Electoral Competition with Targeted Voting Costs*

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Abstract

How do voting laws impact elections? We highlight how laws targeting a specific group of citizens can have weak effects on turnout and vote shares but substantial effects on policy platforms, thereby influencing substantive representation even if there are no observable effects on participation. To parse these effects, we analyze a model of electoral competition with endogenous turnout and targeted voting costs. Each party anticipates the direct effect of raising one side's voting costs: discouraging targeted citizens from voting. Consequently, both platforms shift towards the untargeted group. These platform adjustments mobilize targeted citizens and demobilize the untargeted, muting the net impact on turnout and vote shares—consistent with empirical evidence for small electoral effects. Policy effects, however, hurt targeted citizens and their aligned party. The targeted group's size amplifies these effects. Our results address party competition, participation, representation, and normative and empirical evaluations of voting laws.

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Introduction

In recent years, election law watchdogs have documented a "tidal wave" of new restrictive voting laws in the United States (Brennan Center 2021). Since 2021, state legislatures have enacted numerous laws making it harder for citizens to register to vote, stay registered, or cast their ballots—ranging from strict voter identification requirements to laws that criminalize passing out water to people waiting at the polls (Brennan Center 2021, 2023). A common complaint is that these laws appear *partisan*, with right-leaning state legislatures changing rules in ways designed to make it harder for left-leaning citizens to vote. Voting rights advocates have issued dire warnings about these new laws, stating that "we need to be very, very serious about this moment" and that "[o]ur democracy is in peril" (Johnson 2021). Scholars have also joined the fray, using these laws as a basis to conclude that states have been sites for significant democratic backsliding (Grumbach 2023).

Despite these concerns, the empirical evidence on the electoral impact of restrictive voting laws has been muted. Across a range of policies, researchers studying the impact of these laws on turnout and party vote shares have found little evidence of effects large enough to actually swing elections (Cantoni, Pons and Schafer 2025). Summarizing the available evidence, Grimmer and Hersh (2024) conclude that "the laws have small effects on turnout and essentially no effect on partisan advantage." This suggests a mismatch between the uproar surrounding recent restrictive voting laws and their actual impact. Such seemingly incongruous public and scholarly debates over voting laws are not new either, as earlier expansionary reforms were similarly contested on partisan grounds despite muddled evidence of electoral consequences (Berinsky 2005). One central lesson from those debates is that the electoral consequences of voting laws can depend not only on their direct effects on turnout, but also on indirect effects that can offset or amplify them (Burden et al. 2014).

In this paper, we offer a new perspective on voting laws by studying how they can affect not only voter participation, but also the substantive *representation* of voters. Such representation depends on the policies that parties choose; yet debates about voting laws have largely overlooked their potential indirect effects on party competition over policy—a classic electoral force (Downs 1957). Despite recent evidence that voting laws have affected policy in various contexts (Cascio and Washington 2014; Fujiwara 2015; Bertocchi et al. 2020),[1] our limited theoretical understanding of the role of party competition continues to impede sharper empirical and normative insights into the policy consequences of voting laws. We

[1] Fujiwara (2015) links the adoption of easier voting technology to higher spending on healthcare. Bertocchi et al. (2020) studies how pre-registration policies can increase education spending by increasing participation among young voters. A handful of studies link the Voting Rights Act to improved policy outcomes for Black citizens (Cascio and Washington 2014; Bernini, Facchini and Testa 2023).

analyze how voting laws targeting particular groups, whether they increase or decrease costs, shape both which citizens turn out to vote and which platforms parties adopt.

In short, our theory suggests that even when a voting law does not visibly change turnout, it can still change the policies that parties choose in a way that affects substantive representation. When a law makes it harder for a particular group to vote, parties expect that fewer people from that group will participate. In response, both parties shift their platforms away from the targeted group's preferences, as those voters now matter less in the competition for votes. Importantly, these strategic policy shifts can undo much of the turnout effect we might otherwise expect: the targeted group may become more motivated to vote because the new platforms are worse for them, while the untargeted group may become less motivated. The net result is that turnout may not change much—making the law *appear* to have little electoral impact—even though policy is now less representative of the targeted group's preferences.

To formalize these ideas, we study a game-theoretic model of electoral competition with endogenous turnout and voting costs that vary by citizen ideology. In our model, policy-motivated parties strategically choose their policy platforms based on citizens' policy preferences and potential voting costs. Voters decide whether to vote and for whom, based on the parties' platforms and their own costs of voting. Crucially, by allowing voting costs to vary with citizens' ideological leanings, we can analyze targeted voting laws that differentially affect specific groups of citizens in a partisan way. Moreover, our setup accommodates uncertainty in forecasting exactly how voting laws will affect voting costs, which may shift, spread, or skew in various ways.

Our model highlights how changes to voting costs can affect not only turnout, but also policy platforms. The direct effect of a targeted increase in voting costs discourages turnout among the targeted group, giving the opposing party an electoral advantage. Both parties respond by shifting their platforms away from the targeted group, but for distinct reasons. The advantaged party, now more secure electorally, converts some of that advantage into policy gains by moving toward its ideal point. The disadvantaged party, facing a harder path to victory, moderates further toward the other side to remain competitive. Because both shift in the same direction, targeted voting restrictions move policy away from the targeted group regardless of who wins the election.

In contrast to these policy effects, we show that the effects on relative voter participation and party vote shares can be minimal. Although partisan voting restrictions have direct effects that discourage the targeted group's turnout, the indirect effects on policy platforms feed back into turnout for both groups of citizens, who vote or abstain based on the adjusted platforms. This feedback blunts the relative impact of the direct effects on turnout by

mobilizing the targeted side and demobilizing the untargeted side. Under expressive voting, this occurs because moderate voters in the targeted group find the other side's platform more offensive and their own side's platform more appealing, while moderate voters in the untargeted group experience the opposite. But this core insight—that resulting platform shifts induce offsetting mobilization effects—is more general, holding under various other turnout motivations including affinity voting. The overall effects of partisan voting restrictions therefore include (i) a direct effect on the targeted side's turnout and (ii) indirect effects on policy platforms *and* voter turnout for *both* sides. Ultimately, these direct and indirect effects combine to mute the electoral impact.

The magnitude of these effects varies with the relative sizes of ideological groups in the electorate. For example, platforms shift farther if the targeted group is larger. Thus, we should not expect large effects when that group is small. Additionally, we show that untargeted voting restrictions impacting both groups equally can shift policy toward the smaller group by moving platforms away from the larger group. Thus, even seemingly neutral voting restrictions can bias policy by shifting platforms away from the preferences of the majority of citizens.

Our results have implications for empirical, normative, and theoretical analyses of restrictive voting laws. On the whole, we aim to continue a cycle between theory and evidence in which empirical findings inform theoretical development and models guide empirical inquiry (Canen and Ramsay 2024). Broadly, we *reinterpret* existing empirical patterns, *elaborate* the implications of an alternative theory that would explain these patterns, and *distinguish* implications of different theories (Ashworth, Berry and Bueno de Mesquita 2021).[2] By constructing a simple but flexible representation of asymmetric changes to voting costs that allows parties to strategically adjust to these changes, we parse understudied channels for empirical researchers to explore further. We isolate the party-competition mechanism transparently and parsimoniously with only enough detail to elucidate the core mechanism, rather than piling on details to approximate "real world" complexities (Paine and Tyson 2020). Major changes to our assumptions could yield different results,[3] but our key insights can be generalized.

On the empirical side, the observed impact of voting laws on turnout and vote shares is consistently muted. There are various potential reasons for this regularity. Recent voting laws in the United States may simply have had small direct effects due to weak targeting

[2]See also Spirling and Stewart (2025), who emphasize the value of "offering new explanations" and "elaborating new implications of those explanations" (pg. 1595).

[3]For example, under affinity voting, extreme polarization in the electorate could dampen the incentive for the targeted party to moderate in response to an increase in voting costs due to fears about alienating the party's base.

or limited enforcement (Grimmer and Hersh 2024), or because they have been enacted in states where their marginal impact is low (Berry, Cox and Haile 2025). Earlier expansive laws may have primarily retained already-engaged voters rather than stimulating new participants (Berinsky 2005). And, in general, direct effects on participation may be offset by indirect effects via non-party actors such as the media and interest groups (Burden et al. 2014).

We highlight a distinct party-driven rationale for muted electoral effects: strategic platform adjustment. Essentially, changes to voting costs may induce platform adjustments that preserve electoral competitiveness. Thus, muted electoral effects can coincide with clear and meaningful platform effects. This has a key implication for empirical analyses of voting laws: when parties strategically adjust platforms in response to voting laws, direct effects on turnout and indirect effects through platforms become intertwined. This strategic interdependence can complicate the identification and interpretation of electoral effects, but analyzing effects on policies and platforms can be more straightforward and fruitful. We discuss these considerations more thoroughly in our Empirical Guidance section and suggest several potentially productive paths forward. More broadly, since our mechanism could operate in various contexts, our analysis can help scholars analyze historical settings where restrictions varied more substantially, evaluate counterfactual policy ideas, and understand why parties may invest resources in enacting voting laws even when electoral benefits appear minimal.

On the normative side, we highlight that voting laws can have welfare consequences even if they appear to have minimal electoral impact. Although scholars have sometimes downplayed normative concerns about voting laws in light of observed muted effects on turnout and vote shares (e.g., Cantoni and Pons 2021), focusing on electoral outcomes can overlook critical policy effects and thus obscure welfare and substantive representation consequences. In particular, (i) partisan voting restrictions can meaningfully affect party and voter welfare by shifting platforms away from the preferences of the targeted group, and (ii) even seemingly neutral restrictions can affect welfare by shifting platforms away from the larger group.

Our results also speak to broader, longstanding concerns with participation and representation. Existing studies on restrictive voting laws have largely focused on how they affect participation. Consistent with research linking participation to policies (Peress 2011; Cascio and Washington 2014; Fujiwara 2015; Bertocchi et al. 2020; Lo Prete and Revelli 2021; Oprea, Martin and Brennan 2024), our analysis of the policy consequences of restrictive voting laws highlights representation—and specifically *substantive representation*, meaning the alignment between policy and the interests of the represented (Pitkin 1967)—as another important consideration. These policy consequences are also relevant to legal analyses of voting restrictions based on the *alignment approach* to election law, which emphasizes this alignment as a

relevant factor in determining the constitutional validity of a law (Stephanopoulos 2024).

On the theoretical side, our analysis emphasizes the feedback between voting behavior and politician behavior by endogenizing both turnout and platforms. Models with costly voting typically fix candidates and focus on voters' turnout (Borgers 2004; Krishna and Morgan 2012; Myatt 2015; Herrera, Morelli and Nunnari 2016; Tyson 2016; Arzumanyan and Polborn 2017), while classic studies of electoral competition do not feature abstention (Downs 1957; Wittman 1983; Calvert 1985).[4] By integrating these behaviors, we contribute to understanding how turnout and platforms impact each other (Ledyard 1984; Adams and Merrill III 2003; Hortala-Vallve and Esteve-Volart 2011; Bierbrauer, Tsyvinski and Werquin 2022).

The key gap we fill is parsing competitive consequences of partisan-biased changes to voting costs. Specifically, we highlight how politicians and parties may shift their platforms if voting costs change for particular groups of citizens. In this vein, Bertocchi et al. (2020) also allow voting costs to differ between voting blocs. That is not their main focus, however, and we allow for more general differences in voting costs in order to analyze a broader range of partisan-biased changes.[5] Additionally, they study purely office-motivated candidates, whereas we study policy-motivated candidates. Due largely to this difference, they include incumbency advantage to generate interesting equilibrium behavior. In contrast, we set incumbency aside because it is not central to our interest in studying targeted voting laws.

Model

We analyze a spatial model of an election with policy-motivated parties making binding campaign platforms. There are two groups of citizens, left-leaning citizens and right-leaning citizens. To vote, each citizen must bear a cost and those costs can differ between the two groups. Moreover, both parties are uncertain about the left group's voting costs—capturing, e.g., uncertainty about the impacts of recent voting restrictions targeting those citizens. Additionally, to reflect that voting blocs can differ in size, we allow the groups to have different shares of the population. In the Appendix, we establish our key results under weaker assumptions.

Players. There are two parties, L and R, as well as a unit mass of citizens. Citizens are split into two groups, with a share α in G_L and the remaining $1-\alpha$ in G_R.

Timing. First, the parties L and R simultaneously choose platforms in the one-dimensional

[4]For overviews of theoretical models, see Feddersen (2004) on voting and see Dewan and Shepsle (2011) on electoral competition.

[5]Their appendix analyzes uniform voting costs, a special case of our log-concave costs.

policy space $X = [-1, 1]$.[6] Second, each citizen chooses whether to vote and, if so, which party to vote for. We refer to citizens who turn out to vote as *voters*. Finally, the party with the greater vote share wins the election and enacts its platform.[7]

Preferences. Both parties are purely policy motivated and prefer closer winning platforms. Specifically, each party j has an associated ideal point $\hat{x}_j$ and its utility from elected platform x is $u_j(x) = -|x - \hat{x}_j|$. Throughout, we assume $\hat{x}_L = -1$ and $\hat{x}_R = 1$. Pure policy motivation and linear loss streamline presentation but are not necessary—our main insights are robust to allowing some office motivation and risk aversion.

Citizens have policy preferences and prefer closer winning platforms, but incur a cost if they vote. Each citizen i has an ideal point, $\hat{x}_i$, a voting cost, c_i, and their utility from elected platform x is $u_i(x) = -|x - \hat{x}_i|$, with overall payoff $u_i(x) - c_i \cdot \mathbb{1}\{i \text{ votes}\}$. Like parties, these particular voter preferences are not crucial—our results are robust to partisan attachments and risk aversion.

Crucially, citizens' ideal points and voting costs are related to group membership. In G_R, citizen ideal points are uniformly distributed on $[0, 1]$ and every citizen has voting cost $c_R > 0$. In G_L, citizen ideal points are uniform on $[-1, 0)$ and every citizen has voting cost $c_L \geq 0$. Uniformly distributed ideal points in each group ease presentation but are not required for our main results.

Information. All features are common knowledge except for c_L, the voting cost for citizens in G_L. In particular, citizens in G_L observe c_L before they decide whether to vote, whereas the parties do not know c_L when choosing policies but they share a common belief represented by a distribution function F that has support $[\underline{c}, \overline{c}]$, where $\underline{c} \geq 0$, and associated density f that is log-concave.[8]

We assume uncertainty about only one side's voting costs as a tractable way to capture the essence of targeted voting laws. The prominent criticism that targeted voting restrictions are partisan suggests that the resulting increase in voting costs will disproportionately impact voters on one side. Yet, the exact intensity of that impact is hard to forecast, so parties choose their platforms without knowing how strongly those targeted citizens will be affected. We capture that disparate but uncertain partisan impact in a stark way. Our distributional assumption is quite general, however, and allows small differences in uncertainty.

Strategies and Equilibrium Concept. For each party j, a pure strategy specifies a policy platform $x_j \in X$. For each citizen i, a strategy is a mapping from policy pairs (x_L, x_R) and

[6] These choices could be committing to platforms or choosing candidates' ideal points.

[7] In our setting, ties occur with probability zero generically. To streamline analysis under non-generic conditions, we assume R wins if there is a tie.

[8] We allow $\overline{c} = \infty$ to accommodate common distributions such as the exponential.

their voting cost c_i to their voting decision of whether to vote for L, vote for R, or abstain. An *equilibrium* is a strategy profile in which: (i) for each pair of platforms, each citizen votes for the closer platform if the difference in policy utility exceeds their voting cost and otherwise they abstain, and (ii) each party's platform maximizes its expected payoff given the other party's platform, citizens' voting strategies, and its beliefs about voting costs.

In our main analysis, citizens' voting strategies are strategically equivalent to *expressive voting*—i.e., citizens receive expressive utility from voting but incur their turnout cost (as in, e.g., Hortala-Vallve and Esteve-Volart 2011). This form of voting and abstention is not crucial for our main insights, as shown in the Appendix. It follows in the tradition of analyzing *sincere voting* strategies (Calvert 1985; Wittman 1983) and, since each citizen votes as if they are pivotal, has the spirit of eliminating undominated voting strategies.[9] It induces *abstention by indifference*—i.e., each citizen votes only if their utility difference between the candidates is large enough—which has empirical support (Jessee 2009, 2010; Shor and Rogowski 2018). Our main insights hold more broadly, including with *affinity voting* that induces *abstention by alienation*—i.e., both candidates are too far away from the voter (as in, e.g., Adams and Merrill III 2003; Llavador 2006; Callander and Wilson 2007; Callander and Carbajal 2022).

Additionally, we focus in the main text on conditions ensuring that no citizen will surely vote for the opposing side's candidate. Specifically, (i) no citizen in G_R will vote for L's candidate and (ii) no citizen in G_L will always—i.e., for all realizations of c_L—vote for R's candidate. This property is substantively plausible and fairly innocuous—as we do allow that some citizens in G_L may have positive probability of *crossover voting*.[10] Furthermore, we are not primarily interested in the occurrence of crossover voting, so this streamlines the presentation.

Analysis

In our analysis, we start with citizens' voting behavior by characterizing how party platforms and voting costs determine who votes and which party they vote for. We then consider the parties and characterize the policy platforms they choose in anticipation of the subsequent voting behavior. Using this foundation, we then analyze how voting costs affect various equilibrium features, such as policies, turnout, who wins the election, and party welfare. In particular, we distinguish between *targeted* versus *untargeted* changes to voting costs. Throughout the analysis, we also track how the size of each group of citizens shapes behavior

[9] With a continuum of citizens, no citizen is ever pivotal, so no voter strategy is dominated.

[10] Specifically, if some centrist citizens in G_L are closer to R's candidate, they would vote for them if c_L is low enough but abstain otherwise.

and mediates the effects of voting costs.

Individual Turnout & Vote Choice

Each citizen has a simple voting calculus: they prefer the closer policy, but will not vote unless it is sufficiently closer than the other policy to outweigh their voting cost. Thus, in our baseline setting any equilibrium abstainers within each participating group will be relatively moderate. Remark 1 characterizes citizens' individual turnout and vote choice decisions.

Remark 1. In equilibrium, citizen i abstains if $|u_i(x_L) - u_i(x_R)| \leq c_i$ and otherwise votes for the closer policy. Thus, if $x_L < x_R$, then citizen i abstains if $c_i \geq x_R - x_L$. If $c_i < x_R - x_L$, then i votes for L's candidate if and only if $\hat{x}_i < \frac{x_L + x_R - c_i}{2}$, and votes for R's candidate if and only if $\hat{x}_i > \frac{x_L + x_R + c_i}{2}$.

Voters, Turnout, & Election Outcomes

We can aggregate individual voting behavior to characterize turnout, vote shares, and electoral outcomes. To streamline presentation in the main text, we focus primarily on *partisan voting*—that is, voters in G_L support L, voters in G_R support R, and abstaining citizens are an interval around 0. It drives the key equilibrium properties and, although low realizations of c_L may induce some *crossover voting* from G_L for some platform pairs (x_L, x_R), in equilibrium that is always less likely than partisan voting.

Party L wins if and only if it receives strictly greater vote share than R. Let the share of citizens who vote for party j, given policies (x_L, x_R) and voting costs $c = (c_L, c_R)$, be denoted $\tau_j(x_L, x_R; c)$. Then, with partisan voting and both marginal voters interior to their groups, L wins if and only if:

$$\begin{aligned}\tau_L(x_L, x_R; c) &= \alpha \left| \left(\frac{x_R + x_L}{2} - \frac{c_L}{2} \right) - (-1) \right| \\ &> (1 - \alpha) \left| 1 - \left(\frac{x_R + x_L}{2} + \frac{c_R}{2} \right) \right| = \tau_R(x_L, x_R; c).\end{aligned}$$

Equivalently, since L's turnout is decreasing in c_L, party L wins if and only if c_L is sufficiently low. Remark 2 characterizes the cutpoint on c_L distinguishing whether L wins in equilibrium and, moreover, each party's win probability (see Lemmas A.1–A.3 in the Online Appendix).

Remark 2. In equilibrium, party L wins if and only if

$$c_L < \frac{1}{\alpha} \left(x_R + x_L + (1 - \alpha)\, c_R + 2(2\alpha - 1) \right) \equiv \hat{c}(x_L, x_R), \tag{1}$$

so the probability that L wins is $F(\hat{c}(x_L, x_R))$ and R wins with complementary probability.

Remark 2 highlights that—keeping in mind that we have not yet considered the parties' equilibrium policy choices—party L's electoral prospects are affected by the platforms (x_L, x_R), voting costs for right-leaning citizens (c_R), and G_L's population share (α). First, if x_R shifts rightward, then marginal voters on both sides will behave less favorably toward R. Specifically, (i) marginal voters in G_L become more concerned about R winning, so they become more inclined to turn out and vote for L; and (ii) marginal voters in G_R become less excited about R winning, so they become less inclined to turn out and vote for R. Second, if x_L shifts rightward, then marginal voters on both sides grow more favorable towards L, so turnout increases on the left and decreases on the right. Third, if c_R increases, then marginal voters in G_R will be less inclined to vote and thus turnout decreases among right-leaning citizens. Fourth, if α increases, there are both direct and indirect effects on L's electoral prospects. Most straightforwardly, an increase in α implies a greater share of left-leaning citizens, which improves L's electoral prospects.

Furthermore, the magnitudes of those direct effects are shaped by interaction effects between policies, voting costs, and population shares. Most importantly, α mediates the other effects: as G_L grows, platform shifts have stronger effects on the mobilization of left-leaning citizens but weaker effects on the demobilization of right-leaning citizens. Conversely, platforms and voting costs mediate α's direct effect by altering which citizens turn out on each side.

Party Competition: Electoral Chances and Policy Platforms

We now analyze party platforms. In general, each party j's expected payoff from a platform pair (x_L, x_R) is

$$\mathbb{E}_{c_L}[u_j(x_L; x_R)] = -|x_L - \hat{x}_j| \cdot Pr(L \text{ wins} \mid x_L, x_R) - |x_R - \hat{x}_j| \cdot (1 - Pr(L \text{ wins} \mid x_L, x_R)).$$

In equilibrium, Remark 2 ensures that $Pr(L \mid x_L, x_R) = F(\hat{c}(x_L, x_R))$, so

$$\mathbb{E}_{c_L}[u_L(x_L; x_R)] = u_L(x_R) + (u_L(x_L) - u_L(x_R)) \cdot F(\hat{c}(x_L, x_R)), \text{ and} \tag{2}$$

$$\mathbb{E}_{c_L}[u_R(x_R; x_L)] = u_R(x_R) + (u_R(x_L) - u_R(x_R)) \cdot F(\hat{c}(x_L, x_R)). \tag{3}$$

When choosing its platform, each party balances two familiar competing incentives. On one hand, choosing a more moderate policy increases its probability of winning by attracting more voters from its own side and turning off voters from the other side. On the other hand, doing so also decreases its benefit from winning, since its policy is farther from its own ideal point. The second term in (2) reflects L's expected gain from winning, whereas the second

term in (3) reflects R's expected loss from losing. These terms are symmetric about zero and increase with the policy divergence.[11]

Using (2) and (3), the parties' marginal utilities from moderating are

$$\frac{\partial \mathbb{E}[u_L(x_L; x_R)]}{\partial x_L} = -F(\hat{c}(x_L, x_R)) + \frac{x_R - x_L}{\alpha} f(\hat{c}(x_L, x_R)), \text{ and} \tag{4}$$

$$-\frac{\partial \mathbb{E}[u_R(x_L; x_R)]}{\partial x_R} = -(1 - F(\hat{c}(x_L, x_R))) + \frac{x_R - x_L}{\alpha} f(\hat{c}(x_L, x_R)). \tag{5}$$

In both (4) and (5), the first term reflects the party's downside of moderation—lower policy payoff conditional on winning—weighted by their probability of winning. Victory becomes less sweet, a sting that becomes especially poignant as the party's chances of winning rise. Meanwhile, the second term reflects the upside—greater probability of winning—weighted by the magnitude of the policy gain. Victory becomes more likely, which is especially valuable when parties' platforms are farther apart.

The marginal incentives can differ between the parties only through differences in their downsides of moderation. Specifically, a favored party would have a larger downside because they would be more likely to win and realize the cost of their moderation, while the other party would face less of a downside because they would probably lose anyway. Consequently, a favored party would be less inclined to moderate than its opponent.

Since both parties face the same upside of converging, however, in equilibrium they will be equally likely to win. Formally, combining (4) and (5) yields $F(\hat{c}(x_L^*, x_R^*)) = \frac{1}{2}$, so we say the election is a *toss-up*. This toss-up property is not sensitive to group size or, crucially, voting costs. The key point here is *not* that the election is even ex ante, since straightforward alterations can tilt electoral prospects toward one side or the other without changing the main takeaways.[12] Instead, as established in Proposition 1, the key point is that targeted changes to voting costs have no impact on ex-ante electoral prospects.[13]

Proposition 1. *In equilibrium, neither party's probability of winning varies with F, the distribution of G_L's voting costs.*

Proposition 1 starkly highlights a more general insight: targeted changes to voting costs can have weak effects on electoral prospects. The underlying mechanism here—meaningful

[11] We use *divergence* following the Downsian tradition, since *polarization* has multiple widespread meanings (Esteban and Ray 1994).

[12] For instance, our affinity voting extension in the Appendix generates similar electoral effects but the win probability can be unequal. A favored party can also result from parties with different risk aversion (Farber 1980).

[13] We provide sufficient conditions ensuring that solutions to (4) and (5) characterize a unique equilibrium in the Online Appendix (Assumption A), as discussed below.

policy effects—stems from a familiar tradeoff in models of electoral competition with policy-motivated agents: electoral success versus policy gains. Each party wants to win the election, which pushes platforms to converge, but they also want to maximize the policy benefits that come with victory, which pulls against convergence. In equilibrium, this tension from electoral competition can mute the electoral impact of targeted voting costs because of endogenous policy responses that rebalance each party's chance of winning. These strategic forces highlight that, although empirical work has understandably focused on electoral outcomes and vote shares because they are easier to measure, those observables may obscure important effects.

The toss-up property does not imply that the election outcome is close. Win probability is an *ex ante* feature, whereas the observed margin is *ex post* and depends on realized voting costs. High costs for the left-leaning group dissuade moderate left-leaning citizens, giving the right party an advantage; low costs do the opposite. But *ex ante*, neither party has a greater chance of winning. Furthermore, the ex ante probability of winning (e.g., $F(\hat{c}(x_L^*, x_R^*))$ for L) differs from ex ante expected vote share (e.g., $\mathbb{E}\left[\frac{\tau_L^*}{\tau_R^*+\tau_L^*}\right]$ for L), which we analyze below. Notably, our toss-up property does not imply that expected vote shares are equal.

Nor does the toss-up property imply that both parties are equally well off. Instead, one party is strictly better off unless equilibrium platforms are symmetric around 0. With any asymmetry in platforms, the electoral balance implies that one party's ideal point is closer to the expected policy—which is the midpoint between the platforms—and thus they have a higher expected payoff.[14] Intuitively, differences in voting costs or group size can leave the parties on unequal competitive footing, leading the disadvantaged party to give up more in policy benefits to remain electorally competitive. Meanwhile, the advantaged party can remain electorally competitive without moderating as far.

To analyze party welfare, we can characterize the expected equilibrium policy, $\frac{x_L^*+x_R^*}{2}$, using (i) the definition of $\hat{c}(x_L, x_R)$ in (1) and (ii) the toss-up election property, $F(\hat{c}(x_L^*, x_R^*)) = \frac{1}{2}$. Specifically, (ii) implies that $\hat{c}(x_L^*, x_R^*)$ equals the median of F, denoted $\tilde{c}$, and thus substituting $\tilde{c}$, x_R^*, and x_L^* into (i) and rearranging yields

$$\frac{x_R^* + x_L^*}{2} = \frac{1}{2}(\alpha\tilde{c} - (1-\alpha)c_R) + (1-2\alpha). \tag{6}$$

Unlike win probability, the expected policy depends on both groups' voting costs and their relative size. Thus, even though they do not affect parties' electoral chances, these factors *do* affect welfare. They do so through two distinct channels, as expressed in (6). First, group size has a direct effect, $(1-2\alpha)$, so that enlarging a group shifts platforms toward that

[14] Each party's expected payoff is equal to its utility from the expected policy since $\hat{x}_L < x_L^* < x_R^* < \hat{x}_R$ and both parties are purely policy motivated with absolute loss.

group's aligned party. Second, voting costs have a direct effect that depends on group size: $\frac{1}{2}(\alpha\tilde{c} - (1-\alpha)c_R)$. That is, higher voting costs for the left-leaning group—more precisely, a higher median—shift policy rightward, and higher voting costs for the right-leaning group shift policy leftward, with the magnitude of each effect increasing in the group's size.

Additionally, (4) and (5) imply the divergence between equilibrium platforms is

$$x_R^* - x_L^* = \frac{\alpha}{2\, f(\tilde{c})}. \tag{7}$$

Divergence is driven by uncertainty about the voting costs of citizens in G_L. The key component is $f(\tilde{c})$, which reflects party-level uncertainty about G_L's voting cost on the electoral margin (i.e., around $\tilde{c}$). Greater uncertainty about these costs implies less density around $\tilde{c}$ and thus a lower value of $f(\tilde{c})$, which increases equilibrium platform divergence, and vice versa. Group size again plays a mediating role, as it affects the magnitude of this force and, in turn, scales platform divergence. The uncertainty over costs plays a similar role in inducing divergence as uncertainty over the median voter's ideal point in models such as Wittman (1983), Calvert (1985), and Groseclose (2001).

Together, (6) and (7) pin down equilibrium platforms, expressed in Proposition 2.

Proposition 2. *In equilibrium, the party platforms are:*

$$x_L^* = (1-2\alpha) + \frac{1}{2}(\alpha\tilde{c} - (1-\alpha)c_R) - \frac{\alpha}{4\, f(\tilde{c})} \quad \textit{and} \tag{8}$$

$$x_R^* = (1-2\alpha) + \frac{1}{2}(\alpha\tilde{c} - (1-\alpha)c_R) + \frac{\alpha}{4\, f(\tilde{c})}. \tag{9}$$

Equilibrium platforms are determined by the midpoint and divergence, which is symmetric about the midpoint. Thus, they depend on the same forces that affect those features. The midpoint depends on the levels of voting costs, c_R and $\tilde{c}$, as well as group size, α. Divergence also depends on α, but the key factor is uncertainty about voting costs in G_L. Our characterization highlights that we can distinguish how beliefs about partisan voting costs can affect both platforms in different ways.

Thus far, we have characterized equilibrium behavior by analyzing the unique solution of two necessary first-order conditions. In the Appendix, we provide technical details establishing sufficient conditions on parameters for that solution to be the unique equilibrium. These conditions ensure that neither party has a profitable deviation and that the candidate platforms lie in the interior voting region used above. Notably, they rule out nonlocal deviations that arise from the baseline's linear-loss specification.[15]

[15] With linear loss, a citizen's maximum policy stake from voting is bounded by the distance between the

The sufficient conditions we provide are stronger than required for our platform-response mechanism. In particular, the baseline analysis maintains $\alpha > \frac{1}{2}$, so G_L, the group whose voting costs are uncertain, is larger than G_R. This restriction helps keep the closed-form solution in the relevant interior region, and it is substantively plausible in many applications, where targeted constituencies are large. But it is not crucial for the platform-response mechanism, as we show in the Appendix.

Effects of Voting Costs

We now analyze how changes in voting costs impact equilibrium policies, turnout, vote shares, representativeness, and party welfare. We consider two types of changes in voting costs: (i) *targeted* changes, which affect only one group of citizens, and (ii) *untargeted* changes, which affect both groups. For targeted changes, we consider either a straightforward shift for c_R or a distributional shift for c_L. Specifically, we analyze location shifts of F, which move the median ($\tilde{c}$) one-for-one while fixing the density at the median, $f(\tilde{c})$. For untargeted changes, we uniformly shift both groups' voting costs by ε.[16]

Policies. We start by analyzing how equilibrium platforms vary with voting costs. Proposition 3 follows from Proposition 2 and states these effects.[17] First, increasing the median cost for one group of citizens—a targeted change—shifts policies away from that group. Second, an untargeted increase in voting costs shifts platforms away from the larger group and towards the smaller group.

Proposition 3. *In equilibrium, each party's platform: (i) increases by $\frac{\alpha}{2}$ as $\tilde{c}$ increases, (ii) decreases by $\frac{1-\alpha}{2}$ as c_R increases, and (iii) changes by $\alpha - \frac{1}{2}$ if all voting costs increase equally.*

The key insight in Proposition 3 is that targeted changes to voting costs will shift both parties' policy platforms away from the targeted side. Essentially, the direct effect of targeted voting costs will encourage rightward platform shifts, which have reinforcing equilibrium effects that push further rightward. The direct effect is that citizens in G_L become less inclined to turn out, so L is more concerned with mobilizing them and R is less concerned

two platforms. As a result, a party that deviates far enough can move the election into regions not governed by the local first-order conditions: for some cost realizations, the targeted group may not vote at all, or one party may win regardless of the realized voting cost. The conditions in the Appendix (Assumption A) render such deviations unprofitable.

[16] For c_L, an original distribution F_0 with support $[\underline{c}, \overline{c}]$ is shifted to F_1 with support $[\underline{c} + \varepsilon, \overline{c} + \varepsilon]$, median $\tilde{c}_0 + \varepsilon \equiv \tilde{c}_1$, and $f_0(\tilde{c}_0) = f_1(\tilde{c}_1)$.

[17] Throughout, comparative statics are marginal effects derived by taking partial derivatives.

with mobilizing its own support. Thus, since in equilibrium each party's win probability increases if and only if they shift inward,[18] both parties are more inclined to shift rightward. Such platform adjustments also affect party incentives but, under broad conditions, that equilibrium effect reinforces itself: L shifting rightward makes R more inclined to shift rightward as well, and vice versa. Intuitively, L anticipates more downside from losing while R anticipates less, so L is even more inclined to mobilize voters while R is less. Figure 1 illustrates these effects.

Crucially, Proposition 3 sharpens our core insight from Proposition 1: targeted changes to voting costs can have weak effects on electoral prospects *that are muted due to meaningful policy effects that push away from the targeted side*. Thus, observed electoral effects are attenuated relative to the *ceteris paribus* direct effect and, furthermore, those weak effects can mask important policy consequences. Our baseline model makes this point starkly but this core insight is more general. It does not require linear policy utilities, uniform ideal point distributions, or citizens' expressive motivation for turnout. In the Appendix, we study a smooth version of our model without imposing those assumptions. We show that if the electoral cutoff—the c_L at which parties have equal vote share—increases in each platform, and those platform effects are sufficiently mild both individually and jointly, then the game is log-supermodular. Thus, increases in G_L's voting costs (under location shifts of F) will shift both platforms rightward (Proposition A.7; Vives 2005), thereby muting electoral effects.

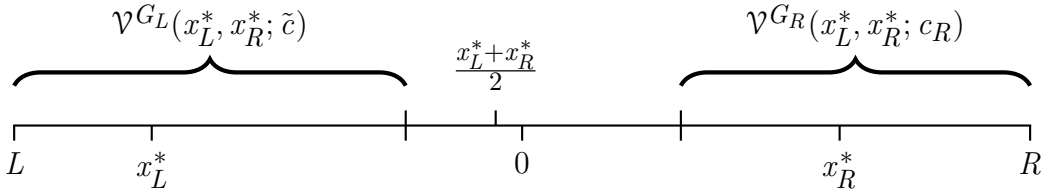
To illustrate how these results might matter in practice, consider a few examples from the empirical literature on the effects of restrictive voting laws. One focus of the literature has been voter ID laws. Using detailed microdata from Texas, Fraga and Miller (2022) find that these laws disproportionately affect Black and Latino voters, who lean Democratic. Our theoretical results thus imply that these laws would shift party platforms rightward. Alternatively, consider same-day registration and pre-registration. Prior research suggests that these policies reduce voting costs for young citizens, who lean Democratic (Grumbach and Hill 2022; Bertocchi et al. 2020). Thus, our model implies that these policies would shift party platforms leftward. Indeed, consistent with this prediction, Bertocchi et al. (2020) find evidence that pre-registration policies raise education spending. Lastly, consider changes to polling place locations and long lines at the polls (e.g., Bagwe, Margitic and Stashko 2023; Pettigrew 2021). If these policies target specific groups of voters (e.g., Democratic-leaning urban voters), we would expect party platforms to shift away from the targeted group's preferences.

Proposition 3 also reveals that group size mediates the effects of voting costs, whether targeted or untargeted. As one group grows, both parties are more sensitive to changes in

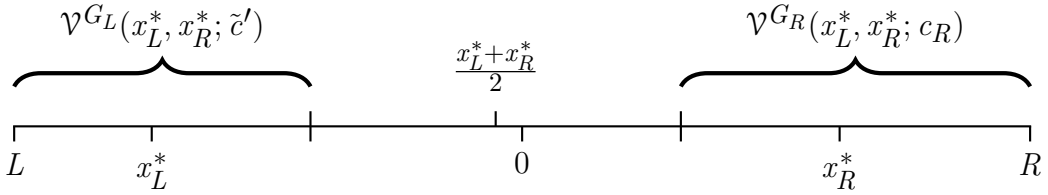
[18] Equilibrium platforms must balance higher win probability against lower policy utility.

Figure 1. Effects of changing G_L's voting costs
(by shifting $\tilde{c}$, the median of F)

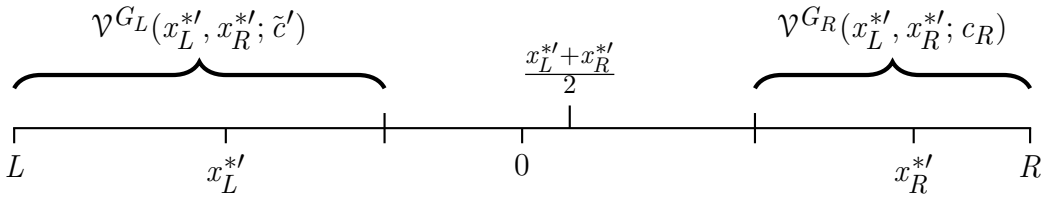
(a) Equilibrium behavior given $\tilde{c}$ & c_R:



(b) Direct effect of $\uparrow \tilde{c}$ to $\tilde{c}'$ on voting behavior:



(c) Equilibrium effects of $\uparrow \tilde{c}$ to $\tilde{c}'$ on platforms and voting behavior:



Note: Figure 1 illustrates how increasing $\tilde{c} = \operatorname{med} F$ affects party platforms and voter behavior. The set of citizens in group G_j who turn out, given platforms and realized cost c_j, is denoted $\mathcal{V}^{G_j}(x_L, x_R; c_j)$, with mass τ^{G_j}; for G_L, the brace is drawn at the displayed cost realization (the median $\tilde{c}$ or $\tilde{c}'$), not the expectation. Figure 1(a) depicts a baseline case with voting cost c_R for G_R and median cost $\tilde{c}$ for G_L. The effects of increasing $\tilde{c}$ to $\tilde{c}'$ are shown in panels (b) and (c): given the platforms in (a), panel (b) shows the direct effect on voting behavior—less turnout in G_L—while panel (c) shows the overall effects once platforms and voting behavior adjust in equilibrium. Both platforms shift rightward by the same amount. **Alt text:** Three number lines showing policy space from L to R. Panel (a): baseline equilibrium. Panel (b): direct effect of increased voting costs shrinks left voter set. Panel (c): equilibrium response shifts both platforms rightward.

voting costs. For example, if G_L is larger than G_R—i.e., $\alpha > \frac{1}{2}$—then changing c_L will shift each party's platform further than if the change targeted G_R. Additionally, an untargeted cost increase when $\alpha > \frac{1}{2}$ will shift both platforms rightward. More broadly, we highlight that policy effects of voting cost changes can vary with groups' sizes and disparities, which suggests potentially important interaction between redistricting and voting laws.

Effects of group size. We now consider how group size affects equilibrium platforms.

Proposition 4 characterizes how G_L's population share affects platforms.

Proposition 4. *In equilibrium, as G_L's population share grows (α increases): L shifts its platform by $\frac{\tilde{c}+c_R}{2} - 2 - \frac{1}{4\,f(\tilde{c})} < 0$, and R shifts its platform by $\frac{\tilde{c}+c_R}{2} - 2 + \frac{1}{4\,f(\tilde{c})}$.*

The total effect of α on equilibrium platforms combines two effects. First, there is a *direct effect* of group size, $\frac{\tilde{c}+c_R}{2} - 2$, that affects both platforms and shifts them leftward (since $\frac{\tilde{c}+c_R}{2} < 2$ under Assumption A) as α increases, and vice versa. Essentially, as G_L grows, a larger proportion of citizens is willing to vote for a relatively extreme left platform, and conversely, a smaller share of citizens is willing to vote for a relatively extreme right platform. The direct effect shifts both platforms leftward. Moreover, larger voting costs attenuate this direct group-size effect, since higher $\tilde{c}$ or c_R makes $\frac{\tilde{c}+c_R}{2} - 2$ less negative. Thus, group size amplifies the effect of cost changes, but voting costs mute the direct effect of group size.

Second, there is an *uncertainty effect* that shifts L's platform leftward by $\frac{1}{4\,f(\tilde{c})}$ and R's platform rightward by that same distance, fueling divergence. As α increases, aggregate turnout in G_L becomes more sensitive to c_L, creating greater uncertainty about the location of the turnout cutoff and causing parties to diverge further.

In addition to studying how group size affects party platforms, we also analyze how it affects the distance between party platforms, or *divergence* $(x_R^* - x_L^*)$. Group size (α) does not generate divergence through its direct effect on platforms. Instead, divergence results from parties' policy motivation and electoral uncertainty. Yet, α does indirectly affect the *magnitude* of equilibrium divergence. By amplifying the uncertainty effect, it increases divergence between equilibrium platforms even as their midpoint shifts leftward. Corollary 1 establishes that, as usual in spatial electoral models, greater electoral uncertainty increases divergence.

Corollary 1. *In equilibrium, $\frac{\partial(x_R^* - x_L^*)}{\partial\alpha} = \frac{1}{2\,f(\tilde{c})}$, so platforms diverge more as the share of citizens in G_L grows.*

Turnout. We now focus on equilibrium turnout. Turnout is widely studied in empirical work on voting laws. It reflects the *ex post* side of the *ex ante* toss-up property that parties are equally likely to win in equilibrium. As previously noted, there may be *ex post* differences in turnout even if elections are *ex ante* toss-ups. Indeed, the groups' vote totals are balanced at the median cost realization—which in equilibrium is the electoral cutoff—but their expected turnout generally differs, so differences in expected turnout need not indicate which party is more likely to win. Our results have implications for empirical studies of how restrictive voting laws affect turnout and vote shares, which we elaborate below in the Empirical Guidance

section. To streamline discussion of our main insights, we focus on partisan voting, so each party's vote total coincides with its group's turnout ($\tau_j = \tau^{G_j}$).[19]

Let $\tau^{G_L,*}$ denote realized equilibrium turnout for G_L and define $\tau^{G_R,*}$ analogously. Given a realized $c_L < x_R^* - x_L^*$, we have $\tau^{G_L,*} = \alpha\big(2(1-\alpha) + \frac{1}{2}(\alpha\,\tilde{c} - (1-\alpha)c_R - c_L)\big)$; once c_L exceeds $x_R^* - x_L^*$, the entire group G_L abstains and $\tau^{G_L,*} = 0$. Let $\rho \equiv F(x_R^* - x_L^*) \in (\frac{1}{2}, 1)$ denote the probability that G_L participates. Group G_R's turnout is the constant $\tau^{G_R,*} = \alpha(1-\alpha)\big(2 - \frac{1}{2}(\tilde{c} + c_R)\big)$. Because there is positive probability that G_L abstains entirely, its expected turnout $\mathbb{E}[\tau^{G_L,*}]$ integrates the realized expression over the participating realizations $\{c_L < x_R^* - x_L^*\}$. Proposition 5 characterizes how equilibrium turnout varies with different changes to voting costs. An increase in G_L's voting costs reduces each group's expected turnout by different amounts.

Proposition 5. *In equilibrium, under a location shift of F and partisan voting: (i) G_R's turnout decreases by $\frac{\alpha}{2}(1-\alpha)$ as either $\tilde{c}$ or c_R increases, whereas (ii) G_L's expected turnout decreases by $\frac{\alpha}{2}(1-\alpha)\rho$ as c_R increases and by $\frac{\alpha}{2}(1-\alpha)\rho + \alpha(1+x_L^*)\,f(x_R^* - x_L^*)$ as $\tilde{c}$ increases.*

First, consider G_R's turnout. It decreases in response to increased voting costs for *either group*. That is, right-leaning citizens will stay at home in greater numbers not only when their own voting cost increases, but also when the *other* group's voting cost increases. This effect arises due to changes in equilibrium platforms. If platforms remained constant, changing $\tilde{c}$ would not affect G_R's turnout. But since a higher voting cost for G_L emboldens R to adopt a more extreme platform, turnout can decrease for both groups. Moreover, the magnitude of this effect is the same regardless of which group's costs increase. That is, G_R's turnout decreases by the same amount in response to an increase in its own voting costs as it does in response to an increase in the other group's voting costs. Finally, the effect is largest if the groups are evenly split in the electorate.

For G_L, voting costs have turnout effects that are similar but somewhat more complicated. Since c_L is a random variable, G_L's turnout is a random variable. Our focus on G_L's expected turnout reflects its prominence in the empirical literature (e.g., linear regression models involve conditional expectations). Proposition 5 implies that an increase in G_L's voting costs will reduce expected turnout for G_L, through two channels: fewer citizens vote in the realizations where G_L participates (the term $\frac{\alpha}{2}(1-\alpha)\rho$), and full abstention becomes more likely (the term $\alpha(1+x_L^*)\,f(x_R^* - x_L^*)$). This net reduction in turnout is muted by equilibrium effects, however, since L shifts its equilibrium platform rightward and counteracts the direct effects of G_L's higher voting costs. Additionally, increasing G_R's voting costs also reduces

[19] We account for low-cost crossover realizations in the Appendix.

G_L's expected turnout. That is, like the other group, G_L's turnout can drop in response to an increase in either group's voting costs, not just its own.

Which group is likely to experience a larger drop in turnout? For increases in G_R's voting costs, the targeted group's expected turnout falls more, because the untargeted group's response is scaled by $\rho \in (\frac{1}{2}, 1)$. For increases in G_L's voting costs, however, the comparison is ambiguous: the difference in the groups' expected turnout responses is $\alpha(1 + x_L^*)\, f(x_R^* - x_L^*) - \frac{\alpha}{2}(1-\alpha)(1-\rho)$, and its sign depends on the primitives. Hence, the effects of targeted voting costs on relative turnout are ex ante unclear.

Finally, beyond targeted increases in voting costs, our model also provides insight into how *untargeted* cost increases might affect turnout. Whenever G_L participates, an untargeted increase in voting costs decreases each group's turnout by the same amount, $\alpha(1-\alpha)$. This effect is especially large when the party supporters are evenly split in the electorate, consistent with evidence that mail-in voting—which effectively decreases voting costs for all voters—seems to increase turnout across the board in evenly divided constituencies (Bonica et al. 2021).[20] Expected responses, however, differ through G_L's participation margin (Proposition 5).

Vote Shares and *Representativeness.* Many scholars are also interested in understanding the relationship between voting costs and two measures that are functions of turnout: (i) vote shares and (ii) the representativeness of voters. First, by *vote shares*, we mean the share of votes cast for one party out of the total votes cast (e.g., $\frac{\tau^{G_L,*}}{\tau^{G_R,*}+\tau^{G_L,*}}$). Scholars have often understood this measure as reflecting electoral competitiveness. Second, by *representativeness* of voters, we mean the similarity between the composition of voters who turn out versus the composition of eligible voters. In our context, representativeness can be measured by how close $\frac{\tau^{G_L,*}}{\tau^{G_R,*}+\tau^{G_L,*}}$ is to α.[21] Scholars have viewed this measure as an indicator of how well policy will align with public interests.

To illustrate how our analysis thus far can shed light on these measures, we fix ideas by focusing on average vote shares and average representativeness. Two widespread intuitions are that increasing voting costs will (i) lead to more imbalanced vote shares, thereby decreasing competitiveness, and (ii) reduce representativeness, thereby decreasing public alignment. Our analysis reveals two insights for these intuitions.

[20] Bonica et al. (2021) find an eight-percentage-point effect for all-mail voting in Colorado that does not meaningfully differ by party. This fairly large effect aligns with our model's predictions since Colorado is roughly evenly split (Colorado Secretary of State 2022).

[21] In our baseline with expressive voting, conditional on a group participating, its extremists vote and abstention occurs among its centrists. It may be the case that one group has a disproportionate share of turnout. This latter aspect of representativeness is our focus.

First, shifting the voting cost distribution does not necessarily change either measure—due to shifts in equilibrium platforms. To illustrate, suppose $\mathbb{E}[c_L] = \tilde{c}$ and let F denote the initial distribution of c_L. Then, if $\text{var}(F)$ is small enough, expected vote share is $\mathbb{E}\left[\frac{\tau^{G_R,*}}{\tau^{G_R,*}+\tau^{G_L,*}}\right] \approx \frac{\tau^{G_R,*}}{\tau^{G_R,*}+\mathbb{E}[\tau^{G_L,*}]} \approx \frac{1}{2}$ for both parties. Moreover, uniformly shifting F rightward has approximately no effect on average vote share or average representativeness. More generally, under a location shift the vote margin is unchanged at every matched participating realization, so any effect on these measures runs through turnout levels and the possibility of full abstention—features of F beyond the median and $f(\tilde{c})$ that pin down platforms. This observation is a stark illustration of a more general point: shifting the cost distribution is not sufficient to observe changes in these measures, as it is important to understand *how* that distribution changes.

Second, changes in average vote share or representativeness can occur without affecting expected policy payoffs. To illustrate, consider an initial cost distribution F that has $\mathbb{E}_F[c_L] = \tilde{c}$, and two different shifts in the cost distribution, F' and F'', that each increase the median cost to $\tilde{c}'$ but differ in that only F' preserves the equality of expected cost and median cost. Formally, $\tilde{c} < \tilde{c}' = \text{median}(F') = \text{median}(F'')$ and $\mathbb{E}_{F'}[c_L] = \tilde{c}' \neq \mathbb{E}_{F''}[c_L]$. Then, after either shift, expected payoffs change in the same way. In contrast, the effects on average vote share and average representativeness generally differ across the two shifts, since these measures depend on features of F beyond those that determine expected payoffs. Thus, changes in either of these measures, or the lack thereof, are not necessarily informative about welfare effects (as measured in our model by policy payoffs).

These observations have implications for interpreting empirical patterns in vote shares. For example, consider voter ID laws. As previously noted, prior research has found that these laws disproportionately affect Black and Latino voters, who lean Democratic (Fraga and Miller 2022). Based on this disparate impact, many scholars have hypothesized that voter ID laws will reduce Democratic turnout more than Republican turnout, thus decreasing Democratic vote shares. Our theoretical results imply that this hypothesis is not necessarily warranted, as an increase in voting costs targeting one group can reduce turnout for both groups, even if the targeting is perfect. We should therefore expect voter ID laws to reduce turnout among Democrats more than Republicans only under certain conditions about the shift in distribution, which could hold in some cases but not in others. This may partly explain the relatively small effects on vote share that have been observed empirically. Critically, however, this does *not* mean that voter ID laws are unimportant for electoral competition—the effects may simply be surfacing in the policies chosen rather than observed vote shares.[22]

[22]These implications for turnout and vote shares also apply to voting laws that facilitate voting, such as same-day registration and pre-registration, which prior research suggests reduce voting costs for young and

Party Welfare. We now study how each party's *ex ante* equilibrium payoff changes with voting costs and group size. By doing so, we shed light on how strongly each party would want to change voting costs under different conditions.

Because the game is constant-sum for the parties, we analyze only one party. From the *ex ante* perspective, L's equilibrium welfare is

$$U_L^* = -1 - \frac{x_L^* + x_R^*}{2} = \frac{1}{2}((1-\alpha)\, c_R - \alpha\tilde{c}) - 2(1-\alpha).$$

Proposition 6 shows how party welfare varies with voting costs and group size.

Proposition 6. *In equilibrium, party L's welfare: (i) decreases by $\frac{\alpha}{2}$ as $\tilde{c}$ increases, (ii) increases by $\frac{1-\alpha}{2}$ as c_R increases, (iii) changes by $\frac{1}{2} - \alpha$ as all voting costs increase equally, and (iv) increases by $2 - \frac{1}{2}(c_R + \tilde{c})$ as α increases. Party R's welfare effects are symmetric.*

Unsurprisingly, each party benefits when their side's voting cost decreases or the other side's voting cost increases. The size of the affected group amplifies these gains and losses, though the magnitude of these effects depends on the functional form of the parties' preferences.

An increase in group size also affects party welfare, by shifting the equilibrium platform midpoint—which alone determines the parties' expected payoffs—toward the growing group, while the aligned party shifts its platform toward its ideal point to convert some of these constituency gains into policy gains. But voting costs moderate this effect: with higher voting costs, citizens are more inclined to stay home, muting the effects of group size. Lastly, an *untargeted* increase in voting costs, expressed in the third comparative static, hurts the party aligned with the larger group and helps the party aligned with the smaller group.

Empirical Guidance

Empirical researchers studying restrictive voting laws have primarily focused on their effects on turnout and vote shares. Our model has implications for empirically analyzing these quantities. A key challenge is that parties may strategically adjust platforms in response to voting laws, so direct and indirect effects become entangled. We discuss what different empirical approaches can identify and suggest productive paths forward.

Consider a hypothetical experiment randomizing voter ID requirements across constituencies. Let $D_i \in \{0, 1\}$ denote treatment assignment for constituency i, $X_i = (x_{L,i}, x_{R,i})$ denote party platforms, and Y_i denote turnout. The causal structure consists of a direct effect ($D_i \to Y_i$) through voting costs and an indirect effect ($D_i \to X_i \to Y_i$) through

thus Democratic-leaning citizens (Grumbach and Hill 2022; Bertocchi et al. 2020).

platform adjustments. Randomization ensures treatment independence: $D_i \perp\!\!\!\perp (Y_i(0), Y_i(1))$, which identifies the total average treatment effect $\text{ATE}_{\text{total}} = \mathbb{E}[Y_i(1) - Y_i(0)] = \mathbb{E}[Y_i|D_i = 1] - \mathbb{E}[Y_i|D_i = 0]$. This estimand captures the overall impact of voting laws on turnout, combining any direct effect through voting costs with any indirect effect operating through platform adjustments. Our model offers a prediction about this total effect: it may be small, even when the direct effect of voting costs on turnout is meaningful.

Researchers may seek to isolate the direct effect of voting costs on turnout, holding platforms fixed. For instance, researchers might target the *average controlled direct effect*: $\text{ACDE}(x') = \mathbb{E}[Y_i(1, x') - Y_i(0, x')]$, where x' is a fixed platform pair common to all constituencies. Identifying and interpreting this quantity has well-known considerations: *sequential unconfoundedness* must hold, and when treatment-mediator interactions are present, the controlled direct effect varies with the reference value at which the mediator is fixed (Acharya, Blackwell and Sen 2016). In our setting, the natural reference values—platforms under treatment versus control—differ precisely because parties optimally respond to voting costs, making the choice of reference value substantively loaded rather than merely technical.

A deeper issue is that platforms are strategically interdependent. The indirect effect, operating via platform repositioning, occurs because parties anticipate the direct effect on turnout. But the direct effect—how the law changes turnout holding platforms fixed—depends on the platforms, which result from strategic adjustment. Furthermore, platforms are themselves interdependent choices: each party's position responds to the other's. Standard mediation approaches require isolating direct and indirect effects, but here direct and indirect effects are determined jointly rather than sequentially, which is a thorny challenge for standard causal frameworks that wholly set aside strategic choices (Heckman and Pinto 2024).

Our analysis suggests several ways forward. First, analyze platform effects. Platform effects are key theoretical quantities and our analysis provides a clear prediction: targeted increases in voting costs shift both parties' platforms away from the targeted side. Standard estimands for the effect of voting laws on platforms, such as $\mathbb{E}[X_i|D_i = 1] - \mathbb{E}[X_i|D_i = 0]$, are identified by randomization without the complications discussed above. Observing platform shifts in the predicted direction, with heterogeneous magnitudes based on the targeted group's size, would provide evidence for the strategic mechanism we highlight. Recent work analyzing policy effects (Fujiwara 2015; Bertocchi et al. 2020) should encourage more work in that vein, which would complement and help interpret findings on turnout and vote shares.

Second, interpret total effects as equilibrium outcomes. A total effect like the $\text{ATE}_{\text{total}}$ bundles direct and indirect channels but is identified under standard assumptions and policy-relevant since it measures a net impact after voting laws change and parties respond. Proposition 1 shows that platform adjustments can offset direct turnout effects, so negligible

total effects on turnout or vote shares, alongside meaningful platform shifts, would be consistent with our theory. Furthermore, they would suggest that focusing on electoral outcomes alone may obscure important policy consequences.

Third, exploit variation in the timing of voting law changes. Unexpected legal changes close to elections, such as court decisions issued weeks before voting, limit parties' ability to adjust platforms. Comparing effects of changes announced months before elections (allowing full adjustment) versus weeks before (preventing adjustment) can reveal whether strategic responses attenuate direct effects in practice. Similar variation may arise from unexpected polling relocations due to natural disasters (Abramson and Thrower 2025).

Fourth, use empirical approaches that account for key aspects of voter and party behavior underlying electoral outcomes. Estimating a model of party and voter behavior defines these quantities precisely and disciplines how they are recovered from data (Canen and Ramsay 2024). For example, Berry, Cox and Haile (2025) study how voting laws affect U.S. House elections by estimating a model of turnout and vote choices that allows general indirect effects such as changes in party platforms or campaign spending. They find that higher voting costs disproportionately discourage Democratic-leaning citizens on the turnout margin, which is consistent with our interest in asymmetric changes to voting costs.

Concluding Discussion

Debates over restrictive voting laws typically emphasize their effects on turnout and vote shares. Our analysis suggests this focus may miss important policy effects: voting restrictions can reshape party platforms, altering policies and representation even when electoral effects are minimal. Using a formal model of electoral competition with endogenous turnout and targeted voting costs, we show that partisan voting restrictions induce parties to shift platforms away from targeted groups' preferences, creating policy consequences regardless of electoral outcomes. Our findings are relevant to both democratic theorists studying participation and representation, as well as empirical scholars analyzing specific laws.

Our model highlights that the key outcomes are not only *who wins* and *who votes*, but also *what policies result*. Thus, our results reframe the interpretation of common empirical patterns. Null or small effects on vote shares need not indicate voting laws lack consequence. Rather, they may reflect platform adjustments that preserve electoral competitiveness while shifting policy away from targeted groups.

We particularly urge empiricists to look at whether platforms and policies change in response to new restrictive voting laws. If restrictive voting laws discourage turnout, and turnout affects policy, then restrictive voting laws should also affect policy. For reasons

outside our model, the policy effects of restrictive laws may be even more severe in practice. Our results rely on the assumption that parties can choose any available platform location. In practice, however, parties may sometimes nominate extreme candidates who are unwilling to choose moderate policies (Nielson and Visalvanich 2017; Hall 2019). In such cases, the candidate's inability to moderate could cause the other party to move its platform in an even more extreme direction.

Our analysis also yields insight into when parties have the sharpest incentives to enact restrictive voting laws. A key takeaway is that increasing a group's voting costs benefits the opposing party more when the targeted group is larger. It is thus unsurprising that as Texas became more purple, the Republican government instituted a sweeping restrictive voting law targeting Democratic-leaning citizens (Tulin and Sanchez 2023). Conversely, reducing voting costs for one party's supporters will benefit that party most when the opposing group is relatively small—a result that holds even when the reduction is untargeted and impacts all citizens. From this perspective, expanding voting access in states with large proportions of Democratic voters can benefit the Democratic Party even without being deliberately partisan.

We focus on voting costs, but these capture only one class of election laws that have drawn attention from scholars and advocates. Although the models would be different, a similarly broad focus on different outcomes—assessing the policy impact and not just turnout—may shed light on the effects of gerrymandering, voter purges, and other election laws.

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Section A.1 characterizes individual voting and aggregates it to characterize election outcomes. Section A.2 defines the platform pair solving the parties' first-order conditions and states sufficient conditions for it to be the unique equilibrium, which is then shown in Sections A.3 and A.4. Sections A.5 and A.6 derive vote totals and turnout together with the platform and welfare comparative statics. The final two sections (Sections A.7 and A.8) address the robustness of our main platform-shift mechanism, first using a smooth reduced-form turnout environment, and then affinity voting.

It is convenient to summarize platform pairs ordered $x_L < x_R$ by their divergence $\Delta \equiv x_R - x_L$, sum $s \equiv x_L + x_R$, and midpoint $m \equiv s/2$, suppressing dependence on (x_L, x_R) to spare notation. The baseline analysis maintains $\alpha \in (\frac{1}{2}, 1)$, which we use in Lemma A.3 and the sufficient conditions in Assumption A. Section A.7 shows that the platform-shift mechanism does not depend on this condition.

A.1 Voting Behavior and Electoral Outcomes

We first characterize how a single citizen votes (Lemma A.1), aggregate these decisions into each group's vote totals (Lemma A.2), and then determine, for each realization of G_L's voting cost, which party wins (Lemma A.3).

Under linear policy loss, every citizen with ideal point $\hat{x} \notin [x_L, x_R]$ has a policy gain from voting that is bounded above by the platform distance Δ. Thus, we sometimes refer to Δ as the *participation cap* (or *cap*).

Lemma A.1. *Let $x_L < x_R$. Citizens with ideal point $\hat{x}$ and voting cost $c \geq 0$ vote for L if and only if $c < \Delta$ and $\hat{x} < \frac{s-c}{2}$, vote for R if and only if $c < \Delta$ and $\hat{x} > \frac{s+c}{2}$, and otherwise abstain.*

PROOF. For a citizen with ideal point $\hat{x}$, the policy-utility gain from x_L rather than x_R is

$$-|\hat{x} - x_L| + |\hat{x} - x_R| = \text{med}\{-\Delta,\ s - 2\hat{x},\ \Delta\},$$

so the gain from voting for either party never exceeds Δ. Voting for L requires this gain to exceed c, which holds if and only if both $\Delta > c$ and $s - 2\hat{x} > c$. The argument for R is symmetric. □

It is convenient to express vote totals using the auxiliary function $\kappa(z) \equiv \min\{1, \max\{0, z\}\}$, which returns the length of $[0, 1] \cap (-\infty, z]$. Lemma A.2 records each group's vote totals given platforms $x_L < x_R$.

Lemma A.2. *Let $x_L < x_R$.*

(i) If $c_R \geq \Delta$, then no citizen in G_R votes. If $c_R < \Delta$, then

$$\tau_R^{G_R} = (1-\alpha)\,\kappa\Big(1 - \frac{s+c_R}{2}\Big), \quad \textit{and} \quad \tau_L^{G_R} = (1-\alpha)\,\kappa\Big(\frac{s-c_R}{2}\Big);$$

so some citizens in G_R vote for L if and only if $s > c_R$, and all of G_R votes for R if and only if $s \leq -c_R$.

(ii) If $c_L \geq \Delta$, then no citizen in G_L votes. If $c_L < \Delta$, then

$$\tau_L^{G_L} = \alpha\,\kappa\Big(1 + \frac{s-c_L}{2}\Big), \quad \textit{and} \quad \tau_R^{G_L} = \alpha\,\kappa\Big(-\frac{s+c_L}{2}\Big);$$

so some citizens in G_L vote for R if and only if $c_L < -s$, and all of G_L votes for L if and only if $c_L \leq s$.

PROOF. Follows immediately from Lemma A.1. If $c_L < \Delta$, then citizens in G_L who vote for L have $\hat{x} < \frac{s-c_L}{2}$, which is a set with mass equal to α times the length of $[-1,0) \cap (-\infty, \frac{s-c_L}{2})$, i.e., $\alpha \cdot \kappa(1 + \frac{s-c_L}{2})$. The G_L citizens who vote for R have $\hat{x} > \frac{s+c_L}{2}$, which is a set with mass equal to $\alpha \cdot \kappa(-\frac{s+c_L}{2})$. If $c_L \geq \Delta$, then every citizen in G_L abstains. The argument for G_R is analogous. □

To characterize the electoral outcome for different realizations of c_L, we use a cutoff that is a piecewise function of the platform sum, s. First, if $s \in [-c_R, c_R]$, then the relevant cutoff is $\hat{c}_0(s) \equiv \frac{s+(1-\alpha)c_R+2(2\alpha-1)}{\alpha}$, which is equivalent to the cutpoint $\hat{c}$ of the main text, but expressed here as a function of $s = x_L + x_R$. If $s < -c_R$, then every participating citizen in G_R votes for R and the cutoff is $\hat{c}_-(s) \equiv s + \frac{2(2\alpha-1)}{\alpha}$. Finally, if $s > c_R$, then some citizens in G_R vote for L and the cutoff is $\hat{c}_+(s) \equiv \frac{(2-\alpha)s+2(2\alpha-1)}{\alpha}$. Collecting these, we define

$$\hat{c}_{\text{reg}}(s) = \begin{cases} \hat{c}_-(s) & \text{if } s < -c_R, \\ \hat{c}_0(s) & \text{if } s \in [-c_R, c_R], \\ \hat{c}_+(s) & \text{if } s > c_R, \end{cases}$$

which is continuous because $\hat{c}_-(-c_R) = \hat{c}_0(-c_R)$ and $\hat{c}_+(c_R) = \hat{c}_0(c_R)$. Since $\alpha > \frac{1}{2}$, both interior cutoffs exceed s, i.e., we have $\alpha \cdot \big(\hat{c}_0(s) - s\big) = (1-\alpha)(s+c_R) + 2(2\alpha-1) > 0$ for every $s \geq -c_R$, and $\alpha \cdot \big(\hat{c}_+(s) - s\big) = 2(1-\alpha)s + 2(2\alpha-1) > 0$ for every $s > 0$. These inequalities are used in the proof of Lemma A.3(iii).

We also define a cutoff in s that identifies the platform sums for which R wins regardless

of c_L. Let $s_{\text{all}} \equiv -\frac{2\alpha-1}{\alpha}$ and $s_{\text{mid}} \equiv -\frac{(1-\alpha)c_R+2(2\alpha-1)}{1+\alpha}$, and define $s_R^0 = \min\{s_{\text{all}}, s_{\text{mid}}\}$, so

$$s_R^0 \equiv \begin{cases} s_{\text{all}} & \text{if } c_R \leq \frac{2\alpha-1}{\alpha}, \\ s_{\text{mid}} & \text{if } c_R > \frac{2\alpha-1}{\alpha}. \end{cases}$$

Lemma A.3. *Let $x_L < x_R$ and $\Delta > c_R$. Then:*

(i) if $s < 0$ and $c_L < \min\{\Delta, -s, s+2\}$, then participating citizens in G_L split between the two parties, and on this range the margin $\tau_L - \tau_R$ is constant in c_L and equals $\alpha \cdot (1+s) - \tau_R^{G_R}$;

(ii) if $s \leq s_R^0$, then R wins for every realization of c_L;

(iii) if $s_R^0 < s \leq 1$, then L wins if and only if $c_L < \min\{\hat{c}_{\text{reg}}(s), \Delta\}$; and

(iv) if $s > 1$, then L wins for every realization of c_L.

PROOF. To begin, we characterize the vote margin on the *aligned range* of c_L in which G_L participates and all of its participating citizens vote for L, given by $M(c_L; s) = \alpha \cdot \left(1 + \frac{s-c_L}{2}\right) + \tau_L^{G_R} - \tau_R^{G_R}$ and strictly decreasing in c_L. Inverting $M(\cdot, s) = 0$ yields the cutoff $\hat{c}_0(s)$ if $s \in [-c_R, c_R]$, the cutoff $\hat{c}_-(s)$ if all of G_R votes for R, and the cutoff $\hat{c}_+(s)$ if some citizens in G_R vote for L. The relevant root is thus $\hat{c}_{\text{reg}}(s)$. Now, we establish (i)–(iv).

(i). If $c_L < \min\{\Delta, -s, s+2\}$, then Lemma A.2 applies and both G_L thresholds lie in $[-1, 0]$, so

$$\tau_L - \tau_R = \alpha \cdot \left(1 + \frac{s - c_L}{2}\right) - \alpha \cdot \left(-\frac{s + c_L}{2}\right) - \tau_R^{G_R} = \alpha \cdot (1+s) - \tau_R^{G_R},$$

which is independent of c_L.

(ii). Consider $s \leq 0$. Party R wins for all c_L if and only if the constant margin from (i) is weakly negative and the margin on the aligned range is weakly negative at its left endpoint, i.e., $\hat{c}_{\text{reg}}(s) \leq -s$. If $s < -c_R$, then $\hat{c}_-(s) \leq -s$ holds if and only if $s \leq s_{\text{all}}$. If $s \in [-c_R, c_R]$, then $\hat{c}_0(s) \leq -s$ holds if and only if $s \leq s_{\text{mid}}$. If $c_R \leq \frac{2\alpha-1}{\alpha}$, then $s_{\text{mid}} \leq -c_R$, so the sure-win region is $\{s \leq s_{\text{all}}\}$. If $c_R > \frac{2\alpha-1}{\alpha}$, then $-c_R \leq s_{\text{mid}} < s_{\text{all}}$, so every $s < -c_R$ is also a sure-win and the sure-win region is $\{s \leq s_{\text{mid}}\}$. In both cases it is exactly $\{s \leq s_R^0\}$, and at $s = s_R^0$ the constant margin is zero, so R wins.

(iii). Suppose $s_R^0 < s \leq 1$. For $c_L \geq \Delta$, G_L abstains and any G_R votes for L are insufficient for a strict majority, so R wins. For $c_L < \Delta$, the margin on the aligned range is strictly decreasing in c_L with root $\hat{c}_{\text{reg}}(s)$. On the split-voting range of part (i), the margin is constant and equals the aligned margin at $c_L = -s$, and since $s > s_R^0$ the equivalences in part (ii) give $\hat{c}_{\text{reg}}(s) > -s$, so this constant margin is strictly positive and the cutoff $\hat{c}_{\text{reg}}(s)$ applies on the entire range. Finally, when $c_L \leq s$ and all of G_L votes for L, the inequalities $\hat{c}_0(s) > s$ for

$s \geq -c_R$ and $\hat{c}_+(s) > s$ for $s > 0$, noted after $\hat{c}_{\text{reg}}$ was defined, place these realizations in $\{c_L < \hat{c}_{\text{reg}}(s)\}$. Therefore L wins if and only if $c_L < \min\{\hat{c}_{\text{reg}}(s), \Delta\}$. *(iv).* If $s > 1$, then no citizen in G_L ever votes for R. For $c_L \geq \Delta$, the margin from G_R alone favors L since either some citizens in G_R vote for L and none vote for R, or the margin equals $(1-\alpha)(s-1) > 0$. For $c_L < \Delta$, G_L only adds votes for L. Thus, L wins for every realization. □

The key point in part (i) is that, for sufficiently low c_L, additional participation by G_L adds votes to both parties in exactly offsetting amounts. The margin then depends only on platforms, so changing one platform can flip a positive-probability set of election outcomes. Below, the sufficient conditions in Section A.2 prevent such deviations from being profitable.

A.2 Party Competition

With voting behavior in hand, we turn to the platform choices. This section defines the candidate platform pair that solves the parties' first-order conditions, states sufficient conditions (Assumption A) under which it is the unique equilibrium, and records the implications of those conditions used in the existence and uniqueness arguments (Lemma A.4).

Define $\Delta^* \equiv \frac{\alpha}{2f(\tilde{c})}$, $s^* \equiv \alpha\tilde{c} - (1-\alpha)c_R - 2(2\alpha-1)$, and $m^* \equiv \frac{s^*}{2}$, along with the candidate platforms $x_L^* \equiv m^* - \frac{\Delta^*}{2}$ and $x_R^* \equiv m^* + \frac{\Delta^*}{2}$, so that we have $x_R^* - x_L^* = \Delta^*$, $x_L^* + x_R^* = s^*$, and $\hat{c}_0(s^*) = \tilde{c}$.

The parties' first-order conditions for $s \in [-c_R, c_R]$ pin down a single candidate platform pair. To see that (x_L^*, x_R^*) solves the first-order conditions, consider platform pairs satisfying $s \in [-c_R, c_R]$ at which $\hat{c}_0(s) < \Delta$. The win probability is then $F(\hat{c}_0(s))$, so

$$U_L(x_L, x_R) = -(1 + x_R) + (x_R - x_L)\, F(\hat{c}_0(s)), \text{ and}$$
$$U_R(x_L, x_R) = -(1 - x_L) + (x_R - x_L)\,\big(1 - F(\hat{c}_0(s))\big).$$

The interior first-order conditions yield $F(\hat{c}_0(s)) = \frac{x_R - x_L}{\alpha} f(\hat{c}_0(s))$ and $1 - F(\hat{c}_0(s)) = \frac{x_R - x_L}{\alpha} f(\hat{c}_0(s))$, which together require $F(\hat{c}_0(s)) = \frac{1}{2}$, so $\hat{c}_0(s) = \tilde{c}$ and $x_R - x_L = \frac{\alpha}{2f(\tilde{c})} = \Delta^*$. Solving $\hat{c}_0(s) = \tilde{c}$ then gives $s = s^*$.

Assumption A provides sufficient conditions in terms of primitives—via Δ^*, s^*, x_L^*, x_R^*, as well as s_R^0 and $\hat{c}_-$ from Section A.1—for (x_L^*, x_R^*) to be the unique equilibrium. Conditions (A1)–(A5) restrict F only through the three scalars $\underline{c}$, $\tilde{c}$, and $f(\tilde{c})$. The conditions (A6) and (A7) restrict the shape of F away from its median.

Assumption A. The primitives (α, c_R, F) satisfy the following conditions: (A1) $\tilde{c} < \Delta^*$; (A2) $2c_R < \Delta^*$; (A3) $-c_R < s^* \leq c_R - \dfrac{\alpha}{1+\alpha}(\Delta^* - \tilde{c})$ and $s^* \leq \underline{c}$; (A4) $\Delta^* + |s^*| \leq 2$; (A5)

$s_R^0 < s^* - \Delta^*/2$ and $s^* + \Delta^*/2 \leq 1$; (A6) $\sup_{t \in [\Delta^*, 1-x_L^*]} t\big(1 - F(t)\big) < \dfrac{\Delta^*}{2}$; and (A7) either $x_R^* \geq 1 - c_R$, or $\sup_{s \in [x_R^* - 1, -c_R]} (2x_R^* - s)\, F\big(\hat{c}_-(s)\big) < \dfrac{\Delta^*}{2}$.

The conditions fall into two categories with distinct roles. Conditions (A1)–(A4) ensure the pair (x_L^*, x_R^*) satisfies $s \in (-c_R, c_R)$ with $\hat{c}_0(s) < \Delta$. In (A3), the condition $s^* \leq \underline{c}$ keeps G_L's marginal voter inside the group at every cost realization (used in Section A.5), and the upper bound on s^* strengthens $s^* < c_R$ enough to prevent L from having a profitable deviation to $s \notin [-c_R, c_R]$ for which the participation cap binds (used when analyzing Party L's deviations in the proof of Proposition A.1).

Conditions (A5)–(A7) then rule out the remaining deviations from the candidate pair characterized by the FOCs on $s \in (-c_R, c_R)$. Three kinds of deviations are noteworthy. First, (A5) precludes deviations by either party into its own *sure-win region*, i.e., $\{s \leq s_R^0\}$ for R or $\{s > 1\}$ for L, in which it wins for every realization of c_L.[1] Second, (A6) precludes deviations by R that increase divergence until the participation cap binds, so that L wins exactly when $c_L < \Delta$. Third, (A7) handles deviations by L that push the platform sum below $-c_R$, where all of G_R turns out for R and the win cutoff becomes $\hat{c}_-$, by ensuring they are either infeasible or unprofitable.

These conditions in Assumption A are sufficient but not necessary for the existence and uniqueness results. They hold jointly on a nonempty open set of primitives.[2] Section A.7 shows that our main insights about the platform-shift mechanism do not require them and extend beyond this baseline setting.

Lemma A.4. *Under Assumption A, we have:*

(i) the density f is nonincreasing on $[\tilde{c}, \bar{c}]$ and $\bar{c} > \Delta^*$, so $\rho \equiv F(\Delta^*) \in (\frac{1}{2}, 1)$ and thus the probability of full abstention by G_L is in $(0, \frac{1}{2})$;

(ii) $|s^*| < c_R$, so $-1 \leq x_L^* < 0 < x_R^* \leq 1$ and the marginal G_R voter at the candidate pair is $\frac{s^* + c_R}{2} \in (0, 1)$; and

(iii) $s^* > s_{\text{mid}} \geq s_R^0$ and $\tilde{c} + s^* > 0$, which implies $\max\{0, -s^*\} < \tilde{c}$.

PROOF. For (i), suppose a maximizer of f lies weakly above $\tilde{c}$. Log-concavity implies quasi-concavity, so f is nondecreasing on $[\underline{c}, \tilde{c}]$, and thus $f(c) \leq f(\tilde{c})$ for $c \leq \tilde{c}$. Then $\frac{1}{2} = \int_{\underline{c}}^{\tilde{c}} f(c)\, dc \leq f(\tilde{c})(\tilde{c} - \underline{c}) \leq f(\tilde{c})\, \tilde{c}$, contradicting (A1). Thus, every maximizer of f lies below $\tilde{c}$, so f is nonincreasing on $[\tilde{c}, \bar{c}]$. Using this monotonicity, we have $\frac{1}{2} = \int_{\tilde{c}}^{\bar{c}} f(c)\, dc \leq$

[1] Deviations into the *opponent's* sure-win region are either infeasible or unprofitable, as shown in the proof of Proposition A.1.

[2] For example, Assumption A holds strictly for the exponential density $f(c) = \lambda e^{-\lambda(c - \underline{c})}$ on $[\underline{c}, \infty)$ with $\lambda = .48$, $\alpha = .85$, $c_R = .10$, and $\underline{c} = .30$. These primitives imply platforms $x_L^* \approx -.852$ and $x_R^* \approx .919$.

$f(\tilde{c})(\bar{c}-\tilde{c})$, so $\bar{c} \geq \tilde{c} + \frac{1}{2f(\tilde{c})} > \Delta^*$, using $\Delta^* = \frac{\alpha}{2f(\tilde{c})}$ and $\alpha < 1$. Together with (A1), this places Δ^* in the interior of the support: $\underline{c} \leq \tilde{c} < \Delta^* < \bar{c}$. Because F is strictly increasing there, $F(\tilde{c}) = \frac{1}{2} < F(\Delta^*) = \rho$ and, since $\Delta^* < \bar{c}$, $\rho < 1$. Thus, $\rho \in (\frac{1}{2}, 1)$.

For (ii), we first establish $|s^*| < c_R$. The lower bound in (A3) gives $s^* > -c_R$ directly. For the other direction, (A1) gives $\Delta^* - \tilde{c} > 0$, so the right-hand side of the upper bound in (A3) is strictly below c_R and thus $s^* < c_R$. Next, (A2) gives $c_R < \frac{\Delta^*}{2}$, so $|s^*| < \frac{\Delta^*}{2}$ and therefore $x_L^* = \frac{s^*-\Delta^*}{2} < 0 < \frac{s^*+\Delta^*}{2} = x_R^*$. The triangle inequality and (A4) imply $\max\{|x_L^*|, |x_R^*|\} \leq \frac{|s^*|+\Delta^*}{2} \leq 1$, so $-1 \leq x_L^* < 0 < x_R^* \leq 1$. Finally, for the marginal G_R voter: $s^* > -c_R$ implies $\frac{s^*+c_R}{2} > 0$, while $s^* < c_R$, (A2), and (A4) together imply $s^* + c_R < 2c_R < \Delta^* \leq 2$, so $\frac{s^*+c_R}{2} < 1$.

For (iii), $s_{\text{mid}} \geq s_R^0$ is immediate because $s_R^0 = \min\{s_{\text{all}}, s_{\text{mid}}\}$. To establish $s^* > s_{\text{mid}}$, recall the first condition in (A5): $s^* > s_R^0 + \frac{\Delta^*}{2}$. If $c_R > \frac{2\alpha-1}{\alpha}$, then $s_R^0 = s_{\text{mid}}$, so (A5) gives $s^* > s_{\text{mid}} + \frac{\Delta^*}{2} > s_{\text{mid}}$. If instead $c_R \leq \frac{2\alpha-1}{\alpha}$, then $s_R^0 = s_{\text{all}}$ and it suffices to show $\frac{\Delta^*}{2} > s_{\text{mid}} - s_{\text{all}}$. A direct calculation, using $c_R \geq 0$ for the inequality, yields $s_{\text{mid}} - s_{\text{all}} = \frac{(1-\alpha)\left[(2\alpha-1)-\alpha c_R\right]}{\alpha(1+\alpha)} \leq \frac{(1-\alpha)(2\alpha-1)}{\alpha(1+\alpha)}$. Meanwhile, (A1) and the lower bound in (A3), which is equivalent to $\tilde{c} > \frac{2(2\alpha-1)}{\alpha} - c_R$, yield $\frac{\Delta^*}{2} > \frac{\tilde{c}}{2} > \frac{2\alpha-1}{\alpha} - \frac{c_R}{2}$. Subtracting the two bounds, we have:

$$\frac{\Delta^*}{2} - (s_{\text{mid}} - s_{\text{all}}) > \frac{2\alpha-1}{\alpha}\left(1 - \frac{1-\alpha}{1+\alpha}\right) - \frac{c_R}{2} = \frac{2(2\alpha-1)}{1+\alpha} - \frac{c_R}{2} > 0,$$

where the final inequality combines $\frac{c_R}{2} \leq \frac{2\alpha-1}{2\alpha}$, which holds for this case, with $\frac{2\alpha-1}{2\alpha} < \frac{2(2\alpha-1)}{1+\alpha}$, which follows because $\alpha > \frac{1}{2}$ implies that $2\alpha - 1 > 0$ and that $\frac{2}{1+\alpha} > \frac{1}{2\alpha}$ for $\alpha > \frac{1}{3}$. Thus, $s^* > s_{\text{all}} + \frac{\Delta^*}{2} > s_{\text{all}} + (s_{\text{mid}} - s_{\text{all}}) = s_{\text{mid}}$ in this case.

For the remaining claims, $\hat{c}_0(s^*) = \tilde{c}$ implies the identities $s^* - s_{\text{mid}} = \alpha(\tilde{c} + s_{\text{mid}})$ and $\tilde{c} + s^* = (1+\alpha)(\tilde{c} + s_{\text{mid}})$. Since $s^* > s_{\text{mid}}$, the first identity gives $\tilde{c} + s_{\text{mid}} > 0$, and the second then gives $\tilde{c} + s^* > 0$. Finally, if $s^* \geq 0$, then $\max\{0, -s^*\} = 0 < \tilde{c}$, where $\tilde{c} > 0$ because F is continuous with $F(\tilde{c}) = \frac{1}{2}$, so $\tilde{c} > \underline{c} \geq 0$. If $s^* < 0$, then $\max\{0, -s^*\} = -s^* < \tilde{c}$ by $\tilde{c} + s^* > 0$. □

Lemma A.5. *Under Assumption A, party L wins at (x_L^*, x_R^*) if and only if $c_L < \tilde{c}$, so* $\Pr(L \text{ wins}) = F(\tilde{c}) = \frac{1}{2}$. *The parties' expected payoffs from the candidate pair are*

$$U_L^* = -(1 + x_R^*) + \frac{\Delta^*}{2} = -1 - \frac{s^*}{2},$$
$$U_R^* = -(1 - x_L^*) + \frac{\Delta^*}{2} = -1 + \frac{s^*}{2}.$$

PROOF. By Lemma A.4(ii), $|s^*| < c_R$, so the candidate pair satisfies $s \in (-c_R, c_R)$, and (A2) implies $\Delta^* > 2c_R > c_R$. By Lemma A.4(iii), $s^* > s_{\text{mid}} \geq s_R^0$, and since $s^* < c_R < 1$ by (A2) and (A4), Lemma A.3(iii) applies. Thus, L wins if and only if $c_L < \min\{\hat{c}_0(s^*), \Delta^*\} = \min\{\tilde{c}, \Delta^*\} = \tilde{c}$, where the last equality uses (A1). Since $F(\tilde{c}) = \frac{1}{2}$, each party wins with probability $\frac{1}{2}$. The value formulas follow because the implemented policy is x_L^* or x_R^* with equal probability and $x_L^* + x_R^* = s^*$. □

Combining Lemmas A.4(iii) and A.5, we have that, since $\max\{0, -s^*\} < \tilde{c}$ and L wins at the candidate pair if and only if $c_L < \tilde{c}$, crossover voting by G_L (i.e., $c_L < -s^*$) can occur only at realizations for which L wins.

Substituting m^* and Δ^* into the definitions of x_L^* and x_R^* yields the closed forms

$$x_L^* = (1-2\alpha) + \frac{1}{2}\big(\alpha\tilde{c} - (1-\alpha)c_R\big) - \frac{\alpha}{4f(\tilde{c})},$$
$$x_R^* = (1-2\alpha) + \frac{1}{2}\big(\alpha\tilde{c} - (1-\alpha)c_R\big) + \frac{\alpha}{4f(\tilde{c})},$$

which are the platforms of Proposition 2 of the main text. Propositions A.1 and A.2 below show that (x_L^*, x_R^*) is the unique equilibrium.

A.3 Existence

We now show that (x_L^*, x_R^*) is an equilibrium and that each party has a unique best response to the other party's platform. There are three key components. First, no deviation that shrinks platform divergence below $\Delta^*/2$ is profitable, even if it wins for sure (Lemma A.6). Second, deviations that maintain an interior electoral cutoff are bounded by a benchmark objective that is single-peaked under log-concavity, with its peak at the candidate pair (Lemma A.7). Third, Assumption A rules out the remaining deviations, which involve the participation cap or the sure-win regions. Proposition A.1 combines these to establish the result.

Lemma A.6. *For every platform pair $(x_L, x_R) \in X^2$, we have $U_L(x_L, x_R) \leq -(1 + x_R) + \max\{x_R - x_L, 0\}$ and $U_R(x_L, x_R) \leq -(1 - x_L) + \max\{x_R - x_L, 0\}$. Consequently, under Assumption A: against x_R^*, any x_L with $x_R^* - x_L < \Delta^*/2$ yields $U_L(x_L, x_R^*) < U_L^* = U_L(x_L^*, x_R^*)$; and against x_L^*, any x_R with $x_R - x_L^* < \Delta^*/2$ yields $U_R(x_L^*, x_R) < U_R^* = U_R(x_L^*, x_R^*)$.*

PROOF. Let $p \in [0, 1]$ be the probability that L's platform is implemented. Then, $U_L(x_L, x_R) = -(1 + x_R) + p\,(x_R - x_L)$ and $U_R(x_L, x_R) = -(1 - x_L) + (1 - p)\,(x_R - x_L)$, so both bounds follow

from $p, 1-p \in [0,1]$. For the consequences, set $x_R = x_R^*$. Any x_L with $x_R^* - x_L < \Delta^*/2$ yields $U_L(x_L, x_R^*) \leq -(1+x_R^*) + \max\{x_R^* - x_L, 0\} < -(1+x_R^*) + \frac{\Delta^*}{2} = U_L(x_L^*, x_R^*) = U_L^*$, where the strict inequality uses $\Delta^* > 0$ and the first equality follows from Lemma A.5. Analogously, setting $x_L = x_L^*$, any x_R with $x_R - x_L^* < \Delta^*/2$ yields $U_R(x_L^*, x_R) < -(1-x_L^*) + \frac{\Delta^*}{2} = U_R(x_L^*, x_R^*) = U_R^*$. □

To bound deviations that maintain an interior electoral cutoff, we compare each party's payoff to a benchmark in which the win probability is governed by cutoff $\hat{c}_0(s)$. Define the benchmark payoffs

$$\bar{U}_R(x_R) \equiv -(1-x_L^*) + \Delta\big(1 - F(\hat{c}_0(s))\big) \quad \text{and} \quad \bar{U}_L(x_L) \equiv -(1+x_R^*) + \Delta\, F(\hat{c}_0(s)),$$

where the first fixes $x_L = x_L^*$ and the second fixes $x_R = x_R^*$, with F extended by 0 below its support and by 1 above it.

Lemma A.7. *The function $\bar{U}_R$ is uniquely maximized at $x_R = x_R^*$, and analogously $\bar{U}_L$ is uniquely maximized at $x_L = x_L^*$.*

PROOF. First, we show the result for Party R. Given $x_L = x_L^*$, both s and Δ increase at rate 1 as x_R increases, while $\hat{c}_0(s)$ increases at rate $\frac{1}{\alpha}$. For $\hat{c}_0(s)$ inside the support, we have

$$\frac{d\,\bar{U}_R}{dx_R} = \big(1 - F(\hat{c}_0)\big)\left(1 - \frac{\Delta}{\alpha}\,\frac{f(\hat{c}_0)}{1-F(\hat{c}_0)}\right).$$

If $x_R < x_L^*$, then $\Delta < 0$ and the derivative, expressed $\big(1-F(\hat{c}_0)\big) - \frac{\Delta}{\alpha} f(\hat{c}_0)$, is the sum of a strictly positive and a nonnegative term, so strictly positive. If $x_R \geq x_L^*$, then $\Delta \geq 0$ and the subtracted term in the bracket is the product of $\frac{\Delta}{\alpha}$, which is nonnegative and strictly increasing, and the hazard rate $f/(1-F)$ evaluated at $\hat{c}_0(s)$, which is positive and nondecreasing in x_R since it is a nondecreasing function (by log-concavity) evaluated at the increasing cutoff. The product is strictly increasing, so the bracket is strictly decreasing, and it equals zero at $x_R = x_R^*$, where $\Delta = \Delta^*$, $\hat{c}_0 = \tilde{c}$, and $F(\tilde{c}) = \frac{1}{2}$ yield $\frac{\Delta^*}{\alpha} \cdot 2 f(\tilde{c}) = 1$. Since $1 - F(\hat{c}_0) > 0$ inside the support, the derivative is strictly positive below x_R^* and strictly negative above it, so $\bar{U}_R$ is single-peaked with its unique maximum at x_R^*. Outside the support, the same conclusion holds by continuity, since the payoff is strictly increasing below the support and constant at a lower bound above it.

Next, consider Party L. Given $x_R = x_R^*$, Δ decreases at rate 1 as x_L increases while $\hat{c}_0(s)$

increases at rate $\frac{1}{\alpha}$. For $\hat{c}_0(s)$ inside the support, we have

$$\frac{d\,\bar{U}_L}{dx_L} = F(\hat{c}_0)\left(\frac{\Delta}{\alpha}\,\frac{f(\hat{c}_0)}{F(\hat{c}_0)} - 1\right).$$

The argument mirrors party R's. For $x_L > x_R^*$, the derivative $-F(\hat{c}_0) + \frac{\Delta}{\alpha} f(\hat{c}_0)$ is strictly negative since $\Delta < 0$. For $x_L \le x_R^*$, the leading term in the bracket is the product of $\frac{\Delta}{\alpha}$, which is nonnegative and strictly decreasing, and the reverse hazard rate f/F at $\hat{c}_0(s)$, which is positive and nonincreasing in x_L by log-concavity, so the bracket is strictly decreasing and equals zero at $x_L = x_L^*$ by an analogous argument. Thus, the derivative is strictly positive below x_L^* and strictly negative above it, so $\bar{U}_L$ is single-peaked with its unique maximum at x_L^*. Outside the support, the payoff is constant at a lower bound below the support and strictly decreasing above it, so the conclusion again follows by continuity. □

Proposition A.1. *Under Assumption A, (x_L^*, x_R^*) is an equilibrium, and each party's best response to the opponent's candidate platform is unique.*

PROOF. By Lemma A.6, no deviation that induces divergence less than $\Delta^*/2$ is profitable, so it suffices to consider deviations with $\Delta \ge \Delta^*/2$. These satisfy $\Delta > c_R$ by (A2), so Lemma A.3 applies throughout. For convenience, we abbreviate $U_R(x_L^*, x_R)$ as $U_R(x_R)$ and $U_L(x_L, x_R^*)$ as $U_L(x_L)$. We consider each party in turn.

First, consider Party R. Fix $x_L = x_L^*$ and consider a deviation x_R with $\Delta = x_R - x_L^* \ge \Delta^*/2$, which must satisfy $s = s^* + \Delta - \Delta^*$. By the first condition of (A5), $s \ge s^* - \Delta^*/2 > s_R^0$, so this deviation avoids the region where R wins for sure. Moreover, $x_R \le 1$ and $x_L^* < 0$ (Lemma A.4(ii)) imply $s < 1$, so it also avoids the region where L wins for sure.

For such deviations, the gap $\hat{c}_{\text{reg}}(s) - \Delta$ is continuous and nondecreasing in Δ. Specifically, since s increases one-for-one with Δ, the gap's slope is 0, $\frac{1}{\alpha} - 1 > 0$, and $\frac{2(1-\alpha)}{\alpha} > 0$ where $\hat{c}_{\text{reg}}$ equals $\hat{c}_-$, $\hat{c}_0$, and $\hat{c}_+$, respectively. At $\Delta = \Delta^*$, we have $s = s^*$ and the gap equals $\tilde{c} - \Delta^* < 0$ by (A1). Moreover, $s^* > -c_R$ by (A3), so every $\Delta \ge \Delta^*$ induces $s > -c_R$, and thus the gap has strictly positive slope. The gap is therefore strictly increasing for $\Delta \ge \Delta^*$ and has at most one zero there. Let $\widetilde{\Delta} > \Delta^*$ denote this zero, setting $\widetilde{\Delta} = \infty$ if the gap remains negative up to $\Delta = 1 - x_L^*$.

If $\Delta \in [\Delta^*/2, \widetilde{\Delta})$, then the gap is strictly negative, so $\hat{c}_{\text{reg}}(s) < \Delta$ and Lemma A.3(iii) implies that L wins if and only if $c_L < \min\{\hat{c}_{\text{reg}}(s), \Delta\} = \hat{c}_{\text{reg}}(s)$. The cutoff $\hat{c}_{\text{reg}}(s)$ weakly exceeds the benchmark cutoff $\hat{c}_0(s)$. Specifically, the two coincide for $s \in [-c_R, c_R]$, while $\hat{c}_-(s) - \hat{c}_0(s) = \frac{(1-\alpha)(-s-c_R)}{\alpha} \ge 0$ for $s < -c_R$ and $\hat{c}_+(s) - \hat{c}_0(s) = \frac{(1-\alpha)(s-c_R)}{\alpha} \ge 0$ for $s > c_R$. Since $\Delta > 0$, it follows that $U_R(x_R) = -(1 - x_L^*) + \Delta\big(1 - F(\hat{c}_{\text{reg}}(s))\big) \le \bar{U}_R(x_R)$, and Lemma A.7 gives $\bar{U}_R(x_R) \le \bar{U}_R(x_R^*) = U_R^*$ (Lemma A.5), with strict inequality for $x_R \ne x_R^*$.

If instead $\Delta \in [\widetilde{\Delta}, 1-x_L^*]$, then the gap is weakly positive, so $\hat{c}_{\text{reg}}(s) \geq \Delta$ and Lemma A.3(iii) implies that L wins if and only if $c_L < \Delta$. Then, since $[\widetilde{\Delta}, 1-x_L^*] \subseteq [\Delta^*, 1-x_L^*]$, condition (A6) implies $U_R(x_R) = -(1-x_L^*) + \Delta\big(1-F(\Delta)\big) < -(1-x_L^*) + \frac{\Delta^*}{2} = U_R^*$. Thus, R has no profitable deviation, and its unique best response to x_L^* is x_R^*.

Next, consider Party L. Fix $x_R = x_R^*$ and consider a deviation x_L with $\Delta = x_R^* - x_L \geq \Delta^*/2$, which must satisfy $s = s^* + \Delta^* - \Delta$. The constraints $x_L \geq -1$ and $\Delta \geq \Delta^*/2$ restrict s to $[x_R^* - 1,\ s^* + \Delta^*/2]$. For such deviations, the gap $\hat{c}_0(s) - \Delta$ is strictly increasing in s, with slope $\frac{1}{\alpha} + 1$, and equals $\tilde{c} - \Delta^* < 0$ at $s = s^*$ by (A1). Its unique zero is $s_c \equiv s^* + \frac{\alpha}{1+\alpha}(\Delta^* - \tilde{c}) > s^*$, and the upper bound in (A3) implies $s_c \leq c_R$. Since $-c_R < s^* < s_c < s^* + \Delta^*/2$, the subranges $[-c_R, s_c]$, $(s_c, s^* + \Delta^*/2]$, and $[x_R^* - 1, -c_R)$ together cover all such deviations, and we consider them in turn.

First, suppose $s \in [-c_R, s_c]$. Then, $s \in [-c_R, c_R]$, and the gap is weakly negative, so $\hat{c}_0(s) \leq \Delta$. If $s > s_R^0$, then Lemma A.3(iii) implies that L wins if and only if $c_L < \min\{\hat{c}_0(s), \Delta\} = \hat{c}_0(s)$, so $U_L(x_L) = \bar{U}_L(x_L)$. If instead $s \leq s_R^0$, then R wins for every realization by Lemma A.3(ii), so $U_L(x_L) = -(1 + x_R^*) \leq \bar{U}_L(x_L)$, using $\Delta > 0$. In either case, Lemma A.7 gives $U_L(x_L) \leq \bar{U}_L(x_L) \leq \bar{U}_L(x_L^*) = U_L^*$ (Lemma A.5), with strict inequality for $x_L \neq x_L^*$.

Second, suppose $s \in (s_c, s^* + \Delta^*/2]$. On this range, the participation cap binds, i.e., $\hat{c}_{\text{reg}}(s) \geq \Delta$. For $s \in (s_c, c_R]$, the gap is strictly positive, so $\hat{c}_0(s) > \Delta$. For $s > c_R$, $\hat{c}_+(s) - \Delta$ is strictly increasing in s and weakly positive at $s = c_R$, where $\hat{c}_+(c_R) = \hat{c}_0(c_R) \geq \Delta$. Since $s_R^0 < s^* < s$ (Lemma A.4(iii)) and $s \leq s^* + \Delta^*/2 \leq 1$ by the second condition of (A5), Lemma A.3(iii) implies that L wins if and only if $c_L < \min\{\hat{c}_{\text{reg}}(s), \Delta\} = \Delta$, so $U_L(x_L) = -(1 + x_R^*) + \Delta\, F(\Delta)$. As x_L increases, Δ decreases, and $t\, F(t)$ is weakly increasing for $t \geq 0$, so this payoff is weakly decreasing in x_L. Its supremum over the range is therefore its limit as s approaches s_c from above. At $s = s_c$, the gap is zero, so $\Delta = \hat{c}_0(s_c)$ and this limit equals $\bar{U}_L$ evaluated at the deviation inducing $s = s_c$. Because $s_c > s^*$, that deviation differs from x_L^*, so Lemma A.7 implies the limit is strictly below $\bar{U}_L(x_L^*) = U_L^*$. Every deviation on this range is therefore strictly unprofitable.

Finally, suppose $s \in [x_R^* - 1, -c_R)$, which is empty if the first condition of (A7) holds, since $x_R^* \geq 1 - c_R$ is equivalent to $x_R^* - 1 \geq -c_R$. Otherwise, the event that L wins is a subset of $\{c_L < \hat{c}_-(s)\}$. For $s > s_R^0$, Lemma A.3(iii) implies that the win event is $\{c_L < \min\{\hat{c}_-(s), \Delta\}\}$, while for $s \leq s_R^0$, Lemma A.3(ii) implies that L never wins. With $\Delta = 2x_R^* - s > 0$, it follows that $U_L(x_L) \leq -(1 + x_R^*) + (2x_R^* - s)\, F\big(\hat{c}_-(s)\big) < -(1 + x_R^*) + \frac{\Delta^*}{2} = U_L^*$, where the strict inequality is the second condition of (A7) and the final equality is Lemma A.5. Thus, L has no profitable deviation, and its unique best response to x_R^* is x_L^*. □

A.4 Uniqueness

Proposition A.2. *Under Assumption A, (x_L^*, x_R^*) is the unique pure-strategy equilibrium of the platform game.*

PROOF. For any implemented policy x, $u_L(x) + u_R(x) = -2$. Thus, taking expectations, $U_L(x_L, x_R) + U_R(x_L, x_R) = -2$ at every platform pair, so the platform game is constant-sum and R's best responses minimize U_L. The equilibria of a two-player constant-sum game are interchangeable. To see this, suppose (x_L', x_R') and (x_L^*, x_R^*) are both equilibria. Then $U_L(x_L^*, x_R') \geq U_L(x_L^*, x_R^*) \geq U_L(x_L', x_R^*) \geq U_L(x_L', x_R') \geq U_L(x_L^*, x_R')$, where the first and third inequalities hold because x_R^* and x_R' are best responses for R, which minimizes U_L, and the second and fourth hold because x_L^* and x_L' are best responses for L. Equality therefore holds throughout, so (x_L', x_R^*) and (x_L^*, x_R') are also equilibria. In particular, x_L' is a best response to x_R^* and x_R' is a best response to x_L^*. Since best responses to the candidate platforms are unique by Proposition A.1, we have $x_L' = x_L^*$ and $x_R' = x_R^*$. □

Since the win probability at the candidate pair is $\frac{1}{2}$ for every F satisfying Assumption A (Lemma A.5), neither party's win probability varies with F. With uniqueness, this delivers Proposition 1 of the main text. The constant-sum identity is also used below in comparing party welfare.

A.5 Vote Totals and Turnout

This section computes equilibrium vote totals and the turnout comparative statics behind Proposition 5 of the main text. Throughout, an asterisk denotes evaluation at the equilibrium platforms (x_L^*, x_R^*). Because $\bar{c} > \Delta^*$ (Lemma A.4(i)), the event $\{c_L \geq \Delta^*\}$ in which all voters in G_L abstain has positive probability.

Proposition A.3. *Under Assumption A, citizens in G_R vote only for R, with turnout $\tau^{G_R,*} = (1-\alpha)\left(1 - \frac{s^*+c_R}{2}\right) > 0$, while in G_L the mass of citizens who vote for L is $\tau_L^{G_L,*}(c_L) = \alpha\left(1 + \frac{s^*-c_L}{2}\right)\mathbf{1}\{c_L < \Delta^*\}$, so $\mathbb{E}[\tau_L^{G_L,*}] = \alpha\int_{\underline{c}}^{\Delta^*}\left(1 + \frac{s^*-c}{2}\right)dF(c)$. In particular, all of G_L abstains if $c_L \geq \Delta^*$.*

PROOF. Apply Lemma A.2 at $s = s^*$ and $\Delta = \Delta^*$. For G_R, the participation case applies because $c_R < \Delta^*$ by (A2). By Lemma A.4(ii), we have $s^* < c_R$, so no citizen in G_R votes for L, and the marginal G_R voter $\frac{s^*+c_R}{2}$ is in $(0,1)$, which yields the positive G_R vote total. For G_L, there are two qualitatively distinct types of realizations for c_L. If $c_L \geq \Delta^*$, then no citizen votes. If $c_L < \Delta^*$, then the set in G_L voting for L is $[-1, \frac{s^*-c_L}{2})$, which has mass

$\alpha \cdot \left(1 + \frac{s^* - c_L}{2}\right)$. The threshold $\frac{s^* - c_L}{2}$ is weakly negative because $c_L \geq \underline{c} \geq s^*$ by (A3), but weakly greater than –1 because $c_L < \Delta^* \leq 2 + s^*$ by (A4). Taking expectations over $\{c_L < \Delta^*\}$ gives $\mathbb{E}[\tau_L^{G_L,*}]$. □

Remark A.1. If $s^* \geq 0$, then no citizen in G_L ever votes for R (Lemma A.2(ii)), so G_L's turnout is $\tau^{G_L,*}(c_L) = \tau_L^{G_L,*}(c_L)$, as in the main text. If $s^* < 0$, then citizens in G_L vote for R exactly when $c_L < -s^*$, with mass $\tau_R^{G_L,*}(c_L) = \alpha\left(-\frac{s^* + c_L}{2}\right)$, where the threshold $-\frac{s^* + c_L}{2}$ lies in $[0, 1]$ on this range because $c_L < -s^*$ and $s^* + c_L \geq s^* > -2$. Since $-s^* < \tilde{c} < \Delta^*$ by Lemma A.4(iii) and (A1), crossover occurs with probability $F(-s^*) < F(\tilde{c}) = \frac{1}{2}$, which is strictly positive if and only if $-s^* > \underline{c}$. Total turnout by G_L is then $\tau^{G_L,*}(c_L) = \tau_L^{G_L,*}(c_L) + \tau_R^{G_L,*}(c_L)$, so:

$$\tau^{G_L,*}(c_L) = \begin{cases} \alpha(1 - c_L), & c_L < -s^*, \\ \alpha\left(1 + \dfrac{s^* - c_L}{2}\right), & -s^* \leq c_L < \Delta^*, \\ 0, & c_L \geq \Delta^*. \end{cases}$$

For location shifts of F, write $F_t(c) = F_0(c - t)$, so the median $\tilde{c}$ shifts one-for-one while $f(\tilde{c})$ and Δ^* remain fixed. Recall that $\rho = F(\Delta^*)$ denotes the probability that G_L has positive turnout. Next, Corollary A.1 establishes the comparative statics of G_R's turnout and of the mass of G_L citizens voting for L. Under partisan voting, $\tau_L^{G_L,*}$ coincides with the main text's G_L turnout $\tau^{G_L,*}$, so these comparative statics establish Proposition 5. In general, however, the crossover mass $\tau_R^{G_L,*}$ from Remark A.1 must also be accounted for in G_L's total turnout.

Corollary A.1. *Under location shifts of F, turnout in G_R satisfies $\frac{\partial \tau^{G_R,*}}{\partial \tilde{c}} = \frac{\partial \tau^{G_R,*}}{\partial c_R} = -\frac{\alpha(1-\alpha)}{2}$, while the expected mass of G_L citizens voting for L, i.e., G_L's expected turnout under partisan voting, satisfies $\frac{\partial \mathbb{E}[\tau_L^{G_L,*}]}{\partial c_R} = -\frac{\alpha(1-\alpha)}{2}\rho$ and $\frac{\partial \mathbb{E}[\tau_L^{G_L,*}]}{\partial \tilde{c}} = -\frac{\alpha(1-\alpha)}{2}\rho - \alpha(1 + x_L^*) f(\Delta^*)$.*

PROOF. First, $\tau^{G_R,*} = (1 - \alpha)\left(1 - \frac{s^* + c_R}{2}\right)$ depends on $(\tilde{c}, c_R)$ only through the sum $s^* + c_R$, where we have $\frac{\partial(s^* + c_R)}{\partial \tilde{c}} = \frac{\partial(s^* + c_R)}{\partial c_R} = \alpha$, using $\frac{\partial s^*}{\partial \tilde{c}} = \alpha$ and $\frac{\partial s^*}{\partial c_R} = -(1 - \alpha)$. Differentiating gives both G_R derivatives.

Next, consider the expected mass of G_L citizens voting for L. For the c_R derivative, neither F nor Δ^* depends on c_R, so differentiating the expectation in Proposition A.3 under

the integral yields

$$\frac{\partial \mathbb{E}[\tau_L^{G_L,*}]}{\partial c_R} = \frac{\alpha}{2}\frac{\partial s^*}{\partial c_R}\int_{\underline{c}}^{\Delta^*} dF(c) = -\frac{\alpha(1-\alpha)}{2}F(\Delta^*) = -\frac{\alpha(1-\alpha)}{2}\rho.$$

For the targeted shift, substituting $u = c - t$ in the expectation yields

$$\mathbb{E}[\tau_L^{G_L,*}](t) = \alpha\int_{\underline{c}_0}^{\Delta^*-t}\left(1+\frac{s^*(t)-u-t}{2}\right)f_0(u)\,du.$$

Differentiating with respect to t yields

$$\frac{d}{dt}\,\mathbb{E}[\tau_L^{G_L,*}] = \alpha\cdot\left(\frac{\alpha-1}{2}F_0(\Delta^*-t) - \left(1+\frac{s^*(t)-\Delta^*}{2}\right)f_0(\Delta^*-t)\right),$$

where the first term collects the integrand's derivative, using $\frac{\partial s^*}{\partial t} = \alpha$, and the second is the boundary term at $u = \Delta^* - t$. Since $F_0(\Delta^*-t) = F_t(\Delta^*) = \rho$, $f_0(\Delta^*-t) = f_t(\Delta^*) = f(\Delta^*)$, and $1 + \frac{s^*-\Delta^*}{2} = 1 + x_L^*$, and the median shifts one-for-one with t, this is the stated expression. □

A.6 Platform and Welfare Comparative Statics

This section proves the platform, group-size, and welfare comparative statics of the main text: Propositions 3, 4, and 6, and Corollary 1. All effects are local in the region satisfying Assumption A, and for shifts in F we continue to use location shifts as above, holding $f(\tilde{c})$ and Δ^* fixed.

Proposition A.4. *In the unique equilibrium,* $\frac{\partial x_L^*}{\partial \tilde{c}} = \frac{\partial x_R^*}{\partial \tilde{c}} = \frac{\alpha}{2}$ *and* $\frac{\partial x_L^*}{\partial c_R} = \frac{\partial x_R^*}{\partial c_R} = -\frac{1-\alpha}{2}$*, and an untargeted common increase in both groups' costs shifts each platform by* $\alpha - \frac{1}{2}$*.*

PROOF. Since $x_L^* = \frac{s^*-\Delta^*}{2}$ and $x_R^* = \frac{s^*+\Delta^*}{2}$ with Δ^* fixed under location shifts, the derivatives follow from $\frac{\partial s^*}{\partial \tilde{c}} = \alpha$ and $\frac{\partial s^*}{\partial c_R} = -(1-\alpha)$. A common cost shift adds these two effects. □

Proposition A.5. *In the unique equilibrium,* $\frac{\partial x_L^*}{\partial \alpha} = \frac{\tilde{c}+c_R}{2} - 2 - \frac{1}{4f(\tilde{c})} < 0$ *and* $\frac{\partial x_R^*}{\partial \alpha} = \frac{\tilde{c}+c_R}{2} - 2 + \frac{1}{4f(\tilde{c})}$*, so equilibrium divergence satisfies* $\frac{\partial(x_R^*-x_L^*)}{\partial \alpha} = \frac{1}{2f(\tilde{c})} > 0$*.*

PROOF. Since $x_L^* = \frac{s^*-\Delta^*}{2}$ and $x_R^* = \frac{s^*+\Delta^*}{2}$, where now both s^* and Δ^* vary with α, the derivatives follow from $\frac{\partial s^*}{\partial \alpha} = \tilde{c} + c_R - 4$ and $\frac{\partial \Delta^*}{\partial \alpha} = \frac{1}{2f(\tilde{c})}$. Moreover, (A1), (A2), and (A4) together imply $\tilde{c} + c_R < \Delta^* + \frac{\Delta^*}{2} = \frac{3}{2}\Delta^* \leq 3$, so $\frac{\tilde{c}+c_R}{2} - 2 < -\frac{1}{2}$, and subtracting $\frac{1}{4f(\tilde{c})} > 0$ preserves the sign: $\frac{\partial x_L^*}{\partial \alpha} < 0$. □

Proposition A.6. *In the unique equilibrium,* $U_L^* = -1 - \frac{s^*}{2}$ *and* $U_R^* = -1 + \frac{s^*}{2}$*, so L's welfare derivatives satisfy* $\frac{\partial U_L^*}{\partial \tilde{c}} = -\frac{\alpha}{2}$, $\frac{\partial U_L^*}{\partial c_R} = \frac{1-\alpha}{2}$*, and* $\frac{\partial U_L^*}{\partial \alpha} = 2 - \frac{\tilde{c}+c_R}{2}$*. Moreover, an untargeted common cost increase changes L's welfare by* $\frac{1-2\alpha}{2}$*. Party R's welfare effects are symmetric.*

PROOF. The value formulas are from Lemma A.5, and the comparative statics follow by differentiating $s^* = \alpha\tilde{c} - (1-\alpha)c_R - 2(2\alpha - 1)$. The untargeted effect adds the $\tilde{c}$ and c_R effects. Party R's effects are symmetric with L's because $U_R^* = -2 - U_L^*$. □

A.7 Smooth Environment

This section establishes that our platform-shift mechanism does not require the stark baseline setting. We provide sufficient conditions under which the platform game is log-supermodular, so that a rightward location shift of F (higher voting costs for G_L) shifts both platforms rightward.

Let $x = (x_L, x_R)$, and let $\mathcal{U}_J(y)$ denote party J's policy payoff from elected platform y. As in the baseline setting, parties are policy-motivated, with ideal points at the endpoints of the policy space X, a compact interval. Their payoff functions $\mathcal{U}_J$ are differentiable, with $\mathcal{U}_L'(y) < 0$ and $\mathcal{U}_R'(y) > 0$ for y strictly between the ideal points. Furthermore, we assume f is log-concave and differentiable; $\tau_L(x; c_L)$ is strictly decreasing in c_L; and τ_L, τ_R are smooth in platforms. Analogous to the baseline setting, we assume Party L wins if and only if $c_L < \hat{c}_x \equiv \hat{c}(x; c_R)$, where now $\hat{c}_x$ is twice differentiable with $\partial\hat{c}_x/\partial x_L > 0$ and $\partial\hat{c}_x/\partial x_R > 0$. Throughout, we restrict attention to platform pairs in a compact, convex sublattice $\mathcal{X} \subset \{(x_L, x_R) \in \text{int}(X)^2 : x_L < x_R\}$ on which $F(\hat{c}_x) \in (0,1)$, so the objects below are well-defined. Note that for such ordered platform pairs, $\mathcal{U}_L'(x_L)\,\mathcal{U}_L'(x_R) > 0$ and $\mathcal{U}_R'(x_R)\,\mathcal{U}_R'(x_L) > 0$, so each party strictly prefers its own platform to its opponent's.

Given platforms x, party J's expected payoff is $U_J(x) = F(\hat{c}_x)\,\mathcal{U}_J(x_L) + \big(1 - F(\hat{c}_x)\big)\,\mathcal{U}_J(x_R)$. Define $\widehat{\Pi}_L \equiv F(\hat{c}_x)\big[\mathcal{U}_L(x_L) - \mathcal{U}_L(x_R)\big]$ and $\widehat{\Pi}_R \equiv \big[1 - F(\hat{c}_x)\big]\big[\mathcal{U}_R(x_R) - \mathcal{U}_R(x_L)\big]$, which are strictly positive on $\mathcal{X}$ because $F(\hat{c}_x) \in (0,1)$ and each party strictly prefers its own platform. Rearranging the expected payoffs yields $U_L = \mathcal{U}_L(x_R) + \widehat{\Pi}_L$ and $U_R = \mathcal{U}_R(x_L) + \widehat{\Pi}_R$. On $\mathcal{X}$, the first term in each of these is independent of the party's own platform. Since ln is strictly increasing, the parties' own-platform rankings, and therefore the best-response map and equilibria within $\mathcal{X}$, coincide with those generated by the objectives $B_J \equiv \ln \widehat{\Pi}_J$.

We first provide sufficient conditions for the game to be log-supermodular. Each objective has increasing differences in the platforms if the relevant cross-partial is nonnegative. For L,

writing $G = f/F$,

$$\frac{\partial^2 B_L}{\partial x_R \partial x_L} = G'(\hat{c}_x)\,\frac{\partial \hat{c}_x}{\partial x_L}\frac{\partial \hat{c}_x}{\partial x_R} + G(\hat{c}_x)\,\frac{\partial^2 \hat{c}_x}{\partial x_R \partial x_L} + \frac{\mathcal{U}'_L(x_L)\,\mathcal{U}'_L(x_R)}{\big[\mathcal{U}_L(x_L) - \mathcal{U}_L(x_R)\big]^2}.$$

Log-concavity of f implies F is log-concave, so the reverse hazard rate G is nonincreasing, $G'(\hat{c}_x) \leq 0$. Thus, $\partial^2 B_L/\partial x_R \partial x_L \geq 0$ if and only if

$$-G'(\hat{c}_x)\,\frac{\partial \hat{c}_x}{\partial x_L}\frac{\partial \hat{c}_x}{\partial x_R} \leq G(\hat{c}_x)\,\frac{\partial^2 \hat{c}_x}{\partial x_R \partial x_L} + \frac{\mathcal{U}'_L(x_L)\,\mathcal{U}'_L(x_R)}{\big[\mathcal{U}_L(x_L) - \mathcal{U}_L(x_R)\big]^2}.$$

Because $\mathcal{U}'_L(x_L)\,\mathcal{U}'_L(x_R) > 0$ and $G(\hat{c}_x) > 0$, the right-hand side is positive if

$$\frac{\partial^2 \hat{c}_x}{\partial x_L \partial x_R} > \underline{\chi} \equiv -\frac{\mathcal{U}'_L(x_L)\,\mathcal{U}'_L(x_R)}{G(\hat{c}_x)\big[\mathcal{U}_L(x_L) - \mathcal{U}_L(x_R)\big]^2},$$

in which case the inequality holds once $\frac{\partial \hat{c}_x}{\partial x_L}\frac{\partial \hat{c}_x}{\partial x_R}$ is small enough, and automatically where $G'(\hat{c}_x) = 0$. An analogous derivation for R, with $H = f/(1-F)$ nondecreasing under log-concavity ($H'(\hat{c}_x) \geq 0$), gives $\partial^2 B_R/\partial x_L \partial x_R \geq 0$ if

$$\frac{\partial^2 \hat{c}_x}{\partial x_L \partial x_R} < \overline{\chi} \equiv \frac{\mathcal{U}'_R(x_R)\,\mathcal{U}'_R(x_L)}{H(\hat{c}_x)\big[\mathcal{U}_R(x_R) - \mathcal{U}_R(x_L)\big]^2},$$

and the same product is small. Since $\underline{\chi} < 0 < \overline{\chi}$, both conditions hold together when (i) the cross-partial $\partial^2 \hat{c}_x/\partial x_L \partial x_R$ lies in $(\underline{\chi}, \overline{\chi})$ and (ii) the slope product $\frac{\partial \hat{c}_x}{\partial x_L}\frac{\partial \hat{c}_x}{\partial x_R}$, weighted by $-G'(\hat{c}_x)$ for L and by $H'(\hat{c}_x)$ for R, is small relative to the remaining positive terms. Note that (i) holds with a linear cutoff (as in our baseline setting), which induces a zero cross-partial.

To analyze increases in G_L's voting costs, we index the cost distribution by a location parameter $t \geq 0$, with $F_t(c) \equiv F_0(c - t)$ and $F_0 = F$, so that higher t shifts voting costs rightward. All objects that depend on F are also indexed accordingly (e.g. $G(\hat{c}_x; t) = f_t(\hat{c}_x)/F_t(\hat{c}_x)$). We require that $F_t(\hat{c}_x) \in (0,1)$ on $\mathcal{X}$. Since this holds at $t = 0$, compactness and continuity imply that it continues to hold for all sufficiently small $t \geq 0$. Additionally, we assume that the joint best-response map leaves $\mathcal{X}$ invariant for every t under consideration.

Proposition A.7. *Suppose that, for every t under consideration, $\frac{\partial^2 B_L}{\partial x_R \partial x_L} \geq 0$ and $\frac{\partial^2 B_R}{\partial x_L \partial x_R} \geq 0$ on $\mathcal{X}$, each party has a unique best response to every opponent platform, and the joint best-response map b_t maps $\mathcal{X}$ into itself. Then, for every such t, the game has a pure-strategy equilibrium in $\mathcal{X}$. Moreover: (i) the least and greatest equilibria in $\mathcal{X}$ are weakly increasing in t; and (ii) following an increase from t to $t' > t$, iterating $b_{t'}$ from any equilibrium in $\mathcal{X}$ of*

the t-game generates a weakly increasing sequence that converges to an equilibrium in $\mathcal{X}$ of the t'-game.

PROOF. Given log-supermodularity, it remains to show that each objective has increasing differences in its own platform and the shift t.[3] Under $F_t(c) = F_0(c - t)$, we have $\frac{\partial F_t}{\partial t} = -f$ and $\frac{\partial f_t}{\partial t} = -f'$. Since $\frac{\partial B_L}{\partial x_L} = G(\hat{c}_x; t)\,\frac{\partial \hat{c}_x}{\partial x_L} + \frac{\mathcal{U}'_L(x_L)}{\mathcal{U}_L(x_L) - \mathcal{U}_L(x_R)}$ and only the first term depends on t,

$$\frac{\partial^2 B_L}{\partial t\,\partial x_L} = \frac{\partial G(\hat{c}_x; t)}{\partial t}\,\frac{\partial \hat{c}_x}{\partial x_L} = \frac{f^2 - f'F}{F^2}\,\frac{\partial \hat{c}_x}{\partial x_L} \geq 0,$$

where $f^2 - f'F \geq 0$ because log-concavity makes the reverse hazard rate nonincreasing. Symmetrically, from $\frac{\partial B_R}{\partial x_R} = -H(\hat{c}_x; t)\,\frac{\partial \hat{c}_x}{\partial x_R} + \frac{\mathcal{U}'_R(x_R)}{\mathcal{U}_R(x_R) - \mathcal{U}_R(x_L)}$, $\frac{\partial^2 B_R}{\partial t\,\partial x_R} = -\frac{\partial H(\hat{c}_x;t)}{\partial t}\,\frac{\partial \hat{c}_x}{\partial x_R} \geq 0$ and $\frac{\partial H}{\partial t} = \frac{-f'(1-F) - f^2}{(1-F)^2} \leq 0$, with the sign again following from log-concavity. Thus, both parties' objectives have increasing differences in their own platform and t.

The cross-partial conditions and uniqueness imply that b_t is weakly increasing in the platform pair, while the preceding derivatives imply that $b_{t'}(x) \geq b_t(x)$ whenever $t' > t$. Because b_t maps the compact lattice $\mathcal{X}$ into itself, the standard results for supermodular games imply the existence of least and greatest fixed points and that these fixed points are weakly increasing in t (Vives 2005).

For (ii), let $x^0 \in \mathcal{X}$ be an equilibrium of the t-game and let $t' > t$. Then $x^1 = b_{t'}(x^0) \geq b_t(x^0) = x^0$. Since $b_{t'}$ is increasing, induction gives $x^{n+1} = b_{t'}(x^n) \geq x^n$ for every n. This sequence converges, since it is weakly increasing in a bounded and compact set. Continuity of payoffs and uniqueness of best responses imply continuity of $b_{t'}$, so its limit is a fixed point and therefore an equilibrium of the t'-game. □

A.8 Affinity Voting

This section presents a setting with affinity voting, in which turnout is driven by citizens' distance to their party's candidate rather than by the policy stakes between the two candidates. We focus on interior equilibria and do not address existence and uniqueness.

Each group has an associated voting propensity, λ_L and λ_R. Let $\lambda_R \geq 0$ be fixed and common knowledge, while λ_L is drawn from a log-concave distribution F on $[\underline{\lambda}, \bar{\lambda}]$ with $\underline{\lambda} \geq 0$ and density f. As in the baseline, parties commit to platforms before λ_L is realized. A citizen

[3] Location shifts are not necessary. The argument requires only that the family $\{F_t\}$ is ordered by likelihood ratio, which implies that the reverse hazard rate rises and the hazard rate falls in t at every cutoff. Location shifts of a log-concave density are likelihood-ratio ordered.

in G_R turns out and votes for R if $|\hat{x}_i - x_R| \leq \lambda_R$ and otherwise abstains, and citizens in G_L behave analogously with λ_L and party L, so each citizen supports only the candidate of their own party.

In equilibrium, if a group's extremists abstained at some platform pair, that group's turnout would not vary with small changes in the party's platform, and the party would profitably shift outward until all its extremists vote. Under conditions inducing interior turnout, we have $\tau_R = (1-\alpha)(1 - x_R + \lambda_R)$ for G_R and $\tau_L = \alpha(1 + x_L + \lambda_L)$ for G_L. Party L wins if and only if $\tau_L > \tau_R$, i.e.,

$$\lambda_L > \frac{1-\alpha}{\alpha}(1+\lambda_R) - 1 - \left(\frac{1-\alpha}{\alpha}\, x_R + x_L\right) \equiv \hat{\lambda},$$

so L wins with probability $1 - F(\hat{\lambda})$. Given platforms (x_L, x_R), the parties' expected payoffs are $U_L = -(1 + x_L) - F(\hat{\lambda})(x_R - x_L)$ and $U_R = -(1 - x_L) + F(\hat{\lambda})(x_R - x_L)$, with the following first-order conditions for x_L and x_R:

$$\begin{aligned} 0 &= -\big(1 - F(\hat{\lambda})\big) + f(\hat{\lambda})(x_R - x_L), \\ 0 &= F(\hat{\lambda}) - \frac{1-\alpha}{\alpha}\, f(\hat{\lambda})(x_R - x_L). \end{aligned}$$

Combining yields $f(\hat{\lambda})(x_R - x_L) = \alpha$ and $F(\hat{\lambda}) = 1 - \alpha$, so in equilibrium the cutoff is $\hat{\lambda}_\alpha \equiv F^{-1}(1-\alpha)$ and divergence is $x_R^* - x_L^* = \alpha / f(\hat{\lambda}_\alpha)$. Solving the system yields the interior candidate

$$\begin{aligned} x_L^* &= (1-\alpha)\lambda_R + (1-2\alpha) - \alpha\hat{\lambda}_\alpha - \frac{\alpha(1-\alpha)}{f(\hat{\lambda}_\alpha)}, \\ x_R^* &= (1-\alpha)\lambda_R + (1-2\alpha) - \alpha\hat{\lambda}_\alpha + \frac{\alpha^2}{f(\hat{\lambda}_\alpha)}, \end{aligned}$$

with equilibrium payoffs

$$\begin{aligned} U_L^* &= \alpha\hat{\lambda}_\alpha - (1-\alpha)(2+\lambda_R), \\ U_R^* &= (1-\alpha)\lambda_R - \alpha(2+\hat{\lambda}_\alpha). \end{aligned}$$

As in the baseline, the game is constant-sum, since $u_L(x) + u_R(x) = -2$ for every implemented policy x, so $U_L^* + U_R^* = -2$. Differentiating, we have $\frac{\partial U_R^*}{\partial \lambda_R} = -\frac{\partial U_L^*}{\partial \lambda_R} = 1 - \alpha > 0$ and $\frac{\partial U_R^*}{\partial \hat{\lambda}_\alpha} = -\frac{\partial U_L^*}{\partial \hat{\lambda}_\alpha} = -\alpha < 0$, as a location shift of F moves $\hat{\lambda}_\alpha$ one-for-one and leaves $f(\hat{\lambda}_\alpha)$

fixed, while

$$\frac{\partial U_R^*}{\partial \alpha} = -\frac{\partial U_L^*}{\partial \alpha} = \frac{\alpha}{f(\hat{\lambda}_\alpha)} - (2 + \lambda_R + \hat{\lambda}_\alpha),$$

using $(F^{-1})'(1-\alpha) = 1/f(\hat{\lambda}_\alpha)$. The equilibrium welfare difference is $U_R^* - U_L^* = 2\big[(1-2\alpha) + (1-\alpha)\lambda_R - \alpha\hat{\lambda}_\alpha\big]$. Equilibrium turnout is

$$\tau_R^* = \alpha(1-\alpha)\left(2 + \lambda_R + \hat{\lambda}_\alpha - \frac{\alpha}{f(\hat{\lambda}_\alpha)}\right),$$
$$\tau_L^* = \alpha\left(2(1-\alpha) + (1-\alpha)\lambda_R + \lambda_L - \alpha\hat{\lambda}_\alpha - \frac{\alpha(1-\alpha)}{f(\hat{\lambda}_\alpha)}\right),$$

so the expected turnout difference is $\mathbb{E}[\tau_R^* - \tau_L^*] = \alpha(\hat{\lambda}_\alpha - \mathbb{E}[\lambda_L])$.

Because $F(\hat{\lambda}_\alpha) = 1-\alpha$ in equilibrium, party L wins with probability α regardless of F and λ_R, so electoral prospects depend only on group size. This is analogous to Proposition 1 of the main text with unequal win probabilities. Changes in voting propensities instead surface in platforms and welfare, as increasing λ_R increases R's welfare by $1-\alpha$ per unit, and a location shift of F increases L's welfare by α per unit. Turnout effects are muted exactly as in the baseline, as increasing λ_R increases G_R's turnout and G_L's expected turnout by the same amount, $\alpha(1-\alpha)$ per unit. The direct mobilization of G_R is partly offset by R shifting outward, while L's rightward moderation mobilizes G_L. And a location shift of F likewise increases both equally, so the expected turnout difference $\mathbb{E}[\tau_R^* - \tau_L^*] = \alpha(\hat{\lambda}_\alpha - \mathbb{E}[\lambda_L])$ is unchanged.

References

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