, 1]$, where $x^\dagger \equiv c_R - x_R^*$.

Define $\hat{c}^\dagger \equiv \hat{c}(x^\dagger, x_R^*) = \frac{(2-\alpha)c_R + 2(2\alpha - 1)}{\alpha}$. Since $F(\tilde{c}) = \frac{1}{2} < \frac{2-\alpha}{2}$ and condition (i) implies $c_R > \underline{c}_R$, we have $\hat{c}^\dagger > q$ and thus $F(\hat{c}^\dagger) > \frac{2-\alpha}{2}$. Moreover, condition (ii) implies $\tilde{c} < \hat{c}^\dagger$.

For $x_L \in (x^\dagger, 1]$, we have $\hat{c}(x_L, x_R^*) \geq \hat{c}^\dagger$. Additionally, G_R-crossover voting implies $\hat{c}(x_L, x_R^*) = \frac{1}{\alpha}\big((2-\alpha)(x_L + x_R^*) + 2(2\alpha - 1)\big)$, so $\frac{\partial \hat{c}}{\partial x_L} = \frac{2-\alpha}{\alpha}$.

The first derivative of L's expected payoff is:

$$\frac{\partial \mathbb{E}[u_L(x_L; x_R^*)]}{\partial x_L} = -F(\hat{c}(x_L, x_R^*)) + (x_R^* - x_L) \cdot \frac{2-\alpha}{\alpha} \cdot f(\hat{c}(x_L, x_R^*)).$$

The second derivative is:

$$\frac{\partial^2 \mathbb{E}[u_L(x_L; x_R^*)]}{\partial x_L^2} = -\frac{2(2-\alpha)}{\alpha} f(\hat{c}(x_L, x_R^*)) + (x_R^* - x_L) \cdot \left(\frac{2-\alpha}{\alpha}\right)^2 \cdot f'(\hat{c}(x_L, x_R^*)).$$

Since mode $F \leq \tilde{c} < \hat{c}^\dagger \leq \hat{c}(x_L, x_R^*)$ for all $x_L \in (x^\dagger, 1]$, we have $f'(\hat{c}(x_L, x_R^*)) \leq 0$ for all such x_L. Therefore $\frac{\partial^2 \mathbb{E}[u_L(x_L; x_R^*)]}{\partial x_L^2} < 0$ for all $x_L \in (x^\dagger, 1]$.

Moreover, at $x_L = x^\dagger$ the right derivative satisfies:

$$\begin{aligned} \frac{\partial \mathbb{E}[u_L(x^\dagger; x_R^*)]}{\partial x_L^+} &= -F(\hat{c}^\dagger) + (x_R^* - x^\dagger) \cdot \frac{2-\alpha}{\alpha} \cdot f(\hat{c}^\dagger) \\ &< -F(\hat{c}^\dagger) + \frac{\alpha}{2f(\tilde{c})} \cdot \frac{2-\alpha}{\alpha} \cdot f(\hat{c}^\dagger) \\ &= -F(\hat{c}^\dagger) + \frac{2-\alpha}{2} \cdot \frac{f(\hat{c}^\dagger)}{f(\tilde{c})} \\ &\leq -F(\hat{c}^\dagger) + \frac{2-\alpha}{2} < 0, \end{aligned}$$

where the first inequality uses $x_R^* - x^\dagger = 2x_R^* - c_R < x_R^* - x_L^* = \frac{\alpha}{2f(\tilde{c})}$, which follows from $x_L^* + x_R^* < c_R$ in Step 1; the second inequality follows because mode $F \leq \tilde{c} < \hat{c}^\dagger$ and log-concavity of f imply $f(\hat{c}^\dagger) \leq f(\tilde{c})$; and the final inequality follows because $F(\hat{c}^\dagger) > \frac{2-\alpha}{2}$.

Since $\frac{\partial^2 \mathbb{E}[u_L(x_L; x_R^*)]}{\partial x_L^2} < 0$ over $x_L \in (x^\dagger, 1]$ and $\frac{\partial \mathbb{E}[u_L(x^\dagger; x_R^*)]}{\partial x_L^+} < 0$, we have $\frac{\partial \mathbb{E}[u_L(x_L; x_R^*)]}{\partial x_L} < 0$ throughout this interval, so L does not have a profitable deviation to any such x_L.

Neither party has a profitable deviation, as desired. ■

To illustrate Proposition 7 with a numerical example, suppose F is an exponential distribution on support $[c_0, \infty)$, where $c_0 \geq 0$, with rate parameter $\lambda > 0$. Then $\tilde{c} = c_0 + \ln(2)/\lambda > c_0 =$ mode F and $f(\tilde{c}) = \lambda/2$. For instance, the conditions in Proposition 7 are satisfied if $\lambda = 1$, $\alpha = 0.75$, $\tilde{c} = 0.743$ (equivalently, $c_0 = 0.05$), and $c_R = 0.12$, since they yield $c_R \in (\underline{c}_R, \overline{c}_R) \approx (-0.18, 0.75)$ and $\tilde{c} \in (\underline{c}_L, \overline{c}_L) \approx (0.59, 0.75)$. These parameters have the following qualitative interpretation, which is reasonable for our substantive application: G_L is larger than G_R and likely to have much higher voting costs. In particular, G_L's voting costs (i) could be as low as 0.05 in their best-case scenario and thus below G_R's voting cost of 0.12, but (ii) are strongly right-skewed with median 0.74, so G_L's voting costs are likely to be much higher than G_R's. Under these parameters, equilibrium platforms are $x_L^* \approx -0.611$ and $x_R^* \approx 0.139$.

Equilibria with Crossover Voting in G_R

We characterize platforms for any equilibrium in which there are citizens in G_R who vote for L. We show that the key insights from our baseline analysis carry over.

For any (x_L, x_R) in which $x_L + x_R - c_R > 0$, i.e., some citizens in G_R vote for L, party L wins if and only if

$$(1-\alpha)\left|1 - \left(\frac{x_R + x_L}{2} + \frac{c_R}{2}\right)\right| \leq \alpha \left| -1 - \left(\frac{x_R + x_L}{2} - \frac{c_L}{2}\right)\right| + (1-\alpha)\left(\frac{x_L + x_R - c_R}{2}\right)$$

$$c_L \leq \frac{1}{\alpha}\Big((2-\alpha)\cdot(x_R + x_L) + 2\cdot(2\alpha - 1)\Big) \equiv \hat{c}^\dagger(x_L, x_R).$$

Therefore $\frac{\partial \mathbb{E}[u_L(x_L; x_R)]}{\partial x_L} = -F(\hat{c}^\dagger(x_L, x_R)) + (x_R - x_L)\cdot \frac{2-\alpha}{\alpha} f(\hat{c}^\dagger(x_L, x_R))$ and $\frac{\partial \mathbb{E}[u_R(x_R; x_L)]}{\partial x_R} = (1 - F(\hat{c}^\dagger(x_L, x_R))) - (x_R - x_L)\cdot \frac{2-\alpha}{\alpha} f(\hat{c}^\dagger(x_L, x_R))$. Thus, the FOCs require any solution (x_L^*, x_R^*) to satisfy

$$(x_R^* - x_L^*)\cdot \frac{f(\hat{c}_*^\dagger)}{F(\hat{c}_*^\dagger)} = \frac{\alpha}{2-\alpha}, \text{ and} \tag{22}$$

$$(x_R^* - x_L^*)\cdot \frac{f(\hat{c}_*^\dagger)}{1 - F(\hat{c}_*^\dagger)} = \frac{\alpha}{2-\alpha}, \tag{23}$$

where we set $\hat{c}_*^\dagger = \hat{c}^\dagger(x_L^*, x_R^*)$ in order to streamline notation. Together, (22) and (23) imply $F(\hat{c}_*^\dagger) = \frac{1}{2}$. Therefore $\hat{c}_*^\dagger = \tilde{c}$, which implies $\frac{x_L^* + x_R^*}{2} = \left(\frac{1}{2-\alpha}\right)\cdot\left(\frac{\alpha}{2}\,\tilde{c} + 1 - 2\alpha\right)$ and $x_R^* - x_L^* =$

$\frac{1}{2f(\tilde{c})} \cdot \frac{\alpha}{2-\alpha}$. Finally, we can combine the previous observations to obtain party platforms:

$$x_L^* = \frac{1}{2(2-\alpha)} \cdot \left(\alpha\tilde{c} + 2(1-2\alpha) - \frac{\alpha}{2f(\tilde{c})}\right), \text{ and}$$
$$x_R^* = \frac{1}{2(2-\alpha)} \cdot \left(\alpha\tilde{c} + 2(1-2\alpha) + \frac{\alpha}{2f(\tilde{c})}\right).$$

Uniqueness

Corollary 2. *If the conditions in Proposition 7 hold, the equilibrium is unique.*

Proof. Both partisan and crossover equilibria must satisfy the toss-up property, yielding $\hat{c}^* = \tilde{c}$ and $\hat{c}_*^\dagger = \tilde{c}$ respectively. For partisan equilibria, this implies $x_L^* + x_R^* = \alpha\tilde{c} - (1-\alpha)c_R - 2(2\alpha-1)$. For crossover equilibria, this implies $x_L^\ddagger + x_R^\ddagger = \frac{\alpha\tilde{c} - 2(2\alpha-1)}{2-\alpha}$.

For a crossover equilibrium to exist, we need $x_L^\ddagger + x_R^\ddagger > c_R$ so that some G_R citizens vote for L. Substituting the expression above yields $\tilde{c} > \frac{(2-\alpha)c_R + 2(2\alpha-1)}{\alpha}$. But the conditions in Proposition 7 require $\tilde{c} < \frac{(2-\alpha)c_R + 2(2\alpha-1)}{\alpha}$, so no crossover equilibrium can exist. Since Proposition 7 implies that the partisan equilibrium exists, it is unique. ■

Affinity Voting

Suppose the two groups of voters, G_L and G_R, each have associated (inverse) voting costs λ_L and λ_R. Let $\lambda_R \geq 0$ be fixed and common knowledge, whereas λ_L is a random variable drawn from a log-concave probability distribution F that has support on the interval $[\underline{\lambda}, \overline{\lambda}]$, where $\underline{\lambda} \geq 0$, and associated density function f.

The timing is analogous to the baseline model: (i) parties make binding campaign commitments, (ii) then uncertainty over λ_L is realized, and (iii) then voters vote.

For each citizen in G_R, suppose they turn out and vote for candidate R if $|\hat{x}_i - x_R| \leq \lambda_R$ and otherwise they abstain. Suppose citizens in G_L behave analogously. Thus, we focus on a setting in which voters support a candidate only if she is from their affiliated party.

Analysis

In equilibrium, with partisan voting there can be abstention among moderates but not extremists. If there were extremists in group G_j abstaining at some platform pair, then that party's turnout would not depend on the platforms. Thus, the party would have no incentive to moderate and shift outward until all extremists vote. Therefore equilibrium turnout in G_R is $(1-\alpha)(1-x_R+\lambda_R)$ and in G_L is $\alpha(1+x_L+\lambda_L)$.

The condition for L to win election with platforms (x_L, x_R) is $(1-\alpha)(1-x_R+\lambda_R) \leq \alpha(1+x_L+\lambda_L)$, so L wins the election if and only if

$$\lambda_L \geq \frac{1-\alpha}{\alpha}(1+\lambda_R) - 1 - \left(\frac{1-\alpha}{\alpha}x_R + x_L\right) \equiv \hat{\lambda}. \tag{24}$$

Thus, $Pr(L \text{ wins} \mid x_L, x_R) = 1 - F(\hat{\lambda})$.

Given a platform pair (x_L, x_R), party L's expected payoff is

$$U_L(x_L, x_R) = -(1-F(\hat{\lambda}))(1+x_L) - F(\hat{\lambda})(1+x_R) \tag{25}$$

$$= -(1+x_L) - F(\hat{\lambda})(x_R - x_L), \tag{26}$$

and R's expected payoff is

$$U_R(x_L, x_R) = -(1-F(\hat{\lambda}))(1-x_L) - F(\hat{\lambda})(1-x_R) \tag{27}$$

$$= -(1-x_L) + F(\hat{\lambda})(x_R - x_L). \tag{28}$$

The FOCs are:

$$0 = \frac{\partial U_L(x_L, x_R)}{\partial x_L} = -(1-F(\hat{\lambda})) + f(\hat{\lambda})(x_R - x_L) \tag{29}$$

$$0 = \frac{\partial U_R(x_L, x_R)}{\partial x_R} = F(\hat{\lambda}) - \frac{1-\alpha}{\alpha} f(\hat{\lambda})(x_R - x_L). \tag{30}$$

Combining the FOCs yields $1 = f(\hat{\lambda})(x_R - x_L) \cdot \frac{1}{\alpha}$, so $x_R - x_L = \frac{\alpha}{f(\hat{\lambda})}$. Substituting back into the first FOC, we have $F(\hat{\lambda}) = 1 - \alpha$ in equilibrium. Let $\hat{\lambda}_\alpha \equiv F^{-1}(1-\alpha)$.

In equilibrium, $x_R^* - x_L^* = \frac{\alpha}{f(\hat{\lambda}_\alpha)}$. A straightforward derivation yields:

$$x_L^* = (1-\alpha)\lambda_R + (1-2\alpha) - \alpha\hat{\lambda}_\alpha - \frac{\alpha(1-\alpha)}{f(\hat{\lambda}_\alpha)} \tag{31}$$

$$x_R^* = (1-\alpha)\lambda_R + (1-2\alpha) - \alpha\hat{\lambda}_\alpha + \frac{\alpha^2}{f(\hat{\lambda}_\alpha)}. \tag{32}$$

The parties' equilibrium values are:

$$U_L^* = -(1 + x_L^*) - (1-\alpha) \cdot \frac{\alpha}{f(\hat{\lambda}_\alpha)} = \alpha\hat{\lambda}_\alpha - (1-\alpha)(2+\lambda_R) \tag{33}$$

$$U_R^* = -(1 - x_L^*) + (1-\alpha) \cdot \frac{\alpha}{f(\hat{\lambda}_\alpha)} = (1-\alpha)\lambda_R - \alpha(2+\hat{\lambda}_\alpha). \tag{34}$$

Denoting $F^{-1} = H$ and $H' = h$, the comparative statics are: $\frac{\partial U_R^*}{\partial \lambda_R} = -\frac{\partial U_L^*}{\partial \lambda_R} = (1-\alpha) > 0$, $\frac{\partial U_R^*}{\partial \hat{\lambda}_\alpha} = -\frac{\partial U_L^*}{\partial \hat{\lambda}_\alpha} = -\alpha < 0$, and $\frac{\partial U_R^*}{\partial \alpha} = -\frac{\partial U_L^*}{\partial \alpha} = \alpha \cdot h(1-\alpha) - (2 + \lambda_R + \hat{\lambda}_\alpha)$.

The welfare difference is: $\Delta^* = U_R^* - U_L^* = 2\big((1-2\alpha) + (1-\alpha)\lambda_R - \alpha\hat{\lambda}_\alpha\big)$.

For equilibrium turnout, we have:

$$\tau_R^* = \alpha(1-\alpha)\left(2 + \lambda_R + \hat{\lambda}_\alpha - \frac{\alpha}{f(\hat{\lambda}_\alpha)}\right) \tag{35}$$

$$\tau_L^* = \alpha\left(2(1-\alpha) + (1-\alpha)\lambda_R + \lambda_L - \alpha\hat{\lambda}_\alpha - \frac{\alpha(1-\alpha)}{f(\hat{\lambda}_\alpha)}\right). \tag{36}$$

Thus, the expected turnout difference in equilibrium is: $\mathbb{E}[\tau_R^* - \tau_L^*] = \alpha(\hat{\lambda}_\alpha - \mathbb{E}[\lambda_L])$.

Setting with Supermodular Competition

In this subsection, we show that our main results are robust in a smooth version of our setting. We establish that: (i) under mild conditions, the platform competition game is log-supermodular, and (ii) increases in voting costs for the left-leaning group (in a stochastic sense) shift both parties' platforms rightward.

Setup and assumptions. Let $x = (x_L, x_R)$ denote the platform pair and let $\mathcal{U}_J(x)$ denote party J's policy payoff from implemented platform x, generalizing the linear loss $u_j(x) = -|x - \hat{x}_j|$ from the baseline model. We assume: (i) $\mathcal{U}_J$ is strictly decreasing in $|x - \hat{x}_J|$ and differentiable for each party J, (ii) f is log-concave and differentiable, (iii) $\tau_L(x; c_L)$ is strictly decreasing in c_L, and (iv) both τ_L and τ_R are sufficiently smooth in platforms. Under these assumptions, party L wins at x if and only if $c_L < \hat{c}(x; c_R) \equiv \hat{c}_x$, and we maintain $\frac{\partial \hat{c}_x}{\partial x_L} > 0$ and $\frac{\partial \hat{c}_x}{\partial x_R} > 0$.

Party J's expected payoff from x is $F(\hat{c}_x) \cdot \mathcal{U}_J(x_L) + (1 - F(\hat{c}_x)) \cdot \mathcal{U}_J(x_R) \equiv \Pi_J(x_L, x_R)$. Letting $\widehat{\Pi}_L(x_L, x_R) \equiv F(\hat{c}_x) \cdot (\mathcal{U}_L(x_L) - \mathcal{U}_L(x_R))$ and $\widehat{\Pi}_R(x_L, x_R) \equiv (1 - F(\hat{c}_x)) \cdot (\mathcal{U}_R(x_R) - \mathcal{U}_R(x_L))$, each party J's problem is equivalent to $\max_{x_J} \ln(\widehat{\Pi}_J(x_L, x_R))$.

Roadmap. We proceed in three steps. First, we derive conditions under which $\ln(\widehat{\Pi}_L)$ and $\ln(\widehat{\Pi}_R)$ each satisfy increasing differences in strategies, making the game log-supermodular. Second, we show that under a location-shift parameterization of F, each party's objective has strictly increasing differences in its own strategy and the shift parameter, implying platforms increase as voting costs for the left-leaning group rise. Third, we apply standard results on supermodular games (Vives 2005) to conclude that equilibria exist and have the desired monotone comparative statics.

Step 1. We first derive conditions that imply $\ln\left(\widehat{\Pi}_L(x_L, x_R)\right)$ has increasing differences, which in our smooth setting is equivalent to $\frac{\partial^2}{\partial x_R \, \partial x_L} \ln\left(\widehat{\Pi}_L(x_L, x_R)\right) \geq 0$ at all x. First, defining $G = \frac{f}{F}$, we have: $\frac{\partial}{\partial x_L} \ln(\widehat{\Pi}_L(x_L, x_R)) = G(\hat{c}_x) \cdot \frac{\partial \hat{c}_x}{\partial x_L} + \frac{\frac{\partial \mathcal{U}_L(x_L)}{\partial x_L}}{\mathcal{U}_L(x_L) - \mathcal{U}_L(x_R)}$ and

$$\frac{\partial^2}{\partial x_R \, \partial x_L} \ln(\widehat{\Pi}_L(x_L, x_R)) = \frac{\partial G(\hat{c}_x)}{\partial \hat{c}_x} \cdot \frac{\partial \hat{c}_x}{\partial x_L} \cdot \frac{\partial \hat{c}_x}{\partial x_R} + G(\hat{c}_x) \cdot \frac{\partial^2 \hat{c}_x}{\partial x_R \, \partial x_L} + \frac{\frac{\partial \mathcal{U}_L(x_L)}{\partial x_L} \cdot \frac{\partial \mathcal{U}_L(x_R)}{\partial x_R}}{(\mathcal{U}_L(x_L) - \mathcal{U}_L(x_R))^2}.$$

Log-concavity of f implies $\frac{\partial G(\hat{c}_x)}{\partial \hat{c}_x} < 0$, so $\frac{\partial^2}{\partial x_R \, \partial x_L} \ln(\widehat{\Pi}_L(x_L, x_R)) \geq 0$ if and only if:

$$\frac{\partial \hat{c}_x}{\partial x_L} \cdot \frac{\partial \hat{c}_x}{\partial x_R} \leq -\left(\frac{\partial G(\hat{c}_x)}{\partial \hat{c}_x}\right)^{-1} \cdot \left(G(\hat{c}_x) \cdot \frac{\partial^2 \hat{c}_x}{\partial x_R \, \partial x_L} + \frac{\frac{\partial \mathcal{U}_L(x_L)}{\partial x_L} \cdot \frac{\partial \mathcal{U}_L(x_R)}{\partial x_R}}{(\mathcal{U}_L(x_L) - \mathcal{U}_L(x_R))^2}\right). \qquad (37)$$

Furthermore, since $\frac{\partial \mathcal{U}_L(x_L)}{\partial x_L} \cdot \frac{\partial \mathcal{U}_L(x_R)}{\partial x_R} > 0$ and $G(\hat{c}_x) > 0$, the RHS of (37) is negative if and only if $\frac{\partial^2 \hat{c}_x}{\partial x_L \, \partial x_R}$ is sufficiently negative. Specifically, if $\frac{\partial^2 \hat{c}_x}{\partial x_L \, \partial x_R} > -\frac{\frac{\partial \mathcal{U}_L(x_L)}{\partial x_L} \cdot \frac{\partial \mathcal{U}_L(x_R)}{\partial x_R}}{G(\hat{c}_x) \cdot (\mathcal{U}_L(x_L) - \mathcal{U}_L(x_R))^2} \equiv \underline{\chi}$, then the RHS of (37) is positive, in which case (37) is satisfied whenever the product $\frac{\partial \hat{c}_x}{\partial x_L} \cdot \frac{\partial \hat{c}_x}{\partial x_R}$ is sufficiently small.

For party R, letting $H = \frac{f}{1-F}$, an analogous derivation yields

$$\frac{\partial^2}{\partial x_L \partial x_R} \ln\left(\widehat{\Pi}_R(x_L, x_R)\right) = -\frac{\partial H(\hat{c}_x)}{\partial \hat{c}_x} \cdot \frac{\partial \hat{c}_x}{\partial x_R} \cdot \frac{\partial \hat{c}_x}{\partial x_L} - H(\hat{c}_x) \cdot \frac{\partial^2 \hat{c}_x}{\partial x_L \, \partial x_R} + \frac{\frac{\partial \mathcal{U}_R(x_R)}{\partial x_R} \cdot \frac{\partial \mathcal{U}_R(x_L)}{\partial x_L}}{(\mathcal{U}_R(x_R) - \mathcal{U}_R(x_L))^2}.$$

Log-concavity of f implies $\frac{\partial H(\hat{c}_x)}{\partial \hat{c}_x} > 0$, so $\frac{\partial^2}{\partial x_L \, \partial x_R} \ln(\widehat{\Pi}_R(x_L, x_R)) \geq 0$ if and only if:

$$\frac{\partial \hat{c}_x}{\partial x_R} \cdot \frac{\partial \hat{c}_x}{\partial x_L} \leq \left(\frac{\partial H(\hat{c}_x)}{\partial \hat{c}_x}\right)^{-1} \cdot \left(- H(\hat{c}_x) \cdot \frac{\partial^2 \hat{c}_x}{\partial x_L \, \partial x_R} + \frac{\frac{\partial \mathcal{U}_R(x_R)}{\partial x_R} \cdot \frac{\partial \mathcal{U}_R(x_L)}{\partial x_L}}{(\mathcal{U}_R(x_R) - \mathcal{U}_R(x_L))^2}\right). \tag{38}$$

Since $\frac{\partial \mathcal{U}_R(x_R)}{\partial x_R} \cdot \frac{\partial \mathcal{U}_R(x_L)}{\partial x_L} > 0$ and $H(\hat{c}_x) > 0$, the RHS is negative if and only if $\frac{\partial^2 \hat{c}_x}{\partial x_L \, \partial x_R}$ is sufficiently positive. Thus, if $\frac{\partial^2 \hat{c}_x}{\partial x_L \, \partial x_R} < \frac{\frac{\partial \mathcal{U}_R(x_R)}{\partial x_R} \cdot \frac{\partial \mathcal{U}_R(x_L)}{\partial x_L}}{H(\hat{c}_x) \cdot (\mathcal{U}_R(x_R) - \mathcal{U}_R(x_L))^2} \equiv \overline{\chi}$, then the RHS of (38) is positive, in which case (38) is satisfied whenever the product $\frac{\partial \hat{c}_x}{\partial x_L} \cdot \frac{\partial \hat{c}_x}{\partial x_R}$ is sufficiently small.

Altogether, the game is supermodular when both conditions (37) and (38) hold. These conditions are jointly satisfied when $\frac{\partial^2 \hat{c}_x}{\partial x_L \, \partial x_R} \in [\underline{\chi}, \overline{\chi}]$ and the product $\frac{\partial \hat{c}_x}{\partial x_L} \cdot \frac{\partial \hat{c}_x}{\partial x_R}$ is sufficiently small. Specifically, when the cross-partial $\frac{\partial^2 \hat{c}_x}{\partial x_L \, \partial x_R}$ is in this interval, the right-hand sides of (37) and (38) are strictly positive, ensuring that both inequalities can be satisfied if $\frac{\partial \hat{c}_x}{\partial x_R} \cdot \frac{\partial \hat{c}_x}{\partial x_L}$ is small enough.

Step 2. We show that equilibrium platforms shift rightward as F increases (i.e., to make higher c_L more likely). Specifically, consider a location-shift parameterization where $F(c; \tilde{c}) = F_0(c - \tilde{c})$ for some base distribution F_0.[25] Note that $\frac{\partial F}{\partial \tilde{c}} = -f$ and $\frac{\partial f}{\partial \tilde{c}} = -f'$. To show that platforms shift rightward as $\tilde{c}$ increases, it remains to prove that $\ln(\widehat{\Pi}_L)$ has strictly increasing differences in $(x_L, \tilde{c})$, and $\ln(\widehat{\Pi}_R)$ has strictly increasing differences in $(x_R, \tilde{c})$.

[25] The shifts in $\tilde{c}$ can be quite general, such as shifting F upward under first-order stochastic dominance or other stochastic orders.

For party L, we verify that $\frac{\partial^2 \ln(\widehat{\Pi}_L)}{\partial x_L \partial \tilde{c}} > 0$. Recall that $\frac{\partial \ln(\widehat{\Pi}_L)}{\partial x_L} = G(\hat{c}_x) \cdot \frac{\partial \hat{c}_x}{\partial x_L} + \frac{\frac{\partial \mathcal{U}_L(x_L)}{\partial x_L}}{\mathcal{U}_L(x_L) - \mathcal{U}_L(x_R)}$, where $G = f/F$, so we have:

$$\frac{\partial^2 \ln(\widehat{\Pi}_L)}{\partial \tilde{c}\, \partial x_L} = \frac{\partial G(\hat{c}_x; \tilde{c})}{\partial \tilde{c}} \cdot \frac{\partial \hat{c}_x}{\partial x_L}.$$

Note that:

$$\begin{aligned}\frac{\partial G(\hat{c}_x;\, \tilde{c})}{\partial \tilde{c}} &= \frac{-f'(\hat{c}_x;\, \tilde{c}) \cdot F(\hat{c}_x;\, \tilde{c}) - f(\hat{c}_x;\, \tilde{c}) \cdot (-f(\hat{c}_x;\, \tilde{c}))}{F(\hat{c}_x;\, \tilde{c})^2} \\ &= \frac{f(\hat{c}_x;\, \tilde{c})^2 - f'(\hat{c}_x) \cdot F(\hat{c}_x;\, \tilde{c})}{F(\hat{c}_x;\, \tilde{c})^2} > 0,\end{aligned}$$

where the inequality follows because log-concavity of f implies log-concavity of F, which in turn implies that the reverse hazard rate f/F is decreasing, with $\frac{d}{dc}\left(\frac{f(c;\tilde{c})}{F(c;\tilde{c})}\right) = \frac{f'(c;\tilde{c}) \cdot F(c;\tilde{c}) - f(c;\tilde{c})^2}{F(c;\tilde{c})^2} < 0$. Thus, since $\frac{\partial \hat{c}_x}{\partial x_L} > 0$ in our setting, we have $\frac{\partial^2 \ln(\widehat{\Pi}_L)}{\partial \tilde{c}\, \partial x_L} = \frac{\partial G}{\partial \tilde{c}} \cdot \frac{\partial \hat{c}_x}{\partial x_L} > 0$, so $\ln(\widehat{\Pi}_L)$ has strictly increasing differences in $(x_L, \tilde{c})$.

For party R, recall that $\frac{\partial \ln(\widehat{\Pi}_R)}{\partial x_R} = -H(\hat{c}_x) \cdot \frac{\partial \hat{c}_x}{\partial x_R} + \frac{\frac{\partial \mathcal{U}_R(x_R)}{\partial x_R}}{\mathcal{U}_R(x_R) - \mathcal{U}_R(x_L)}$, where $H = f/(1-F)$, so we have:

$$\frac{\partial^2 \ln(\widehat{\Pi}_R)}{\partial \tilde{c}\, \partial x_R} = -\frac{\partial H(\hat{c}_x; \tilde{c})}{\partial \tilde{c}} \cdot \frac{\partial \hat{c}_x}{\partial x_R}.$$

Note that:

$$\begin{aligned}\frac{\partial H(\hat{c}_x;\, \tilde{c})}{\partial \tilde{c}} &= \frac{-f'(\hat{c}_x;\, \tilde{c}) \cdot (1 - F(\hat{c}_x;\, \tilde{c})) - f(\hat{c}_x;\, \tilde{c}) \cdot f(\hat{c}_x;\, \tilde{c})}{(1 - F(\hat{c}_x;\, \tilde{c}))^2} \\ &= \frac{-f'(\hat{c}_x;\, \tilde{c}) \cdot (1 - F) - f(\hat{c}_x;\, \tilde{c})^2}{(1 - F(\hat{c}_x;\, \tilde{c}))^2} < 0,\end{aligned}$$

where the inequality follows because log-concavity of f implies log-concavity of $1-F$, which in turn implies that the hazard rate $f/(1-F)$ is increasing, with $\frac{d}{dc}\left(\frac{f(c;\tilde{c})}{1-F(c;\tilde{c})}\right) = \frac{f'(c;\tilde{c}) \cdot (1-F(c;\tilde{c})) + f(c;\tilde{c})^2}{(1-F(c;\tilde{c}))^2} > 0$. Thus, since $\frac{\partial \hat{c}_x}{\partial x_R} > 0$ in our setting, we have $\frac{\partial^2 \ln(\widehat{\Pi}_R)}{\partial \tilde{c}\, \partial x_R} = -\frac{\partial H}{\partial \tilde{c}} \cdot \frac{\partial \hat{c}_x}{\partial x_R} > 0$, so $\ln(\widehat{\Pi}_R)$ has strictly increasing differences in $(x_R, \tilde{c})$.

Step 3. Altogether, we have established that: (i) under the sufficient conditions in Step

1—subsuming our baseline model, in which $\frac{\partial^2 \hat{c}_x}{\partial x_L \partial x_R} = 0$—the game is log-supermodular in strategies (x_L, x_R), and (ii) each party J's objective $\ln(\widehat{\Pi}_J)$ has strictly increasing differences in $(x_J, \tilde{c})$. Under these conditions, standard results on supermodular games imply that PSNE exist and increasing $\tilde{c}$ will: (1) increase platforms in extremal equilibria and (2) from any equilibrium, induce best-reply dynamics that increase platforms (Vives 2005).