Policymaking and Appointments under Electoral and Judicial Constraints*

Peter Bils† Gleason Judd‡ Bradley C. Smith§

Abstract

Many democratic systems supplement periodic elections with checks and balances. Yet, elected executives typically have some influence on one important check, the judicial branch, through their power to nominate justices. How do electoral and judicial constraints influence which policies executives pursue and which justices they nominate? We study a game-theoretic model of electoral accountability in which an executive chooses policy and appoints a justice, who can overturn policy today and (potentially) after the election. We highlight how judicial appointments provide executives a tool for signaling *and* commitment, and also affect their incentives to signal with policy. We characterize how executives combine policy and appointments differently depending on judicial turnover, polarization, office motivation, or ideologies of sitting justices. We find that elections can moderate appointments but can also polarize them; reforms increasing justice turnover can backfire and reduce voter welfare; and distinct forms of polarization can have critically different effects.

*This version updates several formal statements in the published version. It corrects part (ii) of the appendix statement of Lemma 1, makes associated revisions to Definition 2 and the boundary cases in Propositions 1 and 2, and states a scope condition for part of Proposition 3 and for Proposition 4. The main qualitative insights are unchanged. Other edits address typos and clarity. We thank John Duggan, Jean Guillaume Forand, Ryan Hübert, Federica Izzo, Nolan McCarty, Davin Raiha, Tara Slough and audiences at Princeton, Vanderbilt, Stanford, APSA, MPSA, and SPSA for helpful suggestions.

†Depts. of Political Science and Economics, Vanderbilt University, (peter.h.bils@vanderbilt.edu).

‡Dept. of Politics, Princeton University, (gleason.judd@princeton.edu).

§Dept. of Political Science, Vanderbilt University, (bradley.carl.smith@gmail.com)

If men were angels, no government would be necessary. . . A dependence on the people is no doubt the primary control on the government; but experience has taught mankind the necessity of auxiliary precautions. — Madison (1788)

Many political executives have broad powers to shape policy. As such, it is important to understand how political institutions shape their incentives to use those powers according to voter interests. Scholars have emphasized the importance of executive constraints for centuries and advocated for combining elections with "auxiliary precautions." Heeding that advice, democratic statebuilders around the world have combined periodic elections with auxiliary precautions in the form of systems of checks and balances.

A particularly important check on executive power is judicial oversight. Yet, in most democracies, executives appoint judges and thus directly influence the judicial branch. Consequently, the executive may use their influence to appoint friendly justices, undermining the court's ability to constrain incumbent policymaking (Montesquieu, 1748). Alternatively, elections may discipline both executive policymaking and judicial appointments (Hamilton, 1788). Thus, electoral and judicial constraints may interact to shape both policymaking and appointments. Accordingly, in this paper, we ask: how does an executive's anticipation of electoral and judicial constraints influence policymaking and appointments?

We analyze a game-theoretic model of electoral accountability in which the executive makes policy and appoints a justice. First, the executive appoints a justice, anticipating how that justice will constrain policy today and into the future. Next, the executive sets policy, accounting for her own policy preferences, the constraints imposed by the court, and the impact on her electoral prospects. We show how the executive's choices are conditioned by key features of the political environment, such as the value of office, the degree and nature of polarization, and the durability of judicial appointments. In equilibrium, appointments and policy each shape, and are shaped by, the prospect of subsequent policymaking and elections. As such, our findings contribute to a broad literature on the political economy of executive action and highlight judicial appointments as an important form of "strategic pre-action" by executives (Cameron, 2008).

In the model, the incumbent and the challenger are known to ideologically lean in opposite directions, but the other players do not know the extent of each politician's bias, i.e., whether the incumbent is moderate or extremist. The executive considers the impact of her decisions through two channels. First, the incumbent worries that choosing an extreme policy or appointing an extreme justice may be interpreted negatively by the median voter. Second, by changing the composition of the judiciary, appointments directly influence the set of feasible policies that *any* executive officeholder may enact. Specifically, the incumbent faces a tradeoff between appointing a moderate justice, who will constrain her to choosing electorally appealing centrist policies, or appointing an extreme justice, who allows the incumbent to pursue her ideologically preferred policies today and would constrain the challenger if she is elected. Thus, the executive considers (i) how her decisions in office may act as a signal of her ideological bent, *and* (ii) how the appointed justice may act as a commitment device for constraining policies today and into the future.

We show that these considerations lead to three forms of executive behavior in equilibrium. First, in an *informative appointments* equilibrium, a moderate executive chooses a centrist policy and judge, and wins reelection. In contrast, an extremist loses reelection, but adopts her preferred policy and appoints an extreme justice, who then constrains the policies of the incumbent's successor. Second, in a *compromising* equilibrium, the extremist responds to electoral pressures by choosing the same centrist policy and judge as the moderate incumbent, in order to signal ideological congruence with the voter. Finally, in a *tying hands* equilibrium, an extremist incumbent uses the judicial appointment as a commitment device to constrain *her own* future policies in order to win reelection. Using this equilibrium characterization, we study how features of the political environment influence both the incumbent's appointment and policy choices.

We find that the durability of appointments plays a key role in shaping executive behavior and voter welfare. In making policy and choosing an appointee, executives look ahead, anticipating the likelihood of a future vacancy on the court. This shapes the incumbent's expectations about whether the judicial constraint imposed by the current court, and the incumbent's appointee, will persist into the next term. If a vacancy is very likely, then the incumbent cannot use the court as a means to commit to a moderate course of future policy. If a vacancy is very unlikely, then the judicial constraint becomes too strong and undermines electoral accountability in policy. Thus, we find that the voter-optimal vacancy rate is typically intermediate, balancing these two forces.

We also show how elite polarization plays an important role in shaping executive behavior and, in turn, voter-optimal judicial turnover. Moreover, we highlight how polarization's effects can depend critically on its form. In the model, polarization can increase in two ways. First, extremists can shift outward away from center, which we call *ideological divergence*. Second, the proportion of moderates may decrease, which we interpret as an increase in *party extremists*. In expectation, both forms of polarization shift the parties away from the voter and each other, thus increasing between-party polarization. A key difference is that an increase in ideological divergence also increases within-party polarization, which does not vary with an increase in party extremists. We highlight how this difference matters, as the two forms of polarization may have opposing effects on executive behavior in equilibrium. Furthermore, the optimal vacancy rate is a function of polarization. If an extremist is likely, increasing their probability raises the optimal vacancy rate, while greater ideological divergence lowers it.

Finally, we extend the model in three ways. First, we consider a multi-member court to study how the signaling and commitment forces we highlight are affected by the ideological composition of sitting justices. If the expected departing justice is far from the incumbent, then equilibrium policymaking and appointments resemble the baseline model. However, if the expected departing justice is close to the incumbent, then the incumbent is significantly less constrained by electoral pressures than in the baseline model. These differences arise because the location of a vacancy determines the possible locations of a court's median after a new appointment, connecting our work to existing studies that treat appointments as a "move-the-median" game (Krehbiel, 2007; Cameron and Kastellec, 2016). Second, in the Appendix, we incorporate uncertainty over judicial ideology. We find that executive behavior from the baseline model is preserved, as judicial appointments may still be used by the executive as a commitment device to win reelection, even if policy is overruled on the equilibrium path of play. Third, in the Appendix, we also extend the model to incorporate probabilistic review of the executive's policy. Behavior in the equilibria we construct is unchanged if review is sufficiently likely. With infrequent review, however, judicial constraints—and with them the commitment and signaling roles of appointments—can unravel.

Our analysis has several important empirical and policy implications. First, we find conditions under which electoral accountability can alleviate the *counter-majoritarian difficulty* by encouraging the appointment of moderate justices, bringing the court into line with the median voter. Second, we apply our results on the voter-optimal vacancy rate to discuss reforms to the Supreme Court that would alter the frequency of turnover, e.g., term limits for justices. Third, our model highlights the dynamic effect of judicial appointments, offering an explanation for empirical patterns that are difficult to rationalize with static theories.

Although we primarily consider policymaking and appointments in the context of the United States, our model applies more broadly. Our findings apply to contexts where (i) the executive can influence the court's composition, (ii) the court has power to review policy, and (iii) the executive (or the executive's party) is elected by citizens. For example, in France the president is directly elected and appoints 3 members of the constitutional court, which has powers of judicial review. More broadly, 41 countries shared these features in 2020, according to the Comparative Constitutions Project.[1] Although this is a rough approximation, and important institutional features differ across these settings, the underlying strategic forces of

[1] Specifically, in these 41 countries a single elected executive appoints members to the constitutional court. This is a lower bound on relevant countries, as some constitutions (e.g., the US) delegate similar powers to a supreme, rather than constitutional, court.

our model should still be present.

Related Literature

Our findings contribute to a broad literature on the relationship between oversight and political accountability (See, e.g., Patty and Turner, 2021; Turner, 2017, 2021). Judicial review is a prominent source of oversight, but it can encourage *posturing* and undermine electoral accountability by shielding politicians from the effects of ill-advised policies (Fox and Stephenson, 2011). Additionally, judicial constraints do not necessarily prevent political executives from accumulating power over time (Howell et al., forthcoming). Alternatively, by constraining the set of feasible policies, judicial review can strengthen office motivation and enhance electoral accountability (Almendares and Le Bihan, 2015). More broadly, however, this constraining aspect can complicate signaling incentives in spatial policymaking settings (Fox and Stephenson, 2015). Unlike existing work, we allow the elected politician to appoint justices. Thus, we contribute to this literature by studying the effect of judicial oversight on political accountability in an environment where the composition of the judiciary, and consequently the nature of judicial oversight, is subject to control by the politician.

We also contribute to the broader literature on the political economy of elections. A classic strategic consideration is how incumbent actions influence elections by shaping voter expectations about future outcomes.[2] They may provide *signals* about some underlying, but not directly observed, incumbent characteristic (e.g., Duggan, 2000; Maskin and Tirole, 2004; Acemoglu et al., 2013).[3] Alternatively, they may durably alter the policymaking environment and thereby *commit* future officeholders towards certain policies (Persson and Svensson, 1989; Alesina and Tabellini, 1990; Milesi-Ferretti, 1995; Wolitzky, 2013; Callander and Raiha, 2017). In our analysis, a key aspect is that judicial appointments can constrain *both* candidates, and in different ways — e.g., constraining the incumbent to enact policies the voter likes *and* leaving challengers unconstrained to enact policies the voter does not like.[4] A novelty here is tracing how strategic commitment interacts with signaling considerations.

Our paper also addresses three prominent aspects of judicial politics.

First, we contribute to the literature on judicial appointments. In particular, we highlight the role of electoral incentives for the appointing executive, rather than focusing on the role of Senate confirmation (as in, e.g., Rohde and Shepsle, 2007; Krehbiel, 2007; Moraski and

[2] See, e.g., Ashworth (2012) and Duggan and Martinelli (2020) for overviews.

[3] In this vein, our model is especially related to analyses in which politicians have multiple options to influence voter beliefs (Daley and Snowberg, 2011; Ash et al., 2017).

[4] Callander and Raiha (2017) uncover a similar mechanism in a different context, durable public investment, and show that incumbents may strategically make wasteful investments to make challengers less appealing.

Shipan, 1999; Cameron and Kastellec, 2016). Moreover, we account for how appointments and potential electoral turnover can affect each other, addressing the recognized need for incorporating dynamic considerations when studying judicial appointments (Cottrell et al., 2019).[5]

Second, our work relates to theories of executive influence on judicial oversight. Previous work has studied the ways that executives may expand deference by a fixed court (e.g., by *evading* (Vanberg, 2001) or *pressuring* (Clark, 2009)). We instead focus on how executive appointment powers can shape which policies receive judicial deference by shifting the composition of the court. Thus, appointments offer the executive another tool for influencing judicial deference.

Third, the emergence of moderate appointments in our model connects our work to a literature studying the conditions under which respect for judicial decisions can be maintained. In particular, the literature on insurance theory argues that judicial independence insulates risk-averse officeholders against future, turnover-driven policy changes. Consequently, parties may prefer a moderate court because it is easier to sustain judicial independence (Ramseyer, 1994; Stephenson, 2003). Alternatively, we highlight how a moderate court can facilitate re-election by relaxing the incumbent's signaling incentives and making the opposition less appealing. While both theories agree that elections are important for understanding judicial moderation, the underlying mechanisms differ. In our model, appointing a moderate justice *increases* the downside risk for the incumbent by freeing the action of a challenger to pursue extreme policy. Ultimately, by making the challenger relatively less appealing, a moderate appointment allows the incumbent to win re-election. In contrast, insurance theory suggests that parties prefer a moderate court because it *mitigates* the downside risk if the other party wins.

Finally, we contribute to the literature on the political economy of the presidency (Cameron, 2008) by adding to our theoretical understanding of presidential unilateral action. Previous work studies how executive unilateralism is affected by checks and balances (Moe and Howell, 1999; Howell, 2003) and legislative considerations (Bolton and Thrower, 2016; Noble, forthcoming), as well as electoral considerations (Judd, 2017; Howell and Wolton, 2018; Kang, 2020). We shed light on how electoral consequences of unilateral policymaking are conditioned by judicial constraints, in an environment where those constraints depend on judicial appointments by the president.

[5] Closest to our work, Jo et al. (2017) includes exogenous executive turnover that does not depend on the executive's behavior in office.

Model

We study a two-period model featuring an incumbent, I; a challenger, C; a voter, V; and a continuum of potential justices.[6]

In the first period, an incumbent holds office and makes two decisions: (i) she chooses the ideal point of the first-period justice, denoted J_1, and (ii) she proposes a policy x_1 in the policy space $\mathbb{R}$. Next, J_1 chooses whether to strike down the policy. If x_1 is struck down, then J_1 incurs a cost $\phi > 0$ but can issue a ruling that moves the first-period outcome to any policy in $\mathbb{R}$. Though stylized, this captures the notion that justices' decisions are made with the aim of bringing outcomes in line with their policy preferences. Then, V observes x_1, J_1, and the ruling before choosing whether to reelect I or elect the challenger, C.

In the second period, the winner takes office. With probability $\nu \in (0, 1)$, a judicial vacancy opens and the officeholder can choose J_2, the ideal point of the second-period justice. Otherwise, she cannot appoint a new justice and J_1 persists, i.e., $J_2 = J_1$. Once the court is in place, the officeholder proposes second-period policy, x_2. After observing that proposal, J_2 chooses whether to overturn it. Subsequently, payoffs accrue and the game ends.

Each player has an ideal point in the policy space. To simplify notation, we use player i's identity as shorthand for their ideal point. The voter's ideal point is common knowledge, as is the ideal point of the sitting justice in each period. In contrast, politician ideal points are private information. Informally, V knows that I leans right and C leans left, but does not know exactly how far. Formally, we normalize $V = 0$ without loss of generality, so that V knows $I \in \{m, e\}$, where $0 < m < e$, and V's commonly known prior belief places probability $p \in (0, 1)$ on $I = e$ and $1 - p$ on $I = m$. Similarly, V and I share the same commonly known prior belief about C, which puts probability p on $C = -e$ and $1 - p$ on $C = -m$. Qualitatively similar results hold if the support of the type space is not extremely asymmetric, and so we opt for the simpler specification.

In the model, the voter and justices never observe I's type directly but can draw inferences from observed behavior. Thus, the election features pure adverse selection.

In each period, player i's policy utility from x is $u_i(x) = -|x - i|$. To capture exogenous reelection motivations, politicians receive additive benefit $\beta \geq 0$ in each period they hold office. In period t, if policy x_t is proposed then J_t obtains utility $u_{J_t}(x_t)$ for upholding the policy and utility $-\phi$ for striking it down. Finally, to ease presentation, we assume the cost of overturning policy is moderate, i.e., $\phi \in (m, e)$.

For each player, dynamic payoffs are the sum of utility across both periods. To illustrate,

[6]Assuming one voter is without loss of generality, since the median voter is decisive over lotteries in our setting, see Duggan (2014).

if I wins reelection and the policy outcomes are x_1 and x_2, then I's payoff is

$$u_I(x_1) + \beta + u_I(x_2) + \beta.$$

Comments on the Model: We model policymaking and appointments in a spatial setting, following existing studies in which ideological conflict affects the judicial appointments (Moraski and Shipan, 1999; Rohde and Shepsle, 2007; Krehbiel, 2007). Ideology is central to policymaking and elections, and also an important factor in judicial decisionmaking (Martin and Quinn, 2002; Clark et al., forthcoming) and public support for judicial nominees (Gimpel and Wolpert, 1996; Caldeira and Smith Jr, 1996; Sen, 2017).

In the main analysis, we assume that politicians and voters know the judge's ideal point. This assumption allows us to (i) clearly highlight several strategic tradeoffs that executives face when choosing policy and appointing judges and (ii) cleanly compare against existing models of electoral accountability, which emphasize uncertainty about incumbent politicians. Empirically, presidents attempt to minimize their uncertainty about the appointee (Nemacheck, 2008) and voters are knowledgeable about the court (Gibson and Caldeira, 2009a,b).[7] In the appendix, we introduce uncertainty over how the appointed justice will rule and show that the executive faces similar tradeoffs as in the baseline model.

Additionally, the court has only one justice in our main analysis. In an extension, we incorporate additional sitting justices. There, we study how the court's composition can affect the executive's policy choice and appointee, as well as how those decisions may be influenced by expectations about which justice will leave next.

Next, we allow the court to enact any policy if it overturns the executive's chosen policy. In practice, justices often have significant leeway to influence public policy when interpreting laws and writing opinions. They can specify nuanced details (Murphy, 1964) by writing opinions in a way that "...command(s) government agencies to undertake certain policies, sometimes in minute detail." (Canon, 1982, p. 245). Our setup reflects these features and follows existing models of statutory interpretation (see Cameron and Kornhauser (2017) for an overview). The key forces here carry over if the judge cannot craft opinions so finely and must instead choose from a restricted set of policies after overturning the executive.

We also assume that the court always has an opportunity to rule on the executive's policy. While this is true in some contexts — in France, some types of policies are subject to compulsory review — in other contexts review is not guaranteed to occur immediately, if at all. In the appendix, we extend the model to allow probabilistic review and show that the

[7] Voters also appear to be able to recall Senate confirmation votes on Supreme Court nominees (Bass et al., forthcoming), and appointments have become an increasingly contentious and salient issue for voters (Gimpel and Wolpert, 1996, p. 164).

equilibria we construct are robust provided review is sufficiently likely.

Finally, consistent with previous work such as Cameron et al. (2000), we assume the court pays a cost for reviewing the executive's policy. We refer to ϕ as a cost, but it can be interpreted more generally. Crucially, it creates and scales a zone of judicial deference in which policies will not be struck down, as in e.g., Stephenson (2003). Substantively, such deference could reflect various court considerations that discourage justices from hearing and striking down any policy — e.g., opportunity costs of not hearing other cases unrelated to the executive's policies, as well as concerns about future court curbing or public backlash against court activity. Alternatively, it could reflect the court's ability to perceive sufficiently small deviations from its ideal policy, due to, e.g., legal skill or (unmodeled) policy obfuscation.

Strategies and Equilibrium Concept: Informally, a pure strategy profile σ specifies: a policy and appointee in each period for each type of I; a second-period policy and appointee for each type of C; whether V reelects I after seeing the first-period appointee and policy; and the policies each judge would overturn in each period.[8] Additionally, V's belief system is represented by $\mu : \mathbb{R}^2 \to [0, 1]$, where $\mu^{x_1}_{J_1}$ denotes the probability that V places on $I = e$ after observing x_1 and J_1.[9]

We study perfect Bayesian equilibria (PBE), i.e., assessments (σ, μ) such that (i) the strategy profile σ is sequentially rational given μ, and (ii) the belief system μ is derived from σ via Bayes's Rule whenever possible. As is standard in PBE, there is "no signaling what you don't know" (Osborne and Rubinstein, 1994) — since J_1 does not have private information, her ruling does not influence the beliefs of the other players. Throughout, *equilibrium* refers to pure-strategy PBE satisfying *equilibrium dominance*, which we maintain to refine away equilibria supported by unnatural off-path beliefs (Cho and Kreps, 1987).[10]

Analysis

We begin by characterizing the relationship between justice ideology and overturning policy, which is the same in both periods. Then, we characterize second-period executive behavior — who they appoint and which policy they choose. Third, we characterize how electoral outcomes depend on first-period behavior. Fourth, we characterize first-period executive behavior. Building on that foundation, we then characterize which types of incumbents win

[8]We formally define pure strategies in the Appendix.

[9]Defining belief systems for justices is unnecessary.

[10]Formally, say (x_1, J_1) is *equilibrium-dominated* for type θ if every response by V at (x_1, J_1) yields θ strictly less than her equilibrium payoff. At any off-path (x_1, J_1) that is equilibrium-dominated for exactly one type, $\mu^{x_1}_{J_1}$ places probability zero on that type. Beliefs are otherwise unrestricted. This is the equilibrium-dominance requirement underlying the Intuitive Criterion.

reelection, the consequences of polarization, and implications for institutional design.

Judicial Rulings

The strategic calculus for equilibrium rulings is straightforward and analogous across periods.

For the second-period justice, overturning the executive's second-period policy x_2 and instead placing policy at J_2 yields a payoff of $-\phi$, whereas upholding yields $-|x_2 - J_2|$. Thus, the second-period executive's policy is upheld if and only if $|x_2 - J_2| \leq \phi$, so the set of feasible second-period policies consists of all $x_2 \in [J_2 - \phi, J_2 + \phi]$. We refer to this interval as J_2's *acceptance set*.

The first-period justice faces an analogous strategic calculus, since J_1 does not observe I's type and there is "no signaling what you don't know," so J_1's acceptance set is $[J_1 - \phi, J_1 + \phi]$.

Second-period Appointment & Policymaking

Next, we describe a second-period officeholder's equilibrium behavior. To do so, we first define the *constrained optimal policy* for politician-type $\theta \in \{-e, -m, m, e\}$ given judge J:

$$x^*(\theta; J) = \underset{x \in [J-\phi, J+\phi]}{\arg\max} \quad u_\theta(x). \tag{1}$$

We will refer to (1) throughout the analysis, as it also helps us characterize first-period policymaking and judicial appointments.

In the second period, the executive's only policymaking constraint is J_2's acceptance set, so politician-type $\theta \in \{-e, -m, m, e\}$ chooses second-period policy equal to $x^*(\theta; J_2)$. With this in hand, we can characterize the executive's choice of J_2 in the event of a vacancy. Given the absence of electoral considerations, the executive simply appoints a friendly judge so that she can enact her own ideal point. Specifically, politician-type $\theta \in \{-e, -m, m, e\}$ solves $\max_{J_2} \; -|x^*(\theta; J_2) - \theta|$. Thus, any $J_2 \in [\theta - \phi, \theta + \phi]$ is optimal.

The Election

We now study the voter's decision between reelecting I or electing C. Two key factors are (i) how J_1 constrains the candidates differently and (ii) the voter's beliefs about each candidate's extremism after observing (J_1, x_1).

Specifically, the preceding characterization of second-period appointments and policymaking implies that V's continuation value from electing C is

$$\begin{aligned} U_V^C(J_1) = \nu\Big(&\, p\, u_V(-e) + (1-\ p)\, u_V(-m)\Big) \\ &+ (1-\nu)\Big(\, p\, u_V(x^*(-e;J_1)) + (1-\ p)\, u_V(x^*(-m;J_1))\Big), \end{aligned} \tag{2}$$

which depends on J_1 due to constraints on second-period policy if no vacancy opens. Similarly, V's continuation value from re-electing I is

$$\begin{aligned} U_V^I(J_1) = \nu\Big(&\mu_{J_1}^{x_1}\, u_V(e) + (1-\mu_{J_1}^{x_1})\, u_V(m)\Big) \\ &+ (1-\nu)\Big(\mu_{J_1}^{x_1}\, u_V(x^*(e;J_1)) + (1-\mu_{J_1}^{x_1})\, u_V(x^*(m;J_1))\Big), \end{aligned} \tag{3}$$

which depends on J_1 through (i) the constraints on second-period policy in the event of no vacancy and (ii) V's updated beliefs about I's type after observing (J_1, x_1). Thus, V is willing to reelect I in equilibrium if $U_V^I \geq U_V^C$. Otherwise, V strictly prefers to elect C.

Both (2) and (3) reveal that V's continuation value from each candidate depends directly on the first-period appointment, J_1, because a judicial vacancy may not open in the second period. For the rest of this section, we focus on how this constraining effect can render the appointee's signaling effect irrelevant for V's election choice. Specifically, some justices can constrain I favorably enough for V that her vote does not depend on her belief about I.

Definition 1. The election is *safe* if the voter weakly prefers one of the candidates regardless of her beliefs. Otherwise, the election is *competitive*.[11]

In a competitive election, V's choice depends upon her belief about I's type — she prefers to reelect I if, and only if, her belief puts sufficiently high probability on I being moderate. For our analysis, the key observation about competitive elections is that, under V's prior beliefs, I wins reelection only if $J_1 \leq 0$. Below we discuss this observation further, as this will have important consequences for first-period behavior.

For safe elections, we can focus on those that are safe for I. A necessary and sufficient condition for the election to be safe for I is that V prefers to reelect a known extremist over the unknown challenger. Formally,

$$\nu\ u_V(e) + (1-\nu)\, u_V(x^*(e;J_1)) \geq U_V^C(J_1), \tag{4}$$

where, given J_1, the left-hand side gives V's worst-case utility from reelecting I and the right-hand side gives V's expected utility from electing C.

[11] An indifferent voter breaks ties in favor of the safe candidate.

Why can V prefer to reelect an incumbent she knows to be extremist? Given the justice's ideological motivations, any justice who constrains I more will also constrain C less. And if a vacancy opens in the second period, any officeholder will appoint a friendly justice and then enact her own ideal point, as shown earlier. If J_1 is a strong enough constraint on the incumbent, then V may prefer to reelect a known extremist who might be constrained in the second period rather than elect a challenger who may be extreme or moderate, but will certainly be less constrained.

The strength of the judicial constraint affecting whether the election is safe or competitive depends on (i) the first-period justice's ideology, i.e., the location of J_1, and (ii) the vacancy rate, ν. Let $\mathcal{J}^I$ denote the set of J_1 such that the election is safe for I, i.e., such that (4) holds given ν. Additionally, let $\overline{J}^I = \max \mathcal{J}^I$. Lemma 1 provides several useful observations about how the vacancy rate affects the scope for safe elections.[12]

Lemma 1. *There exists a unique vacancy probability* $\overline{\nu}^I \in (0,1)$ *such that: (i)* $\mathcal{J}^I$ *is nonempty if and only if* $\nu \leq \overline{\nu}^I$, *and (ii)* $\overline{J}^I$ *increases towards* 0 *as* ν *decreases over* $(0, \overline{\nu}^I]$.[13]

Lemma 1 highlights how the vacancy rate, ν, plays a key role in the constraining effect of J_1. Higher values of ν reduce the expected impact of J_1 on second-period policymaking because a vacancy is more likely. Consequently, V's decision becomes more sensitive to her beliefs about I's type and the appointment's constraining effect has less bearing on V's election decision. In the other direction, a lower vacancy rate (low ν) facilitates safe elections. It does so by increasing the salience of the constraining effect, since the second-period officeholder probably will not appoint a new justice. Thus, V anticipates that J_1 is likely to persist and also constrain second-period policymaking.

So far, we have highlighted that the vacancy rate, ν, determines *whether safe elections are possible*. Next, we discuss *which* justices make the election safe for I. Broadly, Lemma 1 implies that only some challenger-friendly justices can make the election safe for I. If vacancy is unlikely and J_1 leans sufficiently leftward, then it is likely that the right-leaning I would be constrained to moderate policy if elected, whereas the left-leaning C would be relatively unconstrained. In this case, the election is safe for I because V prefers a constrained extremist incumbent to a potentially moderate, but less-constrained, challenger.[14]

We conclude this section by discussing Lemma 1(*ii*), which characterizes how the vacancy probability affects the scope for competitive elections in the first period. Making the vacancy less likely (decreasing ν) increases the persistence of J_1, so V is more willing to re-elect a

[12] In the appendix, Lemma 1 fully characterizes when elections are safe versus competitive.

[13] As $\nu \downarrow 0$, $\overline{J}^I$ converges to a strictly negative limit, computed in the proof of Lemma A.4. At $\nu = 0$ exactly, (4) holds with equality for all sufficiently extreme J_1, so $\mathcal{J}^I$ is unbounded and $\overline{J}^I$ is not well defined.

[14] By a parallel logic, elections can be safe for C.

known extremist. Thus, J_1 does not have to constrain I as much to ensure a safe election, so $\overline{J}^I$ increases towards 0 as ν decreases.

First-period Appointments & Policymaking

With voter behavior characterized, we now analyze first-period equilibrium behavior in appointments and policymaking. Our model highlights a *dual role of appointments*: signaling and constraining. On one hand, appointments affect the voter's evaluation of the incumbent by providing a signal of the incumbent's ideology. On the other, appointments can directly influence the voter's expectations about future policy by constraining the set of policies that can survive review. As highlighted by the possibility of safe elections, the constraining effect can overwhelm the signaling effect. But if I's appointee does not induce a safe election, then the information provided by I's behavior influences V's vote.

Next, we formally define three mutually exclusive forms of first-period behavior.

Definition 2. A strategy profile has: (i) **compromising** if both types of I appoint identical $J_1 \notin \mathcal{J}^I$, choose the same first-period policy, and win reelection; or (ii) **informative appointments** if each type of I appoints a distinct J_1 and only the moderate wins reelection; or (iii) **tying hands** if both types of I win reelection and type e appoints a $J_1 \in \mathcal{J}^I$.

Our next result demonstrates that I's equilibrium strategy must feature one of these three forms of behavior. Furthermore, we fully characterize the conditions under which each can arise in equilibrium. Under broad conditions, equilibria take one form, and at most two forms coexist as equilibria.[15]

Proposition 1. *Every equilibrium features either compromising, informative appointments, or tying hands. Furthermore, there exists an equilibrium featuring: (i) compromising if and only if $\nu \geq \underline{\nu}$ and $\beta \geq \overline{\beta}^c_\nu$; (ii) informative appointments if and only if either $\nu > \overline{\nu}^I$ or $\beta \leq \overline{\beta}^{th}_\nu$; and (iii) tying hands if and only if $\nu \leq \overline{\nu}^I$ and $\beta \geq \overline{\beta}^{th}_\nu$.*

For each form of the Incumbent's behavior, we can immediately derive several useful observations about equilibrium behavior on the path of play.

Compromising. In these equilibria, V does not update and also must reelect I, as otherwise e would behave distinctly from m. Thus, $J_1 \leq 0$ and $x_1 \leq m$. Because e constrains herself to imitate m, her electoral gain from reelection must compensate. If $\beta = \overline{\beta}^c_\nu$, then e is indifferent between winning reelection after choosing $(J_1, x_1) = (0, m)$ versus her optimal

[15] The only exception, at which all three forms can coexist, is the knife-edge case $\beta = \overline{\beta}^{th}_\nu$. The cutoffs $\underline{\nu}$, $\overline{\beta}^c_\nu$, and $\overline{\beta}^{th}_\nu$ are defined in the Appendix ((A.14), (A.13), and Part (i) of the proof of Proposition 1), with the convention $\overline{\beta}^{th}_\nu = +\infty$ if no safe appointment exists ($\nu > \overline{\nu}^I$).

losing behavior $(J_1, x_1) = (e + \phi, e)$. Increasing β further makes e even more willing to compromise for electoral gain. The compromising equilibrium with $(J_1, x_1) = (0, m)$ still exists but there will also be additional compromising equilibria with $J_1 < 0$ and $x_1 < m$. Note, however, that $(J_1, x_1) = (0, m)$ maximizes the ex ante payoff of both incumbent types among compromising equilibria.

Informative Appointments. In these equilibria, the election is competitive and V learns I's type after seeing (J_1, x_1). Thus, m wins but e loses. Anticipating losing, e chooses $(J_1, x_1) = (e + \phi, e)$, so that she can enact $x_1 = e$ and also constrain any second-period officeholder to enact $x_2 = e$ if no vacancy opens. Pinning down e's behavior helps us characterize m's behavior, as she chooses an appointee and policy that deters e from (i) imitating to win reelection rather than (ii) choosing $(J_1, x_1) = (e + \phi, e)$ and losing. If deterrence requires it, then m's appointee and policy must skew leftward enough to deter imitation by e.[16]

If elections are always competitive ($\nu > \overline{\nu}^I$), then an informative appointments equilibrium always exists. With low vacancy probability ($\nu \leq \overline{\nu}^I$) there are appointees who make the election safe for I. Crucially, this option bounds how far I will skew her appointee in equilibrium. Since reelection is guaranteed at $J_1 = \overline{J}^I$, neither incumbent type will choose a more left-leaning appointee. For low office benefit, i.e., $\beta \leq \overline{\beta}^{th}_{\nu}$, an informative appointments equilibrium exists because m can win reelection without skewing J_1 to $\overline{J}^I$. As β increases, e's incentive to win reelection also increases, so m's choice of J_1 or x_1 must move left in an informative appointments equilibrium to decrease e's policy payoff from imitating m.

Tying Hands. In these equilibria, I appoints her friendliest "safe" judge ($J_1 = \overline{J}^I$) and chooses her constrained optimal first-period policy.[17] Thus, I's behavior may reveal information to V but that information does not influence the election because the appointment's constraining effect makes it irrelevant. Of course, the possibility of a safe election is necessary for tying hands behavior in equilibrium, i.e., the vacancy rate must be low ($\nu \leq \overline{\nu}^I$). Additionally, e must be willing to select $J_1 = \overline{J}^I$ for electoral reasons, i.e., there must be high office benefit ($\beta \geq \overline{\beta}^{th}_{\nu}$).

As noted above, m has no reason to skew J_1 past $\overline{J}^I$: the voter's expectation about J_1's constraint on future policy is favorable enough to ensure reelection, so there is no signaling incentive to skew J_1 farther left. Moreover, because the election is safe, I's policy choice has

[16]There can be multiple equilibrium pairs of appointee and policy for m, as she can pair increasingly skewed policies with less skewed appointees.

[17]Every tying hands equilibrium pins down the extremist's appointment at $\overline{J}^I$. The moderate's is not unique, as she can also win reelection with appointments that reveal her type. Statements about appointment behavior in these equilibria refer to the equilibrium constructed in the proof of Proposition 1, in which both types appoint $\overline{J}^I$.

no electoral consequences. Each incumbent type therefore simply chooses its constrained optimal policy. Thus, it is possible that the types choose different first-period policies, thereby giving V full information. Yet, separation on policy does not jeopardize reelection because J_1's commitment effect decides the election.

Compromising vs. Tying Hands. The potential coexistence of compromising equilibria and tying hands equilibria, as established by Proposition 1, hinges on whether e would rather win reelection by choosing $(J_1, x_1) = (0, m)$ or instead do so with $(J_1, x_1) = (\overline{J}^I, x^*(e; \overline{J}^I))$. The former provides the highest possible payoff among equilibria featuring compromising, so tying hands *must* happen in equilibrium if e prefers the latter.

If $x^*(e; \overline{J}^I) < m$, then compromising equilibria exist whenever office motivation is high enough. Yet, $x^*(e; \overline{J}^I)$ can shift rightward past m as ν decreases because I does not have to skew J_1 as far leftward to ensure safe reelection.[18] Thus, e faces a tradeoff when comparing compromising against tying hands, and this tradeoff favors tying hands if ν is low enough.

Essentially, tying hands provides e with flexibility today, while compromising provides flexibility tomorrow. In the incumbent-optimal compromising equilibrium, e enjoys a weaker constraint on future policymaking but must exercise self-restraint today to enjoy it. By instead tying hands, e faces a tighter constraint tomorrow but can get away with more extreme policy today.

Electoral & Policy Outcomes

Before analyzing polarization and voter welfare, we briefly discuss electoral outcomes and the ideological lean of appointments and policy. Proposition 2 characterizes how I's equilibrium reelection prospects depend on office benefit, β, and judicial turnover, ν.

Proposition 2. *In equilibrium, moderate incumbents win reelection. Extremist incumbents: lose if $\beta < \min\{\overline{\beta}^c_\nu, \overline{\beta}^{th}_\nu\}$, win if $\beta > \overline{\beta}^{th}_\nu$ and $\nu \leq \overline{\nu}^I$, and otherwise can win or lose.*

In equilibrium, I loses only if she is an extremist in an equilibrium featuring informative appointments.[19] Therefore we can understand Proposition 2 simply by considering when such equilibria exist. Conditions that guarantee only informative appointments exist also guarantee that e always loses. In contrast, e always wins if conditions guarantee that such equilibria *do not* exist. Finally, if informative appointments are possible but not guaranteed, then e can win or lose, depending on the particular equilibrium being played.

[18] We formally show this in the appendix in Lemma 1(iv).

[19] Recall the convention $\overline{\beta}^{th}_\nu = +\infty$ for $\nu > \overline{\nu}^I$. At the boundary $\beta = \overline{\beta}^{th}_\nu$, the extremist can win or lose across equilibria.

The conditions on β and ν outlined in Proposition 2 also determine equilibrium appointment and policy outcomes in the first period.[20]

First, if $\beta < \min\{\overline{\beta}^{c}_{\nu}, \overline{\beta}^{th}_{\nu}\}$, then the extremist is not willing to win reelection by incurring the policy cost of appointing a judge who appeals to V. Instead, e appoints $J_1 = e + \phi$ and enacts $x_1 = e$ before losing reelection, so equilibria must feature informative appointments. Thus, m need not skew J_1 all the way to 0 in order to deter election-seeking imitation by e. Consequently, I's first-period appointee and policy are always right-leaning in this case.

Second, if $\beta > \overline{\beta}^{th}_{\nu}$ and $\nu \leq \overline{\nu}^{I}$, equilibria only feature compromising or tying hands. This is because e is especially keen on reelection and low ν guarantees existence of safe elections, limiting how far m will skew J_1 leftward. Thus, in the equilibria we construct, the appointee is always center-left. Specifically, $J_1 \in [-\phi, 0]$, which in turn implies that first-period policy is always center-right because I's constrained optimal policy will be $x_1 \in [0, \phi]$.

Finally, if neither of the above conditions holds, then equilibria featuring informative appointments exist, potentially alongside equilibria featuring compromising. Thus, the appointee can always lean either direction, as e will appoint $J_1 \leq 0$ or $J_1 = e + \phi$ in equilibrium.

Consequences of Polarization

With equilibrium behavior characterized, we turn to consider how changes in polarization influence patterns of appointments. Does polarization lead incumbents to focus on the constraining influence of the judiciary and appoint friendly justices that enable them to implement extreme policy? Or does polarization lead incumbents to appoint moderate justices to win reelection, thereby avoiding the consequences of losing office to an extreme challenger? We show that the answer depends on the particular source of polarization.

Within our framework, there are two natural ways in which polarization can change.

Definition 3. We say there is: (i) an **increase in party extremists** if p increases, and (ii) an **increase in ideological divergence** if e increases.

Recall, that p is the prior probability a politician is an extremist, while e is the ideal point of an extremist. Increasing ideological divergence or party extremists increases polarization by shifting the expected ideal point of I further to the right and the expected ideal point of C left. Thus, in expectation, the parties move further away from V and from each other. However,

[20] For our discussion of these outcomes, we focus on incumbent-optimal equilibria. If m is indifferent over a range of such equilibria, then we select the one that sets J_1 farthest to the right. We do so because this appointee would provide the strongest constraint on C and therefore m would strictly prefer this appointee if there were an arbitrarily small exogenous probability that C holds office in the second period.

increasing ideological divergence also increases within party polarization by increasing the difference between m and e.

Proposition 3 characterizes how polarization affects first-period behavior within each form of equilibrium behavior. Moreover, it shows that these distinct sources of polarization can have opposing effects on appointments and policymaking.

Proposition 3. *In a tying hands equilibrium, increasing party extremists increases the extremist's appointment J_1, while increasing ideological divergence decreases it if divergence is sufficiently large. In an informative appointments equilibrium, increasing party extremists shifts the moderate's appointee weakly further from the extremist, as does increasing ideological divergence if $\nu \geq \frac{1}{1+p}$. In a compromising equilibrium, first-period appointments do not respond to either form of polarization.*

To clarify these differences, we discuss the effect of polarization in each type of equilibrium.[21] Combined with the characterization in Proposition 1, this also describes how polarization affects first-period behavior in any given region of the parameter space.

First, polarization does not affect incumbent behavior in equilibria featuring compromising. Since I's first-period behavior does not vary by type, such equilibria are sustained by V's off-path beliefs, as is typical in pooling equilibria. Thus, first-period behavior is not sensitive to relatively small changes in either form of polarization.

In contrast, polarization does affect first-period behavior in equilibria featuring informative appointments or tying hands. Furthermore, these effects can vary depending on the type of the incumbent, the source of polarization, the value of office, and the vacancy rate. What accounts for this variation? The distinguishing factor is whether polarization affects equilibrium through V's incentives or I's incentives. Below we discuss this distinction in greater detail, considering informative appointments and tying hands equilibria in turn.

In equilibria featuring informative appointments, polarization crucially increases I's value for winning reelection and preventing the challenger from holding office. More precisely, if the deterrence constraint binds, m's first-period behavior is pinned down by e's desire to mimic, which grows with e's value of office. Polarization affects this value endogenously by altering e's continuation value from being replaced by C: as polarization increases, regardless of how it is measured, losing reelection gets worse for e and mimicking m gets more attractive. In turn, m must skew her appointee and/or policy relatively further leftwards to deter imitation. Ideological divergence, however, also raises e's policy cost of imitating m. If vacancies are unlikely, $\nu < \frac{1}{1+p}$, this countervailing force dominates the increased value of reelection,

[21] Appointment statements for informative appointments and compromising equilibria refer to the equilibria constructed in the proof of Proposition 1. For informative appointments, if $\nu < \frac{1}{1+p}$ and office benefit is intermediate, the moderate's appointee shifts strictly closer to the extremist's as divergence increases.

making deterrence easier. If polarization increases through party extremists, then appointees lean weakly farther leftward. But if polarization increases through ideological divergence, then the direct effect of increasing e pushes the extremist's appointee rightward, while the moderate's appointee may shift right or left. Yet, if vacancies are sufficiently likely, $\nu \geq \frac{1}{1+p}$, then the appointee's ideal point shifts (weakly) further away from e. If they are not, then it can shift closer.

In equilibria featuring tying hands, polarization crucially alters V's comparison between I and C. More precisely, I's first-period behavior is pinned down by V's comparison between the *expected challenger* versus the *worst-case incumbent*: it determines $\overline{J}^I$, the rightmost justice who makes V indifferent between these two options. Since in the constructed equilibrium both types' appointment is $\overline{J}^I$, the consequences of polarization flow through the effect on this comparison for V.

In this case, the form of polarization becomes more important. Whereas both forms had a similar effect on incentives under informative appointments, they can have opposing effects under tying hands. First, increasing p makes the expected challenger worse but does not affect the worst-case incumbent. This change makes V more inclined towards safe election, so $\overline{J}^I$ increases towards 0 and therefore J_1 and x_1 shift rightward. Second, greater extremist divergence worsens both the expected challenger *and* the worst-case incumbent. The effect on C is weaker, however, because it is diluted by the probability of being moderate. Consequently, V becomes more concerned about the worst-case incumbent and is less inclined towards safe election. This effect exerts a leftward force on $\overline{J}^I$, and it dominates if divergence is large. Otherwise, the direct impact of shifting e to the right can result in a J_1 that is further right.

Institutional Design

Thus far, we have treated the vacancy rate, ν, as an exogenous feature of the environment. However, there are institutional reforms that could alter ν. For example, implementing a mandatory retirement age for judges would generate more turnover on the court and increase the probability the executive is able to make an appointment.

Our next result considers (i) the effect of ν on voter welfare and (ii) how the welfare-maximizing ν changes with polarization. In order to determine the highest obtainable level of voter welfare, if multiple equilibria exist we select the one that maximizes voter welfare.

Proposition 4. *If β is sufficiently high and $p \leq 1/3$, then: (i) the optimal probability of judicial vacancy is given by $\nu^* \in (0, \overline{\nu}^I]$, (ii) increasing party extremists increases ν^*, and (iii) greater ideological divergence decreases ν^*.*

Proposition 4 highlights the important role that the durability of judicial appointments plays in determining voter welfare.[22] An important factor for the result is that voter welfare is maximized in a tying hands equilibrium. To see this, recall that increasing ν shifts $\overline{J}^I$ leftward, which pulls I's first-period policy toward V. Thus, the optimal level of turnover, ν^*, must balance better first-period outcomes against increasing the probability of a worse second-period outcome. This tradeoff yields that ν^* is also bounded away from 0. If vacancies are too unlikely, then the justice appointed in equilibrium does not sufficiently constrain I's policy choice. Additionally, ν^* is bounded away from 1. As such, durable appointments pave the way for moderation, enhancing voter welfare in the process.

The voter condition that determines the safe incumbent region plays an important role in the optimal vacancy rate. Consequently, both measures of polarization impact ν^*. As discussed earlier, greater ideological divergence makes the voter relatively more concerned about an extremist incumbent than an unknown challenger, while an increase in party extremists makes V relatively more wary of C than she is of a known extremist incumbent. Correspondingly, if e increases then ν^* decreases, while if p increases then ν^* increases.

The relationship between durability of appointments is relevant to concerns over the counter-majoritarian nature of the court, as well as recent calls for judicial reform. We discuss each of these issues further in the Discussion.

Extension: Appointments to Move the Median

Thus far, we have assumed that the appointed justice is decisive in determining the court's review of executive policy. In practice, however, the executive's appointee is just one member of an already existing court. Consequently, the impact of any appointee will typically depend on the ideologies of the sitting justices. In this section, we account for these constraints by assuming the court is composed of multiple judges and that the median justice determines whether the executive's policy is upheld. As noted earlier, such "move-the-median" games are commonly used to model Supreme Court appointments.

Specifically, suppose the court has five justices and a vacancy opens at the beginning of the game, leaving four sitting justices. We assume there are two left-leaning sitting justices, with ideal points L_1 and L_2, and two right-leaning sitting justices, with ideal points R_1 and R_2. Furthermore, we assume their ideal points are ordered as $-e < L_2 < L_1 < -m < 0 < m < R_1 < R_2 < e$, i.e., the justices are more extreme than the moderate politician types, but less extreme than the extremist types.

As before, in the first period I appoints a judge with ideal point $J_1 \in \mathbb{R}$ and chooses a

[22] If an extremist is likely and judicial deference is limited, voter welfare can increase as vacancies become arbitrarily rare, and no vacancy rate in $(0, 1)$ is optimal.

policy $x_1 \in \mathbb{R}$. Unlike before, however, x_1 remains in place only if a majority of justices vote to uphold it. If a majority vote to strike it down, then we assume the first-period policy outcome is the ideal point of the median justice.[23] Thus, the policy is upheld if and only if the median of the court prefers the new policy over getting her ideal policy at a cost of ϕ.

Incorporating multiple justices also affects potential judicial turnover in the second period. We assume that players correctly anticipate *which* justice *might* be replaced next, but do not know *whether* that justice *will* leave. Let $\omega \in \{L_1, L_2, R_1, R_2\}$ denote the justice who may be replaced in the second period. With probability ν, justice ω will leave and the second-period officeholder will appoint a new judge J_2 to the court. With probability $1 - \nu$, there is not a vacancy and the court's composition remains the same. Once the second-period court is in place, the executive chooses a second-period policy. As in the first period, it is upheld only if a majority of justices vote in favor. If it is struck down, the new policy outcome is at the ideal point of the second-period court's median justice.

This extension provides three insights that dovetail with our main results.

Our first insight is that the multi-member nature of the court creates a more nuanced relationship between vacancy, appointments, and electoral outcomes. In particular, the vacancy's location is an important determining factor for the safety of elections. If any vacancy will arise from the side opposite I, with either L_1 or L_2 vacating, then elections are similar to the baseline model: sufficiently low ν allows I the option to choose a justice that guarantees reelection regardless of V's beliefs. In contrast, if any vacancy will arise from the same side as I, with either R_1 or R_2 vacating, then I can achieve electoral safety more easily than in the baseline model. In this case, there exists a range of justices that guarantee reelection for *any* value of ν.

Throughout this extension, we restrict attention to parameters for which, if the potential vacancy is left-leaning, some appointment makes the election safe for the incumbent at $\nu = 0$, while no appointment does so at $\nu = 1$. The Appendix states this restriction formally and verifies that it holds, for example, whenever the sitting justices are symmetric about zero with $R_1 \geq \phi$ and $R_2 < e \leq R_1 + \phi$.

Proposition 5. *For any ν, the set of safe-election appointees if there may be a left-leaning vacancy is a subset of the set of incumbent-safe appointees if there may be a right-leaning vacancy.*

The key factor underlying Proposition 5 is how a potential vacancy's location constrains the scope for future movement of the court's median. In the baseline model, a second-period

[23] A microfoundation for this assumption is that the final ruling is determined by a dynamic bargaining game between justices if they strike down policy, see Cho and Duggan (2009).

vacancy provides free reign to move the court's ideological position. Consequently, vacancies allow any second-period politician to enact any policy. That is no longer true in this extension, as now appointments alter judicial constraints on policy only if they change the court's median. The location of a vacancy determines the "pivots" that constrain where any new median can be. Thus, a vacancy's location also constrains the set of policies that can be enacted after the vacant seat is filled. Importantly, these constraints are consequential for the safety of elections.

If a vacancy will lean left, i.e., opposite I, everyone anticipates that a second-period vacancy would lead to I pulling the second-period median justice rightward in its direction. This makes I less attractive if such a vacancy is likely (high ν). Specifically, safe elections are impossible if ν is high enough, as in the baseline model.

In contrast, a right-leaning vacancy limits I's ability to shift the court further rightward. Thus, I cannot pull policy as extreme if a second-period vacancy opens and, in turn, V's evaluation of I improves. Notably, this force facilitates incumbent-safe elections even for very high values of ν, unlike the baseline model.

Our second insight is that the choice of J_1 affects both (i) the set of policies that are upheld in the second period *and* (ii) where the second-period executive can move the second-period median justice if a vacancy opens. This contrasts with the baseline model, in which constraining policy is the only potential second-period effect of J_1. In this extension, J_1 can now also constrain the location of the second-period median by affecting the court's pivots. To highlight this incentive, consider I's continuation value from choosing a justice with ideal point $J_1 \in [L_2, L_1]$ if C will be elected and the justice who will potentially vacate in the second period is $\omega \in \{L_1, R_1, R_2\}$ and, assuming $L_1 \geq -m - \phi$ so that a moderate challenger facing court median L_1 implements her ideal point, equals:

$$\nu\Big(\, p\, u_I(x^*_{-e}(J_1)) + (1-\ p)\, u_I(-m)\Big) + (1-\nu)\Big(\, p\, u_I(x^*_{-e}(L_1)) + (1-\ p)\, u_I(-m)\Big).$$

Notably, I's choice of J_1 in this range will only affect outcomes if a vacancy *opens.* This contrasts with the baseline model, in which J_1 only affects second-period outcomes if a vacancy *does not open.*

Figure 1 explores this difference further. It depicts I's expected second-period payoff from C holding office, as a function of J_1, with the red line depicting the $\nu = 1$ case and the dashed blue line depicting the $\nu = 0$ case. Thus, the red line captures I's incentive to constrain C's scope to change the second-period median justice, whereas the blue line captures I's incentive to constrain C's policy choices. We see that the first effect explains why I wants to appoint a judge further to the right in the region $[L_2, L_1]$, while the second effect explains

Figure 1. I's Expected Utility from a Challenger

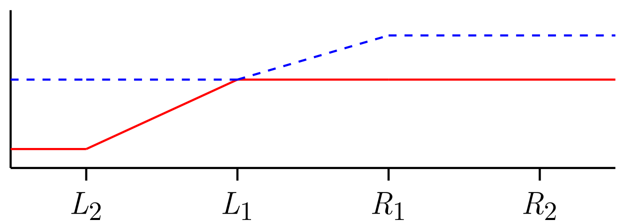

Figure 1 depicts I's expected utility, as a function of the first-period appointee J_1, from C winning the election if a right-leaning justice may leave. The solid line considers the case where $\nu = 1$, while the dotted line is the case where $\nu = 0$.

why I wants to move the judge to the right over the region $[L_1, R_1]$.

Our third insight is that the move-the-median structure of the court can reduce the informativeness of equilibrium appointments. The following result formalizes this.

Proposition 6. *A fully informative equilibrium does not exist if office benefit is large enough.*

Proposition 6 shows that, once move-the-median considerations are accounted for, high office benefit alone is sufficient to eliminate fully informative equilibria. This contrasts with the baseline model, where fully informative equilibria exist whenever tying hands equilibria do not. What accounts for this difference? The key factor is how the move-the-median structure of appointments in the extended model constrains I's ability to shift the court. Crucially, this restricts how the appointee can be used for signaling.

In the baseline model, a fully informative equilibrium requires that extremist incumbents are unwilling to mimic moderates for electoral gain by appointing the same justice. Accordingly, moderate types must skew their judicial appointee leftward enough to avoid imitation by extremists and, without limits on shifting the court, the moderate always can deter imitation by shifting J_1 sufficiently leftward. In the extended model, however, avoiding imitation is not always possible since the move-the-median structure constrains the moderate's ability to move the ideological position of the court. As office benefit increases, making mimicking more attractive, it eventually reaches a level beyond which the moderate cannot deter imitation by shifting J_1 further leftward, due to the ideologies of sitting justices. As such, a fully informative equilibrium cannot be sustained because the moderate is then unable to prevent the extremist from deviating, unlike in the baseline model. Thus, no equilibrium is fully informative.

Discussion

We now turn to a broader discussion of our results. We first discuss the *counter-majoritarian difficulty*. Second, we turn to institutional design, discussing proposed reforms to Supreme Court selection through the lens of our model. Finally, we connect our findings to previous work on strategic pre-action by US Presidents and empirical patterns of appointments.

The Countermajoritarian Difficulty. Although justices are not subject to direct public accountability, the Supreme Court has the power to overturn policies favored by a majority of citizens. Bickel (1986) termed this the *counter-majoritarian difficulty*. Our analysis highlights how having elected executives nominate justices can enable voters to have an *indirect* influence over judicial decisionmaking through their ability to discipline the incumbent. The possibility that elections might create incentives for moderate appointments has been previously raised by legal scholars. Prominently, Stephenson (2003) shows that fear of electoral turnover can lead parties to respect the rulings of the court, even preferring a moderate judiciary that will constrain policy when they are out of office. This is distinct from our model's mechanism, in which incumbents use moderate appointments in order to win reelection through both signaling and commitment, as described previously. It is also consistent with evidence that the court tends to rule in line with public opinion (see, e.g., Dahl, 1957; McGuire and Stimson, 2004). Thus, allowing the executive to appoint justices may alleviate the counter-majoritarian difficulty.

In both *tying hands* and *compromising* equilibria, electoral accountability operates as a force for moderation: incumbents use their appointment to commit to moderate policy, thereby winning reelection. But absent sufficient incentives to cater to the voter, appointment of justices may exacerbate the problem if the incumbent selects an even more extreme justice in order to constrain policymaking by future politicians, as occurs in *informative* equilibria.

According to historical accounts, this indirect effect prevailed in President Eisenhower's appointment of Justice Brennan to the Supreme Court in 1956. Concerned with the upcoming election, Eisenhower wanted to appoint a relatively moderate Democrat to the court in order to appeal to voters and appear less partisan (Yalof, 2001, p. 56). On the other hand, Eisenhower switched gears after winning reelection and nominated moderate-conservative Republicans in Justices Whitaker and Stewart (Yalof, 2001, p. 61). As suggested by our analysis, electoral concerns encouraged Eisenhower to select moderates during his first term. But absent reelection incentives, Eisenhower's second-term nominees were less moderate.

Turnover and Reform. On April 9, 2021, President Joe Biden issued an executive order announcing the formation of the Presidential Commission on the Supreme Court of the United States. Though the Commission ultimately avoided recommending any specific reforms, it considered a number of oft-discussed reform methods including term limits, court expansion, and mandatory retirement ages. Importantly, nearly all of the reforms analyzed would alter the court's rate of turnover. We do not explicitly model these reforms, but our results tying the durability of appointments, ν, to voter welfare provides a framework to assess the costs and benefits from altering rates of turnover in the court's membership.[24]

Our findings suggest that a moderate amount of turnover is optimal whenever the candidate pool is not too extremist, with too-high or too-low rates of turnover encouraging undesirable appointments and policymaking. On one hand, if the turnover rate is too low, extremist politicians are tempted to nominate extremist justices, losing reelection but also locking in extremist policy in the long-term due to the durability of the judicial constraint. On the other hand, if the turnover rate is too high, extremist politicians are unable to win reelection by tying their hands through the appointment of a moderate. The optimal rate of turnover is intermediate and balances these two forces — low enough so that extremists commit to moderate policy, but not so low that it encourages extremists to pursue long-standing and extreme judicial constraint.

Appointments as Strategic Pre-Action. The durability of judicial constraint in our model connects our work to a literature on the political economy of the U.S. presidency. In particular, the power to appoint justices provides the president with an opportunity to durably alter the policymaking environment, engaging in *strategic pre-action* (Cameron, 2008). Other sources of strategic pre-action previously studied include the veto threat, going public, and use of executive orders. As in these models, appointments in our setting have dynamic effects — they constrain policy today and into the future, altering a voter's calculus.

Our analysis contrasts with previous work on appointments, which has typically focused on one-shot, "move-the-median" models. By considering the dynamic effects of appointment choices, our theoretical findings complement existing work by illuminating some otherwise puzzling empirical patterns. Cameron and Kastellec (2016) find an important discrepancy between the predictions of one-shot move-the-median models and the empirical record: nominations that move the court's median *away* from the President's ideal point. Across a variety of move-the-median models, Presidents only ever nominate justices that bring the median weakly toward their own ideal point. Countering this robust theoretical expectation, Cameron and Kastellec find that 15% of appointments are *own goals* moving the court's median farther from the President's ideal point. What might account for this? By modeling the role of electoral accountability in the context of Supreme Court nominations, we have incorporated a potentially important strategic consideration that is absent from the MTM

[24] For further analysis of these proposed reforms see Chilton et al. (Forthcoming), Chilton et al. (2021), and Calabresi and Lindgren (2005). Hemel (2021) is a broader overview.

framework. The previously discussed appointment of Justice Brennan by President Eisenhower suggests that electoral considerations may be an important mechanism behind these own goals. Indeed, the Brennan appointment appears in Cameron and Kastellec (2016) as one of the more egregious examples of an own goal. Additionally, both of Eisenhower's second-term appointments are measured as being ideologically closer to Eisenhower. Although the existence of multiple equilibria makes precise empirical predictions difficult, this example demonstrates that the electoral forces driving moderation in our model also drive presidential decisionmaking on appointments.

Conclusion

We studied how electoral considerations influence judicial appointments and policymaking, and how they, in turn, influence elections. We showed how judicial appointments provide executives with a tool for signaling *and* commitment, while also shaping their incentives to use policy for signaling. Moreover, we find that executives combine policy and appointments differently depending on judicial turnover, polarization, office motivation, and the ideologies of sitting justices. We provide three main substantive findings. First, appointments can solve the "counter-majoritarian difficulty," but otherwise make it worse. Second, policy reforms that increase judicial turnover, e.g., judicial term limits, can backfire and reduce voter welfare. Third, accounting for dynamic electoral incentives produces equilibrium behavior consistent with patterns of judicial appointments.

We close with suggestions for future research. Our findings shed light on some empirical patterns in judicial appointments that are not well explained by previous analyses emphasizing constraints from Senate confirmation. Studying these considerations in tandem, both theoretically and empirically, could be fruitful. A full analysis — incorporating additional constraints due to Senate confirmation — could build on our move-the-median extension. Additionally, we highlight how different forms of polarziation can have different effects on the interplay between elections and judicial appointments. It is also worth considering how an incumbent's action or type might endogenously influence the court's incentive to rule on a case, perhaps through manipulation of the costliness of a ruling. Continued dialogue between theory and empirics is important for developing useful measures of polarization and assessing their consequences for executive policymaking and appointments.

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Appendix

Contents

A Strategies, Beliefs, and Proofs of Main Results

Strategies and Beliefs. A pure strategy for V is a mapping $\rho_V : \mathbb{R}^2 \to \{0,1\}$ specifying whether V reelects I after observing a justice with ideal point J_1 and first-period policy x_1. Next, a pure strategy for I specifies, for each type in $\{m, e\}$: a first-period appointment and policy in $\mathbb{R}^2$, and, for each second-period history — comprising the first-period appointment and policy, the ruling, the election outcome, and the vacancy realization — a second-period appointment (when a vacancy arises and I holds office) and policy. A pure strategy for C likewise specifies, for each type in $\{-m, -e\}$, a second-period appointment and policy at each history where C holds office. Because second-period behavior is solved history by history below, we suppress the dependence on histories in our notation. Finally, a pure strategy for each judge J is a mapping $\rho_J : \mathbb{R}^2 \to \{0,1\}$ specifying for each policy whether J overturns it in each period. Let $\sigma = (\rho_V, \pi_I, \pi_C, \rho_J)$ denote a strategy profile. Additionally, V's belief system is represented by $\mu : \mathbb{R}^2 \to [0,1]$, where $\mu^{x_1}_{J_1}$ denotes the probability that V places on $I = e$ after observing x_1 and J_1. It is not necessary to define belief systems for justices.

If the election is safe for C, then V prefers to elect the unknown challenger over her favorite type of incumbent, i.e.,

$$\nu u_V(m) + (1-\nu) u_V(x_2^*(m; J_1)) \leq U_V^C(J_1). \tag{A.1}$$

We now present an expanded version of Lemma 1.

A.1 Lemma 1

In equilibrium, (i) for each politician $i \in \{I, C\}$, there is a threshold $\overline{\nu}_i \in (0,1)$ on the probability of judicial vacancy such that $\mathcal{J}^i$ is nonempty if and only if $\nu \leq \overline{\nu}_i$; (ii) if $\nu \leq \overline{\nu}_I$, then $\mathcal{J}^I \subseteq [-e-\phi, 0)$ is a compact interval; (iii) if $\nu \leq \overline{\nu}_C$, then $\mathcal{J}^C = [\underline{J}^C, \overline{J}^C] \subseteq (\phi - m, \phi + m]$; and (iv) if ν increases, then $\overline{J}^i$ decreases and $\underline{J}^i$ increases for each $i \in \{I, C\}$.

A.2 Proof of Lemma 1

We prove Lemma 1 in three steps. In Step 1, Lemma A.1 characterizes when the election is safe for C. In Step 2, Lemmas A.2 and A.3 characterize when the election is safe for I. Finally, in Step 3, Lemma A.4 characterizes how the vacancy probability ν affects the conditions producing safe elections.

Step 1. To begin, we characterize the conditions under which the election is safe for C. Let

$$\mathcal{J}^C = \Big[\phi - \frac{m - \nu(pe + (1-p)m)}{1-\nu}, \phi + \frac{m - \nu(pe + (1-p)m)}{1-\nu}\Big]. \tag{A.2}$$

Lemma A.1. *The voter prefers to elect C over a known moderate incumbent if and only if $J_1 \in \mathcal{J}^C$. Furthermore, $\mathcal{J}^C \subseteq (\phi - m, \phi + m]$.*

PROOF. To begin, consider $J_1 \notin [\phi - m, \phi + m]$. Then, we have $|x^*(m; J_1)| \leq |x^*(-m; J_1)|$ and $|x^*(m; J_1)| \leq |x^*(-e; J_1)|$. In the vacancy state, a known moderate yields a loss of m for the voter, whereas an unknown challenger yields expected loss $pe + (1-p)m > m$. Thus, the voter strictly prefers I if she is a known moderate.

To complete the proof, consider $J_1 \in [\phi - m, \phi + m]$. Then, we have $x^*(m; J_1) = m$ and $x^*(-e; J_1) = x^*(-m; J_1) = J_1 - \phi$. If V's beliefs place probability one on I's type being $\theta_I = m$, then she prefers to elect C if and only if

$$-\nu(pe + (1-p)m) - (1-\nu)|J_1 - \phi| \geq -m. \tag{A.3}$$

Rearranging, (A.3) holds if and only if $J_1 \in \mathcal{J}^C$. The interval $\mathcal{J}^C = [\underline{J}^C, \overline{J}^C]$ is nonempty if and only if $\nu \leq \frac{m}{pe+(1-p)m} \equiv \overline{\nu}^C < 1$. Finally, if the interval is nonempty, then $0 \leq \frac{m-\nu(pe+(1-p)m)}{1-\nu}$, so $\mathcal{J}^C \subseteq (\phi - m, \phi + m]$. □

Step 2. Next, we characterize the conditions under which V prefers to reelect a known extremist incumbent. If V's belief places probability one on $\theta_I = e$, then V weakly prefers to reelect I if and only if

$$\begin{aligned} -\nu e - (1-\nu)\,|x^*(e; J_1)| \geq{}& -\nu\big(pe + (1-p)m\big) \\ &- (1-\nu)\Big(p\,|x^*(-e; J_1)| + (1-p)\,|x^*(-m; J_1)|\Big), \end{aligned} \tag{A.4}$$

where the left side is V's expected payoff from a reelected extremist and the right side is $U_V^C(J_1)$. Collecting the vacancy-state terms and dividing by $1-\nu > 0$ shows that (A.4) is equivalent to $h(J_1) \geq T(\nu)$, where

$$\begin{aligned} h(J) &= p\,|x^*(-e; J)| + (1-p)\,|x^*(-m; J)| - |x^*(e; J)|, \\ T(\nu) &= \frac{\nu(1-p)(e-m)}{1-\nu} \geq 0. \end{aligned} \tag{A.5}$$

In (A.5), $h(J)$ is the no-vacancy advantage, from V's perspective, of retaining a known extremist incumbent rather than electing the unknown challenger, and $T(\nu)$ is the challenger pool's advantage in the vacancy state. Thus,

$$\mathcal{J}^I = \{J : h(J) \geq T(\nu)\}, \tag{A.6}$$

so $\mathcal{J}^I$ can be characterized via properties of h.

Lemma A.2. *The function h is continuous and piecewise linear, with $h(J) = 0$ for $J \notin (-e-\phi, e+\phi)$. Moreover, h is nondecreasing over $(-\infty, -\phi]$, strictly decreasing over $[-\phi, 0]$, and nonpositive for all $J \geq 0$, so it attains its maximum at $J = -\phi$. Its maximum value is*

$$h^* \equiv h(-\phi) = p \min\{e, 2\phi\} + (1 - p)m. \tag{A.7}$$

Consequently, for any $\nu \in (0,1)$, $\mathcal{J}^I$ is a compact interval (whenever nonempty) that is contained in $[-e-\phi, 0)$.

PROOF. Each $x^*(\theta; J)$ is the projection of θ onto $[J-\phi, J+\phi]$, so each term of h is continuous and piecewise linear in J, and therefore so is h. For $J \leq -e-\phi$, all of e, $-m$, and $-e$ are weakly above the acceptance set, so every projection equals $J+\phi$ and $h(J) = 0$. Symmetrically, $h(J) = 0$ for $J \geq e+\phi$.

If $J \leq -\phi$, then $x^*(e; J) = J + \phi \leq 0$, so $|x^*(e; J)| = -(J+\phi)$, which has derivative -1 in J. For each $\theta' \in \{e, m\}$, the term $|x^*(-\theta'; J)|$ equals the constant θ' where the challenger $-\theta'$ is unconstrained, and equals $-(J-\phi)$ or $-(J+\phi)$ where she is held at an endpoint of the acceptance set, each of which has derivative -1 in J. Therefore, at every $J \leq -\phi$ at which h is differentiable,

$$h'(J) \geq -p - (1 - p) + 1 = 0,$$

so h is nondecreasing over $(-\infty, -\phi]$.

If $J \in [-\phi, 0]$, then $x^*(e; J) = J + \phi \geq 0$, so $|x^*(e; J)| = J + \phi$, which has derivative 1 in J. The moderate challenger is unconstrained, because $\phi > m$ implies $-m \in [J-\phi, J+\phi]$ for all $J \in [-\phi, 0]$, so $|x^*(-m; J)| = m$ is constant. The term $|x^*(-e; J)|$ equals the constant e where the extremist challenger is unconstrained and equals $-(J-\phi)$ where she is constrained. Therefore, at every $J \in [-\phi, 0]$ at which h is differentiable, $h'(J) \in \{-1, -(1+p)\}$, so h is strictly decreasing over $[-\phi, 0]$.

If $J \in [0, e+\phi]$, then $|x^*(e; J)| = \min\{e, J+\phi\}$, while $|x^*(-e; J)| = |J-\phi| \leq \min\{e, J+\phi\}$ and $|x^*(-m; J)| = |\max\{-m, J-\phi\}| \leq \max\{m, |J-\phi|\} \leq \min\{e, J+\phi\}$, using $m < \phi < e$. Thus, $h(J) \leq 0$ for all $J \in [0, e+\phi]$, and $h(J) = 0$ for all $J \geq e+\phi$ as above.

Together, h attains its maximum at $J = -\phi$. At $J = -\phi$, we have $x^*(e; -\phi) = 0$, the moderate challenger is unconstrained, and the extremist challenger implements $\max\{-e, -2\phi\}$, which yields (A.7).

Finally, fix $\nu \in (0,1)$, so $T(\nu) > 0$. Since $h(J) \leq 0$ for all $J \geq 0$ and $h(J) = 0$ for all $J \notin (-e-\phi, e+\phi)$, the upper contour set $\{J : h(J) \geq T(\nu)\}$ is contained in $(-e-\phi, 0)$. Over $(-\infty, 0]$, h is nondecreasing and then strictly decreasing, so this set is an interval. It is compact by continuity and boundedness. □

To complete Step 2, we show existence of $\overline{\nu}^I \in (0,1)$ such that $\mathcal{J}^I \neq \varnothing$ if and only if $\nu \leq \overline{\nu}^I$. We also characterize the endpoints of $\mathcal{J}^I$.

Lemma A.3. *The set $\mathcal{J}^I$ is nonempty if and only if $\nu \leq \overline{\nu}^I$, where*

$$\overline{\nu}^I = \frac{h^*}{h^* + (1-p)(e-m)} = \begin{cases} \dfrac{pe + (1-p)m}{e}, & \text{if } \phi \geq \frac{e}{2}, \\ \dfrac{2p\phi + (1-p)m}{2p\phi + (1-p)e}, & \text{if } \phi < \frac{e}{2}. \end{cases}$$

PROOF. By Lemma A.2, $\mathcal{J}^I \neq \varnothing$ if and only if $\max_J h(J) = h^* \geq T(\nu)$. Since T is continuous and strictly increasing over $[0,1)$ with $T(0) = 0$ and $T(\nu) \to \infty$ as $\nu \to 1$, there is a unique $\overline{\nu}^I \in (0,1)$ solving $T(\overline{\nu}^I) = h^*$, and $\mathcal{J}^I \neq \varnothing$ if and only if $\nu \leq \overline{\nu}^I$. Solving $T(\overline{\nu}^I) = h^*$ yields $\overline{\nu}^I = \frac{h^*}{h^*+(1-p)(e-m)}$. Substituting (A.7) gives the two cases, which are distinguished by whether the extremist challenger is unconstrained ($e \leq 2\phi$) or constrained ($e > 2\phi$) at $J = -\phi$. The same argument applied to the challenger's comparison establishes the corresponding threshold $\overline{\nu}^C$ for $\mathcal{J}^C$, as characterized in Lemma A.1. □

We now derive the upper endpoint $\overline{J}^I = \max \mathcal{J}^I$ for $\nu \leq \overline{\nu}^I$. Since $h(-\phi) = h^* \geq T(\nu) > 0 > h(0)$ and h is strictly decreasing over $[-\phi, 0]$, $\overline{J}^I$ is the unique $J \in [-\phi, 0)$ that solves $h(J) = T(\nu)$. For all $J \in [-\phi, 0)$, $x^*(e; J) = J + \phi$ and the moderate challenger is unconstrained, so $x^*(-m; J) = -m$. Over this interval, h has at most one kink, at $J = \phi - e$, where $x^*(-e; J)$ switches between $-e$ and $J - \phi$. For $J > \phi - e$, the extremist challenger is constrained, so

$$h(J) = p(\phi - J) + (1-p)m - (J + \phi) = (1-p)(m-\phi) - (1+p)J,$$

and setting $h(J) = T(\nu)$ yields

$$\overline{\gamma}_2(\nu) \equiv \Big(\frac{1-p}{1+p}\Big)\Big(\frac{m - \nu e}{1-\nu} - \phi\Big). \tag{A.8}$$

For $J < \phi - e$, the extremist challenger is unconstrained, so $h(J) = pe + (1-p)m - (J+\phi)$, which yields

$$\overline{\gamma}_1(\nu) \equiv \frac{(1-p)m + (p-\nu)e}{1-\nu} - \phi. \tag{A.9}$$

The applicable case is the one whose root is on its own side of the kink:

$$\overline{J}^I = \begin{cases} \overline{\gamma}_1(\nu), & \text{if } \phi \geq \frac{e}{2} \text{ and } \overline{\gamma}_2(\nu) < \phi - e, \\ \overline{\gamma}_2(\nu), & \text{if } \phi < \frac{e}{2} \text{ or } \overline{\gamma}_2(\nu) \geq \phi - e. \end{cases} \tag{A.10}$$

If $\phi < \frac{e}{2}$, then $\phi - e < -\phi$, so the kink is below the maximizer of h and only the $\overline{\gamma}_2$ case applies for $J \in [-\phi, 0]$. The two expressions are equivalent at $J = \phi - e$, so $\overline{J}^I$ is continuous in ν.

Step 3. Finally, we characterize how $\mathcal{J}^I$ and $\mathcal{J}^C$ change as ν increases.

Lemma A.4. *For $i \in \{I, C\}$, increasing ν increases $\underline{J}^i_\nu$ and decreases $\overline{J}^i_\nu$.*

PROOF. First, we prove the result for $i = C$. After that, we consider $i = I$.

Part 1. Recall $\mathcal{J}^C = [\underline{J}^C, \overline{J}^C]$ as defined in (A.3). Differentiating with respect to ν yields $\frac{\partial \overline{J}^C}{\partial \nu} = -\frac{p(e-m)}{(1-\nu)^2} < 0$ and $\frac{\partial \underline{J}^C}{\partial \nu} = \frac{p(e-m)}{(1-\nu)^2} > 0$. Thus, $\mathcal{J}^C$ shrinks as ν increases.

Part 2. Fix $\nu < \overline{\nu}^I$ and let $\overline{J}^I_\nu = \max \mathcal{J}^I$ and $\underline{J}^I_\nu = \min \mathcal{J}^I$. By Lemma A.2, h is constant in ν and $T(\nu)$ is strictly increasing in ν. Since h is nondecreasing over $J \in (-\infty, -\phi]$ and strictly decreasing over $J \in [-\phi, 0]$, increasing the threshold $T(\nu)$ weakly increases the lower endpoint of the upper contour set and strictly decreases the upper endpoint. Thus, $\underline{J}^I_\nu$ is nondecreasing in ν and $\overline{J}^I_\nu$ is strictly decreasing in ν, so $\mathcal{J}^I$ shrinks as ν increases.

Explicitly, differentiating (A.8) and (A.9) yields

$$\frac{\partial \overline{\gamma}_2}{\partial \nu} = -\Big(\frac{1-p}{1+p}\Big)\frac{e-m}{(1-\nu)^2} < 0 \quad \text{and} \quad \frac{\partial \overline{\gamma}_1}{\partial \nu} = -\frac{(1-p)(e-m)}{(1-\nu)^2} < 0, \tag{A.11}$$

so $\overline{J}^I_\nu$ is strictly decreasing in each case of (A.10), and therefore throughout, since $\overline{J}^I_\nu$ is continuous.

At $\nu = \overline{\nu}^I$, we have $T(\nu) = h^*$, so the unique solution is the maximizer of h and $\overline{J}^I_{\overline{\nu}^I} = -\phi$. Combined with strict monotonicity, $\overline{J}^I_\nu \in [-\phi, 0)$ for $\nu \in (0, \overline{\nu}^I]$. Finally,

$$\lim_{\nu \downarrow 0} \overline{J}^I_\nu = \begin{cases} (1-p)m + pe - \phi & \text{in the } \overline{\gamma}_1 \text{ case,} \\ \Big(\frac{1-p}{1+p}\Big)(m - \phi) & \text{in the } \overline{\gamma}_2 \text{ case,} \end{cases}$$

both of which are strictly negative. Thus, as ν decreases, $\overline{J}^I_\nu$ increases toward a strictly negative limit. $\square$

Proposition 1. *Every equilibrium features either compromising, informative appointments, or tying hands. Furthermore, there exists an equilibrium featuring: (i) compromising if and only if* $\nu \geq \underline{\nu}$ *and* $\beta \geq \overline{\beta}^c_\nu$*; (ii) informative appointments if and only if either* $\nu > \overline{\nu}^I$ *or* $\beta \leq \overline{\beta}^{th}_\nu$*; and (iii) tying hands if and only if* $\nu \leq \overline{\nu}^I$ *and* $\beta \geq \overline{\beta}^{th}_\nu$*.*

A.3 Proof of Proposition 1

The proof has two components. The *classification* component shows that every pure strategy PBE satisfying equilibrium dominance features compromising, informative appointments, or tying hands. The *characterization* component then proves parts (i)–(iii).

We first introduce terminology and preliminary lemmas used in both components. Throughout the proof, we denote the *implemented* policy by x_1, so that every pair (x_1, J_1) we write is feasible, i.e., $x_1 \in [J_1 - \phi, J_1 + \phi]$, and is attained by an upheld proposal.[25]

For a feasible pair (x, J) with $x \in [J - \phi, J + \phi]$, type θ's *policy sacrifice* from winning at (x, J) is

$$\sigma_\theta(x, J) = |x - \theta| + (1 - \nu)\,|x^*(\theta; J) - \theta|. \tag{A.12}$$

Let $L_e = 2\,pe + (1 - p)(e + m)$ and $L_m = p(e + m) + 2(1 - p)m$ denote each type's expected vacancy-state loss from the challenger, and define $D_\theta(\beta) = \beta + \nu L_\theta$, writing $D(\beta) = D_e(\beta)$.

To streamline exposition, we normalize payoffs by subtracting the first-period office benefit β, which I receives regardless of her first-period action. Thus, type θ's normalized payoff if she wins at (x_1, J_1) is $\beta - \sigma_\theta(x_1, J_1)$, and $-\nu L_\theta$ under her optimal losing behavior.

Lemma A.5. *In equilibrium, (i) if a type-*θ *incumbent loses reelection, then she chooses* $(x_1, J_1) = (\theta, \theta + \phi)$*, which yields her normalized payoff* $-\nu L_\theta$*; (ii) a type-*θ *incumbent weakly prefers winning at a first-period pair* (x, J) *to her optimal losing behavior if and only if* $\sigma_\theta(x, J) \leq D_\theta(\beta)$*; and (iii) each type's payoff weakly exceeds her optimal losing payoff,* $-\nu L_\theta$*.*

PROOF. Let θ be I's type. First, we show (i). If she loses after choosing $(\theta, \theta + \phi)$, then first-period policy is her ideal point, θ, and the second-period no-vacancy policy is also θ. Any other losing pair is strictly worse. If $J_1 = \theta + \phi$ and $x_1 \neq \theta$, then first-period policy is strictly worse. If $J_1 < \theta + \phi$, then the second-period no-vacancy policy $x^*(C; J_1) \leq \max\{-m, J_1 - \phi\} < \theta$ is

[25] This is without loss for arguments regarding payoffs. An overturned proposal yields the same outcomes as the upheld proposal $x_1 = J_1$, and our bounds on deviation payoffs depend only on the implemented pair and on whether the incumbent wins. Arguments invoking Bayes' rule instead compare observed actions, so distinct proposals inducing the same implemented policy remain distinct observed actions, and we flag the distinction where it matters in Case 3 of the classification component.

strictly worse. If $J_1 > \theta + \phi$, then the first-period policy is at least $J_1 - \phi > \theta$, again strictly worse.

Next, we show (ii). For a first-period pair (x, J) such that I wins, she receives the office benefit and avoids the expected vacancy loss νL_θ at policy cost $\sigma_\theta(x, J)$. Thus, she prefers winning at (x, J) over any losing action if and only if $\sigma_\theta(x, J) \leq D_\theta(\beta)$.

Finally, we show (iii). Since at most two actions are on the path of play, actions arbitrarily close to $(\theta, \theta+\phi)$ are off path. Each such action yields approximately the optimal losing payoff if the voter elects C, and strictly more if the voter reelects since I would be unconstrained and thus σ_θ would be zero. Thus, each type can secure $-\nu L_\theta$ up to an arbitrarily small loss. $\square$

Lemma A.6. *For all $S \geq 0$, (i) $\min\{\sigma_m(x, J) : \sigma_e(x, J) \geq S\} = (S - (2-\nu)(e-m))^+$; (ii) the minimum is attained by $a(S) = (x_S, J_S)$, where $x_S = \min\{m,\, J_S + \phi\}$, and $J_S = 0$ if $S \leq \sigma_e(m, 0)$ while otherwise J_S is the unique $J < 0$ with $\sigma_e(\min\{m, J+\phi\},\, J) = S$; and (iii) $\sigma_e(a(S)) = \max\{S,\, \sigma_e(m, 0)\}$, which is continuous in S and strictly increasing over $S \geq \sigma_e(m, 0)$.*

PROOF. For (i), the triangle inequality implies $|x - e| \leq |x - m| + (e - m)$, with equality if and only if $x \leq m$. Similarly, $|x^*(e; J) - e| \leq |x^*(m; J) - m| + (e - m)$, with equality whenever $J + \phi \leq m$. Thus $\sigma_e(x, J) \leq \sigma_m(x, J) + (2-\nu)(e-m)$ for every feasible pair, so $\sigma_e(x, J) \geq S$ implies $\sigma_m(x, J) \geq (S - (2-\nu)(e-m))^+$.

Next, we show (ii), which also establishes that $a(S)$ is well defined. Every $J \leq 0$ satisfies $J + \phi < e$, so $|x^*(e; J) - e| = e - J - \phi$. Over $J \in [m - \phi, 0]$ with $x = m$, we have $\sigma_m(m, J) = 0$, while $\sigma_e(m, J) = (e - m) + (1-\nu)(e - J - \phi)$ is continuous and strictly decreasing in J, and takes every value in $[\sigma_e(m, 0),\, (2-\nu)(e-m)]$. Over $J \leq m - \phi$ with $x = J + \phi$, both triangle inequalities hold with equality, so $\sigma_e(J+\phi, J) = (2-\nu)(e - J - \phi)$ is continuous and strictly decreasing in J, takes every value in $[(2-\nu)(e-m), \infty)$, and satisfies $\sigma_m(J+\phi, J) = \sigma_e(J+\phi, J) - (2-\nu)(e-m)$. The two cases coincide at $J = m - \phi$. For $S > \sigma_e(m, 0)$, strict monotonicity then yields a unique J_S solving $\sigma_e(x_S, J_S) = S$. For $S \leq \sigma_e(m, 0)$, the action $a(S) = (m, 0)$ satisfies the constraint because $\sigma_e(m, 0) \geq S$. In every case, $\sigma_m(a(S)) = (S - (2-\nu)(e-m))^+$, which attains the minimum.

Finally, we show (iii). By construction, $\sigma_e(a(S)) = \sigma_e(m, 0)$ for $S \leq \sigma_e(m, 0)$ and $\sigma_e(a(S)) = S$ otherwise. This function is continuous in S and strictly increasing over $S \geq \sigma_e(m, 0)$. $\square$

Let Σ_e denote the extremist's equilibrium sacrifice, so that $\Sigma_e = \sigma_e(x_1^e, J_1^e)$ if she wins reelection and $\Sigma_e = D(\beta)$ if she loses.

Lemma A.7. *In every equilibrium, (i) the moderate wins reelection; and (ii) $\sigma_m(x_1^m, J_1^m) \leq (\Sigma_e - (2-\nu)(e-m))^+$.*

PROOF. By Lemma A.5, $\Sigma_e \leq D(\beta)$ and the extremist's equilibrium payoff equals $\beta - \Sigma_e$. If she wins, this is $\beta - \sigma_e(x_1^e, J_1^e)$. If she loses, it is $-\nu L_e = \beta - D(\beta)$. The proof proceeds in two steps. Step 1 shows that in every equilibrium the moderate's payoff is at least $\beta - (\Sigma_e - (2-\nu)(e-m))^+$. Step 2 derives (i) and (ii).

Step 1. Fix $\varepsilon > 0$ such that $a = a(\Sigma_e + \varepsilon)$ from Lemma A.6 is off the path of play. This is possible for all but at most two values of ε if $\Sigma_e + \varepsilon \geq \sigma_e(m, 0)$, since $S \mapsto a(S)$ is injective there by Lemma A.6(iii). In the remaining case, $a(\Sigma_e + \varepsilon) = a(0) = (m, 0)$ for all small ε.

If $a = (m, 0)$ is on path, it must be the moderate's own action, since the extremist's winning action would have sacrifice $\Sigma_e < \sigma_e(m, 0)$ and her losing action is $(e, e + \phi)$. Then, $\sigma_m(x_1^m, J_1^m) = 0$ and the moderate wins. If she lost, Lemma A.5 would force $(x_1^m, J_1^m) = (m, m + \phi) \neq (m, 0)$. Thus, both claims hold.

Suppose a is off path. Then, the extremist's best conceivable payoff is the maximum of $\beta - \sigma_e(a)$ and her best losing payoff at a. First, we have $\beta - \sigma_e(a) < \beta - \Sigma_e$ since $\sigma_e(a) = \max\{\Sigma_e + \varepsilon, \sigma_e(m, 0)\} > \Sigma_e$ by Lemma A.6(iii). Second, her best losing payoff at a is weakly less than $-(e - x_S) - \nu L_e \leq -(e - m) - \nu L_e < \beta - D(\beta) \leq \beta - \Sigma_e$. Thus, a is equilibrium-dominated for the extremist. If a is also equilibrium-dominated for the moderate, then $\beta - \sigma_m(a)$ is strictly below her equilibrium payoff. Otherwise, equilibrium dominance places probability one on the moderate at a. Since $J_S \leq 0$ while $\mathcal{J}^C \subseteq (\phi - m, \phi + m]$ by Lemma A.1, the voter strictly prefers to reelect a known moderate at a. Deviating to a therefore yields the moderate $\beta - \sigma_m(a)$, and sequential rationality bounds her equilibrium payoff below by it. In either case, the moderate's equilibrium payoff is at least $\beta - \sigma_m(a) = \beta - (\Sigma_e + \varepsilon - (2-\nu)(e-m))^+$. Letting $\varepsilon \to 0$ yields the claim.

Step 2. We first prove (i). If the moderate lost reelection, her payoff would be $-\nu L_m = \beta - D_m(\beta)$, so Step 1 would require $D_m(\beta) \leq (\Sigma_e - (2-\nu)(e-m))^+ \leq (D(\beta) - (2-\nu)(e-m))^+$. But $D_m(\beta) = D(\beta) - \nu(e-m)$ is strictly greater than the right side, since $L_e > e - m$ implies $D(\beta) > \nu(e-m)$ and $\nu < 2 - \nu$, a contradiction. Thus, the moderate wins reelection. Since she wins, her equilibrium payoff is $\beta - \sigma_m(x_1^m, J_1^m)$, and Step 1 yields (ii). $\square$

Lemma A.8. *If type θ appoints some $J_1 \in \mathcal{J}^I$ in equilibrium, then she wins reelection and chooses $x_1 = x^*(\theta; J_1)$. Furthermore, if $\theta = e$, then $J_1 = \overline{J}_\nu^I$.*

PROOF. The election is safe for I at any $J_1 \in \mathcal{J}^I$, so she is reelected regardless of voter beliefs and her policy choice does not affect the election. Sequential rationality then requires $x_1 = x^*(\theta; J_1)$. For $\theta = e$, since $\mathcal{J}^I \subseteq [-e - \phi, 0)$ by Lemma A.2, the extremist is constrained at every safe judge, so her winning payoff $\beta - (2-\nu)(e - J_1 - \phi)$ is strictly increasing over $\mathcal{J}^I$. A deviation to $\overline{J}^I_\nu$ also guarantees reelection, so optimality requires $J_1 = \overline{J}^I_\nu$. $\square$

Lemma A.9. *If the voter's belief at the observed (x_1, J_1) equals her prior, then she weakly prefers reelecting I only if $J_1 \leq 0$ or $J_1 \geq e + \phi$. In the latter case, she is indifferent.*

PROOF. Under the prior, the vacancy-state terms of (3) and (2) coincide, so reelection is weakly preferred if and only if

$$p\big(|x^*(-e;J_1)| - |x^*(e;J_1)|\big) + (1-\ p)\big(|x^*(-m;J_1)| - |x^*(m;J_1)|\big) \geq 0.$$

Fix $J > 0$ and $\theta \in \{m, e\}$. The projections give $|x^*(\theta; J)| = \min\{\theta, J+\phi\}$ for $J \leq \theta + \phi$ and $J - \phi$ for $J \geq \theta + \phi$, while $|x^*(-\theta; J)| = \theta$ for $J \leq \phi - \theta$ and $|J - \phi|$ otherwise. Comparing case by case, $|x^*(-\theta; J)| \leq |x^*(\theta; J)|$ for all $J > 0$, with equality only if $J \leq \phi - \theta$ or $J \geq \theta + \phi$. Since $\phi < e$, the first term in the reelection condition is strictly negative for all $J \in (0, e+\phi)$ and the second term is nonpositive there, while both terms vanish for $J \geq e + \phi$. The claim follows. $\square$

By Lemmas A.5 and A.6, the moderate's lowest cost of deterring imitation by a losing extremist is $c^{sep} \equiv (D(\beta) - (2-\nu)(e-m))^+$. If $\nu \leq \overline{\nu}^I$, so that safe appointments exist, then the extremist's cheapest safe win uses $\overline{J}^I_\nu$ and costs $S_\nu = (2-\nu)(e - \overline{J}^I_\nu - \phi)$. Otherwise, if no safe appointments exist, then we set $S_\nu = +\infty$. Define the following cutoffs on office benefit:

$$\overline{\beta}^{th}_\nu = (2-\nu)(e - \overline{J}^I_\nu - \phi) - \nu(2\,pe + (1-\ p)(e+m)), \text{ and} \tag{A.13}$$

$$\overline{\beta}^{c}_\nu = (e-m) + (1-\nu)(e-\phi) - \nu(2\,pe + (1-\ p)(e+m)). \tag{A.14}$$

The latter is the cost to the extremist of $(x_1, J_1) = (m, 0)$, for which $\sigma_e(m, 0) = (e-m) + (1-\nu)(e-\phi)$. Equivalently, $\overline{\beta}^{th}_\nu = S_\nu - \nu L_e$ and $\overline{\beta}^{c}_\nu = \sigma_e(m,0) - \nu L_e$, so that $\beta \geq \overline{\beta}^{th}_\nu$ if and only if $D(\beta) \geq S_\nu$, and $\beta \geq \overline{\beta}^{c}_\nu$ if and only if $D(\beta) \geq \sigma_e(m, 0)$. For $\nu > \overline{\nu}^I$, the convention $S_\nu = +\infty$ yields $\overline{\beta}^{th}_\nu = +\infty$, consistent with the statement of Proposition 1.

Classification. We prove the classification component in three cases, organized by the extremist's equilibrium behavior. Case 1 covers equilibria in which the extremist loses reelection. Cases 2 and 3 cover equilibria in which she wins, with an appointment inside and outside $\mathcal{J}^I$, respectively. By Lemma A.7, the moderate wins reelection in every equilibrium, so these three cases are exhaustive and induce informative appointments, tying hands, and compromising.

Case 1. Suppose the extremist loses reelection. By Lemma A.5, $(x_1^e, J_1^e) = (e, e+\phi)$, and by Lemma A.7 the moderate wins. Because the voter reelects at the moderate's on-path action, the extremist must not strictly prefer to imitate her, i.e.,

$$-|x_1^m - e| + \beta - (1-\nu)\,|x^*(e; J_1^m) - e| \;\leq\; -\nu L_e, \tag{A.15}$$

where the left side is the extremist's expected utility from choosing (x_1^m, J_1^m) and winning reelection, and the right side is her equilibrium payoff. Rearranging, (A.15) is exactly the *deterrence constraint* $\sigma_e(x_1^m, J_1^m) \geq D(\beta)$, the form we use hereafter. By Lemma A.7 with $\Sigma_e = D(\beta)$, we have $\sigma_m(x_1^m, J_1^m) \leq c^{se\,p}$, while the deterrence constraint and Lemma A.6 give $\sigma_m(x_1^m, J_1^m) \geq c^{se\,p}$. Thus, the moderate wins at a least-cost deterring action, with $\sigma_m(x_1^m, J_1^m) = c^{se\,p}$, and (A.15) holds with equality whenever $c^{se\,p} > 0$. In particular, $J_1^m \neq e + \phi$. Otherwise, $x_1^m \in [e, e+2\phi]$ would give $\sigma_m = \sigma_e + (2-\nu)(e-m) \geq D(\beta) + (2-\nu)(e-m) > c^{se\,p}$. The types therefore appoint distinct judges and only the moderate wins, so the equilibrium features informative appointments.

Case 2. Suppose the extremist wins reelection with $J_1^e \in \mathcal{J}^I$. Lemma A.8 implies $J_1^e = \overline{J}_\nu^I$ and $x_1^e = x^*(e; \overline{J}_\nu^I)$, and Lemma A.7 implies that the moderate also wins. Both types thus win reelection and the extremist appoints into $\mathcal{J}^I$, so the equilibrium features tying hands. If the moderate also appoints some $J_1^m \in \mathcal{J}^I$, then Lemma A.8 yields $x_1^m = x^*(m; J_1^m)$. Alternatively, she may win with an unsafe appointment that reveals her type, since by Lemma A.1 the voter reelects a known moderate at any $J_1 \notin \mathcal{J}^C$. Regardless, the desired classification holds.

Case 3. Suppose the extremist wins reelection with $J_1^e \notin \mathcal{J}^I$. Then, the types must pool on the same pair (x_1, J_1). Otherwise, if the types separate in the first period, then Bayes's rule would place probability one on the extremist at (x_1^e, J_1^e). But since (4) fails strictly outside $\mathcal{J}^I$, the voter would elect C, contradicting that the extremist wins. Let x_1 denote the common implemented policy. Lemma A.9 implies that either $J_1 \leq 0$ or $J_1 \geq e + \phi$. But the latter is impossible, since by feasibility the implemented policy satisfies $x_1 \geq J_1 - \phi \geq e$, so that $\sigma_m(x_1, J_1) = \sigma_e(x_1, J_1) + (2-\nu)(e-m)$, contradicting Lemma A.7. Thus, $J_1 \leq 0$, and since $J_1 \notin \mathcal{J}^I$, the equilibrium features compromising.

It remains to establish that $x_1 \leq m$. If the pooled proposal is overturned, then $x_1 = J_1 \leq 0 < m$. So, to show a contradiction, suppose that the proposal is $x_1 > m$ and that it is upheld. Note that $x_1 \leq J_1 + \phi \leq \phi < e$. Fix $x_1' \in [m, x_1)$. The deviation to the pair (x_1', J_1) is upheld, since $J_1 - \phi < 0 < m \leq x_1' < x_1 \leq J_1 + \phi$, and implements (x_1', J_1). But winning at (x_1', J_1) gives the extremist strictly less than her equilibrium payoff, since $\sigma_e(x_1', J_1) > \sigma_e(x_1, J_1)$, and losing at (x_1', J_1) is even worse by Lemma A.5. So (x_1', J_1) is equilibrium-dominated for the extremist, whereas the moderate is strictly better off with that pair if elected. Therefore, equilibrium dominance places probability one on the moderate at (x_1', J_1) and, since $J_1 \leq 0 \notin \mathcal{J}^C$, the voter reelects. But then the moderate has a strictly profitable deviation — a contradiction. Thus, $x_1 \leq m$.

Cases 1–3 exhaust all equilibria and map each to exactly one clause of Definition 2. The clauses are mutually exclusive. The extremist loses in (ii), both types win with pooling and an appointee outside $\mathcal{J}^I$ in (i), and both types win with the extremist's appointee inside $\mathcal{J}^I$ in (iii). This completes the classification component.

Characterization. We now prove parts (i)–(iii) of Proposition 1.

In the equilibrium constructions below, at any off-path (x_1, J_1) at which beliefs are not pinned down by equilibrium dominance, we assign probability one to the extremist.

Part (i). We proceed in two steps. Step 1 shows that a compromising equilibrium exists if and only if

$$D(\beta) \geq \sigma_e(m, 0) \quad \text{and} \quad S_\nu \geq \sigma_e(m, 0), \tag{A.16}$$

recalling the convention that $S_\nu = +\infty$ if $\nu > \overline{\nu}^I$, so that no safe appointment exists. Step 2 shows that the first inequality in (A.16) holds if and only if $\beta \geq \overline{\beta}^c_\nu$, and that the second holds if and only if $\nu \geq \underline{\nu}$. Along the way, we show that the cutoff $\underline{\nu} \in [0, \overline{\nu}^I)$ is well defined.

Step 1. We first show that (A.16) is necessary. Consider any compromising equilibrium. By the classification component, both types pool on some (x_1, J_1) with $J_1 \leq 0$ and implemented policy $x_1 \leq m$, so $\sigma_e(x_1, J_1) = (e - x_1) + (1 - \nu)(e - J_1 - \phi) \geq \sigma_e(m, 0)$. Since the extremist wins reelection, she must weakly prefer winning at (x_1, J_1) to her optimal losing behavior. By Lemma A.5, this requires $\sigma_e(x_1, J_1) \leq D(\beta)$. Moreover, if safe appointments exist, deviating to $(x^*(e; \overline{J}^I_\nu), \overline{J}^I_\nu)$ guarantees reelection at sacrifice S_ν, so optimality also requires $\sigma_e(x_1, J_1) \leq S_\nu$. If no safe appointment exists, this inequality holds trivially because $S_\nu = +\infty$. Combining these inequalities with $\sigma_e(x_1, J_1) \geq \sigma_e(m, 0)$ yields (A.16).

Conversely, suppose (A.16) holds. We construct a compromising equilibrium in which both types choose $(x_1, J_1) = (m, 0)$, which is upheld since $m < \phi$, and win reelection. On path, the voter's belief equals her prior and, at $J_1 = 0$, the expression in the proof of Lemma A.9 holds with equality, so reelection is sequentially rational. The moderate obtains her ideal policy in both periods and wins reelection, so she does not have a profitable deviation. For the extremist, any deviation results in losing reelection unless it uses a safe appointment, since (4) fails strictly outside $\mathcal{J}^I$. By Lemma A.5, no losing deviation improves on her optimal losing behavior, which is itself unprofitable by the first inequality in (A.16). Furthermore, any deviation with $J_1' \in \mathcal{J}^I$ results in reelection at sacrifice at least $S_\nu \geq \sigma_e(m, 0)$ and is likewise unprofitable.

Finally, the supporting beliefs respect equilibrium dominance. Any action at which a reelected extremist would strictly gain has $\sigma_e < \sigma_e(m,0) \le D(\beta)$, so it is not equilibrium-dominated for her and the extremist belief is admissible. Next, if the refinement instead forces reelection, then the moderate's nonnegative sacrifice leaves her at most at her equilibrium payoff. The constructed profile is therefore a compromising equilibrium.

Step 2. Using $D(\beta) = \beta + \nu L_e$ and (A.14), the first inequality in (A.16) rearranges to $\beta \ge \sigma_e(m,0) - \nu L_e = \overline{\beta}^c_\nu$.

Consider the second inequality, $S_\nu \ge \sigma_e(m,0)$. Suppose first that $\nu \le \overline{\nu}^I$. Substituting $S_\nu = (2-\nu)(e - \overline{J}^I_\nu - \phi)$ and $\sigma_e(m,0) = (e-m) + (1-\nu)(e-\phi)$, the inequality rearranges to

$$\overline{J}^I_\nu \le \frac{m-\phi}{2-\nu}. \tag{A.17}$$

Define $g(\nu) = \frac{m-\phi}{2-\nu} - \overline{J}^I_\nu$, extending $\overline{J}^I_\nu$ to $\nu = 0$ by its right limit, computed at the end of the proof of Lemma A.4, so that (A.17) holds if and only if $g(\nu) \ge 0$. By Lemma A.4, $\overline{J}^I_\nu$ is continuous and strictly decreasing in ν over $[0, \overline{\nu}^I]$, with $\overline{J}^I_{\overline{\nu}^I} = -\phi$, so g is continuous. We establish that g crosses zero at most once, from below, which yields the cutoff.

First, we show $g(\overline{\nu}^I) > 0$. Since $\overline{J}^I_{\overline{\nu}^I} = -\phi$, we have $g(\overline{\nu}^I) = \frac{m-\phi}{2-\overline{\nu}^I} + \phi = \frac{m+\phi(1-\overline{\nu}^I)}{2-\overline{\nu}^I}$, which is positive because $\overline{\nu}^I < 1$.

Second, every zero of g is an upward crossing. Suppose $g(\nu_0) = 0$, i.e., $\overline{J}^I_{\nu_0} = \frac{m-\phi}{2-\nu_0}$. In the $\overline{\gamma}_2$ case of (A.10), using (A.11) and substituting the zero condition,

$$\begin{aligned} g'(\nu_0) &= \frac{m-\phi}{(2-\nu_0)^2} + \Big(\frac{1-p}{1+p}\Big)\frac{e-m}{(1-\nu_0)^2} \\ &= \frac{1-p}{1+p} \cdot \frac{e\big[1+(1-\nu_0)^2\big] - m - \phi(1-\nu_0)^2}{(1-\nu_0)^2(2-\nu_0)} \\ &\ge \frac{1-p}{1+p} \cdot \frac{\phi - m}{(1-\nu_0)^2(2-\nu_0)} > 0, \end{aligned}$$

where the first inequality uses $e > \phi$ and strict positivity uses $p < 1$. In the $\overline{\gamma}_1$ case, the same substitution yields $g'(\nu_0) = \frac{(1-p)(e-m)+(1-\nu_0)^2(e-\phi)}{(1-\nu_0)^2(2-\nu_0)} > 0$. The two expressions for $\overline{J}^I_\nu$ agree where the applicable case changes. Because every zero of g is an upward crossing, g changes sign at most once, and only from negative to positive.

Finally, we define the cutoff. If $g(0) \ge 0$, then $g(\nu) \ge 0$ for all $\nu \in [0, \overline{\nu}^I]$, because g cannot change sign from positive to negative. In this case, set $\underline{\nu} = 0$. If $g(0) < 0$, then continuity and $g(\overline{\nu}^I) > 0$ imply that g has a zero, and the upward-crossing property implies that the zero is unique. In this case, let $\underline{\nu} \in (0, \overline{\nu}^I)$ denote it. In either case, $\underline{\nu} \in [0, \overline{\nu}^I)$ and, over $\nu \le \overline{\nu}^I$, (A.17) holds if and only if $\nu \ge \underline{\nu}$. Thus, over $\nu \le \overline{\nu}^I$, the second inequality holds if and only if $\nu \ge \underline{\nu}$.

Now suppose $\nu > \overline{\nu}^I$. Then $S_\nu = +\infty$, so the second inequality holds, and it is also the case that $\nu > \overline{\nu}^I > \underline{\nu}$. Therefore, for all ν, the second inequality in (A.16) holds if and only if $\nu \geq \underline{\nu}$. This is the cutoff $\underline{\nu}$ in Proposition 1(i), which completes the proof of part (i). In particular, for any $\nu \in [\underline{\nu}, \overline{\nu}^I]$ and $\beta \geq \max\{\overline{\beta}^c_\nu, \overline{\beta}^{th}_\nu\}$, a compromising equilibrium coexists with a tying hands equilibrium (by Part (iii) below).

Part (ii). We prove part (ii) of Proposition 1 in two steps.

Step 1. We begin by proving the first implication. Suppose that either $\nu > \overline{\nu}^I$ or $\beta \leq \overline{\beta}^{th}_\nu$. We construct an informative appointments equilibrium. Let the extremist choose $(x_1^*(e), J_1^*(e)) = (e, e+\phi)$, and let the moderate choose $(x_1^*(m), J_1^*(m)) = a(D(\beta))$, the least-cost deterring action from Lemma A.6. This action has $J_1^*(m) \leq 0$ and satisfies the deterrence constraint $\sigma_e(x_1^*(m), J_1^*(m)) \geq D(\beta)$, with equality whenever $D(\beta) \geq \sigma_e(m, 0)$, so that (A.15) binds whenever $c^{sep} > 0$. It gives the moderate sacrifice $\sigma_m(x_1^*(m), J_1^*(m)) = c^{sep}$. The voter's strategy is sequentially rational. At the moderate's action, her posterior is m and $J_1^*(m) \leq 0$, so $J_1^*(m) \notin \mathcal{J}^C$ and she reelects by Lemma A.1. At the extremist's action, her posterior is e and $e + \phi \notin \mathcal{J}^I$, so she elects C.

Consider type e. A deviation to the moderate's action results in reelection and yields $\beta - \sigma_e(x_1^*(m), J_1^*(m)) \leq \beta - D(\beta)$, her equilibrium payoff, so it is not profitable. Any other deviation is off path, assigned to the extremist by the supporting beliefs, and results in losing reelection unless the appointee is in $\mathcal{J}^I$. By Lemma A.5, no losing action improves on $(e, e+\phi)$. If $\nu > \overline{\nu}^I$, then $\mathcal{J}^I = \varnothing$, and the extremist has no further deviations. If $\nu \leq \overline{\nu}^I$ and $\beta \leq \overline{\beta}^{th}_\nu$, then the remaining deviation is to some $J_1' \in \mathcal{J}^I$, which results in reelection at sacrifice at least S_ν. Such a deviation is not profitable if

$$-(2-\nu)(e - \overline{J}^I_\nu - \phi) + \beta \leq -\nu(2pe + (1-p)(e+m)), \tag{A.18}$$

which holds given our assumption that $\beta \leq \overline{\beta}^{th}_\nu$. Therefore, type e does not have a profitable deviation.

Next, consider type m. Any deviation outside $\mathcal{J}^I$, including to the extremist's action, results in losing reelection. It is unprofitable by Lemma A.5 because $\sigma_m(x_1^*(m), J_1^*(m)) = c^{sep} \leq (D(\beta) - (2-\nu)(e-m))^+ < D(\beta) - \nu(e-m) = D_m(\beta)$. If $\mathcal{J}^I \neq \varnothing$, then a deviation to some $J_1' \in \mathcal{J}^I$ results in reelection at sacrifice at least $(2-\nu)(m - \overline{J}^I_\nu - \phi)^+ = (S_\nu - (2-\nu)(e-m))^+ \geq c^{sep}$, where the inequality uses $D(\beta) \leq S_\nu$. This deviation is therefore not profitable either. Thus, the moderate has no profitable deviation.

Finally, the supporting beliefs respect equilibrium dominance. If $c^{sep} > 0$, then any action (x, J) at which a winning moderate would strictly gain has $\sigma_m(x, J) < c^{sep}$, so Lemma A.6(i) gives $\sigma_e(x, J) \leq \sigma_m(x, J) + (2-\nu)(e-m) < D(\beta)$. The action is not equilibrium-dominated for the extremist, so the extremist belief is admissible. If the refinement instead forces reelection at some action, then $\sigma_e(x, J) > D(\beta)$, so $\sigma_m(x, J) \geq \sigma_e(x, J) - (2 - \nu)(e - m) > c^{se\,p}$, leaving the moderate strictly below her equilibrium payoff. If $c^{se\,p} = 0$, then the moderate wins with zero sacrifice, so no deviation can make her strictly better off. This establishes that if $\nu > \overline{\nu}^I$, or $\nu \leq \overline{\nu}^I$ and $\beta \leq \overline{\beta}_\nu^{th}$, then an informative appointments equilibrium exists.

Step 2. We now prove the second implication, showing that if an informative appointments equilibrium exists, then either $\nu > \overline{\nu}^I$, or $\nu \leq \overline{\nu}^I$ and $\beta \leq \overline{\beta}_\nu^{th}$. Recall from Case 1 of the classification component that in such an equilibrium the extremist chooses $(x_1^e, J_1^e) = (e, e+\phi)$, for a normalized equilibrium payoff of $\beta - D(\beta)$. If $\mathcal{J}^I \neq \varnothing$, then she can deviate to $(x^*(e; \overline{J}_\nu^I), \overline{J}_\nu^I)$, which guarantees reelection at sacrifice exactly S_ν, for a normalized payoff of $\beta - S_\nu$. Unprofitability of this deviation requires $S_\nu \geq D(\beta)$, which is equivalent to $\beta \leq \overline{\beta}_\nu^{th}$. Therefore either $\mathcal{J}^I = \varnothing$, so that $\nu > \overline{\nu}^I$, or $\nu \leq \overline{\nu}^I$ and $\beta \leq \overline{\beta}_\nu^{th}$, as required. This completes the proof of part (ii) of Proposition 1.

Part (iii). We prove part (iii) of Proposition 1 in two steps.

Step 1. We begin by proving the first implication. Suppose that $\nu \leq \overline{\nu}^I$ and $\beta \geq \overline{\beta}_\nu^{th}$. We construct a tying hands equilibrium in which both types appoint $\overline{J}_\nu^I$, with policies $x_1^*(e) = \overline{J}_\nu^I + \phi$ and $x_1^*(m) = \min\{m, \overline{J}_\nu^I + \phi\}$. Both appointments are safe, so both types win reelection, and each equilibrium policy is optimal given the appointment by Lemma A.8. In particular, no policy deviation at $\overline{J}_\nu^I$ is profitable for either type.

Consider type e. First, a deviation to any $J_1' \in \mathcal{J}^I \setminus \{\overline{J}_\nu^I\}$ results in reelection but strictly lower utility, since her winning payoff $\beta - (2 - \nu)(e - J_1' - \phi)$ is strictly increasing over $\mathcal{J}^I$ by Lemma A.8. Second, consider a deviation to some $J_1' \notin \mathcal{J}^I$. The extremist is not reelected following such a deviation, so her payoff from it is highest at $(x_1, J_1') = (e, e + \phi)$ by Lemma A.5. Such a deviation is not profitable if

$$-(2 - \nu)(e - \overline{J}_\nu^I - \phi) + \beta \geq -\nu(2\,pe + (1 - p)(e + m)), \tag{A.19}$$

which holds because we have assumed that $\beta \geq \overline{\beta}_\nu^{th}$.

Next, consider type m. First, a deviation to any $J_1' \in \mathcal{J}^I \setminus \{\overline{J}_\nu^I\}$ results in reelection, but at a policy and no-vacancy continuation weakly to the left of their equilibrium counterparts, which lie weakly to the left of her ideal point. It is therefore not profitable. Second, consider a deviation to some $J_1' \notin \mathcal{J}^I$, after which the moderate loses under the supporting off-path beliefs. By Lemma A.5(ii), no such deviation is profitable if her equilibrium sacrifice, $\sigma_m(x_1^*(m), \overline{J}_\nu^I) = (S_\nu - (2 - \nu)(e - m))^+$, is at most $D_m(\beta)$. Since $\beta \geq \overline{\beta}_\nu^{th}$ gives $S_\nu \leq D(\beta)$, while $\nu(e - m) < (2 - \nu)(e - m)$ and $D(\beta) > \nu(e - m)$ because $L_e > e - m$, we have $(S_\nu - (2 - \nu)(e - m))^+ \leq (D(\beta) - (2 - \nu)(e - m))^+ < D(\beta) - \nu(e - m) = D_m(\beta)$. Thus, the moderate has no profitable deviation.

Finally, the supporting beliefs respect equilibrium dominance. If $x^*(m; \overline{J}^I_\nu) = m$, then the moderate wins with zero sacrifice, so no deviation can make her strictly better off. Otherwise, her equilibrium sacrifice is $S_\nu - (2-\nu)(e-m)$, and by Lemma A.6(i), every feasible pair satisfies $\sigma_e(x, J) \leq \sigma_m(x, J) + (2-\nu)(e-m)$. Therefore any action at which a winning moderate would strictly gain has $\sigma_e(x, J) < S_\nu \leq D(\beta)$. The action is not equilibrium-dominated for the extremist, so the extremist belief is admissible. If equilibrium dominance instead forces reelection at some action, then that action is equilibrium dominated for the extremist. Since her equilibrium payoff is $\beta - S_\nu$, we have $\sigma_e(x, J) > S_\nu$. Therefore $\sigma_m(x, J) \geq \sigma_e(x, J) - (2-\nu)(e-m) > S_\nu - (2-\nu)(e-m)$, so the moderate would obtain strictly less than her equilibrium payoff.

Step 2. We now prove the second implication, showing that if a tying hands equilibrium exists, then $\nu \leq \overline{\nu}^I$ and $\beta \geq \overline{\beta}^{th}_\nu$. Suppose a tying hands equilibrium exists. Then $\mathcal{J}^I$ is nonempty, so $\nu \leq \overline{\nu}^I$. Moreover, the extremist's equilibrium payoff is $\beta - S_\nu$ by Lemma A.8, while a deviation to $(x_1, J_1') = (e, e+\phi)$ yields at least her optimal losing payoff, $-\nu L_e$. Unprofitability of this deviation requires $\beta - S_\nu \geq -\nu L_e$, which is (A.19) and is equivalent to $\beta \geq \overline{\beta}^{th}_\nu$, as required. This completes the proof of part (iii) of Proposition 1.

□

A.4 Proof of Propositions 2 and 3

Proposition 2. *In equilibrium, moderate incumbents win reelection. Extremist incumbents: lose if $\beta < \min\{\overline{\beta}^c_\nu, \overline{\beta}^{th}_\nu\}$, win if $\beta > \overline{\beta}^{th}_\nu$ and $\nu \leq \overline{\nu}^I$, and otherwise can win or lose.*

PROOF. The result follows from the characterization of equilibrium behavior above. □

Proposition 3. *In a tying hands equilibrium, increasing party extremists increases the extremist's appointment J_1, while increasing ideological divergence decreases it if divergence is sufficiently large. In an informative appointments equilibrium, increasing party extremists shifts the moderate's appointee weakly further from the extremist, as does increasing ideological divergence if $\nu \geq \frac{1}{1+p}$. In a compromising equilibrium, first-period appointments do not respond to either form of polarization.*

PROOF. We prove the proposition in three parts, treating tying hands, informative appointments, and compromising equilibria in turn.

Tying hands. In every tying hands equilibrium the extremist appoints $J_1 = \overline{J}^I_\nu$, and in the equilibrium constructed in Part (iii) of the proof of Proposition 1 the moderate does as well. By (A.10), $\overline{J}^I_\nu$ equals $\overline{\gamma}_2$ or $\overline{\gamma}_1$.

First, we show that $\frac{\partial \overline{J}_\nu^I}{\partial p} > 0$ in each case. Differentiating (A.8) and (A.9) yields

$$\frac{\partial \overline{\gamma}_2}{\partial p} = \frac{2}{(1+p)^2}\Big(\phi - \frac{m-\nu e}{1-\nu}\Big) > 0 \qquad \text{and} \qquad \frac{\partial \overline{\gamma}_1}{\partial p} = \frac{e-m}{1-\nu} > 0,$$

where the first inequality holds because $\overline{\gamma}_2 < 0$. Thus, increasing party extremists increases J_1.

Next, we consider the effect of increasing e. Differentiating yields

$$\frac{\partial \overline{\gamma}_2}{\partial e} = -\Big(\frac{1-p}{1+p}\Big)\frac{\nu}{1-\nu} < 0 \qquad \text{and} \qquad \frac{\partial \overline{\gamma}_1}{\partial e} = \frac{p-\nu}{1-\nu}.$$

Thus, J_1 is strictly decreasing in e in the $\overline{\gamma}_2$ case, which by (A.10) applies whenever $\phi < \frac{e}{2}$, that is, if divergence is sufficiently large. In the $\overline{\gamma}_1$ case, the sign of the effect is that of $p - \nu$, so for smaller divergence the effect is ambiguous.

Informative appointments. First, we study the extremist's appointee. In an informative appointments equilibrium, type e chooses $J_1 = e + \phi$, which is increasing in e and constant in p.

Next, we study the moderate's appointee in the equilibrium constructed in Part (ii) of the proof of Proposition 1, $(x_1^*(m), J_1^*(m)) = a(D(\beta))$ from Lemma A.6, and its distance to the extremist's appointee, $\Delta = (e+\phi) - J_1^*(m)$. There are three cases, across which Δ is continuous.

If $D(\beta) \leq \sigma_e(m, 0)$, then $a(D(\beta)) = (m, 0)$, so $\Delta = e + \phi$ is constant in p and increasing in e.

If $\sigma_e(m,0) \leq D(\beta) \leq (2-\nu)(e-m)$, then $x_1^*(m) = m$ and $J_1^*(m)$ solves $\sigma_e(m, J_1^*(m)) = D(\beta)$, i.e., $J_1^*(m) = e - \phi - \frac{D(\beta)-(e-m)}{1-\nu}$, so that

$$\Delta = 2\phi + \frac{D(\beta)-(e-m)}{1-\nu}, \qquad \frac{\partial \Delta}{\partial p} = \frac{\nu(e-m)}{1-\nu} > 0, \qquad \frac{\partial \Delta}{\partial e} = \frac{\nu(1+p)-1}{1-\nu},$$

using $D(\beta) = \beta + \nu[e + m + p(e-m)]$. Thus, Δ is increasing in e in this case if and only if $\nu \geq \frac{1}{1+p}$.

If $D(\beta) \geq (2-\nu)(e-m)$, then $x_1^*(m) = J_1^*(m) + \phi$ and

$$J_1^*(m) = e - \phi - \frac{\nu[e+m+p(e-m)]+\beta}{2-\nu}, \tag{A.20}$$

so that

$$\Delta = 2\phi + \frac{D(\beta)}{2-\nu}, \qquad \frac{\partial \Delta}{\partial p} = \frac{\nu(e-m)}{2-\nu} > 0, \qquad \frac{\partial \Delta}{\partial e} = \frac{(1+p)\nu}{2-\nu} > 0.$$

Since Δ is continuous in (p, e) and weakly increasing in p in every case, increasing party extremists moves the moderate's appointee weakly further from the extremist's. Likewise, if $\nu \geq \frac{1}{1+p}$, then Δ is weakly increasing in e in every case, so increasing ideological divergence also moves the appointees weakly apart. If $\nu < \frac{1}{1+p}$ and $\sigma_e(m, 0) < D(\beta) < (2-\nu)(e-m)$, then Δ is strictly decreasing in e, so the moderate's appointee moves strictly closer to the extremist's.

Compromising. In the compromising equilibrium constructed in Part (i) of the proof of Proposition 1, both types choose $(x_1, J_1) = (m, 0)$. Neither p nor e enters this action, so first-period appointments do not respond to either form of polarization. □

Proposition 4. *If β is sufficiently high and $p \leq 1/3$, then: (i) the optimal probability of judicial vacancy is given by $\nu^* \in (0, \overline{\nu}^I]$, (ii) increasing party extremists increases ν^*, and (iii) greater ideological divergence decreases ν^*.*

A.5 Proof of Proposition 4

Assume β is high enough that a tying hands equilibrium exists whenever $\nu \leq \overline{\nu}^I$, and that the voter-optimal equilibrium is selected. Throughout, write $q \equiv pe + (1-p)m$ and let $G(\nu) = \overline{J}^I_\nu + \phi$, so that G is continuous and strictly decreasing in ν by Lemma A.4, with $G(\overline{\nu}^I) = 0$.

We proceed in four steps. First, we derive voter welfare in a tying hands equilibrium, $W_{TH}(\nu)$, and show that it weakly exceeds voter welfare in any compromising equilibrium at the same ν. Second, we show that no $\nu > \overline{\nu}^I$ is optimal. Third, we show that if $p \leq 1/3$, then W_{TH} is strictly increasing over $(0, \overline{\nu}^I]$, so $\nu^* = \overline{\nu}^I$. Fourth, we derive the comparative statics of ν^*. A remark following the proof characterizes the optimal vacancy rate if an extremist is likely.

Step 1. We first show that, among tying hands equilibria, the equilibrium constructed in Part (iii) of the proof of Proposition 1 weakly maximizes voter welfare. In every tying hands equilibrium, the extremist appoints $\overline{J}^I_\nu$ and, absent a vacancy, implements $G(\nu)$ in each period, so voter welfare can differ across such equilibria only through the moderate's action. Let (x, J) denote the moderate's action in an arbitrary tying hands equilibrium, and write $y = x^*(m; J)$ for her policy absent a vacancy, so that the voter's expected loss from a moderate incumbent is $|x| + \nu m + (1-\nu)|\,y|$. Since the moderate wins reelection, the extremist can imitate (x, J) and win as well, so incentive compatibility requires $\sigma_e(x, J) \geq S_\nu$. Lemma A.6(i) then gives $\sigma_m(x, J) \geq (S_\nu - (2-\nu)(e-m))^+$, and Lemma A.7 gives the reverse inequality. Thus, $\sigma_m(x, J) = (S_\nu - (2-\nu)(e-m))^+ = (2-\nu)\big(m - G(\nu)\big)^+$, using $S_\nu = (2-\nu)(e - G(\nu))$.

Suppose first that $G(\nu) \geq m$. Then $\sigma_m(x, J) = 0$, so $x = y = m$, and the moderate generates the same voter welfare as in the constructed equilibrium. Suppose instead that $G(\nu) < m$. Then $\sigma_e(x, J) \geq S_\nu = \sigma_m(x, J) + (2-\nu)(e-m)$, while the triangle inequalities in the proof of Lemma A.6 give the reverse inequality, so equality holds throughout. Equality requires $x \leq m$ and $y = J + \phi \leq m$, and then $\sigma_e(x, J) = (e - x) + (1-\nu)(e - y) = S_\nu$ rearranges to $x + (1-\nu)\,y = (2-\nu)\,G(\nu)$. Since $\overline{J}^I_\nu \geq -\phi$, we have $G(\nu) \geq 0$, so the triangle inequality gives $|x| + (1-\nu)|\,y| \geq |x + (1-\nu)\,y| = (2-\nu)\,G(\nu)$, with equality at $x = y = G(\nu)$, the moderate's behavior in the constructed equilibrium. In either case, the constructed equilibrium weakly maximizes voter welfare among tying hands equilibria.

In the constructed equilibrium, both types appoint $\overline{J}^I_\nu$. An extremist incumbent then implements $G(\nu)$, while a moderate incumbent implements $x^*(m; \overline{J}^I_\nu) = \min\{m, G(\nu)\}$. Thus,

$$W_{TH}(\nu) = -\nu q - (2-\nu)\big[\,p\,G(\nu) + (1-p)\min\{m, G(\nu)\}\big]. \tag{A.21}$$

The minimum in (A.21) reflects that a moderate incumbent facing $\overline{J}^I_\nu$ is unconstrained whenever $G(\nu) \geq m$, and so implements her ideal point.

At $\nu = \overline{\nu}^I$, we have $G = 0$, so $W_{TH}(\overline{\nu}^I) = -\overline{\nu}^I q$. As $\nu \downarrow 0$, Lemma A.4 yields $G(0) > m$, so $\lim_{\nu\downarrow 0} W_{TH}(\nu) = -2\big[\,p\,G(0) + (1-p)m\big] \equiv W_0$.

Next, we show that voter welfare in a tying hands equilibrium weakly exceeds voter welfare in any compromising equilibrium at the same ν.

Lemma A.10. *Fix $\nu \leq \overline{\nu}^I$ and suppose $\beta \geq \overline{\beta}^{th}_\nu$. Then $W_{TH}(\nu) \geq W_C(\nu)$ for every compromising equilibrium.*

PROOF. Consider a compromising equilibrium with first-period pair (x', J'), in which both types win reelection. We first locate J'. By the classification component of Proposition 1, $J' \leq 0$, $x' \leq m$, and $J' \notin \mathcal{J}^I$, so either $J' < \underline{J}^I_\nu$ or $J' > \overline{J}^I_\nu$. The former is impossible. If $J' < \underline{J}^I_\nu \leq -\phi$, then every acceptable policy and every constrained continuation policy is weakly below $J' + \phi < \overline{J}^I_\nu + \phi$, so both types would strictly gain by deviating to the safe appointment $\overline{J}^I_\nu$, which also wins reelection. Thus, $J' > \overline{J}^I_\nu \geq -\phi$, so $J' + \phi \geq 0$, $x^*(e; J') = J' + \phi$, and voter welfare is

$$W_C(\nu) = -|x'| - \nu q - (1-\nu)\big(\,p(J' + \phi) + (1-p)\,x^*(m; J')\big). \tag{A.22}$$

Denote $G = G(\nu)$ and $G' = J' + \phi > G$. We retain $|x'|$ in the welfare expression since, from $x' \leq m < e$, the incumbent incentive constraints below take the same form whether x' is positive or negative.

In equilibrium, neither type profitably deviates to the safe appointment $\overline{J}^I_\nu$, which wins reelection under any belief. Deviations to other safe appointments are dominated by $\overline{J}^I_\nu$ for both types. Since the equilibrium action and this deviation both win office, the office benefit and vacancy-state terms cancel, leaving

$$-(e - x') - (1-\nu)(e - G') \geq -(2-\nu)(e - G) \quad \iff \quad x' + (1-\nu)G' \geq (2-\nu)G \tag{A.23}$$

for the extremist, and

$$-(m - x') - (1-\nu)\max\{0,\, m - G'\} \geq -(2-\nu)\max\{0,\, m - G\} \tag{A.24}$$

for the moderate.

We compare $W_C(\nu)$ with $W_{TH}(\nu)$ in three cases.

Case 1. Suppose $G \geq m$. The right side of (A.24) is zero while both terms on its left side are nonpositive, so $x' = m$ and $G' \geq m$. Direct computation then yields $W_{TH}(\nu) - W_C(\nu) = p\big[m + (1-\nu)G' - (2-\nu)G\big]$, which is nonnegative by (A.23) evaluated at $x' = m$.

Case 2. Suppose $G < m \leq G'$. Then, (A.24) is $x' \geq m - (2-\nu)(m - G)$, so

$$\begin{aligned} W_{TH}(\nu) - W_C(\nu) &= |x'| + (1-\nu)\big[\,pG' + (1-\ p)m\big] - (2-\nu)G \\ &\geq x' + (1-\nu)\big[\,pG' + (1-\ p)m\big] - (2-\nu)G \\ &\geq (1-\nu)\,p(G' - m) \geq 0. \end{aligned}$$

Case 3. Suppose $G' < m$. Then, $x^*(m; J') = G'$ and $\min\{m, G\} = G$, so

$$\begin{aligned} W_{TH}(\nu) - W_C(\nu) &= |x'| + (1-\nu)G' - (2-\nu)G \\ &\geq x' + (1-\nu)G' - (2-\nu)G \geq 0, \end{aligned}$$

where the final inequality follows from (A.23). In every case, $W_{TH}(\nu) \geq W_C(\nu)$, as required. □

Step 2. Suppose $\nu > \overline{\nu}^I$, so that $\mathcal{J}^I = \varnothing$ and equilibria feature compromising or informative appointments. A compromising equilibrium has vacancy-state loss at least νq, so welfare is at most $-\nu q < -\overline{\nu}^I q = W_{TH}(\overline{\nu}^I)$. In an informative appointments equilibrium, with probability p an extremist plays $(x_1, J_1) = (e, e + \phi)$, generating first-period and continuation losses of e, and with probability $1- p$ the moderate's vacancy-state loss is at least νm. Welfare is therefore below $-\overline{\nu}^I q = W_{TH}(\overline{\nu}^I)$. Thus, no $\nu > \overline{\nu}^I$ is optimal.

Step 3. Consider $\nu \in [0, \overline{\nu}^I]$. Since G is continuous and strictly decreasing with $G(0) > m$ and $G(\overline{\nu}^I) = 0$, there is a unique $\nu_m \in (0, \overline{\nu}^I)$ with $G(\nu_m) = m$, and $G(\nu) \geq m$ if and only if $\nu \leq \nu_m$. Note that $W_{TH}(\nu) = -\nu q - (2-\nu) M(\nu)$ with $M(\nu) = pG(\nu) + (1-p)\min\{m, G(\nu)\}$, so $W'_{TH} = -q + M - (2-\nu)M'$. Partition $[0, \overline{\nu}^I]$ into intervals over which $\overline{J}^I_\nu$ is given by a single case of (A.10) and the minimum in M does not change. Over each such interval, by (A.11), $M'(\nu) = -\frac{\kappa(e-m)}{(1-\nu)^2}$ for an interval-specific constant $\kappa > 0$, so

$$W''_{TH}(\nu) = 2M'(\nu) - (2-\nu)M''(\nu) = \frac{2\kappa(e-m)}{(1-\nu)^3}\big[-(1-\nu) + (2-\nu)\big] = \frac{2\kappa(e-m)}{(1-\nu)^3} > 0,$$

and W_{TH} is strictly convex over each such interval. At ν_m, M' decreases by $(1-p) \cdot |G'|$ as the minimum switches to G, so W'_{TH} jumps upward. Where the applicable case of (A.10) changes, G is continuous while G' decreases from $-\frac{1-p}{1+p}\frac{e-m}{(1-\nu)^2}$ to $-\frac{(1-p)(e-m)}{(1-\nu)^2}$, so W'_{TH} again jumps upward. Thus, W'_{TH} is strictly increasing within each interval and increases at each interval boundary.

If the $\overline{\gamma}_2$ case applies at $\nu = 0$, then $W'_{TH}(0) = \frac{p}{1+p}A$, where $A \equiv (1-3p)e + 2p\phi - (1-p)m$. If the $\overline{\gamma}_1$ case applies at $\nu = 0$, then direct computation yields $W'_{TH}(0) = p(1-p)(e-m) > 0$. If $p \leq 1/3$, then $A = 2p(\phi - m) + (1-3p)(e-m) > 0$, so $W'_{TH}(0) > 0$ in either case. Because W'_{TH} is strictly increasing within each interval and increases at each interval boundary, we have $W'_{TH} > 0$ throughout. Thus, W_{TH} is strictly increasing over $(0, \overline{\nu}^I]$. Combined with Step 2, this establishes $\nu^* = \overline{\nu}^I \in (0,1)$, which yields part (i).

Step 4. If $\nu^* = \overline{\nu}^I$, then the comparative statics of ν^* are those of $\overline{\nu}^I$, as characterized in Lemma A.3. For $\phi \geq \frac{e}{2}$,

$$\frac{\partial \overline{\nu}^I}{\partial p} = \frac{e-m}{e} > 0 \qquad \text{and} \qquad \frac{\partial \overline{\nu}^I}{\partial e} = -\frac{(1-p)m}{e^2} < 0. \tag{A.25}$$

For $\phi < \frac{e}{2}$,

$$\frac{\partial \overline{\nu}^I}{\partial p} = \frac{2\phi(e-m)}{[2p\phi + (1-p)e]^2} > 0 \qquad \text{and} \qquad \frac{\partial \overline{\nu}^I}{\partial e} = -\frac{(1-p)[2p\phi + (1-p)m]}{[2p\phi + (1-p)e]^2} < 0. \tag{A.26}$$

Thus, increasing party extremists increases ν^* and greater ideological divergence decreases ν^*, establishing parts (ii) and (iii). □

Remark A.1. The argument in Step 3 also shows that W'_{TH} changes sign at most once, from negative to positive, so W_{TH} is quasiconvex over $[0, \overline{\nu}^I]$ and has no interior maximum for any p. The optimal vacancy rate is therefore determined by comparing the endpoints. If $W_{TH}(\overline{\nu}^I) \geq W_0$, then $\nu^* = \overline{\nu}^I$ for $p > 1/3$ as well. If $W_0 > W_{TH}(\overline{\nu}^I)$, then voter welfare approaches its supremum as $\nu \downarrow 0$, and no vacancy rate in $(0, 1)$ is optimal. Welfare can be nonmonotone in ν in this case.

B Extension: Appointments to Move the Median

Before proving the results, we introduce notation for the voter's expected utilities from the incumbent and the challenger, given the first-period appointee and the retiring justice. We then characterize the sets of J_1 that are "safe" for the incumbent, as in the baseline model.

Let J_t^{med} be the ideal point of the median justice of the court in period t. In the second period, an executive with ideal point i chooses policy $x_i^*(J_2^{med}) = \underset{x \in [J_2^{med} - \phi,\, J_2^{med} + \phi]}{\arg\max} -|x - i|$. If she is able to appoint a new justice, then she chooses J_2 to maximize $-|x_i^*(J_2^{med}(J_2)) - i|$, and we let $J_2^*(i)$ denote a solution. If the second-period politician cannot appoint, the court median is $J_2^{med}(J_1) = Med(L_1, L_2, R_1, R_2, J_1)$. If she appoints justice J_2 after justice $j \in \{L_1, L_2, R_1, R_2\}$ retires, the court median is $J_2^{med}(J_2 | J_1, j)$.

Two facts are used repeatedly below. First, $x_i^*(M) = \min\{\max\{i,\, M - \phi\},\, M + \phi\}$ and $|x_i^*(M) - i| = \max\{0,\, |i - M| - \phi\}$, so an officeholder who can appoint optimally induces the attainable median closest to her ideal point. Second, if justice j retires, the second-period court consists of the three remaining sitting justices, J_1, and the new appointee J_2. Letting $q_2 \leq q_3$ denote the second and third order statistics of the four justices other than J_2, the set of medians attainable as J_2 varies over $\mathbb{R}$ is $[q_2, q_3]$, and therefore $J_2^{med}(J_2^*(i) | J_1, j) = \min\{\max\{i,\, q_2\},\, q_3\}$.

The voter's expected utility from reelecting an extreme incumbent, given the first-period appointee J_1 and that justice $j \in \{L_1, L_2, R_1, R_2\}$ may retire, is

$$-\nu |x_e^*(J_2^{med}(J_2^*(e) | J_1, j))| - (1 - \nu)|x_e^*(J_2^{med}(J_1))|.$$

The voter's expected utility from electing the challenger, given J_1, is

$$\nu\Big(-\, p|x_{-e}^*(J_2^{med}(J_2^*(-e)|J_1, j))| - (1 - p)|x_{-m}^*(J_2^{med}(J_2^*(-m)|J_1, j))|\Big) \\ + (1 - \nu)\Big(-\, p|x_{-e}^*(J_2^{med}(J_1))| - (1 - p)|x_{-m}^*(J_2^{med}(J_1))|\Big).$$

With these in hand, we next define the safe sets for the incumbent. Throughout, to eliminate uninteresting cases, we maintain the assumption that if a vacancy may arise from L_1 or L_2, some appointment makes the incumbent safe at $\nu = 0$, while no appointment does so at $\nu = 1$. Formally, we assume

$$-|x_e^*(L_1)| > -\,p|x_{-e}^*(L_1)| - (1-\,p)|x_{-m}^*(L_1)|, \text{ and} \tag{A.27}$$

$$-|x_e^*(R_1)| < -\,p|x_{-e}^*(L_2)| - (1-\,p)m. \tag{A.28}$$

For $\nu = 0$, the court's composition is fixed. Then, any $J_1 \leq L_1$ yields median L_1 and (A.27) implies that the voter then prefers a known extremist to the challenger, so the election is safe. For $\nu = 1$, the configuration most favorable to I arises if L_1 may retire and $J_1 \leq L_2$. Then, I can shift the second-period median to R_1, while the extreme challenger cannot shift it further left than L_2 and the moderate challenger implements $-m$. Since (A.28) states that V nonetheless prefers C, no appointment is safe. Note that (A.27) and (A.28) hold, for example, whenever the sitting justices are symmetric about 0 with $\phi \leq R_1$ and $R_2 < e \leq R_1 + \phi$.[26]

Below, both sides of each safe-election condition are affine in ν. Thus, if a condition holds at $\nu = 0$ and fails at $\nu = 1$, then it holds if and only if ν is below a unique cutoff in $(0, 1)$.

Lemma A.11. *Suppose the at-risk justice is left-leaning, $L \in \{L_1, L_2\}$. (i) For $J_1 \leq L_1$, there exist cutoffs $\overline{\nu}^I_{L_2} \in (0,1)$ and $\overline{\nu}^I_{L_1}(J_1) \in (0,1)$ such that the election is safe for the incumbent if and only if $\nu \leq \overline{\nu}^I_{L_2}$ for $L = L_2$ and if and only if $\nu \leq \overline{\nu}^I_{L_1}(J_1)$ for $L = L_1$. (ii) For $J_1 \in [L_1, R_1]$, the election is not safe at $\nu = 1$. (iii) For $J_1 \geq R_1$, the election is never safe.*

PROOF. First, suppose $J_1 \leq L_1$. Here $J_2^{med}(J_1) = L_1$. If the retirement comes from L_2, the attainable second-period medians are $[L_1, R_1]$, so the incumbent shifts the median to R_1, the extreme challenger to L_1, and the moderate challenger to $-m$. The election is safe for I if

$$-\nu|x_e^*(R_1)| - (1-\nu)|x_e^*(L_1)| \geq -\,p|x_{-e}^*(L_1)| - (1-\,p)\big[\nu m + (1-\nu)|x_{-m}^*(L_1)|\big]. \tag{A.29}$$

At $\nu = 0$ this holds strictly, by (A.27). At $\nu = 1$ it fails by (A.28), since $|x_{-e}^*(L_1)| \leq |x_{-e}^*(L_2)|$. Thus, there exists $\overline{\nu}^I_{L_2} \in (0,1)$ such that (A.29) holds if and only if $\nu \leq \overline{\nu}^I_{L_2}$.

If the retirement comes from L_1, the attainable medians are $[\max\{L_2, J_1\},\, R_1]$, and the

[26] Under symmetry with $e \leq R_1 + \phi$, we have $|x_e^*(L_1)| = R_1 - \phi$, $|x_{-e}^*(L_1)| = |x_{-e}^*(L_2)| = e = |x_e^*(R_1)|$, and $|x_{-m}^*(L_1)| = m$ if $R_1 \leq m + \phi$ and otherwise $|x_{-m}^*(L_1)| = R_1 - \phi$. Then, (A.27) is $pe + (1-\,p)|x_{-m}^*(L_1)| > R_1 - \phi$, which holds in either case since $e > \max\{m,\, R_1 - \phi\}$, and (A.28) is $e >\, pe + (1-\,p)m$, which holds since $e > m$.

election is safe for I if

$$
\begin{aligned}
-\nu|x_e^*(R_1)| - (1-\nu)|x_e^*(L_1)| \geq & - p\Big[\nu|x_{-e}^*(\max\{L_2, J_1\})| \\
& \qquad + (1-\nu)|x_{-e}^*(L_1)|\Big] \\
& - (1-\ p)\Big[\nu m + (1-\nu)|x_{-m}^*(L_1)|\Big].
\end{aligned}
$$

Again, this holds strictly at $\nu = 0$, by (A.27), and fails at $\nu = 1$ by (A.28), since $|x_{-e}^*(\max\{L_2, J_1\})| \leq |x_{-e}^*(L_2)|$. Thus, it holds if and only if $\nu \leq \overline{\nu}_{L_1}^I(J_1)$ for a cutoff $\overline{\nu}_{L_1}^I(J_1) \in (0,1)$, which now depends on J_1 through $\max\{L_2, J_1\}$.

Next, suppose $J_1 \in [L_1, R_1]$. Then, $J_2^{med}(J_1) = J_1$ and, for either left-leaning retirement, the attainable medians are $[J_1, R_1]$. Consequently, I shifts the median to R_1, the extreme challenger keeps it at J_1, and the moderate challenger moves it to $\max\{J_1, -m\}$. The election is safe for I if

$$
\begin{aligned}
-\nu|x_e^*(R_1)| - (1-\nu)|x_e^*(J_1)| \geq & - p|x_{-e}^*(J_1)| \\
& - (1-\ p)\Big[\nu\,|x_{-m}^*(\max\{J_1, -m\})| \\
& \qquad + (1-\nu)|x_{-m}^*(J_1)|\Big].
\end{aligned}
\tag{A.30}
$$

At $\nu = 1$, (A.30) fails for every $J_1 \in [L_1, R_1]$. For $J_1 \leq \phi + m$, the RHS is at least $-\ p|x_{-e}^*(L_2)| - (1-\ p)m$, which exceeds $-|x_e^*(R_1)|$ by (A.28). For $J_1 > \phi + m$, the RHS equals $-(J_1 - \phi) > -|x_e^*(R_1)|$, since $J_1 - \phi \leq R_1 - \phi < \min\{e, R_1 + \phi\}$. Thus, a safe election in this case requires $\nu < 1$.

Finally, suppose $J_1 \geq R_1$. Then, $J_2^{med}(J_1) = R_1$ and the attainable medians are $[R_1, \min\{J_1, R_2\}]$, so both challenger types shift the median to R_1 and the incumbent to $\min\{J_1, R_2\}$. The election is safe for I if

$$
-\nu|x_e^*(\min\{J_1, R_2\})| - (1-\nu)|x_e^*(R_1)| \geq -\ p|x_{-e}^*(R_1)| - (1-\ p)|x_{-m}^*(R_1)|.
$$

This never holds, since the LHS is at most $-|x_e^*(R_1)|$, while $|x_{-e}^*(R_1)| = |R_1 - \phi|$ and $|x_{-m}^*(R_1)| \in \{m,\, |R_1 - \phi|\}$ are each strictly below $|x_e^*(R_1)| = \min\{e,\, R_1 + \phi\}$. □

Lemma A.12. *Suppose the at-risk justice is right-leaning, $R \in \{R_1, R_2\}$. For every ν, there exists $\overline{J}_R^I(\nu) \in [L_1, R_1)$ such that the election is safe for the incumbent if and only if $J_1 \leq \overline{J}_R^I(\nu)$.*

PROOF. For either right-leaning retirement, the four justices other than J_2 include L_2, L_1, J_1, and one right-leaning justice. The attainable medians after a vacancy are therefore

$[\max\{L_2, J_1\},\, L_1]$ for $J_1 \leq L_1$ and $[L_1, J_1]$ for $J_1 \in [L_1, R_1]$. For $J_1 \geq R_1$, they are $[L_1,\, \min\{J_1, R_2\}]$ if R_1 retires and $[L_1, R_1]$ if R_2 retires. We consider the same three cases.

First, suppose $J_1 \leq L_1$. The incumbent's best attainable median after a vacancy is $L_1 = J_2^{med}(J_1)$, and the moderate challenger's is likewise L_1, so only the extreme challenger's term varies with ν. The election is safe for I if

$$-|x_e^*(L_1)| \geq -\, p\big[\nu|x_{-e}^*(\max\{L_2, J_1\})| + (1-\nu)|x_{-e}^*(L_1)|\big] - (1-\, p)|x_{-m}^*(L_1)|.$$

This always holds. Since $|x_{-e}^*(\max\{L_2, J_1\})| \geq |x_{-e}^*(L_1)|$, the RHS is maximized at $\nu = 0$, where the inequality is (A.27).

Next, suppose $J_1 \in [L_1, R_1]$. The incumbent's best attainable median after a vacancy is $J_1 = J_2^{med}(J_1)$, the extreme challenger's is L_1, and the moderate challenger's is $\min\{J_1, -m\}$ if $J_1 \leq -m$ and $-m$ otherwise. The election is safe for I if

$$\begin{aligned} -|x_e^*(J_1)| \geq \nu\Big(-\, p|x_{-e}^*(L_1)| - (1-\, p)|x_{-m}^*(\min\{J_1, -m\})|\Big) \\ + (1-\nu)\Big(-\, p|x_{-e}^*(J_1)| - (1-\, p)|x_{-m}^*(J_1)|\Big). \end{aligned} \tag{A.31}$$

Let $G(J_1)$ denote the LHS minus the RHS of (A.31). At $J_1 = L_1$, (A.31) coincides with the condition from Case 1, so $G(L_1) > 0$. At $J_1 = R_1$ it fails. In the ν-term, $p|x_{-e}^*(L_1)| + (1 - p)m < |x_e^*(R_1)|$ by (A.28), and in the $(1-\nu)$-term, $p|x_{-e}^*(R_1)| + (1-\, p)|x_{-m}^*(R_1)| < |x_e^*(R_1)|$, as in the final case of the proof of Lemma A.11. Thus, $G(R_1) < 0$. Moreover, G is continuous and piecewise linear, with every piece on $[L_1, -\phi]$ increasing at rate $p\nu$, every piece on $[-\phi,\, \min\{R_1, \max\{\phi,\, e-\phi\}\}]$ weakly decreasing, and, if $R_1 > \max\{\phi, e-\phi\}$, weakly increasing at rate $1-\nu \geq 0$ on the remainder, where $G \leq G(R_1) < 0$. It follows that the safe appointments in this case are an interval $[L_1,\, \overline{J}_R^I(\nu)]$ with $\overline{J}_R^I(\nu) \in [L_1, R_1)$.

Finally, suppose $J_1 \geq R_1$. The incumbent's best attainable median after a vacancy is $\min\{J_1, R_2\}$, if R_1 retires, or R_1, if R_2 retires; the extreme challenger's is L_1; and the moderate challenger implements $-m$. In either sub-case, the election is safe for I only if

$$-|x_e^*(R_1)| \geq \nu\Big(-\, p|x_{-e}^*(L_1)| - (1-\, p)m\Big) + (1-\nu)\Big(-\, p|x_{-e}^*(R_1)| - (1-\, p)|x_{-m}^*(R_1)|\Big),$$

since the LHS above weakly overstates I's value. This never holds, by the same two comparisons as at $J_1 = R_1$ in the previous case.

Combining the three cases, the incumbent-safe appointments are in $(-\infty,\, \overline{J}_R^I(\nu)]$. $\square$

Proposition 5. *For any ν, the set of safe-election appointees if there may be a left-leaning vacancy is a subset of the set of incumbent-safe appointees if there may be a right-leaning* *vacancy.*

PROOF. By Lemma A.12, if the potential vacancy is right-leaning, then for every ν the set of safe-election appointments is $(-\infty,\, \overline{J}^I_R(\nu)]$ with $\overline{J}^I_R(\nu) \in [L_1, R_1)$. It therefore suffices to show that, for every (J_1, ν), if the election is safe when the potential vacancy is left-leaning then it is safe when the potential vacancy is right-leaning. For $J_1 \leq L_1$ the right-leaning conditions always hold, and for $J_1 \geq R_1$ the left-leaning condition never holds, so the claim is immediate in those regions.

Consider $J_1 \in [L_1, R_1]$. The terms in (A.30) and (A.31) with coefficient $(1-\nu)$ coincide, so it suffices to compare the vacancy terms,

$$S_L(J_1) = -|x^*_e(R_1)| + p|x^*_{-e}(J_1)| + (1-p)\,|x^*_{-m}(\max\{J_1, -m\})|,$$
$$S_R(J_1) = -|x^*_e(J_1)| + p|x^*_{-e}(L_1)| + (1-p)\,|x^*_{-m}(\min\{J_1, -m\})|,$$

and the common no-vacancy term $S_0(J_1) = -|x^*_e(J_1)| + p|x^*_{-e}(J_1)| + (1-p)|x^*_{-m}(J_1)|$. Safety under a left-leaning vacancy is $\nu S_L + (1-\nu)S_0 \geq 0$, while under a right-leaning vacancy it is $\nu S_R + (1-\nu)S_0 \geq 0$. There are three cases.

If $J_1 < -m-\phi$, then $x^*_e(J_1) = x^*_{-m}(J_1) = J_1+\phi < 0$, so $S_0(J_1) = p\big[(J_1+\phi)+|x^*_{-e}(J_1)|\big] > 0$, since $|x^*_{-e}(J_1)|$ equals either $\phi - J_1$ or $e > -J_1$. Moreover $S_R(J_1) = S_0(J_1) + p\big[|x^*_{-e}(L_1)| - |x^*_{-e}(J_1)|\big] \geq S_0(J_1)$, since $|x^*_{-e}(M)| = \min\{e,\, \phi - M\}$ is nonincreasing for $M \leq \phi$. Therefore $\nu S_R + (1-\nu)S_0 > 0$, so the right-leaning condition holds.

If $J_1 \in [-m-\phi,\, \phi+m]$, then $S_R(J_1) \geq S_L(J_1)$. First, $|x^*_e(J_1)| \leq \max\{m, \min\{e, J_1 + \phi\}\} \leq \min\{e,\, R_1+\phi\} = |x^*_e(R_1)|$. Second, $|x^*_{-e}(J_1)| \leq |x^*_{-e}(L_1)|$, which follows from monotonicity for $J_1 \leq \phi$, and from $|x^*_{-e}(J_1)| = J_1 - \phi \leq m < \min\{e,\, \phi - L_1\} = |x^*_{-e}(L_1)|$ for $J_1 \in (\phi,\, \phi+m]$. Third, $|x^*_{-m}(\min\{J_1, -m\})| \geq |x^*_{-m}(\max\{J_1, -m\})|$, since $|x^*_{-m}(M)| \geq m$ for $M \leq -m$ while $|x^*_{-m}(M)| \leq m$ for $M \in [-m,\, \phi+m]$. Thus, left-leaning safety implies right-leaning safety.

If $J_1 \in (\phi+m,\, R_1]$, the left-leaning condition fails. Both challenger types choose $J_1 - \phi > m$ at median J_1, so $S_0(J_1) = (J_1-\phi) - \min\{e,\, J_1+\phi\} < 0$ and $S_L(J_1) = (J_1-\phi) - \min\{e,\, R_1+\phi\} < 0$, so $\nu S_L + (1-\nu)S_0 < 0$. □

Proposition 6. *A fully informative equilibrium does not exist if office benefit is large enough.*

PROOF. Recall that in a fully informative equilibrium each type of I appoints a distinct justice and only the moderate wins reelection. Fix such an equilibrium, let (x_m, J_m) denote type m's first-period action, and consider type e's deviation to (x_m, J_m), after which the voter reelects, as she does on the path of play. Sequential rationality requires $u^*_e(e) \geq u^*_e(m)$, where $u^*_e(e)$ is type e's equilibrium payoff and $u^*_e(m)$ her payoff from the deviation.

Since type e loses reelection on the path of play, $u_e^*(e)$ contains no office benefit, and is bounded above by 0. In contrast, $u_e^*(m) \geq \beta - 3(e - L_1 + \phi)$, since the sitting justices bound how far any appointment can push outcomes to the left. For any feasible J_m, the first-period median is in $[L_1, R_1]$, so the implemented first-period policy is in $[L_1 - \phi,\ R_1 + \phi]$ and type e's first-period loss is at most $e - L_1 + \phi$. In the second period, absent a vacancy the median is again in $[L_1, R_1]$, bounding her loss by $e - L_1 + \phi$. After any retirement, at least two of the four remaining justices are weakly to the right of L_1, so type e can induce a median weakly above L_1, again bounding her loss by $e - L_1 + \phi$.

Combining, an informative appointments equilibrium requires $0 \geq \beta - 3(e - L_1 + \phi)$. Setting $\overline{\beta}^{MTM} = 3(e - L_1 + \phi)$, no fully informative equilibrium exists for $\beta > \overline{\beta}^{MTM}$. $\square$

C Extension: Judicial Uncertainty

Assume that each period there is a mean 0 shock to the judge's ideal point given by $\epsilon_t \in \{-\psi, \psi\}$, with $Pr(\epsilon_t = \psi) = 1/2$. We assume $\frac{\phi}{2} < \psi < \phi$.

In each period t, a judge with ideal point J upholds the incumbent's policy following the positive shock if and only if $x_t \in [J + \psi - \phi, J + \psi + \phi] \equiv A^+(J)$, with $\overline{a}^+ = J + \psi + \phi$ and $\underline{a}^+ = J + \psi - \phi$. Similarly, the judge upholds the policy following the negative shock if and only if $x_t \in [J - \psi - \phi, J - \psi + \phi] \equiv A^-(J)$, with $\overline{a}^+ = J - \psi + \phi$ and $\underline{a}^+ = J - \psi - \phi$. Note, $J + \psi - \phi < J - \psi + \phi$ by assumption that $\psi < \phi$. As before, if the judge overturns the policy then policy is set at the judge's ideal point, i.e., either $J - \psi$ or $J + \psi$.

We first provide some characterization of the second-period incumbent's appointment and policymaking.

Second Period

Assume the officeholder in the second period has ideal point $\hat{x}$. If she is able to appoint a new justice, then she gets her ideal policy, since the acceptance sets of the judge following the positive shock and following the negative shock overlap.

Next, assume the officeholder cannot appoint a new justice. It is optimal for the officeholder to choose either the closest policy to her ideal point that is upheld by the justice following both shocks, or the policy that is closest to her ideal point but is only upheld by the judge following one of the shocks.

We solve for the second period officeholder's precise policy choice by breaking it into cases.

1. If $J_2 < \hat{x} - \psi - \phi$ then the incumbent's optimal policy is either $\overline{a}^-$, which is accepted

following either shock, or $\overline{a}^+$, which is only upheld if there is a positive shock. We have

$$U(\overline{a}^+) = -\frac{1}{2}|J + \psi + \phi - \hat{x}| - \frac{1}{2}|J - \psi - \hat{x}| \geq -|J - \psi + \phi - \hat{x}| = U_{\hat{x}}(\overline{a}^-)$$

which holds by $\psi > \frac{\phi}{2}$.

2. Next, assume $\hat{x} - \psi - \phi < J_2 < \hat{x} + \psi - 2\phi$. The incumbent's optimal policy is either $x = \hat{x}$, which is only accepted if there is a positive shock, or $x_2 = \overline{a}^-$, which is always upheld. Thus, the officeholder chooses $x = \hat{x}$ if and only if

$$U(\overline{a}^+) = -\frac{1}{2}|J - \psi - \hat{x}| \geq -|J - \psi + \phi - \hat{x}| = U_{\hat{x}}(\overline{a}^-),$$

which holds by assumption that $J_2 < \hat{x} + \psi - 2\phi$

3. If $J \in [\hat{x} + \psi - 2\phi, \hat{x} - \psi + \phi]$, then the argument from the previous case implies that the officeholder chooses $x_2 = J - \psi + \phi$, which is always upheld.

4. Next, suppose $J_2 \in [\hat{x} + \psi - \phi, \hat{x} - \psi + \phi]$. In this case, the incumbent chooses $x_2 = \hat{x}$, since this is accepted by the judge following either shock.

5. The proofs for the final three cases follow similar arguments as the first three.

   Suppose $\hat{x} - \psi + \phi < J_2 < \hat{x} - \psi + 2\phi$. In this case, by $\psi > \frac{\phi}{2}$the incumbent chooses $x_2 = J + \psi - \phi$, which is accepted by the judge following either shock.

6. Suppose $\hat{x} - \psi + 2\phi < J_2 < \hat{x} + \psi + \phi$. In this case, the incumbent chooses $x_2 = \hat{x}_m$, which is accepted by the judge following the negative shock and overturned following the positive shock.

7. Finally, assume $\hat{x} + \psi + \phi < J_2$. The incumbent proposes $x_2 = J_2 - \psi - \phi$, which is accepted by the judge following the negative shock and overturned following a positive shock.

C.1 Uncertainty over Justice Ideology

Assuming that in each period after the incumbent politician sets policy, but before the judge rules, the judge experiences an additive shock to her ideal point. These shocks are denoted $\epsilon_t \in \{-\psi, \psi\}$ and drawn i.i.d. across periods, with $Pr(\epsilon_t = \psi) = 1/2$.[27] Thus, as in the

[27]The tying hands result is robust to more general distributions of this shock. We consider this simpler specification for ease of presentation.

baseline model, if the judge overturns the policy then policy is set at the judge's ideal point for the period, $J + \epsilon_t$. In period t, a judge with ideal point J upholds the incumbent's policy following a shock of ϵ_t if and only if $x_t \in [J + \epsilon_t - \phi, J + \epsilon_t - \phi]$.

If voters face uncertainty over the judge's ideology, this may undermine the incumbent's ability to use appointments as a commitment device. We find this is not the case; tying hands equilibria continue to exist in the extended model. However, in the face of uncertainty, a new wrinkle arises; incumbents sometimes gamble, risking judicial action in an attempt to implement policies closer to their own ideal point. This may occur even in a tying hands equilibrium, with the judge overruling policy with positive probability, even while the incumbent wins reelection. The following result establishes this formally.

Proposition A.1. *If ν is sufficiently low and β is sufficiently high, then there exists a tying hands equilibrium in the extended model with judicial uncertainty. Furthermore, if $2\phi - \psi < e < \psi + \phi$, then first-period policy is overturned with positive probability on the path of play in this equilibrium.*

PROOF. The proof proceeds in three steps. First, we characterize the set of safe justices, showing that the maximum judge in this set (1) exists and (2) is such that $J < 0$. Next, we show that if ν is sufficiently low and β is sufficiently high, a tying hands equilibrium exists. Finally, we show that if $2\phi - \psi < e < \psi + \phi$ that first-period policy is overturned with positive probability on the path of play in a tying hands equilibrium.

We begin by characterizing the set of judges that render the election safe for the incumbent. Note that by our characterization of second-period appointments and policymaking above, holding fixed the location of the second period justice J_2, the incumbent sets x_2 to minimize $|\hat{x}_2 - E[x|x_2, J_2]|$, where $E[x|x_2, J_2]$ is the expected policy given the incumbent's policy choice x_2 and the location of the judge J_2.

As an intermediate step, we show that given ideal points $\hat{x}_i \neq \hat{x}_j$, it must be that in equilibrium $|\hat{x}_i - E[x|x^*(\hat{x}_i), J_2]| \leq |\hat{x}_i - E[x|x^*(\hat{x}_j), J_2]|$, where x_i^* and x_j^* are the equilibrium choices of x_2 for players with ideal points $\hat{x}_i$ and $\hat{x}_j$, respectively. For a contradiction, suppose that $|\hat{x}_i - E[x|x^*(\hat{x}_i), J_2]| \leq |\hat{x}_i - E[x|x^*(\hat{x}_j), J_2]|$. However, this contradicts the assumption that x_i^* is an equilibrium policy choice, as $\hat{x}_i$ could deviate to x_j^*. From this, it follows that $E[x|x^*(-m), J| \leq E[x|x^*(-e), J]$ for all J, as $-e < -m$. This implies that the voter's continuation value of a challenger, U_v^c is such that $U_v^c \geq -\nu e - (1 - \nu)|E[x|x^*(-e), J|$. Therefore, a sufficient condition for the incumbent to be unsafe for some J is that $|E[x|x^*(e), J]| \geq |E[x|x^*(-e), J]|$.

Using this, we show that all $J \geq 0$ are unsafe for the incumbent. There are two cases to consider. First, consider $J \geq 0$ such that $E[x|x^*(-e), J] < 0$. Note that in this case

$|E[x|x^*(-e), J]|$ is decreasing in J, $|E[x|x^*(-e), J]|$ is increasing in J, and $|E[x|x^*(-e), 0]| = |E[x|x^*(e), 0]|$. Thus, the sufficient condition holds and no such J is safe. Second, consider $J \geq 0$ such that $E[x|x^*(-e), J] > 0$. The arguments above imply that in this case $E[x|x^*(-e), J] < E[x|x^*(e), J]$. Consequently, we have that $|E[x|x^*(-e), J]| < |E[x|x^*(e), J]|$, which is sufficient for the incumbent to be unsafe. Therefore, all $J \geq 0$ are unsafe for the incumbent.

Now we show that for sufficiently low $\nu > 0$, the set of safe judges is nonempty. A sufficient condition for the incumbent to be safe for a judge J is if a known extremist incumbent is preferred by the voter to a known moderate challenger. This holds if and only if

$$-\nu m - (1-\nu)|E[x|x^*(-m), J]| \leq -\nu e - (1-\nu)|E[x|^*(e), J]|.$$

Rearranging, this holds if and only if

$$\nu \leq \frac{|E[x|x^*(-m), J]| - |E[x|x^*(e), J]|}{e - m + |E[x|x^*(-m), J]| - |E[x|x^*(e), J]|}.$$

To complete the argument note that for all $J < \psi - \phi$ the right hand of this inequality is strictly positive, as $|E[x|x^*(-m), J]| - |E[x|x^*(e), J]|$ by the arguments above. Therefore, for all such J there exists sufficiently small $\nu > 0$ such that the set of safe justices is nonempty.

We now show that the maximum safe judge exists. The set of safe justices is defined by the set of J such that $U_v^c(J) \leq U_v^e(J)$. Because both $U_v^c(J)$ and $U_v^e(J)$ are continuous, we know that this set is closed. Further, we have already shown that the set of safe justices is bounded above by 0. Therefore, a maximum safe judge exists.

We now show that for sufficiently small $\nu > 0$ and sufficiently large β that a tying hands equilibrium exists. Second period behavior in this equilibrium is pinned down by the behavior above. In the first period, the voter reelects if they observe the maximum safe judge paired with any policy choice and elects a challenger otherwise. After an off-path action, the voter places probability 1 on type e. Both types of the incumbent choose the maximum safe justice and set policy that minimizes the distance between the expected first period policy and their ideal point given the location of the judge.

It is immediate that the voter cannot deviate profitably, given her beliefs. We now show that neither type of the incumbent has a profitable deviation in the first period. Note that a deviation to any other safe judge by either type of incumbent is not profitable, as the equilibrium safe judge maximizes the player's utility, subject to winning reelection. Second, a deviation to any unsafe judge results in losing the election, and so such a deviation cannot be profitable for β sufficiently high.

Finally, note that if $2\phi - \psi < e < \psi + \phi$, then our discussion of incumbent policymaking above implies that for all $J \leq 0$, the judge overturns $x^*(e)$ with positive probability. This completes the proof.

□

To conclude, we discuss how the introduction of this additional uncertainty impacts incentives for both compromising and informative appointments.

As before, there can exist an equilibrium in which the executive's choice of policy and appointments is informative, and leads to the extremist being removed from office. Unlike in the baseline model, however, the extremist now may face a tradeoff between appointing the judge that yields the best expected policy payoff today, versus appointing the judge that constrains the challenger most tomorrow.

The introduction of uncertainty about the judge's ideology does not have a significant impact on the form of compromising equilibria, since the incumbent wins reelection by manipulating the voter's beliefs. In particular, if office benefit is high, there continue to exist compromising equilibria in which the incumbent chooses a moderate policy and appoints an (expected) moderate justice. Although the exact characterization of the optimal policy and appointment shift, the executive faces similar strategic considerations with judicial uncertainty.

C.2 Extension: Probabilistic Review

Assume that in each period, after the incumbent sets policy x_t, the judge issues a ruling with exogenous probability $r \in (0, 1)$. If the judge does not rule, then x_t is implemented in period t. If the judge does rule, then he decides to uphold the policy or not, as before.

In the second period, an incumbent with ideal point i can choose a policy x in the judge's acceptance set, $[J_2 - \phi, J_2 + \phi]$ which is always implemented and yields utility $u_i(x)$, or gamble and choose a policy x outside of this set which yields expected utility $ru_{\hat{x}_2}(J_2) + (1-r)u_{\hat{x}_2}(x)$. Clearly, the officeholder chooses the x in $[J_2 - \phi, J_2 + \phi]$ closest to $\hat{x}_2$ if not gambling and chooses $x = \hat{x}_2$ if gambling. Thus, in the second period I gambles if

$$J < I - \frac{\phi}{1-r},$$

and does not gamble otherwise. Similarly, C gambles if

$$J > C + \frac{\phi}{1-r}.$$

Note, we break indifference as needed to avoid best response problems.

We show that it is easier to support a compromising equilibrium in the extended model. This is because the extremist benefits from having the option to gamble conditional on winning reelection and is hurt from giving the challenger the opportunity to gamble conditional on losing the election, relative to the baseline. Note, at $J = 0$, the conditions above imply the m type gambles if and only if the $-m$ type gambles, and same for the e and $-e$ types. Thus, as in the baseline model, if $J = 0$ and the voter retains the prior belief p that I is an extremist, then the voter is indifferent between I and C, so it is optimal to reelect the incumbent. The condition for the e type to use a compromising strategy over choosing the optimal judge and policy when losing reelection is given by

$$\begin{aligned}
&\beta - |e-m| + \max\Big\{-(1-\nu)|e-\phi|, -r(1-\nu)|e-0|\Big\} \\
&\geq (1-\nu)\max\left\{\begin{array}{l}
-r\left|e-\left(-m+\dfrac{\phi}{1-r}\right)\right| - (1-r)\big(p2e+(1-p)(e+m)\big), \\
p\left(-r\left|e-\left(-e+\dfrac{\phi}{1-r}\right)\right| - (1-r)|e+e|\right) \\
\qquad -(1-p)\left|e-\left(-e+\dfrac{\phi}{1-r}-\phi\right)\right|, \\
-\left|e-\left(-e+\dfrac{\phi}{1-r}-\phi\right)\right|
\end{array}\right\} \\
&\quad -\nu\big[p2e+(1-p)(e+m)\big].
\end{aligned}$$

The LHS is weakly lower at $r = 1$ than at $r' < 1$, and strictly for r' sufficiently low, while the RHS is weakly higher at $r = 1$ than for $r' < 1$, and strictly for r' sufficiently low. Thus, the office benefit required to support a compromising equilibrium for $r < 1$ is weakly lower than in the baseline, and strictly lower for r sufficiently low.

We also show the tying hands equilibrium from the baseline game always exists in the extended game if $r < 1$ but above some threshold. Assume a tying hands equilibrium exists in the baseline game and let J' be the judge that is appointed by the extremist in this equilibrium. In the second period, e does not gamble if

$$\begin{aligned}
&J' > e - \frac{\phi}{1-r} \\
&\Leftrightarrow r > \frac{e-J'-\phi}{e-J'},
\end{aligned}$$

and note that the RHS of the final inequality is always strictly less than 0. Moreover, since e is not gambling and $J' < 0$ this implies that the $-e$ and $-m$ types of C also do not gamble. Thus, the analysis from the baseline model carries through exactly for any r that satisfies the above inequality.