Access to Proposers and Influence in Collective Policymaking*

Gleason Judd†

November 11, 2022

Abstract

What are the consequences of access and what is its role in interest group influence? I analyze a model in which interest groups with targeted access can potentially lobby policy proposals by certain politicians. A key result is that access can shape policy outcomes on its own, independently of any lobbying effort. By increasing the potential for lobbying, access leads other politicians to expect that the target's proposals are more likely to favor the group, which changes their own value from bargaining and, in turn, their voting and proposal behavior. These effects of access can benefit the group, but they can also hurt it and potentially even outweigh its gain from better lobbying prospects. For example, moderate groups crave access to relatively extreme politicians but avoid access to a range of more centrist politicians. The results also provide empirical implications for various political expenditures related to access and influence.

Keywords: interest groups, access, lobbying, outside influence, legislative bargaining

*I am indebted to Tasos Kalandrakis for many helpful conversations. I also thank Avi Acharya, Emiel Awad, Peter Bils, Peter Buisseret, Steve Callander, Richard DiSalvo, John Duggan, Wiola Dziuda, Mark Fey, Mike Gibilisco, Cathy Hafer, Matías Iaryczower, Federica Izzo, Kei Kawai, Brenton Kenkel, Chris Li, Zhao Li, Zhaotian Luo, Adam Meirowitz, MaryClare Roche, Larry Rothenberg, Greg Sasso, Brad Smith, Sebastian Thieme, Yannis Vassiliadis, Craig Volden, Alan Wiseman, Stephane Wolton, Jan Zápal, attendees of the 2018 Vanderbilt Frontiers of Formal Theory Conference, the 2022 Kuznets Mini-Conference on Political Distortions and Economic Development at Yale, and audiences at Chicago Harris, NYU, Rochester, and Princeton for suggestions.

†Department of Politics, Princeton University (`gleason.judd@princeton.edu`).

A majority of Americans worry that money has too much influence on US politics, with a chief concern being that interest groups use their resources to skew policy. One of the primary ways an interest group can influence policy is by lobbying, but doing so typically requires *access* — opportunities to engage with policymakers (Wright, 1996). Since access appears to facilitate outside influence, it is a central topic for scholars of interest groups (e.g., Schlozman and Tierney, 1986; Baumgartner, Berry, Hojnacki, Leech and Kimball, 2009).

Despite a growing understanding of how interest groups can acquire access and who they target,[1] we know relatively little about the effects of access and the role it plays in outside influence.[2] A key challenge is that access occurs in a wider political context with highly strategic actors (Baumgartner, 2010; Leech, 2010).[3] One way to address this obstacle is by refining our theoretical understanding of access and influence in such contexts.[4] That is the goal of this paper.

Classic theories of access study how key factors, such as ideological alignment, affect an interest group's desire for access to an isolated politician (e.g., Austen-Smith, 1995; Lohmann, 1995; Hall and Deardorff, 2006). To study access in a broader political context,[5] recent theories incorporate multiple strategic politicians in collective bodies. In this vein, scholars have analyzed untargeted access that facilitates lobbying to shape proposals (Levy and Razin, 2013), as well as targeted access that facilitates lobbying on votes over a fixed proposal (Schnakenberg, 2017; Awad, 2020).

I address an important gap in our understanding of access by studying targeted access that facilitates lobbying to shape proposals in collective policymaking. Shaping bills as they are written in committee is widely seen as the most effective form of lobbying (Schattschneider, 1960; Hall and Wayman, 1990; Kroeger, 2021) and, in order to do so, groups typically must have targeted access, i.e., strong connections with individual politicians (Powell, 2014; Miller, 2021b). Specifically, I study two questions. First, what are the consequences of access

[1]On how groups get access, see, e.g., Blanes i Vidal, Draca and Fons-Rosen (2012); Bertrand, Bombardini and Trebbi (2014); Kalla and Broockman (2015); Fouirnaies and Hall (2018) and McCrain (2018). On who they target, see, e.g., Powell and Grimmer (2016); Fouirnaies (2018); Miller (2021b) and Liu (2021).

[2]For a recent overview of empirical studies of access, see Miller (2021a).

[3]Leech (2010) notes that "[..] organized interests recognize the importance of legislative procedure and agenda setters" and "studies of their targeting choices, and of their lobbying activity more broadly, must consider how procedural context conditions their behavior."

[4]As summarized by Finger (2019), "current approaches to the study of interest groups suffer from theoretical incoherence with regard to how interest groups seek to achieve influence and a lack of consensus around how to operationalize these behaviors" (Finger, 2019, pg. 853). More broadly, Miller (2021a) notes that "though the theoretical linkages between access and other quantities of interest are sometimes unclear, formal theory can help researchers elucidate expectations and guide empirical tests" (Miller, 2021a, pg. 297).

[5]Hall & Deardorff (2006) encourage future work to "incorporate the degree of agreement over specific policies" and explore potential tradeoffs between "a legislator's proximity to their group's ideal policies and the legislator's institutional or partisan ability to get things done" (Hall and Deardorff, 2006, pg. 80).

that may provide chances to lobby the targeted politicians as they draft proposals? Second, which politicians do interest groups want to target?

To do so, I analyze a game-theoretic model of collective policymaking with interest groups. The model has three key features. First, access is targeted and solely provides potential opportunities to exert influence by lobbying — reflecting the standard conceptual distinction between access and lobbying (Wright, 1989). Second, lobbying directly influences policy proposals — capturing the prominent form of lobbying that entails shaping proposals in committee before they are voted on. Third, multiple politicians bargain, potentially for a while, to set policy — reflecting that access does not occur in a vacuum, as other actors may anticipate a group's potential influence over its target today or in the future.

A primary contribution is to expand our understanding of the consequences of targeted access to proposers and its role in interest group influence. Broadly, I show how such access can affect a variety of behaviors on its own and, moreover, that the political context shapes the nature of these effects.

First, I find that an interest group can influence policy outcomes merely by having access, independent of any lobbying effort. That is, I show how access can be sufficient for influence even if it merely creates the possibility of effective lobbying. This finding contrasts with the standard view that access is a critical prerequisite for influence but does not influence behavior on its own (Hansen, 1991).[6]

Second, by unpacking the preceding finding, I show how such access can influence: (i) which policies would pass if proposed, (ii) the target's policy proposal even when the group does not lobby, (iii) policy proposals by non-targets to whom the group has no access and thus cannot lobby, as well as (iv) the group's lobbying expenditures when it does lobby. Additionally, I show how the direction of these effects and their size can depend on the relative extremism of the group and its target, as well as broader factors such as the distributions of ideology and proposal power among the politicians who are bargaining.

A core aspect of the strategic logic for these findings is that access has anticipation effects in equilibrium. Since access creates and increases the potential for lobbying, everyone anticipates the possibility of the target skewing her proposal towards the group. That anticipation alone can change every politician's value of continued bargaining and, in turn, potentially changes how they vote on certain proposals. Essentially, the logic highlights how access can affect behavior via the *law of anticipated reactions* (Friedrich, 1937) and thereby enable interest groups to have influence without actively lobbying, i.e., through the *second face of power* (Bachrach and Baratz, 1962). Although classic studies of influence recognized

[6] For example, Kalla and Broockman (2015) summarize this view clearly, stating that "access to powerful officials is often necessary for influencing policy, even if it is not sufficient."

the importance of anticipation effects (e.g., Simon, 1953) and recent work has sought to account for them in other contexts, such as presidential vetoes (Cameron, 2009), they are absent from theories of access and their role in interest group influence has been overlooked.[7]

Another contribution is to refine our understanding of interest group preferences for access. Once an interest group has access to a legislator it cannot commit to forego chances to lobby if they arise and, for some ideological compositions of legislators, this expectation can result in other legislators proposing policies that are worse for the interest group than those they would propose without the expectation. This indirect effect of access can be good or bad for the group, potentially even negating the beneficial direct effect of making lobbying more likely. I find that the natural intuition of 'more access is better' need not be true, even without access costs or budgets, but in other cases access is even more appealing than previously understood. Specifically, I show how a group's desire for access can depend on its policy preferences relative to the target politician. For example, centrist groups benefit from access to more extreme politicians, but they can be worse off from access to more centrist politicians.

I also make a technical contribution by providing tractable way to incorporate targeted access and lobbying into a rich legislative bargaining framework. The modeling approach (i) distinguishes latent interest group access from actual lobbying, which is observed only when the opportunity arises, and (ii) allows expectations about possible future behavior to shape legislative behavior today. These features are fundamental to the paper's main substantive insights, as well as the anticipation effects in the key strategic mechanism.

The results have several implications for empirical studies of interest groups and access. First, they suggest that empirical studies of influence should expand their scope beyond observed lobbying, as well as account for potential spillover effects on other politicians and activities. Second, they suggest potential relationships between lobbying expenditures and various access-seeking expenditures (e.g., campaign contributions, revolving door hiring). Third, they shed new light on *Tullock's puzzle* — the longstanding empirical regularity that many interest groups are less aggressive than expected in using campaign contributions to pursue access (Tullock, 1972; Ansolabehere, de Figueiredo and Snyder Jr., 2003). Finally, they speak to several other empirical findings that groups often (i) lobby their allies (Ainsworth, 1997; Kollman, 1997; Hojnacki and Kimball, 1998, 1999), (ii) seek access to legislators with substantial agenda power (Powell and Grimmer, 2016; Fouirnaies, 2018), and that (iii) contributing groups are overwhelmingly centrist (Bonica, 2013, p. 301).

[7]For more discussion on these points, see Lowery (2013).

Related Literature

The closest papers are Schnakenberg (2017) and Awad (2020), who also study targeted access in a collective body. Like this paper, they find that (i) targeted access can indirectly affect how non-targeted politicians behave, and (ii) interest groups have a strategic incentive to target ally legislators for access. Despite these broad similarities, there are several important differences.

A fundamental difference is that I study lobbying to shape proposals whereas they study lobbying to affect votes. More precisely, they focus on lobbying that provides information to affect final votes between two exogenous proposals. In contrast, I focus on lobbying that provides resources to shape policy proposals during an interaction that can continue after failed proposals.

By studying a different form of lobbying, the aforementioned similar findings arise from different mechanisms, which then produce distinct additional findings.[8] One distinct finding is whether the interest group can influence behavior without lobbying. In Schnakenberg (2017) and Awad (2020), they cannot — access affects behavior only if the group subsequently lobbies. In this paper, they can — access causes everyone to anticipate the potential for future lobbying, and that anticipation can affect votes and proposals today even without lobbying by the interest group.

A second distinct finding is in *which* allies interest groups want to access. In Schnakenberg (2017) and Awad (2020), groups weakly favor access to moderates because using them as intermediaries can expand what passes, i.e., get policies passed that would have failed if the group had instead lobbied the legislature directly.[9] I find a preference for targeting more extreme allies rather than weakly favoring more centrist allies. By incorporating strategic proposals, I highlight how interest groups can suffer from access that expands what would pass and instead want access that will narrow what can pass.

In order to make these substantive contributions, I also contribute to a theoretical literature incorporating lobbying into legislative bargaining models with strategic proposals and votes. Among various differences, they typically study untargeted access (e.g., Levy and Razin, 2013) or do not emphasize access (e.g., Baron, 2019).[10] Specifically, I extend the

[8]Future work can study how the effects highlighted here interact with the informational effects they emphasize. See Grossman and Helpman (2002) for an extensive overview of canonical informational lobbying models.

[9]In Schnakenberg (2017), groups seek access to allies since they are relatively willing to forward favorable unverifiable information to the other politicians, reducing the cost of persuading a majority. In Awad (2020), groups target verifiable information at moderate allies who, precisely because they are more moderate, can then provide a public cheap-talk message that convinces a majority of legislators under broader conditions.

[10]In addition to its different focus, Baron (2019) studies lobbying directed at votes during bargaining over distributive policy that can continue after passage with endogenous status quo. Closer to this paper,

legislative interaction in Cho and Duggan (2003) to include ideological interest groups who can potentially transfer resources to influence proposals. I extend their equilibrium concept to account for lobbying, prove existence, and show that equilibrium behavior has a clear connection to their characterization — the distribution of equilibrium proposals with lobbying is equivalent to a slightly modified version of the model without lobbying. Moreover, I show that lobbying does not introduce delay in this setting, which extends well-known *no-delay* results, i.e., bargaining always ends immediately, in similar legislative settings without lobbying (e.g., Banks and Duggan, 2006a).

Model

Players. The key players are an interest group, denoted g, and a politician, ℓ. Additionally, there are three other politicians: a left partisan L, a moderate M, and a right partisan R.

Timing. Politicians bargain to set policy in the interval $X \subseteq \mathbb{R}$, which is closed and non-empty. Bargaining occurs over an infinite horizon,[11] with periods discrete and indexed $t \in \{1, 2, \dots\}$. A status quo policy $q \in X$ persists until policy passes. Thereafter, the strategic interaction ends and the passed policy remains forever. During each period t before some proposal passes, bargaining proceeds in the following two stages.

Proposal stage. First, the period-t proposer i_t is drawn from probability distribution $\rho = (\rho_\ell, \rho_L, \rho_M, \rho_R)$, where $\rho_j > 0$ is politician j's recognition probability. If $i_t \neq \ell$, then g is not active and i_t proposes any $x_t \in X$. If $i_t = \ell$, then g can lobby with probability $\alpha \in [0, 1]$, which parameterizes g's *access*. If g is unable to lobby, then ℓ simply proposes any $x_t \in X$. Otherwise, g offers ℓ a binding contract (y_t, m_t) consisting of policy $y_t \in X$ and transfer $m_t \geq 0$.[12] After observing g's offer, ℓ decides whether to accept or reject it. If ℓ accepts, then she proposes $x_t = y_t$ and receives m_t from g. If ℓ rejects, then she can propose any $x_t \in X$ and g keeps m_t.

Voting stage. Next, M decides whether to accept the policy proposal.[13] If M accepts, then bargaining ends with x_t enacted in t and all subsequent periods. If M rejects, then q

Grossman and Helpman (2002) discuss a model in which lobbying can affect a take-it-or-leave-it proposal and the subsequent votes, but their relatively informal analysis does not discuss access and considerations about future bargaining do not play a role.

[11] As usual, this game can alternatively be viewed as having a unknown finite horizon with a constant probability of termination each period.

[12] Assuming that g lobbies whenever possible is without loss of generality, as g can always effectively forgo lobbying by offering ℓ's default proposal without payment.

[13] This stage distills the essence of majoritarian voting in a larger interaction where M is a median voter (Banks and Duggan, 2006b). In the appendix, I show that the median is decisive in such a setting and prove the main results.

persists and active bargaining continues in $t+1$.

Information. All features are common knowledge.

Payoffs. Cumulative dynamic payoffs are the sum of streams of discounted per-period payoffs, with all players sharing the common discount factor $\delta \in (0,1)$. Player i's per-period policy utility from $x \in X$ is $(1-\delta)\, u_g(x_t)$, where $(1-\delta)$ is a normalization for convenience and $u_i(x) = -(\hat{x}_i - x)^2$ with $\hat{x}_i$ denoting i's ideal point.

If lobbying occurs, ℓ accepts g's offer (y_t, m_t), and x_t is the period-t policy,[14] then g's period-t payoff is $(1-\delta)\, u_g(x_t) - m_t$ and ℓ's period-t payoff is $(1-\delta)\, u_\ell(x_t) + m_t$. Thereafter, m_t does not enter per-period payoffs. For complete expressions of dynamic payoffs, see Appendix B.

To sharpen key tradeoffs, I maintain several additional assumptions that are not essential. First, I assume $\hat{x}_M = 0 \in X$, which is a normalization. Additionally, to model L and R as staunchly ideological and opposing partisans, I assume $x_L, x_R \in X$, with $\hat{x}_L < 0 < \hat{x}_R$ and $|q| < \min\{|\hat{x}_L|, \hat{x}_R\}$.

Figure 1: A period with lobbying

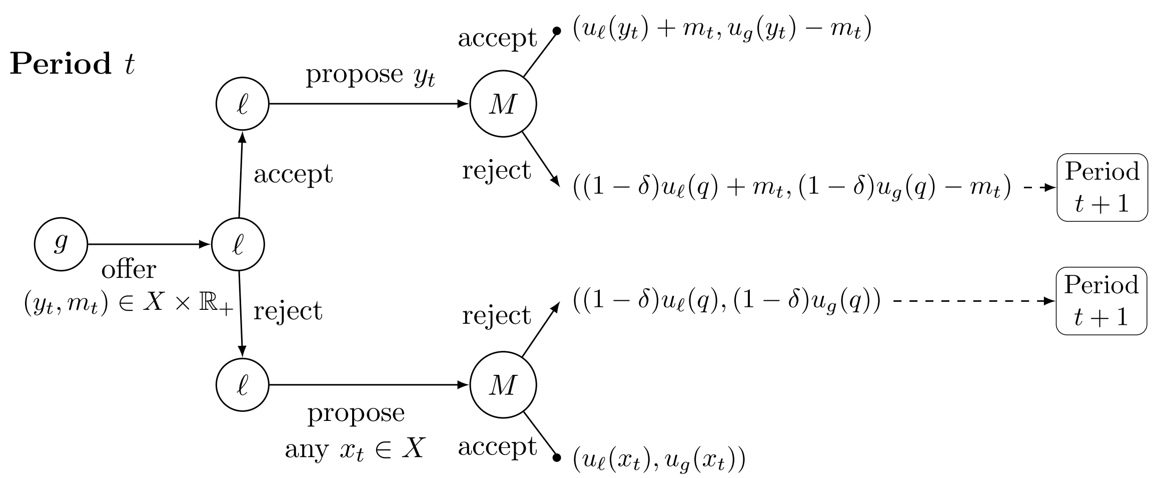

Figure 1 illustrates the within-period interaction if ℓ is recognized and g can lobby. It includes period payoffs following rejection, and cumulative stage payoffs following acceptance. If ℓ is not recognized or g cannot lobby, the within-period interaction is analogous to Figure 1 after ℓ rejects g's offer.

Equilibrium Concept. I study a refinement of stationary subgame perfect Nash equilibrium that builds on standard equilibrium concepts in the legislative bargaining literature (e.g., Banks and Duggan, 2006a). Informally, a *stationary legislative lobbying equilibrium*

[14] If y_t passes, then $x_t = y_t$. Otherwise, $x_t = q$.

satisfies four conditions.[15] First, M passes a proposal if and only if she weakly prefers to do so rather than reject and continue bargaining. Second, if left to their own devices, each politician proposes policy satisfying M and cannot profitably deviate to any other proposal. Third, politician ℓ accepts a lobby offer if and only if she weakly prefers it over the alternative of making her own proposal. Fourth, g offers a policy that will pass and g cannot profitably deviate to any other offer. By stationarity: (i) M's voting decision depends only on the current proposal; (ii) politicians other than ℓ propose independently of preceding play; (iii) ℓ accepts or rejects g's offers based only on the current terms, and ℓ's proposals in lieu of acceptance are independent of preceding play; and (iv) g's offers are independent of preceding play. Although players use strategies that are relatively straightforward behavioral rules, no player can profitably deviate to *any* other strategy.

Before proceeding, I note three conditions on strategies that are without loss of generality and streamline the analysis: (i) M passes proposals when indifferent; (ii) ℓ accepts g's offer when indifferent; and (iii) players use *no-delay* proposal strategies, i.e., each politician proposes passable policy and g offers passable policy. In the appendix, I define *stationary mixed strategy legislative lobbying equilibrium* and show that every such equilibrium is equivalent in outcome distribution to a no-delay stationary pure strategy legislative lobbying equilibrium in which politicians (i) vote in favor of proposals when indifferent and (ii) accept lobby offers when indifferent.[16]

Model Commentary

The model captures a core aspect of access — it weakly increases opportunities to exert influence — since access determines the probability that the group can lobby. Additionally, the model can easily be modified to capture a second potential aspect of access — it weakly increases the effectiveness of lobbying when such opportunities arise — by, e.g., allowing access to increase ℓ's value of transfers from g. Combining these aspects of access does not add substantial insight to the main results because the direct consequence of access is qualitatively the same — it shifts the target's expected proposal towards the group — and thus the indirect effects are also qualitatively analogous.

The key aspect of lobbying that the model captures is the ability to influence proposals. Groups often lobby in committee to shape the language of bills (Schlozman and Tierney, 1986; Kroeger, 2021) and the policy-for-transfer lobbying technology used here provides a

[15]See Appendix B for a formal definition.

[16]Standard arguments (Banks and Duggan, 2006b) imply that proposal strategies must be no delay. Although related, the no-delay property for interest groups is original to this paper. Essentially, lobbying for delay is always too expensive to be worthwhile in equilibrium. Appendix C provides the technical details.

tractable reduced-form representation of various ways that such influence could occur (Powell, 2014). The exact interpretation the lobbying technology in this paper is not central,[17] but the model accommodates two prominent forms. First, there is an *exchange* interpretation that can more broadly reflect the group drafting language (Schattschneider, 1960) or writing a model bill (Kroeger, 2021) to save politicians time, or in exchange for various forms of assistance such as future employment opportunities (Diermeier, Keane and Merlo, 2005) and targeted charitable donations (Bertrand, Bombardini, Fisman and Trebbi, 2020). Second, there is also a *legislative subsidy* interpretation in which the group's lobbying helps a likeminded politician influence her peers on a particular subcommittee whenever it is tasked with writing legislation (Hall and Deardorff, 2006). To streamline discussion, I use the exchange interpretation throughout the analysis.

I do not model lobbying that directly influences how politicians vote on proposals. Thus, I isolate the effects of access that facilitates lobbying over policy content. The main analysis complements standard models of vote buying, which typically study exogenous or take-it-or-leave-it proposals (e.g., Snyder Jr., 1991; Dekel, Jackson and Wolinsky, 2009), by analyzing a setting where politicians make strategic proposals and bargaining continues after failed proposals. In practice, influencing policy content is particularly appealing for interest groups because it is less visible and more intimate. In contrast, consequential vote buying is relatively difficult because, legality aside, it may require groups to coordinate with several politicians, which is like "herding cats" (Milyo, Primo and Groseclose, 2000). I discuss vote buying incentives in the Conclusion and in Appendix E I show that the main results are robust to them.

Finally, in the baseline model, access is targeted at one politician and remains constant throughout bargaining. These assumptions streamline the analysis and can be relaxed somewhat. First, I prove in the appendices that the main results extend to a model allowing more politicians and multiple interest groups that can have access to multiple politicians. Second, stationary access is an analytically convenient way to capture the prevalent view that access is essentially fixed once active policymaking begins (Powell, 2014; Powell and Grimmer, 2016). Of course, access could potentially vary over time, so studying the finer dynamics of access throughout the policymaking process is an interesting avenue for future work.

[17] The lobbying technology is similar to, e.g., Martimort and Semenov (2008); Bils, Duggan and Judd (2021), and an extension in Açemoglu, Egorov and Sonin (2013). See Grossman and Helpman (2002) for an extensive overview and discussion about interpretation. Also see, e.g., Großer, Reuben and Tymula (2013); Powell (2014), and Baron (2019).

Analysis of Equilibrium Legislating and Lobbying

To begin the analysis, I characterize equilibrium behavior in order to introduce how access can affect the strategic calculus for different actors. First, I highlight that equilibrium voting and proposing by politicians has fundamental similarities to related models without lobbying. Then, I characterize equilibrium lobbying and show how it depends on conjectures about voting and non-lobbied proposals. Finally, I combine the preceding qualitative insights in order to sharpen the characterization and more precisely describe how voting, proposing, and lobbying affect each other in equilibrium. Crucially, the characterization explicitly reveals how access — by determining how strongly players anticipate lobbying — will affect voting, proposing, and lobbying.

Since bargaining continues after rejected proposals, there is a feedback between proposals and legislative voting in equilibrium (as in, e.g., Banks and Duggan, 2006a). Optimal policy proposals are shaped by anticipating what M will accept, which depends on M's expectations about future policymaking, which are consistent with proposal strategies in equilibrium. A key step in the analysis shows how access influences M's expectations and thus the acceptance set, thereby affecting proposals that are constrained by the limits of what M will pass.

More precisely, M will pass a proposal if and only if it exceeds her *reservation value* of keeping q for another period and continuing active bargaining. Formally, M's reservation value is $(1-\delta)u_M(q)+\delta V_M^*$, where V_M^* denotes M's equilibrium continuation value immediately after rejecting a proposal.[18] By stationarity, V_M^* is the same each period, so M's reservation value is constant and thus her voting behavior is the same each period. Specifically, the *acceptance set* is $A^*=[-\overline{x}^*,\overline{x}^*]$, where $\overline{x}^*$ is the positive solution to $u_M(x)=(1-\delta)u_M(q)+\delta V_M^*$.

Anticipating what M will pass, each politician (whenever recognized) proposes their favorite policy in A^* (also analogous to Banks and Duggan, 2006a). Clearly, M will simply propose her ideal point, 0. The partisans are constrained by A^* in equilibrium, so L proposes $-\overline{x}^*$ and R proposes $\overline{x}^*$.[19] Finally, if ℓ rejects g's offer or g cannot lobby, then ℓ proposes the policy in A^* closest to $\hat{x}_\ell$, denoted z^*.

Finally, the interest group, g, wants to shift ℓ's proposal as far towards $\hat{x}_g$ as is worth paying for. This strategic calculus depends on its conjectures about voting and non-lobbied proposals. First, shifting ℓ's proposal requires that g compensate her for not instead rejecting and proposing z^*. In equilibrium, g will always make an offer that ℓ accepts, since it can always do weakly better than the trivial acceptable offer of z^* without payment. More

[18] Appendix B contains explicit expressions of continuation values.

[19] This property follows from $|q|<\min\{|\hat{x}_L|,\hat{x}_R\}$ because standard arguments imply $\overline{x}^*<|q|$.

precisely, since g knows ℓ's payoff from proposing z^*, it will compensate her exactly and extract all of the surplus. Stationarity implies that the acceptance set A^* does not depend on today's proposal, so from g's perspective there is a cost of $u_\ell(z^*) - u_\ell(y)$ associated with each policy $y \in A^*$.

Additionally, in principle g could potentially benefit from lobbying for policy outside of A^* if tomorrow's proposer is likely to be an ideological ally who will pass favorable policy for free. Yet, ℓ shares those expectations about future play and therefore must be compensated accordingly in order to propose any policy outside A^*. In equilibrium, the cost of buying delay is never worthwhile for g and it never lobbies for proposals that will be rejected.[20]

In sum, g offers the policy in A^* that provides the best policy payoff given the associated cost. Formally, (y^*, m^*) consists of the policy $y^* = \arg\max_{y \in A^*} u_g(y) + u_\ell(y) - u_\ell(z^*)$ and transfer $m^* = u_\ell(z^*) - u_\ell(y^*)$.[21] Thus, g successfully lobbies ℓ to propose the policy in A^* that maximizes their cumulative policy utility, which is $\hat{y} = \frac{\hat{x}_\ell + \hat{x}_g}{2}$ since they both have quadratic policy utility.

The characterization of equilibrium lobbying implies that the model can be reinterpreted as a one-dimensional bargaining environment in which ℓ has recognition probability $(1 - \alpha)\rho_\ell$ and there is an additional politician at $\hat{y}$ who has recognition probability $\alpha\rho_\ell$. After modifying the legislature to include this additional proposer representing the effect of g's lobbying, politicians propose acceptable bills closest to their ideal point. Applying insights from Cho and Duggan (2003) to this fictitious enlarged legislature implies that this class of equilibria has a unique distribution of equilibrium policies.

Proposition 1 establishes that a stationary legislative lobbying equilibrium exists and all such equilibria have the same outcome distribution. Henceforth, I drop qualifiers and say *equilibrium*. Moreover, it collects the preceding observations to characterize a variety of equilibrium behavior: which policies pass and which will be rejected; which policies various politicians will propose; and which policies the interest group will lobby for and how much it will pay. Figure 2 illustrates a hypothetical equilibrium acceptance set and proposals.

Proposition 1. *A stationary legislative lobbying equilibrium exists and every such equilibrium has the same outcome distribution. In equilibrium,*

(i) the acceptance set is $A^* = [-\overline{x}^*, \overline{x}^*]$, *where* $0 < \overline{x}^* < |q|$;

(ii) M *proposes* 0, R *proposes* $\overline{x}^*$, *and* L *proposes* $-\overline{x}^*$;

(iii) if ℓ *is not lobbied, she proposes the policy* $z^* \in A^*$ *closest to* $\hat{x}_\ell$;

[20] See Appendix C for technical details.

[21] Uniqueness of y^* follows because $u_g + u_\ell$ is strictly concave and A^* is a nonempty closed interval.

(iv) if g can lobby, then it successfully lobbies ℓ to propose the policy $y^* \in A^*$ closest to $\hat{y} = \frac{\hat{x}_\ell + \hat{x}_g}{2}$ using the payment $m^* = u_\ell(z^*) - u_\ell(y^*)$.

Figure 2: Equilibrium characterization

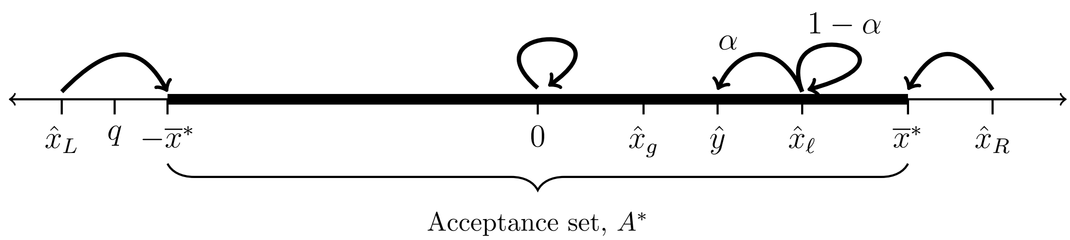

Figure 2 illustrates equilibrium proposals for a hypothetical legislature. Arrows point from politician ideal points to proposals. The bold interval is the acceptance set, A^*. If ℓ is recognized, then with probability α she is lobbied to propose y^*, the policy in A^* closest to $\hat{y} = \frac{\hat{x}_g + \hat{x}_\ell}{2}$, and otherwise she proposes z^*, the policy in A^* closest to $\hat{x}_\ell$. In the depicted legislature, $y^* = \hat{y}$ and $z^* = \hat{x}_\ell$.

Proposition 1 implies that M's equilibrium continuation value is simply the weighted sum of her policy utility from equilibrium proposals, weighted by their probabilities:

$$V_M^* = \rho_M \, u_M(0) + \rho_L \, u_M(-\overline{x}^*) + \rho_R \, u_M(\overline{x}^*) + \rho_\ell \Big(\alpha \, u_M(y^*) + (1-\alpha) \, u_M(z^*) \Big). \tag{1}$$

Substituting (1) into M's indifference condition that defines the boundaries of A^* yields Corollary 1.1, which sharpens our characterization of $\overline{x}^*$.

Corollary 1.1. *In equilibrium, the boundaries of $A^* = [-\overline{x}^*, \overline{x}^*]$ are characterized by*

$$\overline{x}^* = \left(- \frac{(1-\delta) u_M(q) + \delta \, \rho_\ell \Big(\alpha \, u_M(y^*) + (1-\alpha) \, u_M(z^*) \Big)}{1 - \delta(\rho_L + \rho_R)} \right)^{\frac{1}{2}}. \tag{2}$$

Corollary 1.1 implies that the equilibrium acceptance set, A^*, expands if: the status quo (q) shifts away from M, patience (δ) decreases, or total partisan recognition probability ($\rho_L + \rho_R$) increases. These effects are familiar from related models without lobbying (e.g., Banks and Duggan, 2006a). The effects of access, α, are new. Intuitively, greater access causes M to put more weight on the possibility that g might lobby ℓ in the future if today's proposal fails. If lobbying would make ℓ's proposal worse for M, then A^* expands because she is less inclined to keep bargaining, and vice versa. Specifically, (2) reveals that A^* expands if

y^* is farther than z^* from M, and vice versa. Thus, the effect of α on A^* depends critically on how extreme g is relative to ℓ.

Although the effect of access on A^* is original to this paper, it falls under the umbrella of a more general relationship that is familiar from related work without lobbying — the acceptance set expands as the distribution of equilibrium proposals shifts away from M. To be more precise about this general relationship, I next define a notion of changes in *legislative extremism* as a function of α and ρ. The definition compares distributions of unconstrained ideal proposals using first order stochastic dominance, a standard partial order for probability distributions.

Definition 1. For any pair (ρ, α), let $\Lambda(\rho, \alpha)$ be a lottery that puts probability $\alpha\rho_\ell$ on $|\hat{y}|$, probability $(1-\alpha)\,\rho_\ell$ on $|\hat{x}_\ell|$, and probability ρ_j on $|\hat{x}_j|$ for each politician $j \neq \ell$. Say that *legislative extremism increases* if changing (ρ, α) to (ρ', α') is such that: (i) for all $x \in X$, the lottery $\Lambda(\rho', \alpha')$ puts weakly greater probability on x' such that $|x'| \geq |x|$ and (ii) for some $x \in X$, the lottery $\Lambda(\rho', \alpha')$ puts strictly greater probability on x' such that $|x'| > |x|$.

Equivalently, legislative extremism increases if $\Lambda(\rho', \alpha')$ first order stochastically dominates $\Lambda(\rho, \alpha)$. Two distinct special cases in which legislative extremism increases are (i) transferring recognition probability from M to other politicians, or (ii) increasing α if $\hat{y}$ is farther than $\hat{x}_\ell$ from M.

Taking stock, and generalizing our earlier observation, A^* expands as either: legislative extremism increases, δ decreases, or q shifts away from M. By changing the acceptance set, any of these changes will also shift proposals on the boundaries of A^*. Thus, they always affect what L and R will propose. Moreover, they can also shift y^* or z^* if either is constrained by A^*. If so, these changes can also affect g's equilibrium lobby transfer, $m^* = u_\ell(z^*) - u_\ell(y^*)$.

Notably, changes in A^* are the only channel through which m^* can vary, since y^* is either $\hat{y}$ or a boundary of A^*, and analogously for z^*. Building on this observation, Lemma 1 establishes that m^* weakly increases as as either: legislative extremism increases, δ decreases, or q shifts away from M.

Lemma 1. *The interest group's equilibrium payment, m^*, increases as A^* expands.*

Expanding A^* can increase m^* in two distinct ways. First, if y^* is constrained by A^*, then (i) g gets more slack to shift ℓ's proposal farther and (ii) g is willing to pay more to do so. Second, if z^* is constrained by A^*, then (i) ℓ gets more slack to pass more favorable policy if she rejects g's offer and is therefore more inclined to reject any lobby offer, but (ii) g is willing to pay the additional amount required for ℓ to accept.

Next, Proposition 2 builds on the preceding observations to characterize how equilibrium voting, proposals, and expenditures each depend on legislative extremism (α, ρ), the status quo (q), and patience (δ).

Proposition 2. *If either (i) legislative extremism increases, (ii) the status quo policy becomes more extreme, or (iii) patience decreases, then:*

1. *the acceptance set, A^*, expands;*
2. *proposals constrained by A^* become more extreme; and*
3. *the lobby payment, m^*, weakly increases.*

Consequences of Access

Since access (α) affects legislative extremism, Proposition 2 reveals that it can have a variety of effects in equilibrium. Broadly, the direct effect of α on g's lobbying chances can affect ℓ's expected proposal, which can then affect what will pass, what will be proposed, and how many resources will be devoted to lobbying.

Crucially, however, α does not have any effects if subsequent lobbying by g will not shift ℓ's proposal, i.e., if $y^* = z^*$. Such inconsequential lobbying requires that either (i) $\hat{x}_\ell = \hat{x}_g$, or (ii) $\hat{x}_\ell$ and $\hat{y}$ are outside the acceptance set in the same direction. To focus on the interesting case, henceforth I assume $\hat{x}_\ell \neq \hat{x}_g$. In case (ii), the acceptance set is $A^* = [-\overline{x}, \overline{x}]$, where

$$\overline{x} = \left(-\frac{(1-\delta)\, u_M(q)}{1 - \delta(\rho_L + \rho_R + \rho_\ell)} \right)^{\frac{1}{2}}. \tag{3}$$

Although $\overline{x}$ resembles (2), it is defined in terms of primitives and, crucially, does not depend on $\hat{x}_\ell, \hat{x}_g$, or α. Thus, (3) reveals that case (ii) arises if and only if $\max\{\hat{x}_\ell, \hat{y}\} \leq -\overline{x}$ or $\overline{x} \leq \min\{\hat{x}_\ell, \hat{y}\}$. Using this observation, Lemma 2 characterizes the conditions under which access is consequential. Let $\underline{\mathcal{X}}(\hat{x}_g) = \min\{-\overline{x}, -2\overline{x} - \hat{x}_g\}$ and $\overline{\mathcal{X}}(\hat{x}_g) = \max\{\overline{x}, 2\overline{x} - \hat{x}_g\}$, which always satisfy $\underline{\mathcal{X}}(\hat{x}_g) \leq -\overline{x} < \overline{x} \leq \overline{\mathcal{X}}(\hat{x}_g)$.

Lemma 2. *Lobbying affects ℓ's proposal, i.e., $y^* \neq z^*$, if and only if $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), \overline{\mathcal{X}}(\hat{x}_g))$.*

Lemma 2 has two key implications. First, access has no effect if and only if ℓ leans far enough in either direction — fixing $\hat{x}_g$, we have $y^* = z^* = \overline{x}$ if $\hat{x}_\ell$ leans sufficiently rightward, and $y^* = z^* = -\overline{x}$ if $\hat{x}_\ell$ leans sufficiently leftward. Second, if ℓ is not sufficiently extreme, then access will shift ℓ's expected proposal and thereby affect the distribution of equilibrium proposals, i.e., alter legislative extremism.

Combining the second implication of Lemma 2 with Proposition 2 yields Corollary 2.1, which collects catalogs the potential consequences of access and shows how they depend on whether lobbying would make ℓ's proposal more or less extreme.

Corollary 2.1 (Effects of Access)**.** *Suppose $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), \overline{\mathcal{X}}(\hat{x}_g))$. If $|\hat{y}| > |\hat{x}_\ell|$, then as α increases:*

(i) ***target proposal effect*** *– ℓ is more likely to propose y^* and less likely to propose z^*;*

(ii) ***voting effect*** *– the acceptance set, A^*, expands;*

(iii) ***extreme proposal effect*** *– proposals constrained by A^* become more extreme; and*

(iv) ***lobbying expenditure effect*** *– the lobby payment, m^*, weakly increases.*

If $|\hat{y}| < |\hat{x}_\ell|$, then effect (i) is analogous but effects (ii)–(iv) are reversed.

The nature of the indirect effects, *(ii)* – *(iv)*, depends on how extreme g is relative to ℓ, as that determines whether legislative extremism will increase or decrease in α. For example, if $0 < \hat{x}_\ell < \hat{x}_g$, then increasing α will increase legislative extremism so the acceptance set will expand, constrained proposals will shift farther outward, and lobbying expenditures will weakly increase.

The extreme proposal effect is not limited to the partisans, L and R, as it can also change either the lobby proposal, y^*, or ℓ's non-lobby proposal, z^*. It cannot, however, alter both y^* and z^* simultaneously because that would require both y^* and z^* to be constrained. In that case, M would indifferent between them, so the target proposal effect would not affect M's reservation value. Thus, there would be no voting effect and, in turn, no extreme proposal effect on y^* and z^*.

Whom to access?

Thus far, I have shown how (i) access can affect several behaviors by various actors and (ii) the direction of those effects depends on the relative extremism of the group and its target. Since groups appear to have various tools to increase their access, such as campaign contributions or revolving door hiring, I now study who they want to target.

To isolate policy considerations, I allow g to freely choose access.[22] The key insights can be conveyed by studying a one-time choice of access prior to bargaining. Substantively, this captures the possibility that interest groups "may make contributions in anticipation that they may need access to a legislator during a legislative term, rather than when the necessity

[22]The core insights are unchanged by including standard convex cost functions for access.

to purchase influence arises" (Powell and Grimmer, 2016, p. 978). Specifically, I analyze how α affects g's equilibrium value:

$$\rho_M \, u_g(0) + \rho_R \, u_g(\overline{x}^*) + \rho_L \, u_g(-\overline{x}^*) + \rho_\ell \left[\alpha \Big(u_g(y^*) + u_\ell(y^*) - u_\ell(z^*) \Big) + (1-\alpha) \, u_g(z^*) \right]. \tag{4}$$

Although (4) is similar to (1), it sums over g's policy utility and also accounts for g's equilibrium lobbying expenditure, $m^* = u_\ell(z^*) - u_\ell(y^*)$.

I begin with a relatively straightforward observation: the group will not pay for access if ℓ is sufficiently extreme. Essentially, g's lobbying would not shift ℓ's proposal, so access is inconsequential for g. To sharpen this observation, Proposition 3 uses Lemma 2 to characterize a necessary condition for buying access: lobbying must be consequential.

Proposition 3. *The interest group strictly prefers nonzero access only if* $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), \overline{\mathcal{X}}(\hat{x}_g))$.

Since $\underline{\mathcal{X}}(\hat{x}_g) < -\overline{x} < \overline{x} < \overline{\mathcal{X}}(\hat{x}_g)$ always holds, Proposition 3 implies that any group *may* want access to ℓ if $\hat{x}_\ell \in (-\overline{x}, \overline{x})$. In that case, sufficiently low α guarantees that ℓ is unconstrained when proposing, regardless of $\hat{x}_g$, so lobbying would change her proposal and is thus consequential to g.

If lobbying is consequential, i.e., $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), \overline{\mathcal{X}}(\hat{x}_g))$, then inspecting (4) reveals how α can affect g's welfare. First, it affects g's expected lobbying gain when ℓ is recognized, $\alpha[u_g(y^*) + u_\ell(y^*) - u_\ell(z^*) - u_g(z^*)]$, by changing g's lobbying probability and (potentially) its lobbying surplus. The lobbying surplus changes through (i) the target proposal effect, which can change $u_g(y^*) - u_g(z^*)$, and (ii) the lobbying expenditure effect, which can change m^*. Notably, g's lobbying surplus always weakly increases in α — if g is more centrist than ℓ, then g pays weakly less for the same policy; if g is more extreme than ℓ, then g can pass weakly more extreme policy and will do so if that increases its lobbying surplus.

Second, α can also change g's expected policy payoff when a partisan is recognized, $\rho_R \, u_g(\overline{x}^*) + \rho_L \, u_g(-\overline{x}^*)$. This effect flows entirely through the extreme proposal effect. It can be good or bad for g, depending on how extreme g is relative to ℓ and potentially also partisan recognition probability, ρ_L and ρ_R. If both extreme proposals shift towards $\hat{x}_g$, then g benefits. If both shift away, then g is worse off. Finally, if one shifts closer while the other shifts away, then whether g benefits will depend on the relative magnitude of ρ_L and ρ_R.

These two channels may work in opposite directions or together in g's favor. For an example in which they work together, consider $0 < \hat{x}_g < \hat{x}_\ell < \overline{x}^*$. Then, increasing α shifts extreme proposals inward towards g from both sides, so g clearly wants access. More broadly, this holds whenever (i) $\hat{x}_g \in \text{int}\, A^*$ and (ii) A^* shrinks in α, i.e., y^* is more centrist than z^*. Thus, beyond the example above, g also benefits from increasing α if ℓ is in an intermediate range on the opposite side of M.

To see them oppose each other, consider $0 < \hat{x}_\ell < \hat{x}_g < \overline{x}^*$. Then, the extreme proposal effect discourages access because both partisan proposals shift outward away from g.

In the two preceding examples, the extreme proposal effect is unambiguous because $\hat{x}_g$ is strictly inside A^*. In that case, varying α either shifts both partisan proposals away from g or shifts both towards g.

In contrast, if $\hat{x}_g$ is not strictly inside A^*, then the extreme proposal effect depends on proposal power. Specifically, varying α makes one partisan's proposal more favorable for g but also makes the other partisan's proposal less favorable, so the overall extreme proposal effect depends on the relative recognition probability of L and R.

To distinguish these possibilities in terms of primitives, I show that the extreme proposal effect can be unambiguous if and only if $\hat{x}_g$ lies in an interval around M. Notably, the boundaries of this interval are defined by $\overline{x}$, introduced earlier in (3). Thus, I first use it to define useful terminology.

Definition 2. Player j is *moderate* if $\hat{x}_j \in (-\overline{x}, \overline{x})$. Otherwise, j is *extremist*.

Lemma 3 shows that moderate groups can be strictly inside the acceptance set, but extremist groups cannot.

Lemma 3. *If g is moderate, then there exists $\tilde{x} < |\hat{x}_g|$ such that $\hat{x}_\ell \notin (-\tilde{x}, \tilde{x})$ implies $\hat{x}_g \in intA^*$ for α sufficiently small. If g is extremist, then $\hat{x}_g \notin intA^*$ for all $\hat{x}_\ell$ and all α.*

The next two sections leverage the distinction highlighted in Lemma 3 to flesh out a key insight of this analysis: g's incentives to acquire access depend on (i) its own extremism and (ii) its extremism relative to ℓ.

Who do moderate groups want to access?

A key implication of Lemma 3 is that increasing α from zero has an unambiguous extreme proposal effect for moderate groups if ℓ is not too centrist. In turn, we can make two broad observations. First, a moderate g wants access to a range of relatively more extreme politicians on its side of the spectrum, since every effect is beneficial. In contrast, access to slightly more centrist politicians has harmful indirect effects that counteract g's direct benefit from the target proposal effect.

Refining these observations, Proposition 4 shows that moderate groups want to access a range of more extreme politicians *and* an intermediate range of politicians opposite M, but will forgo access to politicians in a relatively more centrist range. Throughout this section, I analyze $\hat{x}_g > 0$ without loss of generality.

Proposition 4. *If $\hat{x}_g \in (0, \overline{x})$, then there are cutpoints satisfying $-\hat{x}_g < x' < x'' < \hat{x}_g$ such that g forgoes access if $\hat{x}_\ell \in (x'', \hat{x}_g)$ but wants access if $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), x') \cup (\hat{x}_g, \overline{\mathcal{X}}(\hat{x}_g))$.*

First, g wants access to ℓ if (i) they are on the same side of M and (ii) ℓ is more extreme but not too extreme, i.e., if $\hat{x}_\ell \in (\hat{x}_g, \overline{\mathcal{X}}(\hat{x}_g))$. In this case, g benefits from every effect of increasing α. If it lobbies, then it will pay weakly less for the same policy. And even if it does not lobby, M's reservation value will increase and thereby shrink A^*, with the resulting partisan proposal effect always benefiting g because $\hat{x}_g \in A^*$ for all α in this case.

Additionally, g wants access if ℓ is in an intermediate interval on the opposite side of M. Specifically, if $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), -\tilde{x}]$, then g is strictly inside A^* at $\alpha = 0$. Since A^* will shrink as α increases, again every effect of increasing α from zero will benefit g. And even if ℓ is slightly more centrist, i.e., $\hat{x}_\ell \in (-\tilde{x}, x')$, then g's expected gain from the the target proposal effect outweighs any expected loss from the other effects.

Next, g forgoes access if ℓ is on the same side of M and slightly more centrist, i.e., $\hat{x}_\ell \in (x'', \hat{x}_g)$. In this case, g will be strictly inside A^* at $\alpha = 0$ and therefore dislike the extreme proposal effect, which shifts partisan proposals outward as depicted in Figure 3. Crucially, if ℓ and g are close enough, then this negative extreme proposal effect dominates the other effects of access.

Intuitively, lobbying will not shift ℓ's proposal very much and g's payoff is not very sensitive to those changes, so the direct benefit is small. Meanwhile, M is more sensitive to those changes, and the acceptance set expands enough that the negative extreme proposal effect is relatively larger.[23] Notably, this case exists for any distribution of proposal power in which L or R is recognized with positive probability. Thus, non-zero partisan proposal power is crucial for g to forgo access, but the magnitude and relative recognition probability of L and R only affect the size of this range.

Finally, in general g's preference for access is unclear if ℓ is in a centrist range, i.e., $\hat{x}_\ell \in (x', x'')$. In this case, the effects of access conflict, as in the previous case, but the

[23] The indirect effects of access on voting and proposals in this paper have connections with spatial models of dynamic bargaining (Baron, 1996; Buisseret and Bernhardt, 2017; Zápal, 2020). There, the policy in place at the end of today becomes the status quo tomorrow, so proposers weigh how today's proposal can affect what can pass tomorrow when someone else might have proposal rights. In equilibrium, politicians pass more centrist policies today in order to make centrist veto players less inclined to pass policy in the future, thus constraining the scale of policy changes by potential future proposers on the other end of the spectrum. In this paper, policymaking ends once a proposal passes, so a group considering access weighs (i) how it will affect the target's proposal if she is recognized, and (ii) how it will affect what happens if the target is not recognized. Since access can indirectly influence which policies pass in equilibrium, incentives to increase or forgo access are affected by a similar desire to constrain potentially extreme proposers.

Figure 3: Forgoing access to more centrist legislators

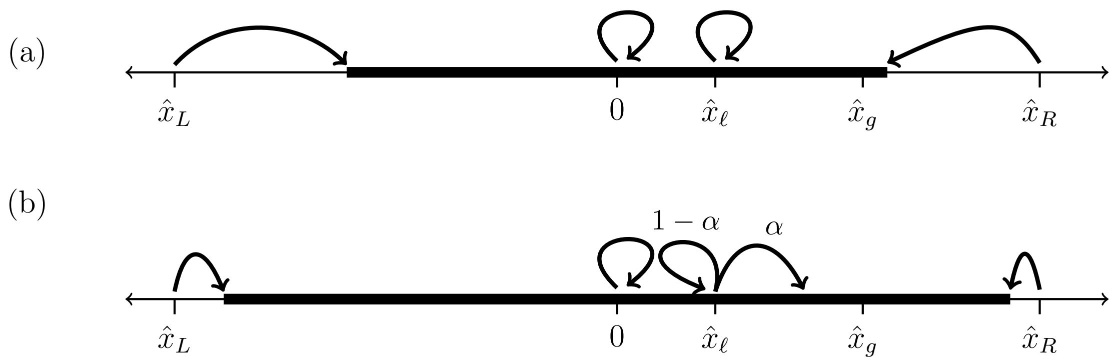

Figure 3 illustrates why a moderate group, g, forgoes access ($\alpha = 0$) if $\hat{x}_\ell \in (x'', \hat{x}_g)$. Part (a) displays equilibrium behavior for $\alpha = 0$. Part (b) illustrates $\alpha > 0$. In each, the bold interval is the acceptance set. Increasing α makes lobbying more likely, which worsens M's expectations, and expands the acceptance set, as shown in (b). Thus, partisan proposals are more extreme. If $\hat{x}_g$ and $\hat{x}_\ell$ are close, then the loss from more extreme partisan proposals dominates and g prefers $\alpha = 0$.

overall effect now depends on partisan recognition probability, specifically either their total or relative magnitude. A stark example is when g is not in A^* at $\alpha = 0$. Then, the extreme proposal effect of increasing α from zero depends on the relative magnitude of ρ_L and ρ_R, since one partisan proposal becomes less favorable for g and the other more favorable.

Who do extreme groups want to access?

Like moderate groups, extreme groups have clear preferences over access if ℓ is aligned with them and extremist. Unlike moderate groups, however, extreme groups never want access in that case because lobbying will not change ℓ's proposal. Formally, $\hat{x}_g \geq \overline{x}$ implies $\overline{\mathcal{X}}(\hat{x}_g) = \overline{x}$ in Lemma 3 and analogously $\hat{x}_g \leq -\underline{x}$ implies $\underline{\mathcal{X}}(\hat{x}_g) = -\overline{x}$.

A key difference is that, since extreme groups are always outside A^*, the direction of the extreme proposal effect always depends on the relative magnitude of ρ_L and ρ_R, regardless of $\hat{x}_\ell$. To overcome this difficulty and shed some light on who extreme groups want to access, Proposition 5 focuses on cases in which one partisan is sufficiently weak. Substantively, this could reflect partisan gatekeeping in which extremists on one side of the spectrum are largely excluded from writing policy. Again, I focus on $\hat{x}_g > 0$ without loss of generality.

Proposition 5. *Suppose $\hat{x}_g > \overline{x}$.*

(i) If ρ_L is small enough, there exists $x' < 0$ such that g wants access if $\hat{x}_\ell \in (x', \overline{x})$.

(ii) If ρ_R is small enough, there exists $x'' \geq -\overline{x}$ such that g wants access if $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), x'')$.

(iii) If $\hat{x}_\ell \geq \overline{x}$, then g does not want access.

In (i), g's opposing partisan is unlikely to propose, so g wants access to a range of moderate politicians including all right-leaning moderates and sufficiently centrist opponents. As long as ℓ does not lean too far leftward, increasing access will worsen M's expectations about future policy and thus expand A^*. Although L's proposal gets worse for g, she is unlikely to propose, so that downside is outweighed by the prospect of better proposals by ℓ and R.

In (ii), g's aligned partisan is unlikely to propose and it wants access to opponents (except those extreme enough to make lobbying trivial) and, if the lobbying surplus is large enough, potentially also to sufficiently centrist aligned moderates. The logic is symmetric to the previous case.

Proposal power and the value of access

Thus far, I have focused primarily on how ideology affects g's incentives to acquire access to ℓ, while noting how partisan proposal power can play a role in those incentives. In this section, I focus on the effects of the target's proposal power. Specifically, I study how ℓ's recognition probability (ρ_ℓ) affects g's *willingness to pay* (WTP) for access, i.e., the marginal effect of α on g's equilibrium value in (4).

Empirical evidence suggests that interest groups prioritize access to legislators who have more proposal power[24] and it is typically taken for granted that greater proposal power makes access more valuable. Yet, the preceding analysis highlights a potentially important subtlety. Although ρ_ℓ increases g's expected lobbying benefit from access, it also amplifies the (possibly negative) extreme proposal effect. Proposition 6 establishes that, despite these potentially competing effects, the standard intuition holds in this paper — if g wants access to ℓ, then g's WTP for access weakly increases with ρ_ℓ.

Proposition 6. *All else equal, the interest group is willing to pay more for access if the target politician has higher recognition probability.*

Proposition 6 is a stark result, reflecting the robustness of the empirical finding that groups prioritize politicians with greater proposal power. It does not depend on the policy preferences of ℓ or g, partisan proposal power, patience, or the status quo. Although these

[24]This is one of the most prominent stylized facts about outside influence and is supported by two robust empirical regularities: (i) legislators on important committees, especially committee chairmen, attract more contributions (Fouirnaies, 2018; Berry and Fowler, 2018), and (ii) lobbyists connected to those legislators command a premium (Blanes i Vidal et al., 2012).

other factors can cause ρ_ℓ to have competing effects, the overall effect is always proportional to ρ_ℓ whenever g's WTP is strictly positive. Since Proposition 6 effectively conditions on g wanting access to ℓ, g's WTP either increases in ρ_ℓ or remains at zero. If g happened to have access to ℓ that it did not want and could act to decrease it, then higher ρ_ℓ increases g's willingness to pay to *reduce* α_ℓ. Regardless, higher ρ_ℓ increases g's marginal value of changing α_ℓ in g's preferred direction, whether that is more access or less.[25]

Discussion

Several of the main findings have implications for empirical studies of access. Broadly, they suggest that a more complete empirical picture of outside influence requires: (i) measuring access in addition to lobbying, (ii) accounting for spillover effects that targeted access can have on other actors and behaviors, and (iii) carefully examining empirical relationships between lobbying expenditures and access targeting.

A key finding is that interest groups can influence behavior merely by having access that *could* lead to lobbying. That is, no lobbying does not imply no influence. This counters the widespread view that access has no influence by itself (e.g., Wright, 1989). Moreover, it reinforces recent critiques emphasizing that empirical relationships between political outcomes and lobbying activity may not provide valid estimates of interest group influence without accounting for influence that does not flow through active lobbying (Lowery, 2013; Powell, 2014; Finger, 2019).

Three additional findings flesh out this point and suggest potential ways that access data can supplement lobbying data to provide a more complete picture of influence.

First, I find that targeted access can also influence behavior by non-targeted politicians. Thus, attempts to recover causal effects of access must especially clear about their estimand and how they can convincingly estimate it with their data. Even if access can be randomized, my analysis highlights how equilibrium effects can (i) prevent expectations about future proposals from being held constant by such randomization and (ii) generate spillover effects that would violate the *stable unit treatment value assumption* (SUTVA) (see, e.g., Imbens and Rubin, 2015). Although some scholars have informally noted potential spillovers,[26] I formally derive and trace a channel that flows entirely through legislative considerations.

Second, by analyzing that channel, I find that access can affect not only policy proposals and votes on those proposals, but also lobbying expenditures. Parsing this effect has impli-

[25] I thank an anonymous reviewer for encouraging discussion of this point.

[26] For example, (Kalla and Broockman, 2015) suggest that other politicians might act differently with the hope of attracting donations from the group as well.

cations for observed measures of access and lobbying expenditures, as well as how potential regulations might "redirect money rather than lessen it" (Powell, 2014). Notably, empirical relationships between measures of access and average lobbying expenditures (across group-legislator pairs) can be misleading if they do not account for relative extremism. For groups connecting to more extreme politicians, my analysis suggests (i) a negative correlation between lobbying expenditures and access, and (ii) that regulating access will redirect money to increase spending on lobbying. For groups connecting to centrists, my analysis suggests (i) a positive correlation between lobbying expenditures and access, and (ii) that regulating access will also decrease lobbying expenditures. Finally, this finding also highlights how a group's lobbying spending can change without any change in their lobbyist's effectiveness at shifting policy.

Third, by accounting for these various effects of access, I find that interest groups may crave access to some politicians but be wary of access to others. This finding has implications for *Tullock's puzzle*, the empirical regularity that groups do not spend that aggressively for access (Tullock, 1972; Ansolabehere et al., 2003). Given evidence that groups can increase access in various ways and the standard intuition that groups want more influence, why do they not spend more? I provide a new logic that emphasizes legislative considerations, unlike existing explanations emphasizing costs or competition (e.g., Chamon and Kaplan, 2013). A key consideration is that increasing your potential for influence can affect what happens if that potential is not realized — an unfavorable effect discourages access, whereas a favorable effect increases the bang for the buck. Either way, these effects suggest that groups may spend less on access-seeking behaviors than expected and that they may not spend anything to target slightly more centrist politicians.

Conclusion

I analyze a model of legislative policymaking in which access provides interest groups with opportunities to lobby policy proposals. The equilibrium analysis sheds new light on the consequences of this prominent form of access by showing how it can endogenously affect voting, proposals, and lobbying. It does so by changing each legislator's expectations about policymaking, and thereby changing which policies can pass in equilibrium. Essentially, the potential for *future* lobbying can influence *today's* proposal and lobbying expenditures.

The analysis also sheds light on how much access interest groups want to particular legislators who may be involved in writing policy. Moderate groups forgo access to a range of more centrist legislators since such connections endogenously increase policy extremism enough to outweigh the perk of better lobbying prospects. On the other hand, these groups crave access to more extreme legislators because it facilitates lobbying and also *reduces* policy extremism.

By developing our theoretical expectations for the consequences of a link between access and lobbying proposals, I highlight how such a link can affect policy and shape observed data. The analysis here emphasizes how such access can have indirect effects due to legislative considerations, i.e., what other politicians will vote for and what they will propose if given the opportunity. Although the channel I emphasize is prominent, other important channels are likely present in various situations. Whenever we cannot disentangle multiple channels empirically, we need to be aware that they may oppose or complement each other. To understand these relationships and potentially suggest avenues to disentangle various influence tactics, future work should study how the legislative forces highlighted here interact with other channels of outside influence such as vote buying, informational lobbying, and efforts to influence who gets elected.

For example, consider lobbying to influence votes, or "vote buying." Although I abstract from it in order to isolate lobbying over policy details, the analysis can inform how access to proposers strengthens or weakens incentives to buy key votes. First, however, note that groups may not want vote buying capabilities *ex ante*, for reasons analogous to those encouraging strategic delegation in spatial settings (e.g., Klumpp, 2007; Gailmard and Hammond, 2011) — forgoing vote buying can constrain some potential proposers in a way that benefits the group. Then, the analysis here can apply directly. Second, access to a potential proposer affects how willing veto players are to reject proposals and therefore changes the cost of influencing votes. For example, accessing a slightly more centrist politician increases the group's cost of shifting the opposite end of the acceptance set inward, but it also decreases the cost of shifting the closer end outward. Yet, as in the main analysis, *ex ante* a moderate group wants to constrain extremists on both ends. I show in Appendix E that Proposition 4 extends to a setting with lobbying over votes. Future work should more fully analyze lobbying votes, lobbying policy, and incentives for access.

References

Açemoglu, Daron, Georgy Egorov, and Konstantin Sonin, "A Political Theory of Populism," *Quarterly Journal of Economics*, 2013, *128* (2), 771–805.

Ainsworth, Scott H., "The Role of Legislators in the Determination of Interest Group Influence," *Legislative Studies Quarterly*, 1997, pp. 517–533.

Ansolabehere, Stephen, John M. de Figueiredo, and James M. Snyder Jr., "Why is There So Little Money in U.S. Politics?," *Journal of Economic Perspectives*, 2003, *17* (1), 105–130.

Austen-Smith, David, "Campaign Contributions and Access," *American Political Science Review*, 1995, *89* (03), 566–581.

Awad, Emiel, "Persuasive Lobbying with Allied Legislators," *American Journal of Political Science*, 2020, *64* (4), 938–951.

Bachrach, Peter and Morton S. Baratz, "Two Faces of Power," *American Political Science Review*, 1962, *56* (4), 947–952.

Banks, Jeffrey S. and John Duggan, "A General Bargaining Model of Legislative Policy-Making," *Quarterly Journal of Political Science*, 2006, *1* (1), 49–85.

_ **and** _ , "A Social Choice Lemma on Voting Over Lotteries with Applications to a Class of Dynamic Games," *Social Choice and Welfare*, 2006, *26* (2), 285–304.

Baron, David P., "A Dynamic Theory of Collective Goods Programs," *American Political Science Review*, 1996, *90* (02), 316–330.

_ , "Lobbying Dynamics," *Journal of Theoretical Politics*, 2019, *31* (3), 403–452.

_ **and Ehud Kalai**, "The Simplest Equilibrium of a Majority-Rule Division Game," *Journal of Economic Theory*, 1993, *61* (2), 290–301.

Baumgartner, Frank R., "Interest Groups and Agendas," in "The Oxford Handbook of American Political Parties and Interest Groups" 2010.

_ **, Jeffrey M. Berry, Marie Hojnacki, Beth L. Leech, and David C. Kimball**, *Lobbying and policy change: Who wins, who loses, and why*, University of Chicago Press, 2009.

Berry, Christopher R. and Anthony Fowler, "Congressional Committees, Legislative Influence, and the Hegemony of Chairs," *Journal of Public Economics*, 2018, *158*, 1–11.

Bertrand, Marianne, Matilde Bombardini, and Francesco Trebbi, "Is it Whom You Know or What You Know? An Empirical Assessment of the Lobbying Process," *American Economic Review*, 2014, *104* (12), 3885–3920.

_ , _ , **Raymond Fisman, and Francesco Trebbi**, "Tax-Exempt Lobbying: Corporate Philanthropy as a Tool for Political Influence," *American Economic Review*, 2020, *110* (7), 2065–2102.

Bils, Peter, John Duggan, and Gleason Judd, "Lobbying and Policy Extremism in Repeated Elections," *Journal of Economic Theory*, 2021, *193*, 105223.

Blanes i Vidal, Jordi, Mirko Draca, and Christian Fons-Rosen, "Revolving Door Lobbyists," *American Economic Review*, 2012, *102* (7), 3731–3748.

Bonica, Adam, "Ideology and Interests in the Political Marketplace," *American Journal of Political Science*, 2013, *57* (2), 294–311.

Buisseret, Peter and Dan Bernhardt, "Dynamics of Policymaking: Stepping Back to Leap Forward, Stepping Forward to Keep Back," *American Journal of Political Science*, 2017, *61* (4), 820–835.

Cameron, Charles M., "The Presidential Veto," in George C. Edwards III and William G. Howell, eds., *The Oxford Handbook of the American Presidency*, Oxford University Press, 2009, pp. 362—-82.

Chamon, Marcos and Ethan Kaplan, "The Iceberg Theory of Campaign Contributions: Political Threats and Interest Group Behavior," *American Economic Journal: Economic Policy*, 2013, *5* (1), 1–31.

Cho, Seok-Ju and John Duggan, "Uniqueness of Stationary Equilibria in a One-Dimensional Model of Bargaining," *Journal of Economic Theory*, 2003, *113* (1), 118–130.

Dekel, Eddie, Matthew O. Jackson, and Asher Wolinsky, "Vote Buying: Legislatures and Lobbying," *Quarterly Journal of Political Science*, 2009, *4* (2), 103–128.

Diermeier, Daniel, Michael Keane, and Antonio Merlo, "A Political Economy Model of Congressional Careers," *The American Economic Review*, 2005, *95* (1), 347–373.

Duggan, John, "Majority Voting over Lotteries: Conditions for Existence of a Decisive Voter," *Economics Bulletin*, 2014, *34* (1), 263–270.

Finger, Leslie K., "Interest Group Influence and the Two Faces of Power," *American Politics Research*, 2019, *47* (4), 852–886.

Fouirnaies, Alexander, "When Are Agenda Setters Valuable?," *American Journal of Political Science*, 2018, *62* (1), 176–191.

— **and Andrew B. Hall**, "How Do Interest Groups Seek Access to Committees?," *American Journal of Political Science*, 2018, *62* (1), 132–147.

Friedrich, Carl J., *Constitutional Government and Politics*, Harper: New York, 1937.

Gailmard, Sean and Thomas Hammond, "Intercameral bargaining and intracameral organization in legislatures," *The Journal of Politics*, 2011, *73* (2), 535–546.

Großer, Jens, Ernesto Reuben, and Agnieszka Tymula, "Political Quid Pro Quo Agreements: An Experimental Study," *American Journal of Political Science*, 2013, *57* (3), 582–597.

Grossman, Gene M. and Elhanan Helpman, "Special Interest Politics," *MIT Press Books*, 2002.

Hall, Richard L. and Alan V. Deardorff, "Lobbying as Legislative Subsidy," *American Political Science Review*, 2006, *100* (01), 69–84.

— **and Frank W. Wayman**, "Buying Time: Moneyed Interests and the Mobilization of Bias in Congressional Committees," *American Political Science Review*, 1990, *84* (03), 797–820.

Hansen, John Mark, *Gaining Access: Congress and the Farm Lobby, 1919-1981*, University of Chicago Press, 1991.

Hojnacki, Marie and David C. Kimball, "Organized interests and the decision of whom to lobby in Congress," *American Political Science Review*, 1998, *92* (04), 775–790.

— **and** — , "The Who and How of Organizations' Lobbying Strategies in Committee," *Journal of Politics*, 1999, *61* (04), 999–1024.

Imbens, Guido W. and Donald B. Rubin, *Causal Inference in Statistics, Social, and Biomedical Sciences*, Cambridge University Press, 2015.

Kalandrakis, Tasos, "A Priori Bounds on Legislative Bargaining Agreements," *American Journal of Political Science*, 2021, *Forthcoming.*

Kalla, Joshua L. and David E. Broockman, "Campaign Contributions Facilitate Access to Congressional Officials: A Randomized Field Experiment," *American Journal of Political Science*, 2015, *60* (3), 545–558.

Klumpp, Tilman, "Strategic Voting and Conservatism in Legislative Elections," *Unpublished manuscript*, 2007.

Kollman, Ken, "Inviting Friends to Lobby: Interest Groups, Ideological Bias, and Congressional Committees," *American Journal of Political Science*, 1997, *41* (2), 519–544.

Kroeger, Mary A., "Groups as Lawmakers: Group-Sponsored Bills in the California State Legislature," `https://drive.google.com/file/d/1gXkbtG8vSs4LYQnOcQEUpT2lq_eH7jig/view` 2021.

Leech, Beth L., "Lobbying and influence," in L. Sandy; Jeffrey M. Berry Maisel and George C. Edwards III, eds., *The Oxford Handbook of American Political Parties and Interest Groups*, Oxford University Press, 2010.

Levy, Gilat and Ronny Razin, "Dynamic Legislative Decision Making when Interest Groups Control the Agenda," *Journal of Economic Theory*, 2013, *148* (5), 1862–1890.

Liu, Huchen, "Campaign Contributions and Access to Congressional Offices: Patterns in Foreign Lobbying Data," *Political Research Quarterly*, 2021, *75* (3).

Lohmann, Susanne, "Information, Access, and Contributions: A Signaling Model of Lobbying," *Public Choice*, 1995, *85* (3-4), 267–284.

Lowery, David, "Lobbying influence: Meaning, Measurement and Missing," *Interest Groups & Advocacy*, 2013, *2* (1), 1–26.

Martimort, David and Aggey Semenov, "Ideological Uncertainty and Lobbying Competition," *Journal of Public Economics*, 2008, *92* (3), 456–481.

McCrain, Joshua, "Revolving Door Lobbyists and the Value of Congressional Staff Connections," *The Journal of Politics*, 2018, *80* (4), 1369–1383.

Miller, David R., "Empirical Approaches to the Study of Access," *Interest Groups & Advocacy*, 2021, *10* (3), 1–17.

_ , "On Whose Door to Knock? Organized Interests' Strategic Pursuit of Access to Members of Congress," *Legislative Studies Quarterly*, 2021, *47* (1), 157–192.

Milyo, Jeffrey, David Primo, and Timothy Groseclose, "Corporate PAC campaign contributions in perspective," *Business and Politics*, 2000, *2* (1), 75–88.

Powell, Eleanor Neff and Justin Grimmer, "Money in Exile: Campaign Contributions and Committee Access," *The Journal of Politics*, 2016, *78* (4), 974–988.

Powell, Lynda W., "The Influence of Campaign Contributions on the Legislative Process," *Duke Journal of Constitutional Law & Public Policy*, 2014, *9* (1), 75–101.

Schattschneider, E.E., *The Semi-Sovereign People*, Holt, Rhinehart and Winston, New York, 1960.

Schlozman, Kay Lehman and John T. Tierney, *Organized Interests and American Democracy*, Harper- Collins, 1986.

Schnakenberg, Keith E., "Informational Lobbying and Legislative Voting," *American Journal of Political Science*, 2017, *61* (1), 129–145.

Simon, Herbert A., "Notes on the Observation and Measurement of Political Power," *Journal of Politics*, 1953, *15* (4), 500–516.

Snyder Jr., James M., "On Buying Legislatures," *Economics & Politics*, 1991, *3* (2), 93–109.

Tullock, Gordon, "The Purchase of Politicians," *Western Economic Journal*, 1972, *10* (3), 354–55.

Wright, John R., "PAC Contributions, Lobbying, and Representation," *Journal of Politics*, 1989, *51* (3), 713–729.

_ , *Interest Groups and Congress: Lobbying, Contributions, and Influence*, Allyn & Bacon, 1996.

Zápal, Jan, "Simple Markovian Equilibria in Dynamic Spatial Legislative Bargaining," *European Journal of Political Economy*, 2020, *63*, 101816.

Appendix (online only)

Table of Contents

A Proof of Proposition 4 for Baseline Model

First, I prove the key parts of Proposition 4 for the model presented in the main text. The logic is similar to the full proof in Appendix B, but the details are easier to digest. Let subscripts on equilibrium objects denote dependence on α, e.g., A^*_α denotes the equilibrium acceptance set given α.

Consider $\hat{x}_g \in (0, \overline{x})$. I show that (i) there exists $x'' < \hat{x}_g$ such that g strictly prefers $\alpha = 0$ for all $\hat{x}_\ell \in (x'', \hat{x}_g)$, and (ii) if $\hat{x}_\ell \in (\hat{x}_g, \overline{x})$, then g strictly prefers $\alpha > 0$. The argument for the other regions in which g strictly prefers $\alpha > 0$ is similar to (ii) and can be found in the full proof provided in Appendix B.

We can show that there exists $\tilde{x} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (\tilde{x}, \overline{x})$ implies $\hat{x}_g, \hat{x}_\ell \in A^*_\alpha$ for all α (for details, see Lemma 3 in Appendix B and arguments in Appendix D). Then, Proposition 1 implies $m^* = -u_\ell(\hat{y})$ and, moreover, that the equilibrium outcome distribution is equivalent to a lottery λ_α putting probability ρ_M on 0, ρ_R on $\overline{x}^*_\alpha$, ρ_L on $-\overline{x}^*_\alpha$, $\alpha\,\rho_\ell$ on $\hat{y} = \frac{\hat{x}_\ell + \hat{x}_g}{2}$, and $(1-\alpha)\rho_\ell$ on $\hat{x}_\ell$. Since u_g is quadratic, we can therefore express g's equilibrium value from α access using the mean ($\mathbb{E}[\lambda_\alpha]$) and variance ($\mathbb{V}[\lambda_\alpha]$) of λ_α:

$$\begin{aligned}\mathcal{U}_g(\alpha; \hat{x}_\ell) &= u_g(\mathbb{E}[\lambda_\alpha]) - \mathbb{V}[\lambda_\alpha] - \alpha\,\rho_\ell\, m^* && (5)\\ &= -\hat{x}_g^2 + 2\hat{x}_g\left[\alpha\,\rho_\ell(\hat{y} - \hat{x}_\ell) + \rho_\ell\,\hat{x}_\ell + (\rho_R - \rho_L)\overline{x}^*_\alpha\right] \\ &\qquad - \left[\alpha\,\rho_\ell(\hat{y}^2 - \hat{x}_\ell^2) + \rho_\ell\,\hat{x}_\ell^2 + (\rho_R + \rho_L)(\overline{x}^*_\alpha)^2\right] - \frac{\alpha\,\rho_\ell}{4}(\hat{x}_\ell - \hat{x}_g)^2. && (6)\end{aligned}$$

Differentiating (6) with respect to α yields:

$$\begin{aligned}\frac{\partial \mathcal{U}_g(\alpha; \hat{x}_\ell)}{\partial \alpha} &= 2\hat{x}_g\left[\rho_\ell(\hat{y} - \hat{x}_\ell) + (\rho_R - \rho_L)\frac{\partial \overline{x}^*_\alpha}{\partial \alpha}\right] \\ &\qquad - \rho_\ell(\hat{y}^2 - \hat{x}_\ell^2) - 2\overline{x}^*_\alpha(\rho_R + \rho_L)\frac{\partial \overline{x}^*_\alpha}{\partial \alpha} - \frac{\rho_\ell}{4}(\hat{x}_\ell - \hat{x}_g)^2 && (7)\\ &\propto (\hat{x}_g - \hat{x}_\ell)^2 - \frac{\delta(\hat{x}_g - \hat{x}_\ell)(3\hat{x}_g + \hat{x}_\ell)}{2[1 - \delta(\rho_L + \rho_R)]}\left[\rho_L\left(1 + \frac{\hat{x}_g}{\overline{x}^*_\alpha}\right) + \rho_R\left(1 - \frac{\hat{x}_g}{\overline{x}^*_\alpha}\right)\right], && (8)\end{aligned}$$

where (8) follows from factoring out ρ_ℓ and simplifying after substituting $\hat{y} = \frac{\hat{x}_\ell + \hat{x}_g}{2}$ and $\frac{\partial \overline{x}^*_\alpha}{\partial \alpha} = \frac{\delta\,\rho_\ell(\hat{x}_g - \hat{x}_\ell)(3\hat{x}_g + \hat{x}_\ell)}{8\overline{x}^*_\alpha[1 - \delta(\rho_L + \rho_R)]}$. There are two cases.

Case 1: If $\hat{x}_g > \hat{x}_\ell$, then (8) is proportional to

$$(\hat{x}_g - \hat{x}_\ell) - \frac{\delta(3\hat{x}_g + \hat{x}_\ell)}{2[1 - \delta(\rho_L + \rho_R)]}\left[\rho_L\left(1 + \frac{\hat{x}_g}{\overline{x}^*_\alpha}\right) + \rho_R\left(1 - \frac{\hat{x}_g}{\overline{x}^*_\alpha}\right)\right]. \qquad (9)$$

As $\hat{x}_\ell \uparrow \hat{x}_g$, (9) converges to

$$-\frac{2\delta \hat{x}_g}{1-\delta(\rho_L+\rho_R)}\left[\rho_L\left(1+\frac{\hat{x}_g}{\overline{x}_1^*}\right)+\rho_R\left(1-\frac{\hat{x}_g}{\overline{x}_1^*}\right)\right]<0, \tag{10}$$

where the inequality follows because $\frac{\hat{x}_g}{\overline{x}_1^*} \in (0,1)$. Thus, continuity of (9) in $\hat{x}_\ell$ implies existence of $x'' < \hat{x}_g$ such that $\mathcal{U}_g(\alpha;\hat{x}_\ell)$ strictly decreases in α for all $\hat{x}_\ell \in (x'', \hat{x}_g)$, as desired.

Case 2: If $\hat{x}_\ell \in (\hat{x}_g, \overline{x})$, then (8) is proportional to

$$(\hat{x}_\ell-\hat{x}_g)+\frac{\delta(3\hat{x}_g+\hat{x}_\ell)}{2[1-\delta(\rho_L+\rho_R)]}\left[\rho_L\left(1+\frac{\hat{x}_g}{\overline{x}_\alpha^*}\right)+\rho_R\left(1-\frac{\hat{x}_g}{\overline{x}_\alpha^*}\right)\right]>0, \tag{11}$$

where the inequality follows because $\frac{\hat{x}_g}{\overline{x}_\alpha^*} \in (0,1)$ for all $\alpha \in [0,1]$. Thus, $\mathcal{U}_g(\alpha;\hat{x}_\ell)$ strictly increases in α.

B Extended Model: More Politicians and Interest Groups

I prove Propositions 1–5 in a model that relaxes restrictions on the number of legislators and interest groups. There are three disjoint sets of players: n^V (finite and odd) voting legislators in N^V; $n^L \geq 3$ committee members in N^L; and $n^G \leq n^L$ interest groups in N^G. Let $N = N^V \cup N^L \cup N^G$.

Throughout, voting legislators are denoted by i and called *voters*. I denote committee members by ℓ and interest groups by g. Each $\ell \in N^L$ is associated with only one group, g_ℓ. Each $g \in N^G$ can have access to multiple $\ell \in N^L$ and this set is $N_g^L \subseteq N^L$. Let $\alpha_\ell \in [0,1]$ denote g_ℓ's access to ℓ.[27]

Legislative bargaining occurs over an infinite number of periods $t \in \{1,2,\dots\}$. The policy space is a non-empty, closed interval $X \subseteq \mathbb{R}$. Let $\rho = (\rho_1,\dots,\rho_{n^L}) \in \Delta([0,1])^{n^L}$ be the distribution of recognition probability.[28] In each period t, bargaining proceeds as follows. If no policy has passed before t, then ℓ proposes with probability $\rho_\ell > 0$. All players observe the period-t proposer, ℓ_t. With probability $1-\alpha_\ell$, g_{ℓ_t} cannot lobby and ℓ_t freely proposes any $x_t \in X$. With probability α_ℓ, g_{ℓ_t} can lobby and offers ℓ_t a binding contract $(y_t, m_t) \in X \times \mathbb{R}_+$. Next, ℓ_t accepts or rejects. Let $a_t \in \{0,1\}$ denote ℓ_t's period-t acceptance decision, where $a_t = 1$ indicates acceptance and $a_t = 0$ if either ℓ_t rejects or g_{ℓ_t} is unable to lobby in t. If ℓ_t accepts, then ℓ_t is committed to propose $x_t = y_t$ in t and g_{ℓ_t} transfers m_t

[27] An independent legislator is accommodated by $\alpha_\ell = 0$.

[28] Where $\Delta([0,1])^{n^L}$ denotes the n^L-dimensional unit simplex.

to ℓ_t. If ℓ_t rejects, then she can propose any $x_t \in X$ and g_{ℓ_t} keeps m_t. All players observe x_t. There is a simultaneous vote by $i \in N^V$ using simple majority rule. If x_t passes, then bargaining ends with x_t enacted in t and all subsequent periods. If x_t fails, then q is enacted in t and bargaining proceeds to $t+1$.

Each player $j \in N$ has quadratic policy utility with ideal point $\hat{x}_j \in X$. To align with the main text, M denotes the median voter. As in the main text, I assume $\hat{x}_M = 0 \in X$ and $q \neq 0$. Additionally, I assume there exists $\ell \in N^L$ that is on the same side of q as M and such that: $\alpha_\ell < 1$ or g_ℓ is also on that same side of q. For example, if $q > 0$, then some $\ell \in N^L$ satisfies $\hat{x}_\ell < q$ and at least one of the following holds: $\alpha_\ell < 1$ or $\hat{x}_{g_\ell} \leq q$.

Players discount streams of per-period utility by common discount factor $\delta \in (0,1)$. For convenience, I normalize per-period policy utility by $(1-\delta)$. Let $I_t^\ell \in \{0,1\}$ equal one if and only if ℓ is the period-t proposer *and* g_{ℓ_t} can lobby in t. Given a sequence of offers $(y_1, m_1), (y_2, m_2), \ldots$, a sequence of proposers $\ell_1, \ell_2, \ldots$ a sequence of acceptance decisions $a_1, a_2, \ldots$, and a sequence of independent policy proposals $x_1, x_2, \ldots$ such that bargaining continues until t, the discounted sum of per-period payoffs for $i \in N^V$ is

$$(1-\delta^{t-1})u_i(q) + \delta^{t-1}\Bigg[(1-a_t)\, u_i(x_t) + a_t u_i(y_t)\Bigg];$$

for $\ell \in N^\ell$,

$$\sum_{t'=1}^{t-1} \delta^{t'-1}[(1-\delta)\, u_\ell(q) + I_{t'}^\ell a_{t'} m_{t'}] + \delta^{t-1}\Bigg[(1-a_t)\, u_\ell(x_t) + a_t\Bigg(u_\ell(y_t) + I_t^\ell m_t\Bigg)\Bigg];$$

and for $g \in N^g$,

$$\sum_{t'=1}^{t-1} \delta^{t'-1}\Bigg[(1-\delta)\, u_g(q) - a_{t'} m_{t'} \sum_{\ell \in N_g^\ell} I_{t'}^\ell\Bigg] + \delta^{t-1}\Bigg[(1-a_t)\, u_g(x_t) + a_t\Bigg(u_g(y_t) - m_t \sum_{\ell \in N_g^\ell} I_{t'}^\ell\Bigg)\Bigg].$$

The model in the main text is a special case featuring one voter with ideal point $\hat{x}_M$; four committee members with ideal points $\hat{x}_L, \hat{x}_M, \hat{x}_\ell$, and $\hat{x}_R$; and one group at $\hat{x}_g$ with access $\alpha_\ell \geq 0$ and $\alpha_j = 0$ for all $j \neq \ell$.

Strategies and Equilibrium Concept: I study a refinement of stationary subgame perfect equilibrium. First, I formalize mixed strategies to express continuation values. I then define pure strategies and the equilibrium concept: *no-delay stationary legislative lobbying equilibrium*. In Appendix C, I define *stationary mixed strategy legislative lobbying equilibria* and show that they must be equivalent in outcome distribution to the equilibrium concept defined here. Thus, the outcome distribution characterized in Proposition 1 applies even more broadly.

Let $\Delta(X)$ be the set of probability measures on X. Let $W = X \times \mathbb{R}_+$ denote the lobby-offer space and $\Delta(W)$ denote the set of probability measures on W. A stationary mixed strategy for $g \in N^G$ is a probability measure $\lambda_g \in \Delta(W)^{|N_g^L|}$ over g's offers $(y, m) \in W$ to each $\ell \in N_g^L$. A stationary mixed legislative strategy for $\ell \in N_g^L$ is a pair (π_ℓ, φ_ℓ); where $\pi_\ell \in \Delta(X)$ is a probability measure over ℓ's independent proposals and $\varphi_\ell : W \to [0, 1]$ specifies the probability ℓ accepts each $(y, m) \in W$. Finally, voter i's stationary mixed strategy $\nu_i : X \to [0, 1]$ specifies the probability i votes for each $x \in X$.

Let λ denote a profile of interest group strategies, (π, φ) a profile of committee member strategies, and ν a profile of voter strategies. A stationary strategy profile is $\sigma = (\lambda, \pi, \varphi, \nu)$. Under σ, let $\overline{\nu}_\sigma(x)$ be the probability x passes if proposed.

Let $w = (y, m) \in W$ denote an arbitrary lobby offer. Define

$$\xi_\ell(\alpha, \sigma) = (1 - \alpha_\ell) + \alpha_\ell \int_W [1 - \varphi_\ell(y, m)]\lambda_g^\ell(dw), \tag{12}$$

which is the probability under σ that ℓ makes an independent policy proposal, conditional on being recognized. Given σ, each $i \in N^V$ has continuation value

$$\begin{aligned} V_i(\sigma) = \sum_{\ell \in N^L} \rho_\ell \Bigg(&\alpha_\ell \int_W \varphi_\ell(y, m) \Big[\overline{\nu}_\sigma(y) u_i(y) + [1 - \overline{\nu}_\sigma(y)][(1 - \delta)u_i(q) + \delta V_i(\sigma)]\Big] \lambda_{g_\ell}^\ell(dw) \\ &+ \xi_\ell(\alpha, \sigma) \int_X \Big[\overline{\nu}_\sigma(x) u_i(x) + [1 - \overline{\nu}_\sigma(x)][(1 - \delta)u_i(q) + \delta V_i(\sigma)]\Big] \pi_\ell(dx) \Bigg), \end{aligned} \tag{13}$$

the continuation value of $\ell \in N^L$ is

$$\begin{aligned} \widetilde{V}_\ell(\sigma) = \sum_{j \neq \ell} \rho_j \Bigg(&\alpha_j \int_W \varphi_j(y, m) \Big[\overline{\nu}_\sigma(y) u_\ell(y) + [1 - \overline{\nu}_\sigma(y)][(1 - \delta)u_\ell(q) + \delta \widetilde{V}_\ell(\sigma)]\Big] \lambda_{g_j}^j(dw) \\ &+ \xi_j(\alpha, \sigma) \int_X \Big[\overline{\nu}_\sigma(x) u_\ell(x) + [1 - \overline{\nu}_\sigma(x)][(1 - \delta)u_\ell(q) + \delta \widetilde{V}_\ell(\sigma)]\Big] \pi_j(dx) \Bigg) \\ + \rho_\ell \Bigg(&\alpha_\ell \int_W \varphi_\ell(y, m) \Big[\overline{\nu}_\sigma(y) u_\ell(y) + [1 - \overline{\nu}_\sigma(y)][(1 - \delta)u_\ell(q) + \delta \widetilde{V}_\ell(\sigma)] + m\Big] \lambda_{g_\ell}^\ell(dw) \\ &+ \xi_\ell(\alpha, \sigma) \int_X \Big[\overline{\nu}_\sigma(x) u_\ell(x) + [1 - \overline{\nu}_\sigma(x)][(1 - \delta)u_\ell(q) + \delta \widetilde{V}_\ell(\sigma)]\Big] \pi_\ell(dx) \Bigg), \end{aligned} \tag{14}$$

and the continuation value of $g \in N^G$ is

$$\begin{aligned}
\widehat{V}_g(\sigma) = \sum_{\ell \notin N_g^L} \rho_\ell \Bigg(& \alpha_\ell \int_W \varphi_\ell(y,m) \Big[\overline{\nu}_\sigma(y) u_g(y) + [1-\overline{\nu}_\sigma(y)][(1-\delta)u_g(q) + \delta \widehat{V}_g(\sigma)] \Big] \lambda_{g_\ell}^\ell(dw) \\
& + \xi_\ell(\alpha,\sigma) \int_X \Big[\overline{\nu}_\sigma(x) u_g(x) + [1-\overline{\nu}_\sigma(x)][(1-\delta)u_g(q) + \delta \widehat{V}_g(\sigma)] \Big] \pi_\ell(dx) \Bigg) \\
+ \sum_{\ell \in N_g^L} \rho_\ell \Bigg(& \alpha_\ell \int_W \varphi_\ell(y,m) \Big[\overline{\nu}_\sigma(y) u_g(y) + [1-\overline{\nu}_\sigma(y)][(1-\delta)u_g(q) + \delta \widehat{V}_g(\sigma)] - m \Big] \lambda_g^\ell(dw) \\
& + \xi_\ell(\alpha,\sigma) \int_X \Big[\overline{\nu}_\sigma(x) u_g(x) + [1-\overline{\nu}_\sigma(x)][(1-\delta)u_g(q) + \delta \widehat{V}_g(\sigma)] \Big] \pi_\ell(dx) \Bigg). \qquad (15)
\end{aligned}$$

A stationary pure strategy for $g \in N^G$ is $(y_g, m_g) \in X^{|N_g^L|} \times \mathbb{R}_+^{|N_g^L|}$, where y_g is g's profile of policy offers and m_g is g's profile of monetary offers. A stationary pure strategy for $\ell \in N^L$ is (z_ℓ, a_ℓ); where $z_\ell \in X$ specifies ℓ's independent proposal, and $a_\ell : X \times \mathbb{R} \to \{0,1\}$ equals one iff ℓ accepts g_ℓ's offer. Finally, for each $i \in N^V$, $v_i : X \to \{0,1\}$ equals one iff i supports the proposal.

Given a profile of stationary pure strategies σ, the set of policies that pass is constant across periods, so denote it $A(\sigma)$. For $\ell \in N^L$, define

$$\widetilde{U}_\ell(x;\sigma) = \begin{cases} u_\ell(x) & \text{if } x \in A(\sigma) \\ (1-\delta)u_\ell(q) + \delta \widetilde{V}_\ell(\sigma) & \text{else.} \end{cases} \qquad (16)$$

Formally, a pure strategy profile $\sigma = (y, m, z, a, v)$ is a *no-delay stationary legislative lobbying equilibrium* if it satisfies five conditions. First, for all $g \in N^G$ and $\ell \in N_g^L$, (y_g^ℓ, m_g^ℓ) satisfies

$$y_g^\ell \in \underset{y \in A(\sigma)}{\arg\max} \quad u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell) \qquad (17)$$

and

$$m_g^\ell = u_\ell(z_\ell) - u_\ell(y_g^\ell). \qquad (18)$$

Second, for all $\ell \in N^L$ and $(y,m) \in W$, $a_\ell(y,m) = 1$ iff

$$\widetilde{U}_\ell(y;\sigma) + m \geq \widetilde{U}_\ell(z_\ell;\sigma). \qquad (19)$$

Third, for each $\ell \in N^L$,

$$z_\ell \in \underset{x \in A(\sigma)}{\arg\max}\ u_\ell(x). \tag{20}$$

Finally, for each $i \in N^V$, $v_i(x) = 1$ iff

$$u_i(x) \geq (1-\delta)u_i(q) + \delta V_i(\sigma), \tag{21}$$

i.e., voting strategies are *stage-undominated* (Baron and Kalai, 1993; Banks and Duggan, 2006a).

B.1 Proof of Proposition 1

Proof. There are four parts. Part 1 shows existence of a fixed point that maps a profile of (i) no-delay stationary lobby-offer strategies and (ii) no-delay stationary proposal strategies to itself as the solution to optimization problems for $g \in N^G$ and $\ell \in N^L$. Part 2 uses the fixed point to construct a strategy profile σ. Part 3 verifies that σ satisfies (17) – (21). Part 4 shows there is a unique equilibrium outcome distribution.

Part 1: Let $(y, z) = (y_1, \ldots, y_{n^L}, z_1, \ldots, z_{n^L}) \in X^{2n^L}$ and for each $j \in N$ define

$$r_j(y, z) = \sum_{\ell \in N^L} \rho_\ell \bigg(\alpha_\ell u_j(y_\ell) + (1-\alpha_\ell) u_j(z_\ell) \bigg). \tag{22}$$

Set $A(r(y, z)) = \{x \in X | u_M(x) \geq (1-\delta)u_M(q) + \delta r_M(y, z)\}$, which is non-empty, compact, and convex because $\delta \in (0, 1)$, $q \neq 0$, and u_M is strictly concave. Moreover, $A(r(y, z))$ is continuous in (y, z).

For each $\ell \in N^L$, define

$$\widetilde{\phi}_\ell(y, z) = \underset{y_\ell \in A(r(y,z))}{\arg\max}\ u_{g_\ell}(y_\ell) + u_\ell(y_\ell), \tag{23}$$

which is unique for all (y, z) because $A(r(y, z))$ is non-empty, compact and convex, and the objective function is strictly concave and continuous. Because $A(r(y, z))$ is continuous, the Theorem of the Maximum implies continuity of $\widetilde{\phi}_\ell(y, z)$. Next, define

$$\phi_\ell(y, z) = \underset{z_\ell \in A(r(y,z))}{\arg\max}\ u_\ell(z_\ell), \tag{24}$$

which is also unique for all (y, z), and continuous by the Theorem of the Maximum.

Define the mapping $\Phi : X^{2n^L} \to X^{2n^L}$ as $\Phi(y,z) = \prod_{\ell \in N^L} \widetilde{\phi}_\ell(y,z) \times \prod_{\ell \in N^L} \phi_\ell(y,z)$, which is a product of continuous functions and thus continuous. By Brouwer's theorem, a fixed point $(y^*, z^*) = \Phi(y^*, z^*)$ exists because Φ is a continuous function mapping a non-empty, compact, and convex set into itself.

Part 2: Define a stationary pure strategy profile σ as follows. First, for all $g \in N^G$ and $\ell \in N^L_g$, set $y^\ell_g = y^*_\ell$ and $m^\ell_g = u_\ell(z^*_\ell) - u_\ell(y^*_\ell)$. Next, for $\ell \in N^L$, set $z_\ell = z^*_\ell$ and define

$$a_\ell(y,m) = \begin{cases} 1 & \text{if } u_\ell(y) + m \geq u_\ell(z^*_\ell), \text{ for } y \in A(r(y^*, z^*)) \\ 1 & \text{if } (1-\delta)u_\ell(q) + \delta(r_\ell(y^*, z^*) + \rho_\ell \alpha_\ell m^\ell_g) + m \geq u_\ell(z^*_\ell), \text{ for } y \notin A(r(y^*, z^*)) \\ 0 & \text{else.} \end{cases} \tag{25}$$

Finally, for each $i \in N^V$ define v_i so that $v_i(x) = 1$ if $u_i(x) \geq (1-\delta)u_i(q) + \delta r_\ell(y^*, z^*)$ and $v_i(x) = 0$ otherwise.

Part 3: I verify that σ satisfies (17)–(21) and no player has a profitable deviation.

First, I verify (21) to show $A(\sigma) = A(r(y^*, z^*))$. Note that for each $g \in N^G$ and all $\ell \in N^L_g$, we have $y^\ell_g \in A(r(y^*, z^*))$ and $a_\ell(y^\ell_g, m^\ell_g) = 1$. Moreover, $z_\ell \in A(r(y^*, z^*))$ for all $\ell \in N^L$. Thus, voter i's continuation value under σ is $V_i(\sigma) = \sum_{\ell \in N^L} \rho_\ell[\alpha_\ell u_i(y^*_\ell) + (1 - \alpha_\ell)u_i(z^*_\ell)] = r_i(y^*, z^*)$. Thus, each voter i's strategy satisfies (21). Duggan (2014) implies that M is decisive over lotteries, so $A(\sigma) = A(r(y^*, z^*))$.

To check (17), consider $g \in N^G$ and $\ell \in N^L_g$. Since $a_\ell(z_\ell, 0) = 1$, focusing on offers that ℓ accepts is without loss of generality. Because $A(\sigma) = A(r(y^*, z^*))$, (23) implies $\widetilde{\phi}_\ell(y^*, z^*) = \underset{y_\ell \in A(\sigma)}{\arg\max}\, u_{g_\ell}(y_\ell) + u_\ell(y_\ell) - u_\ell(z^*_\ell)$. Thus, (17) holds because $\widetilde{\phi}_\ell(y^*, z^*) = y^*_\ell = y^g_\ell$. Finally, Lemma C.3 implies that g does not have a profitable deviation to any $y \notin A(\sigma)$.

It is immediate that m^ℓ_g satisfies (18) and g does not have a profitable deviation.

To check (19), note that ℓ's expected dynamic payoff from rejecting g_ℓ's offer is $\widetilde{U}_\ell(z_\ell; \sigma) = u_\ell(z^*_\ell)$. Thus, ℓ weakly prefers to accept any (y, m) satisfying $y \in A(r(y^*, z^*))$ if and only if $u_\ell(y) + m \geq u_\ell(z^*_\ell)$. If $y \notin A(r(y^*, z^*))$, then ℓ weakly prefers to accept (y, m) if and only if $(1-\delta)u_\ell(q) + \delta(r_\ell(y^*, z^*) + \rho_\ell \alpha_\ell m^\ell_g) + m \geq u_\ell(z^*_\ell)$. Thus, a_ℓ satisfies (19).

To check (20), note that (24) implies $\phi_\ell(y^*, z^*) = \underset{x \in A(\sigma)}{\arg\max}\, u_\ell(x)$ because $A(\sigma) = A(r(y^*, z^*))$. Thus, (20) holds because $\phi_\ell(y^*, z^*) = z^*_\ell = z_\ell$ for each $\ell \in N^L$. The no-delay property implies $x \notin A(\sigma)$ is not a profitable deviation for any $\ell \in N^L$.

Part 4. Let σ and σ' be stationary legislative lobbying equilibria. It suffices to show that $(y_g, m_g) = (y'_g, m'_g)$ for all $g \in N^G$ and $z_\ell = z'_\ell$ for all $\ell \in N^L$. Arguments analogous to

Proposition 1 in Cho and Duggan (2003) imply that $y_g = y'_g$ for all $g \in N^G$ and $z_\ell = z'_\ell$ for all $\ell \in N^L$. Thus, $A(\sigma) = A(\sigma')$. Fix ℓ. Since σ and σ' are no-delay, ℓ's expected dynamic payoff from rejecting g_ℓ's offer is $u_\ell(z_\ell)$ under both σ and σ'. Because equilibrium lobby offers always make targeted legislators indifferent, g's equilibrium payment equals $u_\ell(z_\ell) - u_\ell(y_g^\ell)$ in σ and σ'. Thus, $m_g = m'_g$ for all $g \in N^G$, completing the proof.

□

B.2 Proof of Lemma 1

Proof. Let $A^* = [-\overline{x}^*, \overline{x}^*]$ denote the equilibrium acceptance set. There are two cases.

Case 1. Suppose $\hat{x}_\ell \in A^*$, which implies $z_\ell = \hat{x}_\ell$. There are two subcases. First, if $\hat{y}_\ell \in A^*$, then $y_g^\ell = \hat{y}_\ell$ and (18) implies $m_g^\ell = u_\ell(\hat{x}_\ell) - u_\ell(\hat{y}_\ell)$, so m_g^ℓ is constant as $\overline{x}^*$ increases because $z_\ell = \hat{x}_\ell$ and $y_g^\ell = \hat{y}_\ell$ are unchanged. Second, consider $\hat{y}_\ell \notin A^*$, which requires $\hat{x}_{g\ell} \notin [-\overline{x}^*, \overline{x}^*]$ since $\hat{x}_\ell \in A^*$. Without loss of generality, assume $\hat{x}_{g\ell} > \overline{x}^*$. Thus, $z_\ell = \hat{x}_\ell$ and $y_g^\ell = \overline{x}^*$, so (18) implies $m_g^\ell = u_\ell(\hat{x}_\ell) - u_\ell(\overline{x}^*)$, which increases with $\overline{x}^*$.

Case 2. Suppose $\hat{x}_\ell \notin A^*$. Without loss of generality, assume $\hat{x}_\ell > z_\ell = \overline{x}^*$. There are three subcases. First, if $\hat{y}_\ell < -\overline{x}^*$, then $y_g^\ell = -\overline{x}^*$ and (18) implies $m_g^\ell = u_\ell(\overline{x}^*) - u_\ell(-\overline{x}^*)$, so m_g^ℓ increases with $\overline{x}^*$ because $-\overline{x}^* < \overline{x}^* < \hat{x}_\ell$. Second, if $\hat{y}_\ell \in A^*$, then $y_g^\ell = \hat{y}_\ell$ is constant as $\overline{x}^*$ increases, so (18) implies $m_g^\ell = u_\ell(\overline{x}^*) - u_\ell(\hat{y}_\ell)$, which increases with $\overline{x}^*$. Third, if $\hat{y}_\ell \geq \overline{x}^*$, then $y_g^\ell = \overline{x}^*$, so (18) implies $m_g^\ell = u_\ell(\overline{x}^*) - u_\ell(\overline{x}^*) = 0$, which is constant.

Altogether, m_g^ℓ weakly increases in $\overline{x}^*$. □

B.3 Proof of Proposition 2

Given Lemma 1, it suffices to show that A^* expands as: legislative extremism increases, $|q|$ increases, or δ decreases.

- *Legislative extremism.* Follows from Part 1 of Proposition 8 in Kalandrakis (2021).
- *Status quo exremism, $|q|$.* Follows from Proposition 6 in Kalandrakis (2021).
- *Discount factor, δ.* Letting $C_\ell = \mathbb{I}\{\hat{x}_\ell \in \text{int } A^*\}$ and $\widetilde{C}_\ell = \mathbb{I}\{\hat{y}_\ell \in \text{int } A^*\}$ for all $\ell \in N^L$, we have:

$$\overline{x}^* = \left(- \frac{(1-\delta)u_M(q) + \delta \Sigma_{\ell \in N^L} \rho_\ell \left[(1-\alpha_\ell) C_\ell \, u_M(\hat{x}_\ell) + \alpha_\ell \widetilde{C}_\ell \, u_M(\hat{y}_\ell) \right]}{1 - \delta \Sigma_{\ell \in N^L} \rho_\ell \left[(1-\alpha_\ell)(1-C_\ell) + \alpha_\ell (1-\widetilde{C}_\ell) \right]} \right)^{\frac{1}{2}}. \tag{26}$$

If $\hat{x}_\ell, \hat{y}_\ell \notin \{-\overline{x}^*, \overline{x}^*\}$ for all $\ell \in N^L$, then

$$\frac{\partial \overline{x}^*}{\partial \delta} \propto u_M(q)\Big[1 - \Sigma_{\ell \in N^L}\rho_\ell[(1-\alpha_\ell)(1-C_\ell) + \alpha_\ell(1-\widetilde{C}_\ell)]\Big] \\ - \Sigma_{\ell \in N^L}\rho_\ell\Bigg[(1-\alpha_\ell)C_\ell\, u_M(\hat{x}_\ell) + \alpha_\ell \widetilde{C}_\ell\, u_M(\hat{y}_\ell)\Bigg] \tag{27}$$

$$= \Sigma_{\ell \in N^L}\rho_\ell\Bigg[(1-\alpha_\ell)C_\ell\,[u_M(q) - u_M(\hat{x}_\ell)] + \alpha_\ell \widetilde{C}_\ell[u_M(q) - u_M(\hat{y}_\ell)]\Bigg] \tag{28}$$

$$< 0, \tag{29}$$

where (29) follows because $\overline{x}^* < |q|$ implies that $u_M(q) - u_M(\hat{x}_\ell) < 0$ if $C_\ell = 1$ and similarly $u_M(q) - u_M(\hat{y}_\ell) < 0$ if $\widetilde{C}_\ell = 1$.

If there exists $\ell \in N^L$ such that $\hat{x}_\ell$ or $\hat{y}_\ell$ is in $\{-\overline{x}^*, \overline{x}^*\}$, then (26) has right and left derivatives, which are both negative by an analogous argument.

B.4 Proofs of Lemma 2 & Proposition 3

Consider $\ell \in N^L$ and refer to g_ℓ as g for convenience. The results fix $\hat{x}_g$ and vary $\hat{x}_\ell$. Throughout, assume $\hat{x}_g > 0$, as the other case is symmetric. Recall $\hat{y}_\ell = \frac{\hat{x}_g + \hat{x}_\ell}{2} = \arg\max_{y \in X} u_g(y) + u_\ell(y)$.

Let $\sigma(\alpha_\ell; \hat{x}_\ell)$ denote an equilibrium given $\hat{x}_\ell$ and α_ℓ, and denote the corresponding social acceptance set as $A(\alpha_\ell; \hat{x}_\ell)$, with upper bound $\overline{x}(\alpha_\ell; \hat{x}_\ell)$. That is, $A(\alpha_\ell; \hat{x}_\ell)$ corresponds to A^* from the main text but makes explicit the dependence on α_ℓ and $\hat{x}_\ell$.

First, I state a lemma that partitions whether $\hat{x}_\ell \in \mathrm{int}A(0; \hat{x}_\ell)$ and plays a key role in proving Lemma 3 and Proposition 3.

Lemma B.1. *For all $\ell \in N^L$, there exists $\overline{x}_\ell \in (0, q]$ such that $\hat{x}_\ell \in intA(0; \hat{x}_\ell)$ if $\hat{x}_\ell \in (-\overline{x}_\ell, \overline{x}_\ell)$ and otherwise $A(0; \hat{x}_\ell) = [-\overline{x}_\ell, \overline{x}_\ell]$.*

The proof of Lemma B.1 proceeds in a series of Lemmas that are provided in Appendix D. An outline of the argument is that I first define a function $\xi^\ell : \mathbb{R}_+ \to \mathbb{R}$ constructed so that $\xi^\ell(x) > 0$ if and only if $x \in \mathrm{int}A(0; x)$. Then, I show that there is a unique $\overline{x}_\ell \in (0, q]$ such that $\xi^\ell(x) > 0$ if and only if $x \in [0, \overline{x}_\ell)$. It then follows that $\hat{x}_\ell \in (-\overline{x}_\ell, \overline{x}_\ell)$ implies $\hat{x}_\ell \in \mathrm{int}A(0; \hat{x}_\ell)$, and otherwise $A(0; \hat{x}_\ell) = [-\overline{x}_\ell, \overline{x}_\ell]$.

Using Lemma B.1, I prove Lemma 2 and Proposition 3. Let y^*_0 denote y^* for $\alpha_\ell = 0$ and define z^*_0 analogously.

Lemma 2. *In equilibrium, $y^*_0 \neq z^*_0$ if and only if $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), \overline{\mathcal{X}}(\hat{x}_g))$.*

Proof. If $\hat{x}_\ell \in (-\overline{x}_\ell, \overline{x}_\ell)$, then Lemma B.1 implies $z_0^* = \hat{x}_\ell$. Then, $\hat{x}_\ell \neq \hat{x}_g$ implies $y_0^* \neq z_0^*$.

Otherwise, z_0^* is the boundary of $A_0^* = [-\overline{x}, \overline{x}]$ closer to $\hat{x}_\ell$. For $\hat{x}_\ell \leq -\overline{x}$, we have $y_0^* > -\overline{x}$ if and only if $\hat{x}_\ell > \underline{\mathcal{X}}(\hat{x}_g)$. Analogously, for $\hat{x}_\ell \geq \overline{x}$ we have $y_0^* < \overline{x}$ if and only if $\hat{x}_\ell < \overline{\mathcal{X}}(\hat{x}_g)$. □

Proposition 3. *Interest group g strictly prefers $\alpha_\ell > 0$ only if $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), \overline{x}(\hat{x}_g))$.*

Proof. Suppose $\hat{x}_\ell \notin (\underline{\mathcal{X}}(\hat{x}_g), \overline{x}(\hat{x}_g))$. Lemma 2 implies $y_0^* = z_0^*$. Thus, $A_{\alpha_\ell}^*$ is constant in α_ℓ. It follows that the equilibrium outcome distribution is constant in α_ℓ, so g is indifferent. □

B.5 Proof of Lemma 3

Lemma 3. *If $\hat{x}_g \in (0, \overline{x}_\ell)$, then there exists $x' \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \notin (-x', x')$ implies $\hat{x}_g \in intA(0; \hat{x}_\ell)$. If $\hat{x}_g \notin (0, \overline{x}_\ell)$, then $\hat{x}_g \notin intA(\alpha_\ell; \hat{x}_\ell)$ for all $\hat{x}_\ell$ and α_ℓ.*

Proof. Consider $\hat{x}_g \in (0, \overline{x}_\ell)$. If $\hat{x}_\ell = \hat{x}_g$, then Lemma B.1 implies $\hat{x}_g \in \mathrm{int}A(0; \hat{x}_\ell)$. By symmetry, $\hat{x}_\ell = -\hat{x}_g$ also implies $\hat{x}_g \in \mathrm{int}A(0; \hat{x}_\ell)$. Recall that $A(0; \hat{x}_\ell)$ strictly expands as $\hat{x}_\ell$ shifts away from 0 over $(-\overline{x}_\ell, \overline{x}_\ell)$. Because there is a unique equilibrium outcome distribution, Theorem 3 of Banks and Duggan (2006a) implies $A(0; \hat{x}_\ell)$ is continuous in $\hat{x}_\ell$. Thus, there exists $x' \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \notin (-x', x')$ implies $\hat{x}_g \in \mathrm{int}A(0; \hat{x}_\ell)$.

To complete the proof, consider $\hat{x}_g \geq \overline{x}_\ell$. Lemma B.1 implies $\hat{x}_g \notin \mathrm{int}A(0; \hat{x}_\ell) = (\overline{x}_\ell, \overline{x}_\ell)$ for all $\hat{x}_\ell \geq \hat{x}_g$. Thus, $A(\alpha_\ell; \hat{x}_\ell) \subset A(0; \hat{x}_g)$ for all $(\alpha_\ell, \hat{x}_\ell)$, so $\hat{x}_g \notin \mathrm{int}A(\alpha_\ell; \hat{x}_\ell)$. □

B.6 Preliminary Results for Propositions 4 & 5

Next, Lemmas B.2–B.5 establish properties used to prove Propositions 4 and 5.

Lemma B.2. *Suppose $\hat{x}_g \in (0, \overline{x}_\ell)$. There exists $\tilde{x} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (\tilde{x}, \hat{x}_g)$ implies $\hat{x}_g \in intA(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0, 1]$.*

Proof. Consider $\hat{x}_g \in (0, \overline{x}_\ell)$. By Lemma 3, there exists $x' \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (x', \hat{x}_g)$ implies $\hat{x}_g \in \mathrm{int}A(0; \hat{x}_\ell)$. Then $0 < \hat{x}_\ell < \hat{x}_g$ implies $A(0; \hat{x}_\ell) \subset A(\alpha_\ell; \hat{x}_\ell)$. □

For each $j \in N^L \backslash \{\ell\}$, define

$$\begin{aligned} E_j^{LB}(\alpha_\ell; \hat{x}_\ell) &= \mathbb{I}\{\hat{x}_j \leq -\overline{x}(\alpha_\ell; \hat{x}_\ell)\}, \\ E_j^{UB}(\alpha_\ell; \hat{x}_\ell) &= \mathbb{I}\{\hat{x}_j \geq \overline{x}(\alpha_\ell; \hat{x}_\ell)\}, \text{ and} \\ C_j(\alpha_\ell; \hat{x}_\ell) &= \mathbb{I}\{\hat{x}_j \in \mathrm{int}A(\alpha_\ell; \hat{x}_\ell)\}. \end{aligned}$$

Define $\widetilde{E}_j^{LB}(\alpha_\ell; \hat{x}_\ell)$, $\widetilde{E}_j^{UB}(\alpha_\ell; \hat{x}_\ell)$, and $\widetilde{C}_j(\alpha_\ell; \hat{x}_\ell)$ analogously using $\hat{y}_j$. Let $I_g^j \in \{0, 1\}$ indicate whether $j \in N_g^L$.

Assumption B.1. *There exists $j \in N^L \backslash \{\ell\}$ such that $\alpha_j < 1$ and $\hat{x}_j \notin A(0; \hat{x}_g)$.*

Assumption B.2. *There exists $j \in N^L \backslash \{\ell\}$ such that $\alpha_j > 0$ and $\hat{y}_j \notin A^*(0; \hat{x}_g)$.*

Next, define

$$\upsilon_1^g(\alpha_\ell; \hat{x}_\ell) = \rho_\ell \left(\alpha_\ell \Big[u_g(\hat{y}_\ell) + u_\ell(\hat{y}_\ell) - u_\ell(\hat{x}_\ell) \Big] + (1 - \alpha_\ell)\, u_g(\hat{x}_\ell) \right) \tag{30}$$

and

$$\begin{aligned} \upsilon_2^g(\alpha_\ell; \hat{x}_\ell) = \sum_{j \neq \ell} \rho_j \Bigg(& \Big[\alpha_j\, \widetilde{E}_j^{LB}(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j)\, E_j^{LB}(\alpha_\ell; \hat{x}_\ell) \Big]\, u_g(-\overline{x}(\alpha_\ell; \hat{x}_\ell)) \\ & + \Big[\alpha_j\, \widetilde{E}_j^{UB}(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j)\, E_j^{UB}(\alpha_\ell; \hat{x}_\ell) \Big]\, u_g(\overline{x}(\alpha_\ell; \hat{x}_\ell)) \\ & + \alpha_j \Big[\widetilde{C}_j(\alpha_\ell; \hat{x}_\ell)\, u_g(\hat{y}_j) - I_g^j\, m_g^j(\alpha_\ell; \hat{x}_\ell) \Big] + (1 - \alpha_j)\, C_j(\alpha_\ell; \hat{x}_\ell)\, u_g(\hat{x}_j) \Bigg). \end{aligned} \tag{31}$$

Lemma B.3. *If $\hat{x}_\ell \neq \hat{x}_g$, then $\frac{\partial \upsilon_1^g(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} > 0$.*

Proof. Suppose $\hat{x}_\ell \neq \hat{x}_g$. Then $\frac{\partial \upsilon_1^g(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} = \frac{\rho_\ell}{2}(\hat{x}_g - \hat{x}_\ell)^2 > 0$, by (30) and $\hat{y}_\ell = \frac{\hat{x}_\ell + \hat{x}_g}{2}$. □

Lemma B.4. *Suppose $0 \leq \hat{x}_\ell < \hat{x}_g < \overline{x}_\ell$. If at least one of Assumption B.1 or B.2 holds, then $\upsilon_2^g(\alpha_\ell; \hat{x}_\ell)$ strictly decreases in α_ℓ.*

Proof. It suffices to show that

$$\begin{aligned} & \Big[\alpha_j\, \widetilde{E}_j^{LB}(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j)\, E_j^{LB}(\alpha_\ell; \hat{x}_\ell) \Big]\, u_g(-\overline{x}(\alpha_\ell; \hat{x}_\ell)) \\ & \quad + \Big[\alpha_j\, \widetilde{E}_j^{UB}(\alpha_\ell; \hat{x}_\ell) + (1 - \alpha_j)\, E_j^{UB}(\alpha_\ell; \hat{x}_\ell) \Big]\, u_g(\overline{x}(\alpha_\ell; \hat{x}_\ell)) \\ & \quad + \alpha_j \Big[\widetilde{C}_j(\alpha_\ell; \hat{x}_\ell)\, u_g(\hat{y}_j) - I_g^j\, m_g^j(\alpha_\ell; \hat{x}_\ell) \Big] + (1 - \alpha_j)\, C_j(\alpha_\ell; \hat{x}_\ell)\, u_g(\hat{x}_j) \end{aligned} \tag{32}$$

decreases in α_ℓ for all $j \in N^L \backslash \{\ell\}$ and strictly decreases for some j.

Without loss of generality, consider $\hat{x}_j \geq 0$. Since $0 \leq \hat{x}_\ell < \hat{x}_g$, we know $\overline{x}(\alpha_\ell; \hat{x}_\ell)$ increases in α_ℓ. There are two implications. First, $\hat{x}_g \in (0, \overline{x})$ implies $\hat{x}_g < \overline{x}(0; \hat{x}_\ell)$ by Lemma 3, so $u_g(\overline{x}(\alpha_\ell; \hat{x}_\ell))$ and $u_g(-\overline{x}(\alpha_\ell; \hat{x}_\ell))$ both decrease in α_ℓ. Second, exactly one of the following holds: $E_j^{UB}(\alpha_\ell; \hat{x}_\ell) = 1$ for all α_ℓ; $C_j(\alpha_\ell; \hat{x}_\ell) = 1$ for all α_ℓ; or there is a unique $\overline{\alpha}_\ell^j \in [0, 1]$ such that $\alpha_\ell \in [0, \overline{\alpha}_\ell^j]$ implies $E_j^{UB}(\alpha_\ell; \hat{x}_\ell) = 1$ and $\alpha_\ell \in (\overline{\alpha}_\ell^j, 1]$ implies $C_j(\alpha_\ell; \hat{x}_\ell) = 1$. An analogous observation holds for $\widetilde{E}_j^{UB}(\alpha_\ell;\hat{x}_\ell)$ and $\widetilde{C}_j(\alpha_\ell;\hat{x}_\ell)$. Thus,

$$E_j^{LB}(\alpha_\ell;\hat{x}_\ell)\,u_g(-\overline{x}(\alpha_\ell;\hat{x}_\ell)) + E_j^{UB}(\alpha_\ell;\hat{x}_\ell)\,u_g(\overline{x}(\alpha_\ell;\hat{x}_\ell)) + C_j(\alpha_\ell;\hat{x}_\ell)\,u_g(\hat{x}_j) \tag{33}$$

and

$$\widetilde{E}_j^{LB}(\alpha_\ell;\hat{x}_\ell)\,u_g(-\overline{x}(\alpha_\ell;\hat{x}_\ell)) + \widetilde{E}_j^{UB}(\alpha_\ell;\hat{x}_\ell)\,u_g(\overline{x}(\alpha_\ell;\hat{x}_\ell)) + \widetilde{C}_j(\alpha_\ell;\hat{x}_\ell)\,u_g(\hat{y}_j) \tag{34}$$

both decrease in α_ℓ. Furthermore, because Assumptions B.1 or B.2 holds, at least one of (33) and (34) strictly decreases for some $j \in N^L\backslash\{\ell\}$. Lemma 1 implies $m_g^j(\alpha_\ell;\hat{x}_\ell)$ weakly increases in α_ℓ for all $j \in N_g^L$. Altogether, (32) decreases in α_ℓ for all $j \in N^L\backslash\{\ell\}$ and strictly decreases for some j, as desired. □

Lemma B.5. *Assume $\hat{x}_g \in (0,\overline{x}_\ell)$. If at least one of Assumption B.1 or B.2 holds, then there exists $x' < \hat{x}_g$ such that $\upsilon_1^g(\alpha_\ell;\hat{x}_\ell) + \upsilon_2^g(\alpha_\ell;\hat{x}_\ell)$ strictly decreases in α_ℓ for all $\hat{x}_\ell \in (x',\hat{x}_g)$.*

Proof. I show $|\frac{\partial \upsilon_2^g(\alpha_\ell;\hat{x}_\ell)}{\partial \alpha_\ell}| > \frac{\partial \upsilon_1^g(\alpha_\ell;\hat{x}_\ell)}{\partial \alpha_\ell}$ for $\hat{x}_\ell < \hat{x}_g$ sufficiently close to $\hat{x}_g$.

By Lemma B.2, there exists $\tilde{x} \in [0,\hat{x}_g)$ such that $\hat{x}_\ell \in (\tilde{x},\hat{x}_g)$ implies $\hat{x}_g \in \operatorname{int}A(\alpha_\ell;\hat{x}_\ell)$ for all $\alpha_\ell \in [0,1]$. Fix $\hat{x}_\ell \in (\tilde{x},\hat{x}_g)$ and $\alpha_\ell \in [0,1]$.

First, I characterize a lower bound on $|\frac{\partial \upsilon_2(\alpha_\ell;\hat{x}_\ell)}{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)}|$. Define

$$\begin{aligned}\Gamma = \sum_{j\neq\ell}\rho_j\Bigg(&\left[\alpha_j\,\widetilde{E}_j^{LB}(\hat{x}_g) + (1-\alpha_j)\,E_j^{LB}(\hat{x}_g)\right]\frac{\partial u_g(-\overline{x}(\tilde{x}))}{\partial \overline{x}(\tilde{x})}\\ &+\left[\alpha_j\,\widetilde{E}_j^{UB}(\hat{x}_g) + (1-\alpha_j)\,E_j^{UB}(\hat{x}_g)\right]\frac{\partial u_g(\overline{x}(\tilde{x}))}{\partial \overline{x}(\tilde{x})}\Bigg).\end{aligned} \tag{35}$$

Note $\Gamma < 0$ because (i) $\hat{x}_g \in (-\overline{x}(\tilde{x}),\overline{x}(\tilde{x}))$ implies $\frac{\partial u_g(\overline{x}(\tilde{x}))}{\partial \overline{x}(\tilde{x})} < 0$ and $\frac{\partial u_g(-\overline{x}(\tilde{x}))}{\partial \overline{x}(\tilde{x})} < 0$, and (ii) at least one of Assumptions B.1 and B.2 hold.

I claim $\frac{\partial \upsilon_2(\alpha_\ell;\hat{x}_\ell)}{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)} < \Gamma$, where

$$\begin{aligned}\frac{\partial \upsilon_2(\alpha_\ell;\hat{x}_\ell)}{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)} = \sum_{j\neq\ell}\rho_j\Bigg(&\left[\alpha_j\,\widetilde{E}_j^{LB}(\alpha_\ell;\hat{x}_\ell) + (1-\alpha_j)\,E_j^{LB}(\alpha_\ell;\hat{x}_\ell)\right]\frac{\partial u_g(-\overline{x}(\alpha_\ell;\hat{x}_\ell))}{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)}\\ &+\left[\alpha_j\,\widetilde{E}_j^{UB}(\alpha_\ell;\hat{x}_\ell) + (1-\alpha_j)\,E_j^{UB}(\alpha_\ell;\hat{x}_\ell)\right]\frac{\partial u_g(\overline{x}(\alpha_\ell;\hat{x}_\ell))}{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)}\\ &- I_g^j\,\alpha_j\,\frac{\partial m_g^j(\alpha_\ell;\hat{x}_\ell)}{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)}\Bigg).\end{aligned} \tag{36}$$

Three steps show the claim. First, note $\hat{x}_\ell \in (\tilde{x},\hat{x}_g)$ implies $\overline{x}(\hat{x}_g) \geq \overline{x}(\alpha_\ell;\hat{x}_\ell)$. Thus, we have

$\widetilde{E}_j^{UB}(\hat{x}_g) \leq \widetilde{E}_j^{UB}(\alpha_\ell; \hat{x}_\ell)$, $E_j^{UB}(\hat{x}_g) \leq E_j^{UB}(\alpha_\ell; \hat{x}_\ell)$, $\widetilde{E}_j^{LB}(\hat{x}_g) \leq \widetilde{E}_j^{LB}(\alpha_\ell; \hat{x}_\ell)$, and $E_j^{LB}(\hat{x}_g) \leq E_j^{LB}(\alpha_\ell; \hat{x}_\ell)$ for all $j \neq \ell$. Second, $\hat{x}_g < \overline{x}(\tilde{x}) < \overline{x}(\alpha_\ell; \hat{x}_\ell)$ implies $\frac{\partial u_g(\overline{x}(\alpha_\ell;\hat{x}_\ell))}{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)} < \frac{\partial u_g(\overline{x}(\tilde{x}))}{\partial \overline{x}(\tilde{x})} < 0$ and symmetrically $\frac{\partial u_g(-\overline{x}(\alpha_\ell;\hat{x}_\ell))}{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)} < \frac{\partial u_g(-\overline{x}(\tilde{x}))}{\partial \overline{x}(\tilde{x})} < 0$. Third, $\frac{\partial m_g^j(\alpha_\ell;\hat{x}_\ell)}{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)} \geq 0$ for all $j \in N_g^L$ by Lemma 1.

For almost all $\alpha_\ell \in [0,1]$, $\frac{\partial \upsilon_2(\alpha_\ell;\hat{x}_\ell)}{\partial \alpha_\ell} = \frac{\partial \upsilon_2(\alpha_\ell;\hat{x}_\ell)}{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)} \frac{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)}{\partial \alpha_\ell}$. Define $\mathbf{C}_j(\alpha_\ell; \hat{x}_\ell) = [(1-\alpha_j)(1 - C_j(\alpha_\ell;\hat{x}_\ell)) + \alpha_j(1 - \widetilde{C}_j(\alpha_\ell;\hat{x}_\ell))]$. Then,

$$\frac{\partial \upsilon_2(\alpha_\ell;\hat{x}_\ell)}{\partial \alpha_\ell} < \Gamma \frac{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)}{\partial \alpha_\ell} \tag{37}$$

$$= \Gamma \frac{\delta \rho_\ell \Big[u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell)\Big]}{2\overline{x}(\alpha_\ell;\hat{x}_\ell)\Big[1 - \delta\Big(\sum_{j \in N^\ell} \rho_j \mathbf{C}_j(\alpha_\ell;\hat{x}_\ell)\Big)\Big]} \tag{38}$$

$$< \Gamma \frac{\delta \rho_\ell}{2\overline{x}_\ell}\Big[\frac{1}{4}(\hat{x}_g - \hat{x}_\ell)(3\hat{x}_\ell + \hat{x}_g)\Big], \tag{39}$$

where (37) follows from $\frac{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)}{\partial \alpha_\ell} > 0$ and $0 > \Gamma > \frac{\partial \upsilon_2(\alpha_\ell;\hat{x}_\ell)}{\partial \overline{x}(\alpha_\ell;\hat{x}_\ell)}$; (38) from applying the implicit function theorem to $\overline{x}(\alpha_\ell;\hat{x}_\ell)$, which is possible for almost all $\alpha_\ell \in [0,1]$; and (39) because $\Gamma[u_M(\hat{x}_\ell) - u_M(\hat{y}_\ell)] < 0$, $\delta \sum_{j \in N^\ell} \rho_j \mathbf{C}_j^\ell(\alpha_\ell;\hat{x}_\ell) \in (0,1)$, $0 < \overline{x}(\alpha_\ell;\hat{x}_\ell) < \overline{x}_\ell$, and simplifying using $\hat{y}_\ell = \frac{\hat{x}_g + \hat{x}_\ell}{2}$.

By Lemma B.3, $\frac{\partial \upsilon_1(\alpha_\ell;\hat{x}_\ell)}{\partial \alpha_\ell} = \frac{\rho_\ell}{2}(\hat{x}_g - \hat{x}_\ell)^2$. Thus, for generic α_ℓ, (39) implies that $\frac{\partial \upsilon_1(\alpha_\ell;\hat{x}_\ell)}{\partial \alpha_\ell} + \frac{\partial \upsilon_2(\alpha_\ell;\hat{x}_\ell)}{\partial \alpha_\ell} < 0$ if

$$\frac{\rho_\ell}{2}(\hat{x}_g - \hat{x}_\ell)^2 + \frac{\delta \rho_\ell \Gamma}{2\overline{x}_\ell}\left[\frac{1}{4}(\hat{x}_g - \hat{x}_\ell)(3\hat{x}_\ell + \hat{x}_g)\right] < 0,$$

which holds for $\hat{x}_\ell > \hat{x}_g\left(\frac{4\overline{x}_\ell + \delta\Gamma}{4\overline{x}_\ell - 3\delta\Gamma}\right)$. Define $x' = \max\left\{\tilde{x}, \hat{x}_g\left(\frac{4\overline{x}_\ell + \delta\Gamma}{4\overline{x}_\ell - 3\delta\Gamma}\right)\right\}$. Note $x' < \hat{x}_g$ because (i) $\tilde{x} < \hat{x}_g$ and (ii) $\delta\Gamma < 0$ implies $\frac{4\overline{x}_\ell + \delta\Gamma}{4\overline{x}_\ell - 3\delta\Gamma} < 1$. Thus, $\hat{x}_\ell \in (x', \hat{x}_g)$ implies $\frac{\partial \upsilon_1(\alpha_\ell;\hat{x}_\ell)}{\partial \alpha_\ell} + \frac{\partial \upsilon_2(\alpha_\ell;\hat{x}_\ell)}{\partial \alpha_\ell} < 0$ for generic α_ℓ. Continuity implies $\upsilon_1^g(\alpha_\ell;\hat{x}_\ell) + \upsilon_2^g(\alpha_\ell;\hat{x}_\ell)$ strictly decreases in α_ℓ for such $\hat{x}_\ell$. □

B.7 Proof of Proposition 4

Proposition 4 *Suppose $\hat{x}_g \in (0, \overline{x}_\ell)$. If either Assumption B.1 or B.2 holds, then there are cutpoints satisfying $-\hat{x}_g < x' < x'' < \hat{x}_g$ such that:*

(i) $\alpha_\ell^* = 0$ if $\hat{x}_\ell \in (x'', \hat{x}_g)$, and

(ii) $\alpha_\ell^* > 0$ if $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), x') \cup (\hat{x}_g, \overline{\mathcal{X}}(\hat{x}_g))$.

Proof. Part (i). By Lemma B.2, there exists $\tilde{x} \in [0, \hat{x}_g)$ such that $\hat{x}_\ell \in (\tilde{x}, \hat{x}_g)$ implies $\hat{x}_g \in A(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0,1]$. By Lemma B.5, there exists $x' < \hat{x}_g$ such that $\hat{x}_\ell \in (x', \hat{x}_g)$ implies $\upsilon_1^g(\alpha_\ell; \hat{x}_\ell) + \upsilon_2^g(\alpha_\ell; \hat{x}_\ell)$ strictly decreases in α_ℓ. Let $x'' = \max\{\tilde{x}, x'\}$ and consider $\hat{x}_\ell \in (x'', \hat{x}_g)$. Then, for all $\alpha_\ell \in [0,1]$, we have $z_\ell = \hat{x}_\ell \in A(\alpha_\ell; \hat{x}_\ell)$ and $y_g^\ell = \hat{y}_\ell \in A(\alpha_\ell; \hat{x}_\ell)$. Thus, g's equilibrium value from α access is $\mathcal{U}_g(\alpha_\ell; \hat{x}_\ell) = \upsilon_1^g(\alpha_\ell; \hat{x}_\ell) + \upsilon_2^g(\alpha_\ell; \hat{x}_\ell)$ for all $\alpha_\ell \in [0,1]$, so g strictly prefers $\alpha_\ell = 0$. This establishes (i).

Part (ii). First, consider $\hat{x}_\ell \in (\hat{x}_g, \overline{\mathcal{X}}(\hat{x}_g))$. It suffices to show that g's ex ante expected utility strictly increases as α_ℓ increases from zero. There are two subcases.

1. If $\hat{x}_\ell < \overline{x}_\ell$, then $\mathcal{U}_g(\alpha_\ell; \hat{x}_\ell) = \upsilon_1^g(\alpha_\ell; \hat{x}_\ell) + \upsilon_2^g(\alpha_\ell; \hat{x}_\ell)$ for sufficiently small α_ℓ. By Lemma B.3, $\frac{\partial \upsilon_1^g(\alpha_\ell; \hat{x}_\ell)}{\partial \alpha_\ell} > 0$. To complete this case, I argue that $\upsilon_2^g(\alpha_\ell; \hat{x}_\ell)$ increases for sufficiently small α_ℓ. Under the maintained assumptions, $\hat{x}_g \in (-\overline{x}(0; \hat{x}_\ell), \overline{x}(0; \hat{x}_\ell))$ and $\hat{y}_\ell \in (\hat{x}_g, \overline{x}(0; \hat{x}_\ell))$. Thus, $\overline{x}(\alpha_\ell; \hat{x}_\ell)$ strictly decreases for sufficiently small α_ℓ. Therefore $u_g(-\overline{x}(\alpha_\ell; \hat{x}_\ell))$ and $u_g(\overline{x}(\alpha_\ell; \hat{x}_\ell))$ are strictly increasing for such α_ℓ. Lemma 1 implies $m_g^j(\alpha_\ell; \hat{x}_\ell)$ weakly decreases in α_ℓ for all $j \in N_g^L \backslash \{\ell\}$. Thus, $\upsilon_2^g(\alpha_\ell; \hat{x}_\ell)$ strictly increases over sufficiently small α_ℓ.

2. If $\hat{x}_\ell > \overline{x}_\ell$, then $\overline{x}(0; \hat{x}_\ell) = \overline{x}_\ell$. Thus, $\mathcal{U}_g(0; \hat{x}_\ell)$ is

$$\begin{aligned}
&\rho_\ell \Bigg(\alpha_\ell \Big[u_g(\hat{y}_\ell) + u_\ell(\hat{y}_\ell) - u_\ell(\overline{x}_\ell) \Big] + (1-\alpha_\ell)\, u_g(\overline{x}_\ell) \Bigg) \\
&\quad + \sum_{j \neq \ell} \rho_j \Bigg(\Big[\alpha_j\, \widetilde{E}_j^{LB}(0; \hat{x}_\ell) + (1-\alpha_j)\, E_j^{LB}(0; \hat{x}_\ell) \Big] u_g(-\overline{x}_\ell) \\
&\qquad + \Big[\alpha_j\, \widetilde{E}_j^{UB}(0; \hat{x}_\ell) + (1-\alpha_j)\, E_j^{UB}(0; \hat{x}_\ell) \Big] u_g(\overline{x}_\ell) \\
&\qquad + \alpha_j\, \widetilde{C}_j(0; \hat{x}_\ell)\, u_g(\hat{y}_j) + (1-\alpha_j)\, C_j(0; \hat{x}_\ell)\, u_g(\hat{x}_j) \\
&\qquad - I_g^j\, \alpha_j\, m_g^j(0; \hat{x}_\ell) \Bigg).
\end{aligned} \tag{40}$$

Arguments similar to subcase 1 show that (40) strictly increases in α_ℓ at $\alpha_\ell = 0$.

To complete the proof for Part (ii), consider $\hat{x}_\ell < 0$. For $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), -\hat{x}_g)$, analogous arguments show that $\mathcal{U}_g(\alpha_\ell; \hat{x}_\ell)$ strictly increases at $\alpha_\ell = 0$. Finally, since g's ex-ante expected payoff is continuous in $\hat{x}_\ell$, there exists $x' > -\hat{x}_g$ such that $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), x')$ implies $\alpha_\ell^* > 0$. $\square$

B.8 Proof of Proposition 5

Proposition 5 *Assume $\hat{x}_g \geq \overline{x}_\ell$.*

(i) If $\sum_{i\in N^L} \rho_i \Bigg[(1-\alpha_i)\,\mathbb{I}\{\hat{x}_i \leq -\overline{x}\} + \alpha_i\,\mathbb{I}\{\hat{y}_i \leq -\overline{x}\}\Bigg]$ is sufficiently small, then there exists $x' < 0$ such that $\hat{x}_\ell \in (x', \overline{x})$ implies $\alpha^*_\ell > 0$.

(ii) If $\sum_{i\in N^L} \rho_i \Bigg[(1-\alpha_i)\,\mathbb{I}\{\hat{x}_i \geq \overline{x}\} + \alpha_i\,\mathbb{I}\{\hat{y}_i \geq \overline{x}\}\Bigg]$ is sufficiently small, then there exists $x'' \geq -\overline{x}$ such that $\hat{x}_\ell \in (\underline{\mathcal{X}}(\hat{x}_g), x'')$ implies $\alpha^*_\ell > 0$.

Proof. I prove (i), as (ii) is analogous. Consider $\hat{x}_\ell \in [0, \overline{x}_\ell)$ and assume $\sum_{i\in N^L} \rho_i[(1-\alpha_i)\,\mathbb{I}\{\hat{x}_i \leq -\overline{x}\} + \alpha_i\,\mathbb{I}\{\hat{y}_i \leq -\overline{x}\}] = 0$. I show that g's ex-ante expected payoff strictly increases at $\alpha_\ell = 0$. The desired result will then follow because g's ex-ante expected payoff is continuous in $\sum_{i\in N^L} \rho_i[(1-\alpha_i)\,\mathbb{I}\{\hat{x}_i \leq -\overline{x}\} + \alpha_i\,\mathbb{I}\{\hat{y}_i \leq -\overline{x}\}]$.

We have $\hat{x}_\ell \in [0, \overline{x}(0;\hat{x}_\ell))$ and $\hat{y}_\ell > \hat{x}_\ell$. Therefore $0 \leq z_\ell(0;\hat{x}_\ell) = \hat{x}_\ell < y^\ell_g(0;\hat{x}_\ell) \leq \hat{y}_\ell$. Furthermore, $-\overline{x}(0;\hat{x}_\ell)$ is not proposed with positive probability because $\sum_{i\in N^L} \rho_i[(1-\alpha_i)\,\mathbb{I}\{\hat{x}_i \leq -\overline{x}\} + \alpha_i\,\mathbb{I}\{\hat{y}_i \leq -\overline{x}\}] = 0$. Thus, we have

$$\begin{aligned}\mathcal{U}_g(0;\hat{x}_\ell) = \rho_\ell \Bigg(&\alpha_\ell \Big[u_g(y^\ell_g(0;\hat{x}_\ell)) + u_\ell(y^\ell_g(0;\hat{x}_\ell)) - u_\ell(\hat{x}_\ell)\Big] + (1-\alpha_\ell)\,u_g(\hat{x}_\ell)\Bigg) \\ &+ \sum_{j\neq\ell} \rho_j \Bigg(\Big[\alpha_j\,\widetilde{E}^{UB}_j(0;\hat{x}_\ell) + (1-\alpha_j)\,E^{UB}_j(0;\hat{x}_\ell)\Big]\,u_g(\overline{x}(0;\hat{x}_\ell)) \\ &\qquad + \alpha_j\Big[\widetilde{C}_j(0;\hat{x}_\ell)\,u_g(\hat{y}_j) - I^j_g\,m^j_g(0;\hat{x}_\ell)\Big] + (1-\alpha_j)\,C_j(0;\hat{x}_\ell)\,u_g(\hat{x}_j)\Bigg). \end{aligned} \tag{41}$$

Three steps show (41) strictly increases at $\alpha_\ell = 0$.

- First, $0 \leq \hat{x}_\ell < y^\ell_g(0;\hat{x}_\ell) \leq \hat{y}_\ell$ implies $y^\ell_g(0;\hat{x}_\ell)$ weakly increases in α_ℓ. Therefore $u_g(y^\ell_g(\alpha_\ell;\hat{x}_\ell))$ weakly increases and $u_\ell(y^\ell_g(\alpha_\ell;\hat{x}_\ell))$ weakly decreases. Because u is quadratic and $\hat{x}_\ell < y^\ell_g(0;\hat{x}_\ell) \leq \hat{y}_\ell = \frac{\hat{x}_g+\hat{x}_\ell}{2} < \hat{x}_g$, it follows that $u_g(y^\ell_g(\alpha_\ell;\hat{x}_\ell))$ increases weakly faster than $u_\ell(y^\ell_g(\alpha_\ell;\hat{x}_\ell))$ decreases. Therefore $u_g(y^\ell_g(0;\hat{x}_\ell)) + u_\ell(y^\ell_g(0;\hat{x}_\ell)) - u_\ell(\hat{x}_\ell)$ weakly increases in α_ℓ. Furthermore, $\hat{x}_\ell < y^\ell_g(0;\hat{x}_\ell) \leq \hat{y}_\ell < \hat{x}_g$ also implies $u_g(y^\ell_g(0;\hat{x}_\ell)) + u_\ell(y^\ell_g(0;\hat{x}_\ell)) - u_\ell(\hat{x}_\ell) - u_g(\hat{x}_\ell) \geq 0$. It follows that $\alpha_\ell \Big[u_g(y^\ell_g(0;\hat{x}_\ell)) + u_\ell(y^\ell_g(0;\hat{x}_\ell)) - u_\ell(\hat{x}_\ell)\Big] + (1-\alpha_\ell)\,u_g(\hat{x}_\ell)$ weakly increases at $\alpha_\ell = 0$.
- Second, $\overline{x}(0;\hat{x}_\ell)$ strictly increases in α_ℓ because $0 \leq z_\ell < y^\ell_g(0;\hat{x}_\ell) \leq \overline{x}(0;\hat{x}_\ell)$. Since $\overline{x}(0;\hat{x}_\ell) < \hat{x}_g$, it follows that $u_g(\overline{x}(0;\hat{x}_\ell))$ increases at $\alpha_\ell = 0$.

- Third, $m_g^j(0;\hat{x}_\ell)$ weakly increases in α_ℓ for all $j \in N_g^L$ by Lemma 1. But it strictly increases only for $j \in N_g^L$ such that $\hat{y}_j > \overline{x}(0;\hat{x}_\ell)$. Thus, g's lobbying surplus weakly increases in α_ℓ for all $j \in N_g^L$.

□

B.9 Proof of Proposition 6

To state and prove Proposition 6, I modify the baseline model to compare WTP across distinct legislator-group pairs. Specifically, I replace ℓ with two legislators, ℓ_1 and ℓ_2, and replace g with two groups, g_1 and g_2. To isolate differences in proposal power, assume $\hat{x}_{g_1} = \hat{x}_{g_2}$ and $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$, but $\rho_{\ell_1} < \rho_{\ell_2}$. These modifications do not qualitatively change the equilibrium characterization. Two identical pairs avoid potential complications that can arise if one group has access to two legislators, where access to one legislator can affect offers to the other.

Proof. Fix $\alpha \in [0,1]$. It suffices to show that $\frac{\partial \mathcal{U}_{g_1}(\alpha_1;\hat{x}_\ell)}{\partial \alpha_1}|_{\alpha_1=\alpha} \geq 0$ implies $\frac{\partial \mathcal{U}_{g_2}(\alpha_2;\hat{x}_\ell)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq \frac{\partial \mathcal{U}_{g_1}(\alpha_1;\hat{x}_\ell)}{\partial \alpha_1}|_{\alpha_1=\alpha}$. Because $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$ and $\hat{x}_{g_1} = \hat{x}_{g_2}$, we have $y_{g_1} = y_{g_2}$ and $z_{\ell_1} = z_{\ell_2}$. Thus, $m_{g_1} = m_{g_2}$. Denote $y = y_{g_1}$, $z = z_{\ell_1}$, and $m = m_{g_1}$. Assume $\frac{\partial \mathcal{U}_{g_1}(\alpha_1;\hat{x}_\ell)}{\partial \alpha_1}|_{\alpha_1=\alpha} \geq 0$. There are five cases.

- *Case 1:* Suppose $z = \hat{x}_\ell$ and $y = \hat{y}$. Then,

$$\begin{aligned}
\frac{\partial \mathcal{U}_{g_1}(\alpha_1;\hat{x}_\ell)}{\partial \alpha_1}|_{\alpha_1=\alpha} &= \rho_{\ell_1}\Bigg(u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell)\Bigg) \\
&\quad - \frac{\partial \overline{x}_\alpha}{\partial \alpha_1}\Bigg(\rho_L \frac{\partial u_{g_1}(-\overline{x}_\alpha)}{\partial \overline{x}_\alpha} - \rho_R \frac{\partial u_{g_1}(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}\Bigg) \\
&= \rho_{\ell_1}\Bigg[u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \\
&\quad - \frac{\delta[u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\frac{\partial u_M(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}[1-\delta(\rho_L+\rho_R)]}\Bigg(\rho_L \frac{\partial u_{g_1}(-\overline{x}_\alpha)}{\partial \overline{x}_\alpha} - \rho_R \frac{\partial u_{g_1}(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}\Bigg)\Bigg]
\end{aligned} \tag{42}$$

$$\begin{aligned}
&\leq \rho_{\ell_2}\Bigg[u_{g_1}(\hat{y}) + u_{\ell_1}(\hat{y}) - u_{g_1}(\hat{x}_\ell) - u_{\ell_1}(\hat{x}_\ell) \\
&\quad - \frac{\delta[u_M(\hat{y}) - u_M(\hat{x}_\ell)]}{\frac{\partial u_M(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}[1-\delta(\rho_L+\rho_R)]}\Bigg(\rho_L \frac{\partial u_{g_1}(-\overline{x}_\alpha)}{\partial \overline{x}_\alpha} - \rho_R \frac{\partial u_{g_1}(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}\Bigg)\Bigg]
\end{aligned} \tag{43}$$

$$= \frac{\partial \mathcal{U}_{g_2}(\alpha_2; \hat{x}_\ell)}{\partial \alpha_2}|_{\alpha_2=\alpha}, \tag{44}$$

where (42) follows from $\frac{\partial \overline{x}_\alpha}{\partial \alpha_1} = \rho_{\ell_1} \frac{\delta[u_M(\hat{y})-u_M(\hat{x}_\ell)]}{\frac{\partial u_M(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}[1-\delta(\rho_L+\rho_R)]}$; (43) because (i) $\rho_{\ell_2} > \rho_{\ell_1}$ and (ii) $\frac{\partial \mathcal{U}_{g_2}(\alpha_2;\hat{x}_\ell)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq 0$ implies that the bracketed expression in (42) is positive; and (44) because $\hat{x}_{\ell_1} = \hat{x}_{\ell_2}$, $\hat{x}_{g_1} = \hat{x}_{g_2}$, and $\frac{\partial \overline{x}_\alpha}{\partial \alpha_2} = \rho_{\ell_2} \frac{\delta[u_M(\hat{y})-u_M(\hat{x}_\ell)]}{\frac{\partial u_M(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}[1-\delta(\rho_L+\rho_R)]}$.

- *Case 2:* Suppose $z = \overline{x}_\alpha$ and $y = \hat{y}$. In this case, $\frac{\partial \overline{x}_\alpha}{\partial \alpha_1} = \rho_{\ell_1} \frac{\delta[u_M(\hat{y})-u_M(\overline{x}_\alpha)]}{\frac{\partial u_M(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}(1-\delta[\rho_L+\rho_R+(1-\alpha)(\rho_{\ell_1}+\rho_{\ell_2})])}$ and $\frac{\partial \overline{x}_\alpha}{\partial \alpha_2} = \rho_{\ell_2} \frac{\delta[u_M(\hat{y})-u_M(\overline{x}_\alpha)]}{\frac{\partial u_M(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}(1-\delta[\rho_L+\rho_R+(1-\alpha)(\rho_{\ell_1}+\rho_{\ell_2})])}$. Arguments analogous to Case 1 show $\frac{\partial \mathcal{U}_{g_2}(\alpha_2;\hat{x}_\ell)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq \frac{\partial \mathcal{U}_{g_1}(\alpha_1;\hat{x}_\ell)}{\partial \alpha_1}|_{\alpha_1=\alpha}$. The argument for $z = -\overline{x}_\alpha$ and $y = \hat{y}$ is symmetric.

- *Case 3:* Suppose $z = \hat{x}_\ell$ and $y = \overline{x}_\alpha$. In this case, $\frac{\partial \overline{x}_\alpha}{\partial \alpha_1} = \rho_{\ell_1} \frac{\delta[u_M(\overline{x}_\alpha)-u_M(\hat{x}_\ell)]}{\frac{\partial u_M(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}(1-\delta[\rho_L+\rho_R+\alpha(\rho_{\ell_1}+\rho_{\ell_2})])}$ and $\frac{\partial \overline{x}_\alpha}{\partial \alpha_2} = \rho_{\ell_2} \frac{\delta[u_M(\overline{x}_\alpha)-u_M(\hat{x}_\ell)]}{\frac{\partial u_M(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}(1-\delta[\rho_L+\rho_R+\alpha(\rho_{\ell_1}+\rho_{\ell_2})])}$. Arguments analogous to Case 1 show $\frac{\partial \mathcal{U}_{g_2}(\alpha_2;\hat{x}_\ell)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq \frac{\partial \mathcal{U}_{g_1}(\alpha_1;\hat{x}_\ell)}{\partial \alpha_1}|_{\alpha_1=\alpha}$. The argument for $z = \hat{x}_\ell$ and $y = -\overline{x}_\alpha$ is symmetric.

- *Case 4:* Suppose $z = \overline{x}_\alpha$ and $y = -\overline{x}_\alpha$. In this case, $\frac{\partial \overline{x}_\alpha}{\partial \alpha_1} = \rho_{\ell_1} \frac{\delta[u_M(-\overline{x}_\alpha)-u_M(\overline{x}_\alpha)]}{\frac{\partial u_M(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}[1-\delta(\rho_L+\rho_R+\rho_{\ell_1}+\rho_{\ell_2})]}$ and $\frac{\partial \overline{x}_\alpha}{\partial \alpha_2} = \rho_{\ell_2} \frac{\delta[u_M(-\overline{x}_\alpha)-u_M(\overline{x}_\alpha)]}{\frac{\partial u_M(\overline{x}_\alpha)}{\partial \overline{x}_\alpha}[1-\delta(\rho_L+\rho_R+\rho_{\ell_1}+\rho_{\ell_2})]}$. Arguments analogous to Case 1 show $\frac{\partial \mathcal{U}_{g_2}(\alpha_2;\hat{x}_\ell)}{\partial \alpha_2}|_{\alpha_2=\alpha} \geq \frac{\partial \mathcal{U}_{g_1}(\alpha_1;\hat{x}_\ell)}{\partial \alpha_1}|_{\alpha_1=\alpha}$. The argument for $z = -\overline{x}_\alpha$ and $y = \overline{x}_\alpha$ is symmetric.

- *Case 5:* Suppose $z = \overline{x}_\alpha$ and $y = \overline{x}_\alpha$. Then, $\frac{\partial \mathcal{U}_{g_2}(\alpha_2;\hat{x}_\ell)}{\partial \alpha_2}|_{\alpha_2=\alpha} = \frac{\partial \mathcal{U}_{g_1}(\alpha_1;\hat{x}_\ell)}{\partial \alpha_1}|_{\alpha_1=\alpha} = 0$. The argument for $z = -\overline{x}_\alpha$ and $y = -\overline{x}_\alpha$ is symmetric.

□

C Equivalence of Outcome Distribution

A stationary strategy profile $\sigma = (\lambda, \pi, \varphi, \nu)$ is a *stationary legislative lobbying equilibrium* if it satisfies four conditions. First, for all $g \in N^G$ and $\ell \in N_g^L$, λ_g^ℓ places probability one on

$$\arg\max_{(y,m)} \overline{\nu}_\sigma(y)u_g(y) + [1-\overline{\nu}_\sigma(y)][(1-\delta)u_g(q) + \delta\widehat{V}_g(\sigma)] - m$$
$$\text{s.t. } \overline{\nu}_\sigma(y)u_\ell(y) + [1-\overline{\nu}_\sigma(y)][(1-\delta)u_\ell(q) + \delta\widetilde{V}_\ell(\sigma)] + m$$
$$\geq$$

$$\int_X \left[\overline{\nu}_\sigma(x) u_\ell(x) + [1 - \overline{\nu}_\sigma(x)][(1-\delta)u_\ell(q) + \delta \widetilde{V}_\ell(\sigma)] \right] \pi_\ell(dx). \tag{45}$$

Second, for all $\ell \in N^L$ and $(y, m) \in W$,

$$\begin{gathered} \overline{\nu}_\sigma(y) u_\ell(y) + [1 - \overline{\nu}_\sigma(y)][(1-\delta)u_\ell(q) + \delta \widetilde{V}_\ell(\sigma)] + m \\ > \\ \int_X \left[\overline{\nu}_\sigma(x) u_\ell(x) + [1 - \overline{\nu}_\sigma(x)][(1-\delta)u_\ell(q) + \delta \widetilde{V}_\ell(\sigma)] \right] \pi_\ell(dx). \end{gathered} \tag{46}$$

implies $\varphi_\ell(y, m) = 1$ and the opposite strict inequality implies $\varphi_\ell(y, m) = 0$. Third, for all $\ell \in N^L$,

$$\pi_\ell \left(\arg\max_{x \in X} \overline{\nu}_\sigma(x) u_\ell(x) + [1 - \overline{\nu}_\sigma(x)][(1-\delta)u_\ell(q) + \delta \widetilde{V}_\ell(\sigma)] \right) = 1. \tag{47}$$

Finally, for all $i \in N^V$ and $x \in X$, $u_i(x) > (1-\delta)u_i(q) + \delta V_i(\sigma)$ implies $\nu_i(x) = 1$ and the opposite strict inequality implies implies $\nu_i(x) = 0$.

Lemma C.1 shows that surplus lobby payments never happen in equilibrium. The proof is straightforward and omitted.

Lemma C.1. *In every stationary legislative lobbying equilibrium, for all $\ell \in N^L$ every $(y, m) \in supp(\lambda_g^\ell)$ satisfies*

$$\begin{gathered} \overline{\nu}_\sigma(y) u_\ell(y) + [1 - \overline{\nu}_\sigma(y)][(1-\delta)u_\ell(q) + \delta \widetilde{V}_\ell(\sigma)] + m \\ = \\ \int_X \left[\overline{\nu}_\sigma(x) u_\ell(x) + [1 - \overline{\nu}_\sigma(x)][(1-\delta)u_\ell(q) + \delta \widetilde{V}_\ell(\sigma)] \right] \pi_\ell(dx). \end{gathered} \tag{48}$$

From (12), recall $\xi_\ell(\alpha; \sigma) = (1 - \alpha_\ell) + \alpha_\ell \int_W [1 - \varphi_\ell(y, m)] \lambda_g^\ell(dw)$. Define

$$\hat{\chi}(X') = \sum_{\ell \in N^L} \rho_\ell \left(\xi_\ell(\alpha; \sigma) \int_{X'} \overline{\nu}_\sigma(x)\, \pi_\ell(dx) + \alpha_\ell \int_{X' \times \mathbb{R}_+} \varphi_\ell(y, m)\, \overline{\nu}_\sigma(y)\, \lambda_g^\ell(dw) \right), \tag{49}$$

the probability some $x \in X' \subseteq X$ is passed in a given period under σ. Next, define

$$\check{\chi} = \sum_{\ell \in N^L} \rho_\ell \left(\xi_\ell(\alpha; \sigma) \int_X [1 - \overline{\nu}_\sigma(x)]\, \pi_\ell(dx) + \alpha_\ell \int_W \varphi_\ell(y, m)\, [1 - \overline{\nu}_\sigma(y)]\, \lambda_g^\ell(dw) \right), \tag{50}$$

the probability of a failed proposal in a given period under σ.

Following Banks and Duggan (2006a), each player's continuation value can be expressed as a function of a common lottery over policy, denoted χ^σ. Using (49) and (50), define χ^σ so that for all measurable $X' \subseteq X$: (i) if $q \notin X'$, then $\chi^\sigma(X') = \frac{\hat{\chi}(X')}{1-\delta\check{\chi}}$, and (ii) if $q \in X'$, then $\chi^\sigma(X') = \frac{\hat{\chi}(X')+(1-\delta)\check{\chi}}{1-\delta\check{\chi}}$.

Set $V^{den}(\sigma) = 1 - \delta\check{\chi}$ and define

$$V_i^{num}(\sigma) = \sum_{\ell \in N^L} \rho_\ell \Bigg(\xi_\ell(\alpha;\sigma) \int_X \Big[\overline{\nu}_\sigma(x)\, u_i(x) + [1-\overline{\nu}_\sigma(x)]\,(1-\delta)u_i(q) \Big]\, \pi_\ell(dx) \\ + \alpha_\ell \int_W \varphi(y,m) \Big[\overline{\nu}_\sigma(y)\, u_i(x) + [1-\overline{\nu}_\sigma(y)]\,(1-\delta)u_i(q) \Big]\, \lambda_g^\ell(dw) \Bigg).$$

For each $i \in N^V$, i's continuation value defined in (13) satisfies $V_i(\sigma) = \frac{V_i^{num}(\sigma)}{V^{den}(\sigma)}$. Then we can express $V_i(\sigma)$ as a lottery over policy, $V_i(\sigma) = \int_X u_i(x)\chi^\sigma(dx)$.

The policy lottery χ^σ is common to all players, but committee members may receive payment and interest groups may make payments. Define

$$\hat{m}_\ell(\sigma) = \rho_\ell\, \alpha_\ell \int_W m\, \varphi_\ell(y,m)\, \lambda_g^\ell(dw), \tag{51}$$

which is ℓ's expected lobby payment in each period until passage. For $\ell \in N^L$, re-arranging (14) yields

$$\begin{aligned} \widetilde{V}_\ell(\sigma) &= \frac{V_\ell^{num}(\sigma) + \hat{m}_\ell(\sigma)}{V^{den}(\sigma)} \\ &= \int_X u_\ell(x)\chi^\sigma(dx) + \frac{\hat{m}_\ell(\sigma)}{V^{den}(\sigma)}. \end{aligned} \tag{52}$$

Similarly, for $g \in N^G$ rearranging (15) yields

$$\begin{aligned} \widehat{V}_g(\sigma) &= \frac{V_g^{num}(\sigma) - \sum_{\ell \in N_g^L} \hat{m}_\ell(\sigma)}{V^{den}(\sigma)} \\ &= \int_X u_g(x)\chi^\sigma(dx) - \sum_{\ell \in N_g^L} \frac{\hat{m}_\ell(\sigma)}{V^{den}(\sigma)}. \end{aligned} \tag{53}$$

Finally, define

$$\widetilde{U}_\ell(\sigma) = \int_X \Big[\overline{\nu}_\sigma(x)u_\ell(x) + \Big(1 - \overline{\nu}_\sigma(x)\Big)\Big((1-\delta)u_\ell(q) + \delta\widetilde{V}_\ell(\sigma)\Big)\Big]\pi_\ell(dx), \tag{54}$$

which is ℓ's expected dynamic payoff under σ conditional on being recognized as the proposer and rejecting g_ℓ's offer.

Lemma C.2. *There does not exist a stationary legislative lobbying equilibrium σ such that χ^σ is degenerate on q.*

Proof. Let σ denote an equilibrium. To show a contradiction, assume $\chi^\sigma(q) = 1$. Thus, $V_M(\sigma) = u_M(q)$, which implies $u_M(q) \geq (1-\delta)u_M(q) + \delta V_M(\sigma)$ and therefore $q \in A(\sigma)$. Without loss of generality, assume $q > 0$.

By assumption, there exists $\ell \in N^L$ such that $\hat{x}_\ell < q$ and at least one of $\hat{x}_{g_\ell} \leq q$ or $\alpha_\ell < 1$ holds. If $\alpha_\ell < 0$, then it is straightforward to show that ℓ must have a profitable deviation, a contradiction.

For the other case, suppose $\hat{x}_\ell < q$, $\hat{x}_{g_\ell} \leq q$, and $\alpha_\ell = 1$. Note that $u_{g_\ell}(y) + u_\ell(y) - \widetilde{U}_\ell(\sigma)$ is g_ℓ's expected dynamic payoff from any offer (y, m) such that $\overline{\nu}_\sigma(y) = 1$, $\varphi_\ell(y, m) = 1$, and ℓ is indifferent between accepting and rejecting. We have $\hat{y}_\ell = \arg\max_{y \in X} u_{g_\ell}(y) + u_\ell(y) - \widetilde{U}_\ell(\sigma)$ and $\hat{y}_\ell < q$. Strict concavity and continuity imply existence of $\varepsilon > 0$ and $y^\varepsilon < q$ such that $\overline{\nu}_\sigma(y^\varepsilon) = 1$, $\varphi_\ell(y^\varepsilon, \widetilde{U}_\ell(\sigma) - u_\ell(y^\varepsilon) + \varepsilon) = 1$, and

$$u_{g_\ell}(y^\varepsilon) + u_\ell(y^\varepsilon) - \widetilde{U}_\ell(\sigma) - \varepsilon > u_{g_\ell}(q) + u_\ell(q) - \widetilde{U}_\ell(\sigma) \tag{55}$$

$$\geq u_{g_\ell}(q) + u_\ell(q) - \widetilde{U}_\ell(\sigma) - \delta \left(\sum_{j \in N_g^L} \frac{\hat{m}_j(\sigma)}{V^{den}(\sigma)} - \frac{\hat{m}_\ell(\sigma)}{V^{den}(\sigma)} \right), \tag{56}$$

where (56) follows from $\sum_{j \in N_g^L} \frac{m_j(\sigma)}{V^{den}(\sigma)} \geq \frac{m_\ell(\sigma)}{V^{den}(\sigma)}$. The RHS of (55) is weakly greater than g_ℓ's expected payoff from lobbying ℓ to q if $\overline{\nu}_\sigma(q) = 1$; and (56) is weakly greater than g_ℓ's expected payoff from lobbying ℓ to any y' such that $\overline{\nu}_\sigma(y') = 0$. Thus, g_ℓ must have a profitable deviation, a contradiction. $\square$

Lemma C.3. *Let σ denote a stationary legislative lobbying equilibrium. For all $\ell \in N^L$ there exists $(y, m) \in X \times \mathbb{R}_+$ such that $\overline{\nu}_\sigma(y) = 1$ and g_ℓ strictly prefers (y, m) to any (y', m') such that $\overline{\nu}_\sigma(y') = 0$.*

Proof. Fix an equilibrium σ. Let χ^q denote a probability distribution degenerate on q. Define the continuation distribution following rejection under σ as $\chi = (1-\delta)\chi^q + \delta\chi^\sigma$, which is non-degenerate because $\delta \in (0, 1)$ and $\chi^\sigma(q) < 1$ by Lemma C.2.

For every player $k \in N$, the expected dynamic policy payoff from a rejected policy proposal satisfies

$$(1-\delta)u_k(q) + \delta V_k(\sigma) = \int_X u_k(x)\, \chi(dx).$$

Let x^σ denote the mean of χ. Since u is strictly concave and χ is non-degenerate, Jensen's Inequality implies

$$u_k(x^\sigma) > \int_X u_k(x)\,\chi(dx) = (1-\delta)u_k(q) + \delta V_k(\sigma). \tag{57}$$

Consider $\ell \in N^L$. First, assume $\varphi_\ell(y,m) = 1$ whenever ℓ is indifferent. The condition for g_ℓ to strictly prefer (y,m) such that $\overline{\nu}_\sigma(y) = 1$, rather than (y',m') such that $\overline{\nu}_\sigma(y') = 0$, is

$$u_{g_\ell}(y) + u_\ell(y) - \widetilde{U}_\ell(\sigma) > (1-\delta)u_{g_\ell}(q) + \delta\widehat{V}_{g_\ell}(\sigma) + (1-\delta)u_\ell(q) + \delta\widetilde{V}_\ell(\sigma) - \widetilde{U}_\ell(\sigma).$$

Equivalently,

$$u_{g_\ell}(y) + u_\ell(y) > (1-\delta)u_{g_\ell}(q) + \delta\widehat{V}_{g_\ell}(\sigma) + (1-\delta)u_\ell(q) + \delta\widetilde{V}_\ell(\sigma). \tag{58}$$

Notice that

$$\begin{aligned} \widehat{V}_{g_\ell}(\sigma) + \widetilde{V}_\ell(\sigma) &= V_{g_\ell}(\sigma) - \sum_{\ell' \in N_g^L} \frac{\hat{m}_{\ell'}(\sigma)}{V^{den}(\sigma)} + V_\ell(\sigma) + \frac{\hat{m}_\ell(\sigma)}{V^{den}(\sigma)} && (59)\\ &\leq V_{g_\ell}(\sigma) - \frac{\hat{m}_\ell(\sigma)}{V^{den}(\sigma)} + V_\ell(\sigma) + \frac{\hat{m}_\ell(\sigma)}{V^{den}(\sigma)} && (60)\\ &= V_{g_\ell}(\sigma) + V_\ell(\sigma), && (61)\end{aligned}$$

where (59) follows from substituting for $\widetilde{V}_\ell(\sigma)$ and $\widehat{V}_g(\sigma)$ using (52) and (53); and (60) from $\sum_{\ell' \in N_g^L} \frac{\hat{m}_{\ell'}(\sigma)}{V^{den}(\sigma)} \geq \frac{\hat{m}_{\ell'}(\sigma)}{V^{den}(\sigma)}$.

By (57), $\overline{\nu}_\sigma(x^\sigma) = 1$ follows because $u_M(x^\sigma) > (1-\delta)u_M(q) + \delta V_M(\sigma)$. Furthermore, (57) implies $u_{g_\ell}(x^\sigma) > (1-\delta)u_{g_\ell}(q) + \delta V_{g_\ell}(\sigma)$ and $u_\ell(x^\sigma) > (1-\delta)u_\ell(q) + \delta V_\ell(\sigma)$. Thus, (61) implies that (58) holds because

$$\begin{aligned} u_{g_\ell}(x^\sigma) + u_\ell(x^\sigma) &> (1-\delta)u_{g_\ell}(q) + \delta V_{g_\ell}(\sigma) + (1-\delta)u_\ell(q) + \delta V_{g_\ell}(\sigma)\\ &\geq (1-\delta)u_{g_\ell}(q) + \delta\widehat{V}_{g_\ell}(\sigma) + (1-\delta)u_\ell(q) + \delta\widetilde{V}_\ell(\sigma).\end{aligned}$$

Next, assume $\varphi_\ell(x^\sigma, m) < 1$ for m such that ℓ is indifferent between accepting (x^σ, m) and rejecting. For sufficiently small $\varepsilon > 0$, $\varphi_\ell(x^\sigma, m+\varepsilon) = 1$ and the preceding argument implies g_ℓ strictly prefers $(x^\sigma, m+\varepsilon)$ over any (y', m') such that $\overline{\nu}_\sigma(y') = 0$. □

Lemma C.4. *Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential voting.*

Proof. Let σ be an equilibrium. By Duggan (2014), M is decisive. Quadratic utility and

$\hat{x}_M = 0 \neq q$ together imply $A(\sigma) = \{x \in X | u_M(x) \geq (1-\delta)u_M(q) + \delta V_M(\sigma)\}$ is a closed, non-empty interval symmetric about 0. Let $A(\sigma) = [-\overline{x}(\sigma), \overline{x}(\sigma)]$. Then $x \in (-\overline{x}(\sigma), \overline{x}(\sigma))$ implies $\overline{\nu}_\sigma(x) = 1$.

Fix $\ell \in N^L$. By Lemma C.2, $\chi^\sigma(q) < 1$. Lemma C.3 implies existence of $(y, m) \in W$ such that $\overline{\nu}_\sigma(y) = 1$ and g_ℓ strictly prefers (y, m) over all (y', m') with $\overline{\nu}_\sigma(y') = 0$. Thus, $y \in A(\sigma)$ for all $(y, m) \in \text{supp}(\lambda_{g_\ell})$. Without loss of generality, assume $\overline{\nu}_\sigma(-\overline{x}(\sigma)) < 1$. It suffices to check two cases.

- *Case 1:* If $\hat{x}_\ell \leq -\overline{x}(\sigma)$ and $u_\ell(-\overline{x}(\sigma)) > (1-\delta)u_\ell(q) + \delta\widetilde{V}_\ell(\sigma)$, then $x \in A(\sigma)$ for all $x \in \text{supp}(\pi_\ell)$. Because u_ℓ is strictly concave and continuous, and $\overline{\nu}_\sigma(-\overline{x}(\sigma)) < 1$, there exists $\varepsilon > 0$ such that ℓ has a profitable deviation to $-\overline{x}(\sigma) + \varepsilon$, a contradiction.
- *Case 2:* Assume $\hat{y}_\ell \leq -\overline{x}(\sigma)$. Continuity, Lemma C.3, and $\overline{\nu}_\sigma(-\overline{x}(\sigma)) < 1$ imply existence of $\varepsilon, \varepsilon' > 0$ such that g_ℓ has a profitable deviation to $(y', m') = (-\overline{x}(\sigma) + \varepsilon, \widetilde{U}_\ell(\sigma) - u_\ell(-\overline{x}(\sigma) + \varepsilon) + \varepsilon')$, a contradiction.

It follows that either σ must involve deferential voting, or σ is equivalent in outcome distribution to an equilibrium with deferential voting. □

Lemma C.5. *Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to an equilibrium with deferential acceptance strategies.*

Proof. Let σ denote an equilibrium. By Lemma C.4, we can assume $\overline{\nu}_\sigma(x) = 1$ iff $x \in A(\sigma)$. Fix $\ell \in N^L$ and define $y^*_{g_\ell} = \underset{y \in A(\sigma)}{\arg\max}\, u_{g_\ell}(y) + u_\ell(y) - \widetilde{U}_\ell(\sigma)$, which is uniquely defined, and $m^*_{g_\ell} = \widetilde{U}_\ell(\sigma) - u_\ell(y^*_{g_\ell})$.

By Lemma C.2, $\chi^\sigma(q) < 1$. For sufficiently small $\varepsilon > 0$, Lemma C.3 implies g strictly prefers $(y^*_{g_\ell}, m^*_{g_\ell} + \varepsilon)$ over every (y', m') such that $y' \notin A(\sigma)$. Thus, if π_ℓ is not degenerate on $y^*_{g_\ell}$ and $\varphi_\ell(y^*_{g_\ell}, m^*_{g_\ell}) < 1$, then there exists $\varepsilon > 0$ such that g_ℓ has a profitable deviation to $(y^*_{g_\ell}, m^*_{g_\ell} + \varepsilon)$, a contradiction. Thus, σ must satisfy either (i) $\pi_\ell(y^*_{g_\ell}) = 1$, or (ii) $\lambda^\ell_g(y^*_{g_\ell}, m^*_{g_\ell}) = 1$ and $\varphi_\ell(y^*_{g_\ell}, m^*_{g_\ell}) = 1$, as desired. □

A strategy profile σ is *no-delay* if $\overline{\nu}_\sigma(x) = 1$ for all $x \in \text{supp}(\pi_\ell)$ and $\overline{\nu}_\sigma(y) = 1$ for all $(y, m) \in \text{supp}(\lambda^\ell_g)$.

Lemma C.6. *Every stationary legislative lobbying equilibrium is no-delay.*

Proof. Fix an equilibrium σ. By Lemma C.2, $\chi^\sigma(q) < 1$. Thus, Lemma C.3 implies g strictly prefers some $(y, m) \in W$ such that $\overline{\nu}_\sigma(y) = 1$. Lemma C.4 implies we can assume $\overline{\nu}_\sigma(x) = 1$ iff $x \in A(\sigma)$. Lemma C.5 implies we can assume all $\ell \in N^L$ use deferential acceptance strategies.

For each $\ell \in N^L$, the preceding observations and Lemma C.1 imply λ_g^ℓ puts probability one on (y^*, m^*) such that $y^* = \arg\max_{y \in A(\sigma)} u_{g_\ell}(y) + u_\ell(y) - u_\ell(z_\ell; \sigma)$, which is unique. Lemmas C.4 and C.5 imply we can assume $\overline{\nu}_\sigma(y^*) = 1$ and $\varphi_\ell(y^*, m^*) = 1$.

It remains to verify that $z_\ell \notin A(\sigma)$ cannot be optimal for any $\ell \in N^L$. To show a contradiction, assume proposing $z_\ell \notin A(\sigma)$ is optimal for some $\ell \in N^L$. Let $z^* = \arg\max_{x \in A(\sigma)} u_\ell(x)$. There are two steps. Step 1 establishes useful properties of ℓ's preferences over lotteries. Step 2 shows a contradiction.

Step 1: Recall the continuation lottery induced by σ, denoted $\chi = (1-\delta)\chi^q + \delta\chi^\sigma$ with mean x^σ. Jensen's inequality implies $u_i(x^\sigma) > \int_X u_i(x)\,\chi(dx) = (1-\delta)u_i(q) + \delta V_i(\sigma)$ for all $i \in N$, so $x^\sigma \in \text{int}A(\sigma)$.

Next, let χ^{z^*} denote the policy lottery nearly equivalent to χ, but shifting probability $\frac{\delta\,\rho_\ell\,\alpha_\ell}{V^{den}(\sigma)}$ from y^* to z^*. Let x^{z^*} denote the mean of χ^{z^*}. For all $i \in N$, Jensen's inequality implies

$$u_i(x^{z^*}) > \int_X u_i(x)\,\chi^{z^*}(dx) = (1-\delta)u_i(q) + \delta V_i(\sigma) - \frac{\delta\,\rho_\ell\,\alpha_\ell u_i(y^*)}{V^{den}(\sigma)} + \frac{\delta\,\rho_\ell\,\alpha_\ell u_i(z^*)}{V^{den}(\sigma)}.$$

Moreover, x^{z^*} is located weakly between x^σ and z^*, implying $x^{z^*} \in A(\sigma)$.

Step 2: Since $z_\ell \notin A(\sigma)$ is optimal, Lemma C.1 implies

$$\begin{aligned} m^* &= (1-\delta)u_\ell(q) + \delta\widetilde{V}_\ell(\sigma) - u_\ell(y^*) \\ &= (1-\delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta\hat{m}_\ell(\sigma)}{V^{den}(\sigma)} - u_\ell(y^*). \end{aligned} \tag{62}$$

Using (51), $\hat{m}_\ell(\sigma)$ is expressed recursively as

$$\begin{aligned} \hat{m}_\ell(\sigma) &= \rho_\ell\,\alpha_\ell\left((1-\delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta\hat{m}_\ell(\sigma)}{V^{den}(\sigma)} - u_\ell(y^*)\right) \\ &= \frac{\rho_\ell\,\alpha_\ell V^{den}(\sigma)}{V^{den}(\sigma) - \delta\rho_\ell\,\alpha_\ell}\left((1-\delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*)\right). \end{aligned} \tag{63}$$

Because $z_\ell \notin A(\sigma)$ is optimal,

$$u_\ell(z^*) \leq (1-\delta)u_\ell(q) + \delta\widetilde{V}(\sigma) \tag{64}$$

$$= (1-\delta)u_\ell(q) + \delta V_\ell(\sigma) + \frac{\delta\,\rho_\ell\,\alpha_\ell[(1-\delta)u_\ell(q) + \delta V_\ell(\sigma) - u_\ell(y^*)]}{V^{den}(\sigma) - \delta\rho_\ell\,\alpha_\ell}, \tag{65}$$

where (65) follows from the definition of $\widetilde{V}_\ell(\sigma)$ and using (63) to substitute for $\hat{m}_\ell(\sigma)$.

Next, we have $V^{den}(\sigma) - \delta\rho_\ell\,\alpha_\ell \geq 1 - \delta\sum_{j\in N^L}\rho_j(1-\alpha_j) - \delta\rho_\ell\,\alpha_\ell > 0$, where the first inequality follows because Lemma C.3 implies all lobby offers are accepted and passed under σ, so $V^{den}(\sigma) \geq 1 - \delta\sum_{j\in N^L}\rho_j(1-\alpha_j)$; and the second inequality follows from $\delta[\rho_\ell\alpha_\ell + \sum_{j\in N^L}\rho_j(1-\alpha_j)] < 1$. Rearranging and simplifying (65),

$$
\begin{aligned}
0 &\leq V^{den}(\sigma)\Big((1-\delta)u_\ell(q) + \delta V_\ell(\sigma)\Big) - \delta\rho_\ell\,\alpha_\ell u_\ell(y^*) - u_\ell(z^*)\Big(V^{den}(\sigma) - \delta\rho_\ell\,\alpha_\ell\Big) \\
&\propto (1-\delta)u_\ell(q) + \delta V_\ell(\sigma) - \frac{\delta\rho_\ell\,\alpha_\ell[u_\ell(y^*) - u_\ell(z^*)]}{V^{den}(\sigma)} - u_\ell(z^*) \\
&= \int_X u_\ell(x)\,\chi^{z^*}(dx) - u_\ell(z^*),
\end{aligned}
$$

a contradiction because $u_\ell(z^*) \geq u_\ell(x^{z^*}) > \int_X u_\ell(x)\chi^{z^*}(dx)$.

□

Lemma C.7. *Every stationary legislative lobbying equilibrium is such that λ_g is degenerate for all $g \in N^G$ and π_ℓ is degenerate for all $\ell \in N^L$.*

Proof. Let σ denote an equilibrium. By Duggan (2014), $A_M(\sigma) = A(\sigma)$, which is nonempty, compact and convex.

First, consider $g \in N^g$ and $\ell \in N^L_g$. Recall $\widetilde{U}_\ell(\sigma)$ from (54). Lemmas C.1 and C.6 imply λ^ℓ_g puts probability one on the unique (y^*, m^*) satisfying $y^* = \arg\max_{y\in A(\sigma)} u_g(y) + u_\ell(y) - \widetilde{U}_\ell(\sigma)$, and $m^* = \widetilde{U}_\ell(\sigma) - u_\ell(y^*)$.

Second, consider $\ell \in N^L$. Lemma C.6 implies π_ℓ puts probability one on $x^* = \arg\max_{x\in A(\sigma)} u_\ell(x)$, which is unique. □

Proposition 1.2 *Every stationary legislative lobbying equilibrium is equivalent in outcome distribution to a no-delay stationary legislative lobbying equilibrium with deferential acceptance and deferential voting.*

Proof. Follows from Lemmas C.4 - C.7. □

D Partitioning Moderates & Extremists

Consider $\ell \in N^L$. First, I define a function ζ^ℓ that relates to M's equilibrium voting decision. Then, Lemmas D.3 - D.6 characterize ζ^ℓ. Finally, Lemma 3 delivers a partitional characterization on $\hat{x}_g$ that facilitates Proposition 4.

Preliminaries to define ζ^ℓ. Recall $\overline{x}(0) = \overline{x}(\hat{x}_g)$ for $\hat{x}_g = 0$. Let $\hat{D}^{\ell,y} = \{\hat{y}_j : |\hat{y}_j| > \overline{x}(0), j \neq \ell\}$ and $\hat{D}^{\ell,x} = \{\hat{x}_j : |\hat{x}_j| > \overline{x}(0), j \neq \ell\}$. Next, set $D^{\ell,y} = \{|y| : y \in \hat{D}^{\ell,y}\}$ and $D^{\ell,x} = \{|x| : x \in \hat{D}^{\ell,x}\}$. Define D^ℓ as the unique elements of $D^{\ell,y} \cup D^{\ell,x} \cup \{\overline{x}(0)\}$. Let $K^\ell + 1 = |D^\ell|$. Denote the k-th element of D^ℓ as d^ℓ_k. Index elements $k = 0, \ldots, K^\ell$ of D^ℓ in ascending order so that $d^\ell_0 = \overline{x}(0)$ and $k' > k$ implies $d^\ell_{k'} > d^\ell_k$.

For each k and $j \neq \ell$, let $C^k_j = \mathbb{I}\{\hat{x}_j \in [-d^\ell_k, d^\ell_k]\}$ and $\widetilde{C}^k_j = \mathbb{I}\{\hat{y}_j \in [-d^\ell_k, d^\ell_k]\}$. Define

$$I^k_j = (1-\alpha_j)C^k_j u_M(\hat{x}_j) + \alpha_j \widetilde{C}^k_j u_M(\hat{y}_j)$$

and

$$O^k_j = (1-\alpha_j)(1-C^k_j) + \alpha_j(1-\widetilde{C}^k_j),$$

suppressing dependence on ℓ. Let

$$\mathring{x}^\ell_k = \left(\frac{1}{\delta\rho_\ell}\left[(1-\delta)u_M(q) + \delta\sum_{j\neq\ell}\rho_j I^k_j - u_M(d^\ell_k)\left(1-\delta\sum_{j\neq\ell}O^k_j\right)\right]\right)^{\frac{1}{2}}. \tag{66}$$

Because $d^\ell_0 = \overline{x}(0)$, rearranging (66) yields $\mathring{x}^\ell_0 = 0$.

Lemma D.1. *For all $\ell \in N^\ell$ and each $k = 0, \ldots, K^\ell$, we have*

$$\delta\sum_{j\neq\ell}\rho_j I^{k+1}_j - u_M(d^\ell_{k+1})(1-\delta\sum_{j\neq\ell}\rho_j O^{k+1}_j) = \delta\sum_{j\neq\ell}\rho_j I^k_j - u_M(d^\ell_{k+1})(1-\delta\sum_{j\neq\ell}\rho_j O^k_j).$$

Proof. Consider $\ell \in N^L$ and fix $k < K^\ell$. Then,

$$\begin{aligned}
&\delta\sum_{j\neq\ell}\rho_j I^{k+1}_j - u_M(d^\ell_{k+1})(1-\delta\sum_{j\neq\ell}\rho_j O^{k+1}_j)\\
&= \delta\sum_{j\neq\ell}\rho_j I^{k+1}_j - u_M(d^\ell_{k+1})(1-\delta\sum_{j\neq\ell}\rho_j O^{k+1}_j) + \delta u_M(d^\ell_{k+1})\sum_{j\neq\ell}\rho_j O^k_j - \delta u_M(d^\ell_{k+1})\sum_{j\neq\ell}\rho_j O^k_j\\
&= \delta\sum_{j\neq\ell}\rho_j I^{k+1}_j - u_M(d^\ell_{k+1})(1-\delta\sum_{j\neq\ell}\rho_j O^k_j) + \delta u_M(d^\ell_{k+1})\sum_{j\neq\ell}\rho_j (O^{k+1}_j - O^k_j)
\end{aligned}$$

$$= \delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k) + \delta \sum_{j \neq \ell} \rho_j (I_j^k - I_j^{k+1}) \tag{67}$$

$$= \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k), \tag{68}$$

where (67) follows because $u_M(d_{k+1}^\ell) \sum_{j \neq \ell} \rho_j (O_j^{k+1} - O_j^k) = \sum_{j \neq \ell} \rho_j (I_j^k - I_j^{k+1})$ by construction. □

Lemma D.2. *For all $\ell \in N^\ell$, $\mathring{x}_k^\ell$ strictly increases in k.*

Proof. Consider $\ell \in N^L$ and fix $k < K^\ell$. Lemma D.1 and $0 > u_M(d_k^\ell) > u_M(d_{k+1}^\ell)$ together imply

$$\delta \sum_{j \neq \ell} \rho_j I_j^{k+1} - u_M(d_{k+1}^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k+1}) > \delta \sum_{j \neq \ell} \rho_j I_j^k - u_M(d_k^\ell)(1 - \delta \sum_{j \neq \ell} \rho_j O_j^k). \tag{69}$$

Thus, $\mathring{x}_k^\ell < \mathring{x}_{k+1}^\ell$ follows from (66). □

Definition of ζ^ℓ. For $k = 0, \ldots, K^\ell$, define $\overline{x}_k^\ell : \mathbb{R}_+ \to \mathbb{R}_+$ as

$$\overline{x}_k^\ell(x) = \left(- \frac{(1-\delta) u_M(q) + \delta \rho_\ell u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_\ell^k}{1 - \delta \sum_{j \neq \ell} \rho_j O_\ell^k} \right)^{\frac{1}{2}}, \tag{70}$$

and $\zeta_k^\ell : \mathbb{R}_+ \to \mathbb{R}$ as

$$\zeta_k^\ell(x) = u_M(x) - \left((1-\delta) u_M(q) + \delta \rho_\ell u_M(x) + \delta \sum_{j \neq \ell} \rho_j I_j^k + \delta u_M(\overline{x}_k^\ell(x)) \sum_{j \neq \ell} \rho_j O_j^k \right).$$

By construction, $\overline{x}_k^\ell(\mathring{x}_k^\ell) = d_k^\ell$ for all k. Adopt the convention $d_{K^\ell+1}^\ell = \infty$. Define the piecewise function $\zeta^\ell : \mathbb{R}_+ \to \mathbb{R}$ as

$$\zeta^\ell(x) = \zeta_k^\ell(x) \quad \text{if } x \in [d_k^\ell, d_{k+1}^\ell).$$

Lemma D.3. *For all $\ell \in N^L$, $\zeta^\ell(0) > 0$ and $\zeta^\ell(q) \leq 0$.*

Proof. Consider $\ell \in N^L$. First, we have

$$\begin{aligned} \zeta^\ell(0) &= \zeta_0^\ell(0) \\ &= u_M(0) - \left((1-\delta) u_M(q) + \delta \rho_\ell u_M(0) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(\overline{x}_0^\ell(0)) \sum_{j \neq \ell} \rho_j O_j^0 \right) \end{aligned}$$

$$= -\left((1-\delta)u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^0 + \delta u_M(d_0^\ell) \sum_{j \neq \ell} \rho_j O_j^0 \right) \tag{71}$$

$$> 0,$$

where (71) follows from $u_M(0) = 0$ and $\overline{x}_0^\ell(0) = d_0^\ell$.

Next, I show $\zeta^\ell(q) \leq 0$. Let k' denote the largest k such that $\mathring{x}_k^\ell \leq q$. There are three steps.

- *Step 1:* Because $\overline{x}^{k'}(\mathring{x}_{k'}^\ell) = d_{k'}^\ell$, we have

$$u_M(d_{k'}^\ell) = \frac{(1-\delta)u_M(q) + \delta\rho_\ell u_M(\mathring{x}_{k'}^\ell) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \tag{72}$$

$$\geq \frac{(1-\delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta \sum_{j \neq \ell} \rho_j I_j^{k'}}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \tag{73}$$

$$\geq \frac{(1-\delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta u_M(d_{k'}^\ell) \sum_{j \neq \ell} \rho_j [(1-\alpha_j)C_j^{k'} + \alpha_j \widetilde{C}_j^{k'}]}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}} \tag{74}$$

$$= \frac{(1-\delta)u_M(q) + \delta\rho_\ell u_M(q) + \delta u_M(d_{k'}^\ell)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'})}{1 - \delta \sum_{j \neq \ell} \rho_j O_j^{k'}}, \tag{75}$$

where (72) follows from rearranging (70); (73) from $\mathring{x}_{k'}^\ell \leq q$; (74) because for all j the construction of $I_j^{k'}$ implies $I_j^{k'} \geq u_M(d_{k'}^\ell)[(1-\alpha_j)C_j^{k'} + \alpha_j \widetilde{C}_j^{k'}]$; and (75) because $\sum_{j \neq \ell} \rho_j [(1-\alpha_j)C_j^{k'} + \alpha_j \widetilde{C}_j^{k'}] = 1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}$ by construction.

Rearranging and simplifying (75) yields $u_M(d_{k'}^\ell) \geq \frac{(1-\delta+\delta\rho_\ell)u_M(q)}{1-\delta+\delta\rho_\ell} = u_M(q)$. Thus,

$$\sum_{j \neq \ell} \rho_j I_j^{k'} = \sum_{j \neq \ell} \rho_j \left[(1-\alpha_j)C_j^{k'} u_M(\hat{x}_j) + \alpha_j \widetilde{C}_j^{k'} u_M(\hat{y}_j) \right] \tag{76}$$

$$\geq u_M(d_{k'}^\ell) \sum_{j \neq \ell} \rho_j \left[(1-\alpha_j)C_j^{k'} + \alpha_j \widetilde{C}_j^{k'} \right] \tag{77}$$

$$= u_M(d_{k'}^\ell)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}) \tag{78}$$

$$\geq u_M(q)(1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}), \tag{79}$$

where (76) follows from the definition of $I_j^{k'}$; (77) from $u_M(\hat{x}_j) \geq u_M(d_{k'}^\ell)$ if $C_j^{k'} = 1$ and $u_M(\hat{y}_j) \geq u_M(d_{k'}^\ell)$ if $\widetilde{C}_j^{k'} = 1$; (78) because $\sum_{j \neq \ell} \rho_j [(1-\alpha_j)C_j^{k'} + \alpha_j \widetilde{C}_j^{k'}] = 1 - \rho_\ell - \sum_{j \neq \ell} \rho_j O_j^{k'}$ by construction; and (79) from $u_M(d_{k'}^\ell) \geq u_M(q)$.

- *Step 2:* We have

$$
\begin{aligned}
u_M(\overline{x}^{\ell}_{k'}(q)) &= \frac{(1-\delta)u_M(q)+\delta\rho_\ell u_M(q)+\delta\sum_{j\neq\ell}\rho_j I_j^{k'}}{1-\delta\sum_{j\neq\ell}\rho_j O_j^{k'}} \\
&\geq \frac{(1-\delta)u_M(q)+\delta\rho_\ell u_M(q)+\delta u_M(q)(1-\rho_\ell-\sum_{j\neq\ell}\rho_j O_j^{k'})}{1-\delta\sum_{j\neq\ell}\rho_j O_j^{k'}} \qquad (80)\\
&= u_M(q), \qquad (81)
\end{aligned}
$$

where (80) follows from Step 1, and (81) from simplifying.

- *Step 3:* To see $\zeta^\ell(q)\leq 0$, note

$$
\begin{aligned}
\zeta^\ell(q) &= u_M(q) - \left((1-\delta)u_M(q)+\delta\rho_\ell u_M(q)+\delta\sum_{j\neq\ell}\rho_j I_j^{k'}+\delta u_M(\overline{x}^{k'}(q))\sum_{j\neq\ell}\rho_j O_j^{k'}\right)\\
&\leq u_M(q) - \left((1-\delta)u_M(q)+\delta\rho_\ell u_M(q)+\delta u_M(q)(1-\rho_\ell-\sum_{j\neq\ell}\rho_j O_j^{k'})+\delta u_M(q)\sum_{j\neq\ell}\rho_j O_j^{k'}\right) \qquad (82)\\
&= 0, \qquad (83)
\end{aligned}
$$

where (82) follows from Steps 1 and 2.

□

Lemma D.4. *For all $\ell\in N^L$, ζ^ℓ is continuous.*

Proof. Consider $\ell\in N^L$ and fix k. Because $\overline{x}^\ell_k(x)$ is continuous, ζ^ℓ is continuous over $(\mathring{x}^\ell_k,\mathring{x}^\ell_{k+1})$. Due to the piecewise construction of ζ^ℓ, suffices to show $\zeta^\ell_k(\mathring{x}^\ell_{k+1})=\zeta^\ell_{k+1}(\mathring{x}^\ell_{k+1})$.

First, I establish $d^\ell_{k+1}=\overline{x}^\ell_k(\mathring{x}^\ell_{k+1})$. Rearranging (66) for $k+1$ yields

$$
\begin{aligned}
0 &= u_M(d^\ell_{k+1})\left(1-\delta\sum_{j\neq\ell}\rho_j O_j^{k+1}\right)-(1-\delta)u_M(q)-\delta\rho_\ell u_M(\mathring{x}^\ell_{k+1})-\delta\sum_{j\neq\ell}\rho_j I_j^{k+1}\\
&= u_M(d^\ell_{k+1})\left(1-\delta\sum_{j\neq\ell}\rho_j O_j^{k}\right)-(1-\delta)u_M(q)-\delta\rho_\ell u_M(\mathring{x}^\ell_{k+1})-\delta\sum_{j\neq\ell}\rho_j I_j^{k}, \qquad (84)
\end{aligned}
$$

where (84) follows from Lemma D.1. Thus, $u_M(d^\ell_{k+1})=\frac{(1-\delta)u_M(q)+\delta\rho_\ell u_M(\mathring{x}^\ell_{k+1})+\delta\sum_{j\neq\ell}\rho_j I_j^k}{1-\delta\sum_{j\neq\ell}\rho_j O_j^k}$, so $d^\ell_{k+1}=\overline{x}^\ell_k(\mathring{x}^\ell_{k+1})$.

Then,

$$\zeta_k^\ell(\mathring{x}_{k+1}^\ell) = u_M(\mathring{x}_{k+1}^\ell) - \left((1-\delta)u_M(q) + \delta\rho_\ell u_M(\mathring{x}_{k+1}^\ell) + \delta \sum_{j\neq\ell} \rho_j I_j^k + \delta u_M(\overline{x}_k^\ell(\mathring{x}_{k+1}^\ell)) \sum_{j\neq\ell} \rho_j O_j^k \right)$$

$$= u_M(\mathring{x}_{k+1}^\ell) - \left((1-\delta)u_M(q) + \delta\rho_\ell u_M(\mathring{x}_{k+1}^\ell) + \delta \sum_{j\neq\ell} \rho_j I_j^{k+1} + \delta u_M(\overline{x}_{k+1}^\ell(\mathring{x}_{k+1}^\ell)) \sum_{j\neq\ell} \rho_j O_j^{k+1} \right) \tag{85}$$

$$= \zeta_{k+1}^\ell(\mathring{x}_{k+1}^\ell), \tag{86}$$

where (85) follows from Lemma D.1 because $d_{k+1}^\ell = \overline{x}_k^\ell(\mathring{x}_{k+1}^\ell)$. □

Lemma D.5. *For all $\ell \in N^L$, ζ^ℓ is strictly decreasing.*

Proof. Consider $\ell \in N^L$ and fix k. The proof shows that the derivative of ζ^ℓ is strictly negative at every $x \in (\mathring{x}_k^\ell, \mathring{x}_{k+1}^\ell)$. Continuity then implies that ζ^ℓ is strictly decreasing.

Consider $x \in (\mathring{x}_k^\ell, \mathring{x}_{k+1}^\ell)$. Then,

$$\zeta^\ell(x) = u_M(x) - \left((1-\delta)u_M(q) + \delta\rho_\ell u_M(x) + \delta \sum_{j\neq\ell} \rho_j I_j^k + \delta u_M(\overline{x}_k^\ell(x)) \sum_{j\neq\ell} \rho_j O_j^k \right),$$

and

$$\frac{\partial \zeta^\ell(x)}{\partial x} = -2x + 2x\delta\rho_\ell + \frac{2x\delta\rho_\ell(\delta\sum_{j\neq\ell}\rho_j O_j^k)}{1-\delta\sum_{j\neq\ell}\rho_j O_j^k} \tag{87}$$

$$\propto \delta\rho_\ell + \delta\sum_{j\neq\ell}\rho_j O_j^k - 1 \tag{88}$$

$$< 0, \tag{89}$$

where (87) follows from $\frac{\partial u_M(\overline{x}_k^\ell(x))}{\partial \overline{x}_k^\ell(x)} \frac{\partial \overline{x}_k^\ell(x)}{\partial x} = -\frac{2x\delta\rho_\ell}{1-\delta\sum_{j\neq\ell}\rho_j O_j^k}$; and (89) because $\delta \in (0,1)$ and $\rho_\ell + \sum_{j\neq\ell}\rho_j O_j^k \leq 1$. □

Lemma D.6. *For all $\ell \in N^\ell$, there is a unique $\overline{x}_\ell \in (0, q]$ such that $\zeta^\ell(x) > 0$ for all $x \in [0, \overline{x}_\ell)$, $\zeta^\ell(\overline{x}_\ell) = 0$, and $\zeta^\ell(x) < 0$ for all $x > \overline{x}_\ell$.*

Proof. Consider $\ell \in N^L$. Lemma D.3 implies $\zeta^\ell(0) > 0$ and $\zeta^\ell(q) \leq 0$. By Lemma D.5, ζ^ℓ is strictly decreasing. Thus, there is a unique $\overline{x}_\ell \in (0, q]$ such that $\zeta^\ell(x) > 0$ for all $x \in [0, \overline{x}_\ell)$ and $\zeta^\ell(x) < 0$ for all $x > \overline{x}_\ell$. Lemma D.4 implies $\zeta^\ell(\overline{x}_\ell) = 0$. □

Lemma 3. *For all $\ell \in N^L$, $\hat{x}_g \in (-\overline{x}_\ell, \overline{x}_\ell)$ implies $\hat{x}_g \in intA(\hat{x}_g)$. Otherwise, $A(\hat{x}_g) = [-\overline{x}_\ell, \overline{x}_\ell]$.*

Proof. Consider $\ell \in N^L$ with associated $g \in N^G$. Assume $\hat{x}_\ell = \hat{x}_g$.

Part 1. First, suppose $\hat{x}_g \in (-\overline{x}_\ell, \overline{x}_\ell)$ and assume $\hat{x}_g \geq 0$ without loss of generality. I show $\hat{x}_g \in \text{int}A(\hat{x}_g)$. Let k' be the largest k such that $\mathring{x}^\ell_k \leq \hat{x}_g$. Define the strategy profile σ' such that it puts probability ρ_ℓ on $\hat{x}_g$ and for each $j \neq \ell$ it (i) puts probability $(1-\alpha_j)\rho_j$ on: $\hat{x}_j$ if $\hat{x}_j \in [-d^\ell_{k'}, d^\ell_{k'}]$, $\overline{x}^\ell_{k'}(\hat{x}_g)$ if $\hat{x}_j > d^\ell_{k'}$, or $-\overline{x}^\ell_{k'}(\hat{x}_g)$ if $\hat{x}_j < -d^\ell_{k'}$; and (ii) puts probability $\alpha_j\rho_j$ on: $\hat{y}_j$ if $\hat{y}_j \in [-d^\ell_{k'}, d^\ell_{k'}]$, $\overline{x}^\ell_{k'}(\hat{x}_g)$ if $\hat{y}_j > d^\ell_{k'}$, or $-\overline{x}^\ell_{k'}(\hat{x}_g)$ if $\hat{y}_j < -d^\ell_{k'}$. By construction, $\overline{x}(\sigma') = \overline{x}^\ell_{k'}(\hat{x}_g)$. Furthermore, proposal strategies are optimal given $A(\sigma') = [-\overline{x}(\sigma'), \overline{x}(\sigma')]$.

I now check optimality for M. Because $\hat{x}_g \in [\mathring{x}^\ell_{k'}, \mathring{x}^\ell_{k'+1})$, we have $\overline{x}(\sigma') = \overline{x}^\ell_{k'}(\hat{x}_g) \in [d^\ell_{k'}, d^\ell_{k'+1})$. Thus, M optimally accepts all offers by $j \neq \ell$. Next, I verify $\hat{x}_g \in \text{int}A(\sigma')$. By Lemma D.6, $\hat{x}_g \in (-\overline{x}_\ell, \overline{x}_\ell)$ implies $\zeta(\hat{x}_g) > 0$, which is equivalent to

$$u_M(\hat{x}_g) > \frac{(1-\delta)u_M(q) + \delta\rho_\ell u_M(\hat{x}_g) + \delta\sum_{j\neq\ell}\rho_j I^{k'}_j}{1-\delta\sum_{j\neq\ell}\rho_j O^{k'}_j}.$$

Under σ', this is equivalent to $\hat{x}_g \in \text{int}A(\sigma')$.

Thus, σ' is equivalent to the equilibrium $\sigma(\hat{x}_g)$ and $\hat{x}_g \in \text{int}A(\hat{x}_g)$, as desired.

Part 2. Assume $\hat{x}_g \notin (-\overline{x}_\ell, \overline{x}_\ell)$ and suppose $\hat{x}_g \geq 0$ without loss of generality. I verify $A(\hat{x}_g) = [-\overline{x}_\ell, \overline{x}_\ell]$ in two steps. Step 1 shows $\overline{x}(\hat{x}_g) \geq \overline{x}_\ell$. Step 2 shows $\overline{x}(\hat{x}_g) \leq \overline{x}_\ell$.

Step 1. Suppose $\overline{x}(\hat{x}_g) < \overline{x}_\ell$. Let k' be the largest k such that $\mathring{x}^\ell_{k'} \leq \overline{x}(\hat{x}_g)$. Because $\hat{x}_g > \overline{x}_\ell > \overline{x}(\hat{x}_g)$, it follows that $\sigma(\hat{x}_g)$ puts probability ρ_ℓ on $\overline{x}(\hat{x}_g)$. Thus, $u_M(\overline{x}(\hat{x}_g)) = \frac{(1-\delta)u_M(q)+\delta\sum_{j\neq\ell}\rho_j I^{k'}_j}{1-\delta\rho_\ell-\delta\sum_{j\neq\ell}\rho_j O^{k'}_j}$ and rearranging yields $\zeta(\overline{x}(\hat{x}_g)) = 0$. Lemma D.6 implies $\overline{x}(\hat{x}_g) = \overline{x}_\ell$, a contradiction.

Step 2. Suppose $\overline{x}(\hat{x}_g) > \overline{x}_\ell$. If $\hat{x}_g \geq \overline{x}(\hat{x}_g)$, then the argument from Step 1 shows a contradiction. Assume $\hat{x}_g < \overline{x}(\hat{x}_g)$. Let k' be the largest k such that $\mathring{x}^\ell_{k'}(\hat{x}_g) \leq \overline{x}(\hat{x}_g)$. Then $\sigma(\hat{x}_g)$ puts probability ρ_ℓ on $\hat{x}_g$. Next, M optimally accepts $\hat{x}_g$ under $\sigma(\hat{x}_g)$ iff $u_M(\hat{x}_g) \geq \frac{(1-\delta)u_M(q)+\delta\rho_\ell u_M(\hat{x}_g)+\delta\sum_{j\neq\ell}\rho_j I^{k'}_j}{1-\delta\sum_{j\neq\ell}\rho_j O^{k'}_j}$. Rearranging, this condition is equivalent to $\zeta(\hat{x}_g) \geq 0$. By Lemma D.6, this requires $\hat{x}_g \leq \overline{x}_\ell$, a contradiction. □

E Extension with Vote Buying

To incorporate vote buying into the baseline model, I assume there are two interest groups, g_1 and g_2, that share the same ideal point, denoted $\hat{x}_g$. Group g_1 can lobby to buy votes — specifically, following a proposal in any period t, g_1 can offer to transfer $m_t \geq 0$ to M in exchange for controlling her vote that period. Group g_2 can potentially lobby to shape proposals as in the baseline model — by offering (y_t, m_t) to ℓ if the opportunity arises in any period t.

I study whether g_2 wants access to ℓ that creates the possibility of lobbying her proposal. Since they share the same ideology and I abstract from budgets, g_1 will lobby votes exactly as g_2 would. By separating their lobbying roles, however, g_2 does not need to consider effects on potential vote buying costs when evaluating access *ex ante*. Thus, this extension provides a closer analogue to the main analysis of endogenous access to proposers.

Throughout the analysis, we implicitly assume that $\hat{x}_g \in (0, \overline{x})$ and that $\hat{x}_\ell$ is sufficiently close to $\hat{x}_g$. Let $\Upsilon_{m,g}(x) = u_g(x) + u_M(x)$. The boundaries of A^* are the roots of

$$\Upsilon_{m,g}(x) = (1-\delta)\Upsilon_{m,g}(q) + \delta[\hat{V}_g^* + V_M^*]. \tag{90}$$

Since $\Upsilon_{m,g}(x) = 2x(\hat{x}_g - x) - \hat{x}_g^2$ and

$$\hat{V}_g^* + V_M^* = \rho_M \Upsilon_{m,g}(0) + \rho_\ell[\alpha \Upsilon_{m,g}(y^*) + (1-\alpha)\Upsilon_{m,g}(z^*)] + \rho_L \Upsilon_{m,g}(\underline{x}^*) + \rho_R \Upsilon_{m,g}(\overline{x}^*) - \alpha \rho_\ell m^*,$$

we can more explicitly re-express (90) as:

$$\begin{aligned} x(\hat{x}_g - x) = \frac{1}{1-\delta(\rho_L+\rho_R)} \Bigg(& (1-\delta)(\hat{x}_g - q)q \\ & + \delta\Bigg(\rho_\ell\Big[(1-\alpha)(\hat{x}_g - z^*)z^* + \alpha\big((\hat{x}_g - y^*)y^* - \frac{1}{2}m^*\big)\Big]\Bigg)\Bigg). \end{aligned} \tag{91}$$

Denoting the RHS of (91) by $\Omega(\hat{x}_\ell)$, which is always strictly negative, the two solutions are given by $\overline{x}^* = \frac{1}{2}\Big(\hat{x}_g + (\hat{x}_g^2 - 4\Omega(\hat{x}_\ell))^{\frac{1}{2}}\Big)$ and $\underline{x}^* = \frac{1}{2}\Big(\hat{x}_g - (\hat{x}_g^2 - 4\Omega(\hat{x}_\ell))^{\frac{1}{2}}\Big)$.

Next, as in Appendix A, we can use a mean-variance expression for g's equilibrium value:

$$\begin{aligned} \mathcal{U}_g(\alpha; \hat{x}_\ell) = -\hat{x}_g^2 + 2\hat{x}_g & \Big[\alpha\rho_\ell(\hat{y} - \hat{x}_\ell) + \rho_\ell \hat{x}_\ell + \rho_R \overline{x}^* + \rho_L \underline{x}^*\Big] \\ & - \Big[\alpha\rho_\ell(\hat{y}^2 - \hat{x}_\ell^2) + \rho_\ell \hat{x}_\ell^2 + \rho_R(\overline{x}^*)^2 + \rho_L(\underline{x}^*)^2\Big] - \alpha\rho_\ell\, m^*, \end{aligned} \tag{92}$$

where we substitute $y^* = \hat{y}$ and $z^* = \hat{x}_\ell$, which can be shown to follow from $\hat{x}_g \in (0, \overline{x})$ and $\hat{x}_\ell$ sufficiently close to $\hat{x}_g$.

Then, we have:

$$\frac{\partial \mathcal{U}_g(\alpha;\hat{x}_\ell)}{\partial \alpha} = 2\hat{x}_g\left[\rho_\ell(\hat{y}-\hat{x}_\ell)+(\rho_R-\rho_L)\frac{\partial \overline{x}^*}{\partial \alpha}\right] - \rho_\ell(\hat{y}^2-\hat{x}_\ell^2) - 2\frac{\partial \overline{x}^*}{\partial \alpha}[\rho_R\overline{x}^* - \rho_L\underline{x}^*] - \frac{\rho_\ell}{4}(\hat{x}_\ell-\hat{x}_g)^2 \tag{93}$$

$$\propto (\hat{x}_g-\hat{x}_\ell)^2 + \frac{\rho_R(\overline{x}^*-\hat{x}_g)+\rho_L(\hat{x}_g-\underline{x}^*)}{2(1-\delta(\rho_L+\rho_R))(\hat{x}_g^2-4\Omega(\hat{x}_\ell))^{\frac{1}{2}}}\cdot(\hat{x}_\ell-\hat{x}_g)(5\hat{x}_g-\hat{x}_\ell), \tag{94}$$

where (94) follows from substituting for $\hat{y}$ and $\frac{\partial \overline{x}^*}{\partial \alpha} = \rho_\ell \frac{(\hat{x}_g-\hat{x}_\ell)(5\hat{x}_g-\hat{x}_\ell)}{8(1-\delta(\rho_L+\rho_R))\sqrt{\hat{x}_g^2-4\Omega(\hat{x}_\ell)}}$, then simplifying.

There are two cases.

Case 1: Suppose $\hat{x}_\ell > \hat{x}_g$. Then, both terms in (94) are strictly positive, so $\mathcal{U}_g$ strictly increases in α. Thus, g strictly prefers positive access.

Case 2: Suppose $\hat{x}_\ell < \hat{x}_g$. Then,

$$\frac{\partial \mathcal{U}_g(\alpha;\hat{x}_\ell)}{\partial \alpha} \propto (\hat{x}_g-\hat{x}_\ell) - \frac{\rho_R(\overline{x}^*-\hat{x}_g)+\rho_L(\hat{x}_g-\underline{x}^*)}{2(1-\delta(\rho_L+\rho_R))(\hat{x}_g^2-4\Omega(\hat{x}_\ell))^{\frac{1}{2}}}\cdot(5\hat{x}_g-\hat{x}_\ell). \tag{95}$$

Then, we have $\lim_{\hat{x}_\ell \uparrow \hat{x}_g} \frac{\partial \mathcal{U}_g(\alpha;\hat{x}_\ell)}{\partial \alpha} = -\frac{\rho_R(\overline{x}^*-\hat{x}_g)+\rho_L(\hat{x}_g-\underline{x}^*)}{(1-\delta(\rho_L+\rho_R))(\hat{x}_g^2-4\Omega(\hat{x}_\ell))^{\frac{1}{2}}}\cdot 2\hat{x}_g < 0$. Thus, g strictly prefers zero access if $\hat{x}_\ell$ is more centrist and sufficiently close.