Electoral Competition into Collective Policymaking*

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Abstract

Parties holding institutional power regularly struggle in elections. To analyze how collective policymaking shapes electoral competition, we model a majoritarian election for an office that participates in legislative bargaining. The election affects policy through the winner's proposals and, in equilibrium, by altering what other officeholders would pass: a moderate winner constrains partisan extremists, while an extreme winner enables them. In centrist constituencies, the party whose aligned officeholders hold less institutional power has stronger incentives to converge and is therefore favored. In partisan-leaning constituencies, the constituency-aligned party is favored. These results yield a party-driven logic for *midterm loss* and *majority-party disadvantage* that accommodates party strongholds even without intrinsic voter partisanship. The analysis also addresses why majority parties consolidate procedural advantages despite electoral costs, and why candidate polarization persists under intense majority competition.

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Introduction

Parties with institutional power in government often struggle in elections. In the US, the president's party has lost House seats in 20 of the last 22 midterm elections. This pattern of *midterm loss* also befalls governors' parties at the state level (Folke and Snyder, 2012) and incumbent presidents' parties during non-concurrent legislative elections in other presidential systems (Kedar, 2009). More broadly, majority-party status is persistently associated with weaker electoral performance, including in US state legislatures (Feigenbaum et al., 2017).

These patterns have sparked interest in understanding *why* these parties struggle electorally.[1] One potentially important factor is the configuration of government: institutional structures, distribution of procedural rights, or partisan and ideological composition. This factor is central to theories of *partisan balancing*, which argue that moderate voters seek to elect officeholders from different parties because they collectively produce moderate policy outcomes.[2] In these accounts, midterm losses arise because voters know the president's party during non-concurrent elections, so moderates are more inclined to vote for the opposing party's legislative candidates. Crucially, these theories focus on voters and largely set aside the role of parties in elections.[3]

Yet electoral outcomes are shaped by the behavior of voters *and* parties.[4] In the US, parties play a key role in selecting candidates (Bawn et al., 2012) and can field diverse candidates tailored to different constituencies (Ansolabehere et al., 2001), while majority parties can organize institutional rights. Classic theories suggest that these parties would have incentives to adjust for electoral gain, whether through their candidates' positions (Downs, 1957; Wittman, 1983; Calvert, 1985) or their allocation of institutional rights (Cox and McCubbins, 2005). This raises questions about observed electoral disadvantages. Why do parties holding institutional power consistently suffer electoral losses when they could adjust their electoral or in-government strategies? And, at the same time, why do some constituencies remain *party strongholds* that routinely elect one party's candidates regardless of that party's institutional power (Krasa and Polborn, 2018)?[5]

To sharpen theoretical understanding of these patterns, we study a fundamental question:

[1] Midterm loss is widely recognized: "For some reason, the president—whoever the president is—the midterms are tough. Why would they be tough? If we're doing great, they should be easy" (President Trump, April 8, 2025).

[2] See Alesina and Rosenthal (1989, 1996); Fiorina (1996), and Kedar (2009).

[3] Prominent alternative explanations for midterm loss also focus on voters: coattail effects (Hinckley, 1967; Campbell, 1985), turnout changes (Campbell, 1987), referendum voting on the executive (Tufte, 1975), and loss aversion (Patty, 2006). See Folke and Snyder (2012) for discussion and empirical evidence.

[4] Kedar (2009, p. 192): "electoral processes take (at least) two to tango—voters and parties."

[5] The Economist (2023) calls the coexistence of safe districts and persistent alternation in party control "the great mystery of American politics."

how does the prospect of collective policymaking shape electoral competition? We analyze how policymaking institutions and constituency characteristics jointly affect voter behavior and party strategy, including which candidates they nominate and how they allocate legislative rights. In doing so, we provide a unified strategic logic for distinct partisan electoral advantages including *midterm losses* and *party strongholds*.

Our core innovation is a tractable game-theoretic framework that integrates majoritarian electoral competition with post-election collective policymaking, explicitly linking voters, party competition, and collective policymaking.[6] Two policy-motivated parties each nominate a candidate, and voters elect one by majority rule. The winning candidate participates—alongside other officeholders—in collective policymaking, modeled as legislative bargaining structured by proposal and veto rights (Banks and Duggan, 2000). When choosing their candidates, parties are uncertain about voter preferences but anticipate how the electoral outcome feeds into policymaking.

In this framework, the election can affect policy outcomes through two channels: directly, through the election winner's own policy proposals, or indirectly, by altering proposals made by other officeholders. The indirect channel operates via two mechanisms. First, the election may affect the allocation of proposal or veto rights—e.g., by changing the location of a pivotal legislator or which party holds majority control. Second, even without altering other officeholders' preferences and institutional rights, the winner's ideology affects their bargaining strategies.

Our baseline model holds the institutional environment fixed to isolate the effects of the winner's ideology. The winner bargains with three other officeholders: two *partisan extremists* located at the party ideal points and a *veto player* located between them. This parsimonious structure represents various collective policymaking settings, including legislatures or separation-of-powers systems.[7] Interpreted narrowly, the baseline resembles elections—such as midterms or special elections—where powerful officeholders are already in place and the winner affects what those officeholders can achieve.[8] The framework applies more broadly, however, to settings with uncertainty about concurrent elections.

We establish existence of an essentially unique equilibrium, and characterize the candidates, their win probabilities, and policy outcomes. We then show how they vary with institutional rights and constituency characteristics. We apply this analysis to study midterm loss and

[6]As Krehbiel et al. (2005, p. 113) note: "these multistage choices are substantively complex and analytically unwieldy, particularly if modeled explicitly and considered in total, from citizen preferences through government outcomes."

[7]In a legislative interpretation, the veto player is a pivotal legislator. In a separation-of-powers setting, the veto player may be the president or a moderate legislator, depending on the ideological configuration. In either interpretation, partisan extremists represent the majority and minority parties-in-government.

[8]For instance, during midterm elections, the president and two-thirds of senators are fixed.

party strategy in allocating legislative rights.

Our baseline reveals a key mechanism operating through the winner's impact on the veto player's bargaining position. When the winner is more extreme—further from the veto player—the veto player's continuation value worsens, expanding the set of proposals that can pass in equilibrium. An extreme winner thus enables partisan extremists to pass more extreme proposals; a moderate winner constrains them.

The winner's indirect effects shape voter and party preferences over candidates. Players evaluate candidates based on how far a candidate's location is from (i) the player's own ideal point (*candidate proximity*) *and* (ii) the veto player (*candidate extremism*). The extremism consideration arises endogenously from the winner's indirect effects on extremist proposals. Proximity concerns dominate—each player's ideal winner shares their ideal point—but extremism affects how players compare pairs of candidates.

How the voter and parties assess candidate extremism differs. A voter near the veto player dislikes extremism, since it enables both partisan extremists to pass more extreme proposals. Parties, by contrast, evaluate extremism based on the distribution of institutional rights: a party benefits from extremism when its aligned extremist holds greater proposal rights than the opposing extremist, and is harmed otherwise. We call the party aligned with the stronger extremist the *strong-extremist party*, and its opponent the *weak-extremist party*.

In the election, parties face a classic tradeoff: converging toward the opposing party improves electoral chances but yields less favorable policy if they win. We show that preferences over candidates satisfy a single-crossing condition, so there is a unique indifferent voter location; a party wins if and only if the realized median voter is on its side of that location. Furthermore, the indifferent voter location is always sufficiently close to the veto player, so decisive voters dislike extremism. Thus, although both parties can always improve their win probability by shifting their candidate toward their opponent, the winner's indirect effects can make this tradeoff asymmetric across parties. When convergence by either party reduces candidate extremism, the strong-extremist party—which dislikes constraining its allies in government—is less inclined to converge than the weak-extremist party, which benefits from constraining its powerful opponents. If one party converges far enough that its candidate crosses the veto player's location, however, then further convergence by this party *increases* candidate extremism and voters reward such convergence less.

Due to the potential asymmetries in parties' electoral tradeoffs, equilibrium characterization depends on constituency characteristics—specifically, whether parties believe the median voter will be near the veto player (*centrist constituency*) or lean strongly toward one party (*partisan-leaning constituency*). In each case, policymaking institutions generate systematic but distinct electoral patterns.

In a centrist constituency, parties' equilibrium candidates are on opposite sides of the veto player. The weak-extremist party is favored to win, as stronger moderation incentives translate into an electoral advantage. This *partisan balancing* pattern intensifies as extremist proposal rights become more unequal. Importantly, the result is party-driven: it emerges even if voters evaluate candidates on proximity alone. Moreover, greater extremist proposal rights can *reduce* candidate polarization: voters more strongly reward moderation, inducing both parties to converge.

In a partisan-leaning constituency, parties' equilibrium candidates are on the same side of the veto player. The party aligned with the electorate's lean is favored to win. This *party stronghold* pattern is voter-driven: voters discount further convergence by the party whose candidate has crossed the veto player. We thus provide a logic for strongholds that does not require any intrinsic partisan attachments. Moreover, greater total extremist proposal rights strengthens strongholds, as the favored party becomes even more likely to win. Both candidates also shift toward the veto player, yet under mild conditions on the voter distribution candidate divergence nonetheless grows because the disadvantaged party's candidate moderates faster.

We apply this baseline analysis to two substantive questions: why does the president's party lose seats at midterms? And why do parties concentrate proposal rights among extremists despite electoral costs?

First, we identify a novel, party-driven mechanism for midterm loss. We consider a single district election in a presidential year, when the president's party is unknown, compared to a midterm year, when the president is known.[9] We capture presidential power through a fixed share of proposal rights allocated to the president's party extremist. The model predicts midterm losses in centrist districts. The president's party has weaker incentives to moderate during midterms than in the presidential year, since moderation constrains a known co-partisan rather than a potential out-partisan president.[10] By contrast, we find no midterm loss in partisan-leaning districts, where extremists' relative proposal rights do not affect electoral outcomes. We additionally show midterm losses in centrist districts are more likely when the president's victory is more surprising.

Our analysis also addresses why majority parties seek to consolidate proposal rights despite electoral costs. We consider parties' preferences over shifting proposal rights from the veto player toward their aligned extremist. We show policy-motivated parties prefer to empower their extremist because policymaking gains outweigh electoral disadvantages in

[9]This application demonstrates that our framework can incorporate exogenous uncertainty over the policymaking environment—here, the outcome of a concurrent presidential election.

[10]This logic has a distant connection to Crain and Tollison (1976)'s argument that legislators from the governor's opposition party work harder to win seats.

centrist districts. This helps explain why majority status is electorally costly: majority party leaders prefer to concentrate proposal rights among ideologically committed members rather than moderates, even if it hurts the party's electoral prospects.

We relax the assumption of a fixed institutional environment in two extensions. First, we allow extremist proposal rights to depend on the winner's party, capturing elections that impact relative party power in policymaking (e.g., which party holds majority control). Our core results are robust: the weak-extremist party remains advantaged in a centrist constituency, while the constituency-aligned party is favored to win in a partisan-leaning one. Varying the strength of this link between elections and extremist proposal rights, we show candidates may remain quite polarized even as competition for majority control intensifies. Second, we study elections where the winner becomes the veto player. In that setting, the strong-extremist party is favored to win in centrist districts.

A final extension studies the role of voter sophistication about policymaking. We show that partisan balancing can arise even when voters evaluate candidates solely on proximity, but strongholds weaken with more proximity-oriented voters.

Related Literature

We integrate majoritarian electoral competition with collective policymaking to analyze how institutional rights shape both legislative bargaining and electoral outcomes.[11] Our framework connects to canonical models of electoral competition (Downs, 1957; Wittman, 1983; Calvert, 1985) while incorporating policymaking institutions. Unlike models that integrate electoral competition with reduced-form policymaking (Grofman, 1985; Krasa and Polborn, 2018; Desai and Tyson, 2025) or candidate entry with voting over exogenous proposals (Patty and Penn, 2019), we model policymaking as sequential bargaining structured by proposal and veto rights (Banks and Duggan, 2000).

Our approach identifies how policymaking institutions produce systematic partisan advantages. This mechanism is distinct from previously studied sources, including differences in risk aversion (Farber, 1980), candidate valence (Groseclose, 2001; Aragones and Palfrey, 2002), policy implementation costs (Xefteris and Zudenkova, 2018), and national-party platforms (Krasa and Polborn, 2018). In our framework, advantages arise endogenously from collective policymaking rather than from exogenous local or national factors (Eyster and Kittsteiner, 2007; Krasa and Polborn, 2018; Zhou, 2025).

[11] Various models analyze proportional representation elections into legislative bargaining (Austen-Smith and Banks, 1988; Baron and Diermeier, 2001; Cho, 2014) or simultaneous majoritarian elections with spillovers (Zhou, 2025).

Krasa and Polborn (2018) also study electoral competition into collective bodies. In their model, local candidates compete simultaneously across many districts and voters care about both their district candidate's platform and the majority party's national platform. Although they allow national platforms to depend on winners' ideologies, they do not explicitly model collective policymaking. We focus on a single election, isolating how institutional constraints shape electoral incentives.[12] This allows us to explain both party strongholds and partisan balancing through policymaking considerations—identifying where each occurs and why parties maintain arrangements despite electoral costs.

Theories of partisan balancing (Alesina and Rosenthal, 1989, 1996; Kedar, 2009) and divided government (Fiorina, 1996) emphasize voter-driven mechanisms. Our results highlight a party-driven mechanism that generates similar patterns even when voters evaluate candidates on proximity alone. Other party-driven theories address divided government—Jacobson (1990) studies how strategic candidate entry responds to electoral conditions, and Ingberman and Villani (1993) show that risk-averse parties can sustain divergence by solving a coordination problem—but do not analyze electoral competition. Our framework also connects to thermostatic models of public responsiveness, which document that opinion shifts against the policy direction of the current government (Wlezien, 1995; Soroka and Wlezien, 2010). Our analysis identifies a complementary channel: voters who anticipate policymaking consequences favor candidates who constrain powerful extremists, while parties aligned with greater institutional power have weaker incentives to offer such candidates.

Finally, we contribute to the legislative bargaining literature (Baron and Ferejohn, 1989; Banks and Duggan, 2000; McCarty, 2000; Kalandrakis, 2006), which traditionally analyzes how institutional rights shape policy outcomes with fixed participants. We shed light on how policymaking institutions affect who is selected to participate. Models featuring delegation and selection have addressed this question (Harstad, 2010; Gailmard and Hammond, 2011; Kang, 2017), but we endogenize a participant through electoral competition. A key feature of our analysis is that we allow general delay costs during bargaining, unlike prior work that either precludes delay (Klumpp, 2010) or assumes it is costless (e.g., Beath et al., 2016). This generates the extremism considerations that shape how players evaluate candidates beyond ideological proximity, as voters and parties care about how candidates affect what extremists can pass.

[12] Other models of legislative elections across multiple districts with preference-aggregated policy include Hinich and Ordeshook (1974); Austen-Smith (1984, 1986); Morelli (2004). Elsewhere, elections are based on national party platforms determined via collective choice among legislative incumbents (Snyder, 1994; Snyder and Ting, 2002; Ansolabehere et al., 2012) or centralized party leadership (Callander, 2005).

Model

Our model integrates electoral competition with legislative bargaining to study how collective policymaking institutions affect electoral outcomes.

Players. The key players are two electoral parties, L and R; a voter, v; and a continuum of potential candidates. Three additional players participate exclusively in policymaking: a veto player $\mathcal{M}$ and two partisan extremists, $\mathcal{L}$ and $\mathcal{R}$.

Timing. The game has two phases: electoral competition and collective policymaking.

Electoral phase. Parties L and R simultaneously nominate candidates, ℓ and r. Voter v observes the candidates and elects one.

Policymaking phase. The policymaking phase is sequential bargaining with random recognition among four players: the elected candidate $e \in \{\ell, r\}$ and players $\mathcal{M}$, $\mathcal{L}$, and $\mathcal{R}$. At time $t = 1, 2, \ldots$, a proposer is selected according to recognition distribution $\rho = (\rho_e, \rho_{\mathcal{M}}, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$, where $\rho_i \in [0, 1]$ denotes player i's recognition probability, $\sum \rho_i = 1$, and $\rho_e > 0$. The selected player proposes policy $x_t \in X = [-\overline{X}, \overline{X}]$. Veto player $\mathcal{M}$ either accepts, ending bargaining, or rejects, continuing bargaining to time $t + 1$.[13]

Preferences. Players have spatial policy preferences represented by absolute loss utility. When policy $x \in X$ is enacted, player i with ideal point i receives per-period utility $u_i(x) = -|i - x|$. We fix the veto player at $\mathcal{M} = 0$ and set $\mathcal{L} = -\overline{X}$ and $\mathcal{R} = \overline{X}$ to capture partisan extremists in government. Electoral parties are located at their aligned extremists' positions: $L = -\overline{X}$ and $R = \overline{X}$.

Cumulative payoffs sum per-period utilities discounted by a common factor $\delta \in (0, 1)$ and are normalized by $1 - \delta$. Players receive a common benefit of agreement $c > 2\overline{X}$, with disagreement utility normalized to zero. Specifically, if policy x passes at time t, the cumulative payoff to player i is $\delta^{t-1} \cdot (c - |i - x|)$.

Information. All features of the game are common knowledge except the voter's ideal point, v, which is not observed by either party. Instead, parties L and R share a common prior belief that v is distributed according to cumulative distribution function F with density f, which is log-concave, differentiable, and has full support.[14]

[13] Our bargaining subgame is a special case of Banks and Duggan (2000) and Cardona and Ponsati (2011). As usual, it is equivalent to an unknown finite horizon with constant termination probability.

[14] Many commonly used probability distributions, including the Normal distribution, satisfy these assumptions.

Equilibrium Concept. We study strategy profiles that are (i) pure strategy subgame perfect equilibria in the election phase and (ii) stationary subgame perfect equilibria in the policymaking phase for any elected candidate $e \in X$.

Parameter Restrictions. We maintain two assumptions throughout the main analysis.

Assumption 1. Suppose $\delta \in (\overline{\delta}, 1)$, where $\overline{\delta} = \frac{c - \overline{X}}{c - (\rho_{\mathcal{L}} + \rho_{\mathcal{R}} + \rho_e) \cdot \overline{X}} \in (0, 1)$.

Assumption 1 ensures bargaining delays are costly but not prohibitive, and the veto player rejects proposals at either party's ideal point in equilibrium. The latter property is not necessary but highlights our key forces.

Assumption 2. Suppose $\rho_{\mathcal{L}} + \rho_{\mathcal{R}} < \frac{1}{2\delta}$.

Assumption 2 limits extremist proposal rights, so moderates retain meaningful proposal rights and players' continuation values over e satisfy the *ally principle*: if they could unilaterally appoint the winner, each player would choose one sharing their ideal point. Consequently, parties moderate only for electoral reasons.

For exposition, we maintain the stronger Assumption 2a for most of the analysis. We relax it in Appendix F.

Assumption 2a. Suppose $\rho_e + \rho_{\mathcal{L}} + \rho_{\mathcal{R}} < \frac{1}{2\delta}$.

Model Discussion. In our model, parties select candidates who will bargain strategically with other officeholders if elected.[15] We model bargaining as a *minimal legislative process* (Baron, 1994), structured by proposal and veto rights. Our bargaining environment can be interpreted in various ways to capture key aspects of legislative or separation-of-powers settings.[16] To emphasize institutional factors, we focus on stationary, sequentially rational strategies (Baron and Kalai, 1993). Our bargaining setting corresponds to a *bad status quo* environment (Banks and Duggan, 2000, 2006), where delay costs enter through discounting rather than explicit status quo policies. Substantively, this represents policy issues lacking a clear status quo (Diermeier and Vlaicu, 2011) or where existing policy has decayed sufficiently to spur action (Callander and Martin, 2017).[17]

In the election stage, parties are uncertain about voter ideal points, following classic models (Wittman, 1983; Calvert, 1985; Roemer, 1994).[18] Parties are purely policy-motivated

[15] Baron and Diermeier (2001) highlight the merits of this approach.

[16] For extensive discussions of our bargaining environment and its interpretations, see Baron and Ferejohn (1989); Baron (1991); McCarty (2000); Kalandrakis (2006), and Eraslan and Evdokimov (2019).

[17] See Diermeier and Vlaicu (2011) for further discussion.

[18] See Ashworth and Bueno de Mesquita (2009) and Duggan (2014) for discussions of uncertainty about voter preferences.

and can select any candidate position, emphasizing the role of policymaking conditions rather than electoral frictions.[19] Our results do not require electoral parties to share ideal points with extremist politicians, only that both are sufficiently far from $\mathcal{M}$.[20] Lastly, the voter fully anticipates how candidates affect bargaining outcomes. We relax this feature in an additional extension to study which of our results are voter-driven versus party-driven.[21]

In the baseline model, we study a single election into an otherwise fixed collective body, setting aside dynamic considerations (Forand, 2014) and simultaneous elections (Callander, 2005; Krasa and Polborn, 2018; Zhou, 2025). This applies directly to non-concurrent elections—such as special or midterm elections—where powerful officeholders are already in place. This baseline setting also incorporates uncertainty over concurrent elections through voter and party expectations over post-election institutional rights, as our midterm loss analysis demonstrates.

With the collective body fixed, the election outcome affects policymaking only through the winner's ideal point. This facilitates a clean comparison with standard models (Wittman, 1983; Calvert, 1985) and allows us to isolate how the winning candidate's ideology can have indirect effects on the behavior of other officeholders that shape electoral competition. In practice, a single election may affect collective policymaking through additional channels, such as by reallocating institutional power between parties. Substantively, midterm loss and majority-party disadvantage have arisen across eras and institutional settings that differed widely in the degree to which a single election impacts relative party power, suggesting these patterns do not require this channel.

In two extensions, we incorporate such channels by allowing the election to affect institutional rights. First, we allow the proposal rights vector ρ to depend on the election by introducing variable extremist proposal rights allocated to the partisan extremist aligned with the winner's party. This captures an election that affects the balance of power between parties in policymaking—for instance, by determining majority control of a legislature. Second, we analyze a setting in which the veto player's location $\mathcal{M}$ depends on the election outcome. Specifically, we model an election in which the winner becomes the veto player, such as elections for executives or pivotal legislators.

[19] Allowing small win motivation would not substantially change our main insights, though it would require a different existence argument due to payoff discontinuities (Reny, 2020).

[20] Our framework allows for other configurations of electoral party ideal points, which may alter parties' convergence incentives through the policy channel.

[21] Varying voter sophistication is uncommon. Existing work typically fixes voters as sophisticated or naive. An exception is Merrill III and Adams (2007).

Analysis

We first characterize equilibrium policymaking, showing how the election winner affects policy outcomes through direct and indirect channels. We then characterize preferences over election winners and electoral competition. We apply these results to midterm loss and parties' allocation of proposal rights. Extensions allow election outcomes to affect institutional rights and vary voter sophistication.

Equilibrium Policymaking and the Election Winner's Effects

The policymaking subgame has a unique equilibrium (Cardona and Ponsati, 2011): each proposer offers the policy closest to their ideal point that veto player $\mathcal{M}$ will accept. This *acceptance set* forms a symmetric interval around $\mathcal{M} = 0$. Its radius varies with the winner's ideal point, e, through its effect on $\mathcal{M}$'s continuation value. Specifically, the equilibrium acceptance set $A(e) = [-\overline{x}(e), \overline{x}(e)]$ has radius:

$$\overline{x}(e) = \begin{cases} \frac{\delta\rho_e\,|e|+(1-\delta)\,c}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else,} \end{cases} \tag{1}$$

where $\overline{x} = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)}$ and $\rho_E = \rho_{\mathcal{L}} + \rho_{\mathcal{R}}$ denotes total extremist proposal rights. Assumption 1 implies $\mathcal{L}$ and $\mathcal{R}$ are outside $A(e)$ for any e, so both extremists propose its nearest boundary.

Lemma 1 (Cardona and Ponsati (2011))**.** *For each $e \in X$, the equilibrium acceptance set is $A(e) = [-\overline{x}(e), \overline{x}(e)]$ and the unique policy lottery assigns:*

a. *probability $\rho_{\mathcal{M}}$ to 0 (the veto player's ideal point),*
b. *probability $\rho_{\mathcal{L}}$ to $-\overline{x}(e)$ (the leftmost policy in the acceptance set),*
c. *probability $\rho_{\mathcal{R}}$ to $\overline{x}(e)$ (the rightmost policy in the acceptance set), and*
d. *probability ρ_e to $\min\{\overline{x}, \max\{-\overline{x}, e\}\}$ (the election winner's proposal).*

The winner affects policymaking through two channels. The *direct channel* operates through the winner's own proposal when recognized: they propose their ideal point e if it lies within the acceptance set, or the nearest boundary otherwise. The *indirect channel* operates through the winner's effect on $\mathcal{M}$'s acceptance set $A(e)$, which determines what extremists can pass when recognized. Remark 1 characterizes how the acceptance set—and thus the indirect channel—varies with e.

Remark 1. *The radius of the equilibrium acceptance set, $\overline{x}(e)$, is continuous in e and: (i) equal to $\overline{x}$ for all $e \notin (-\overline{x}, \overline{x})$, (ii) strictly decreasing over $e \in (-\overline{x}, 0)$, and (iii) strictly increasing over $e \in (0, \overline{x})$.*

A key takeaway from Remark 1 is that moderation begets moderation, while extremism enables extremism. A more centrist election winner—one closer to $\mathcal{M} = 0$—improves $\mathcal{M}$'s continuation value by proposing a more centrist policy when recognized. This shrinks the acceptance set, constraining what extremists can pass. Conversely, a more extreme winner worsens $\mathcal{M}$'s continuation value: the acceptance set expands, enabling extremists to pass more extreme policies. Figure 1 illustrates.

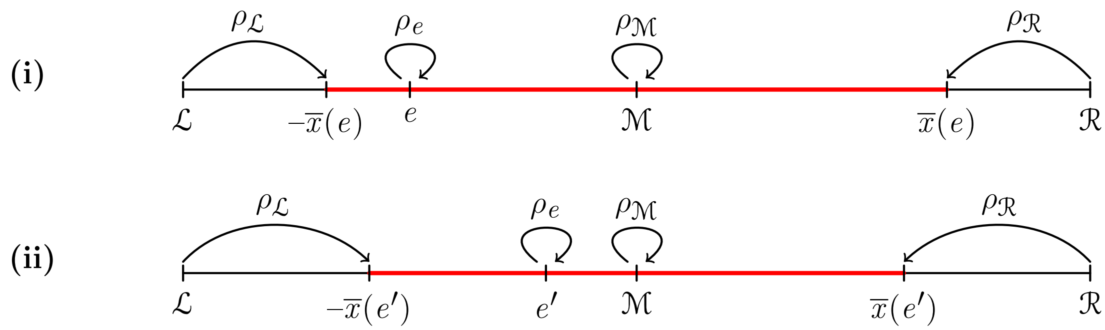


Figure 1. Effect of the winner's ideal point, e versus e', on equilibrium policymaking. A winner closer to $\mathcal{M}$ proposes more moderate policy and constrains proposals by extremists $\mathcal{L}$ and $\mathcal{R}$.

Preferences over Election Winners

We now characterize players' preferences over election winners. We first establish general properties, then sharpen parties' preferences and identify the indifferent voter location for any candidate pair.

Lemma 1 implies that player i's continuation value from electing a winner with ideal point e is:

$$\mathcal{U}_i(e) = \rho_e \cdot u_i(x_e(e)) + \rho_{\mathcal{L}} \cdot u_i(-\overline{x}(e)) + \rho_{\mathcal{R}} \cdot u_i(\overline{x}(e)) + \rho_{\mathcal{M}} \cdot u_i(0), \tag{2}$$

where $x_e(e) = \min\{\overline{x}, \max\{-\overline{x}, e\}\}$. The direct channel affects the first term: *proximity* between e and i determines i's utility from the winner's proposal. The indirect channel affects the second and third terms: the winner's *extremism*—their distance from $\mathcal{M} = 0$—determines the acceptance set radius, and thus i's utility from partisan extremist proposals. Together, proximity and extremism considerations shape how players rank potential election winners.

How the indirect channel affects player i depends on their ideal point. A moderate player $i \in (-\overline{x}(0), \overline{x}(0))$—one interior to $A(e)$ for any e— benefits from election winner centrism, which constrains both extremists. This taste for moderation intensifies with total extremist proposal rights ρ_E. An extreme player $i \notin (-\overline{x}, \overline{x})$—one always outside the acceptance set—faces a tradeoff: election winner extremism yields more favorable proposals from the aligned partisan extremist but less favorable proposals from the opposing extremist. The net preference over winner extremism depends on relative partisan extremist rights: extreme players favor winner extremism when their aligned partisan extremist holds greater proposal rights than the opposing extremist, and favor moderation otherwise. This preference also intensifies with total extremist rights ρ_E.[22]

Assumption 2 ensures that proximity concerns dominate. Each player's optimal election winner shares their ideal point, so the *ally principle* holds. Lemma 2 formalizes these properties.

Lemma 2. *For player i: $\mathcal{U}_i$ is piecewise linear, constant over $e \le -\overline{x}$ and $e \ge \overline{x}$, and single-peaked. If $i \in (-\overline{x}, \overline{x})\backslash\{0\}$, then $\mathcal{U}_i$ decreases weakly slower from its unique maximizer i toward $\mathcal{M} = 0$ than away from $\mathcal{M}$ and, in a neighborhood of i, strictly slower if $\rho_E > 0$. If $i \notin (-\overline{x}, \overline{x})$, then $\mathcal{U}_i$ is maximized by any e on its side of $(-\overline{x}, \overline{x})$ and strictly decreases as e shifts away over $(-\overline{x}, \overline{x})$.*

Parties. Each party $P \in \{L, R\}$ is outside $(-\overline{x}, \overline{x})$, so Lemma 2 implies that its continuation value from election winner e equals its utility from the mean of the policy lottery induced by e:

$$\mu_e = \rho_e \cdot x_e(e) + \rho_{\mathcal{L}} \cdot (-\overline{x}(e)) + \rho_{\mathcal{R}} \cdot (\overline{x}(e)) + \rho_{\mathcal{M}} \cdot 0. \tag{3}$$

This equivalence holds because parties have linear-loss utility and the policy lottery's support lies between their ideal points. Assumption 2 implies that μ_e strictly increases over $e \in (-\overline{x}, \overline{x})$, so $\mathcal{U}_P$ strictly decreases as e shifts away from P. Consequently, parties moderate only for electoral reasons.

When partisan extremist proposal rights are unequal ($\rho_{\mathcal{L}} \neq \rho_{\mathcal{R}}$), however, the *rate* at which $\mathcal{U}_P$ decreases differs depending on which direction e shifts. This asymmetry arises from extremism considerations: shifting e toward $\mathcal{M}$ constrains both partisan extremists, while shifting e away enables both.

Lemma 3 formalizes this asymmetry.

[22] Preferences over winner extremism for players in the intermediate regions, $i \in (-\overline{x}, -\overline{x}(0)) \cup (\overline{x}(0), \overline{x})$, are more complex since e determines whether they are inside or outside $A(e)$. However, these players necessarily lie inside $A(e)$ when e is sufficiently close to their ideal point, so their continuation value $\mathcal{U}_i$ exhibits the same asymmetry favoring moderation around their ideal point as more centrist players.

Lemma 3. *For each party $P \in \{L, R\}$, we have $\mathcal{U}_P(e) = u_P(\mu_e)$. If $\rho_{\mathcal{L}} > \rho_{\mathcal{R}}$, then*

$$\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} < -\rho_e < \frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}. \tag{4}$$

If $\rho_{\mathcal{L}} < \rho_{\mathcal{R}}$, the inequalities are reversed. If $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, they are equalities.

Lemma 3 establishes a key property: unequal partisan extremist rights generate asymmetric party preferences over election winner extremism. The party aligned with the stronger partisan extremist—the *strong-extremist party*—benefits from election winner extremism enabling its powerful ally. The party aligned with the weaker partisan extremist—the *weak-extremist party*—benefits from election winner moderation constraining its powerful opponent.

Unique Indifferent Voter Location. When comparing candidates, the voter considers both their proximity and extremism. The voter's preference between ℓ and r depends on their own location, the candidates' positions, and the distribution of proposal rights. Assumptions 1 and 2 ensure that preferences satisfy a single-crossing property: for any candidate pair (ℓ, r), there exists a unique ideal point $\iota_{\ell,r}$ that is indifferent between them. Lemma 4 characterizes $\iota_{\ell,r}$ and identifies which candidate the voter prefers.

Lemma 4. *For candidate pair $-\overline{x} \leq \ell < r \leq \overline{x}$, the voter is indifferent if and only if v is equal to:*

$$\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}\left(\frac{\ell+r}{2} - \delta\rho_E\Big(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\Big)\right), \tag{5}$$

which satisfies $\iota_{\ell,r} \in \big(\max\{\ell, -\overline{x}(r)\}, \min\{r, \overline{x}(\ell)\}\big)$. The voter chooses ℓ if $v < \iota_{\ell,r}$ and chooses r otherwise.

Lemma 4 establishes how extremism considerations shape the indifferent voter location. Without extremist proposal rights ($\rho_E = 0$), voters evaluate candidates on proximity alone, and the indifferent voter location is simply the midpoint between candidates, $\iota_{\ell,r} = \frac{\ell+r}{2}$. With extremist proposal rights ($\rho_E > 0$), a voter near the veto player benefits from candidate moderation, because it constrains partisan extremists. This shifts the indifferent voter location toward the more extreme candidate, amplifying the electoral rewards from candidate moderation. This effect strengthens with total extremist proposal rights ρ_E. Assumption 2a implies that $\iota_{\ell,r}$ is inside the equilibrium acceptance set if either candidate is elected, ensuring that parties benefit electorally from candidate moderation.

Electoral Calculus

Party P's continuation value from a candidate pair (ℓ, r) is:

$$V_P(\ell, r) = Pr(L \text{ wins} \mid \ell, r) \cdot \mathcal{U}_P(\ell) + (1 - Pr(L \text{ wins} \mid \ell, r)) \cdot \mathcal{U}_P(r).$$

From Lemma 3, $\mathcal{U}_P(\ell) = u_P(\mu_\ell)$ and $\mathcal{U}_P(r) = u_P(\mu_r)$. Since party L wins if $v < \iota_{\ell,r}$, we have $Pr(L \text{ wins} \mid \ell, r) = F(\iota_{\ell,r})$. Lemma 5 expresses party payoffs directly in terms of the indifferent voter location and expected policy means, sharpening the characterization of parties' continuation values.

Lemma 5. *For each party $P \in \{L, R\}$, the continuation value from a candidate pair satisfying $\ell < r$ is:*

$$V_P(\ell, r) = F(\iota_{\ell,r}) \cdot u_P(\mu_\ell) + \big(1 - F(\iota_{\ell,r})\big) \cdot u_P(\mu_r), \tag{6}$$

which is continuous and strictly quasiconcave in P's own candidate.

Lemma 5 highlights that parties face a classic tradeoff: converging further toward the other party's candidate improves electoral chances, but worsens policy outcomes conditional on winning. Policymaking institutions shape this tradeoff through their effects on expected policies (μ_ℓ and μ_r) and the indifferent voter location ($\iota_{\ell,r}$). The lemma also establishes that each party's payoffs are strictly quasiconcave in its own candidate, which facilitates equilibrium existence.[23]

Electoral Competition

We now characterize equilibrium properties. Players' concerns about proximity and extremism depend on the distribution of proposal rights, which combines with the voter distribution to create potentially asymmetric convergence incentives. Our analysis traces how these forces jointly determine candidate locations and electoral outcomes. Throughout the electoral analysis we restrict candidates to $[-\overline{x}, \overline{x}]$, which is without loss since any outside candidate induces the same policy lottery and payoffs as the nearest endpoint (Lemmas 1 and 2).

Proposition 1. *An equilibrium exists. Every equilibrium satisfies $-\overline{x} \leq \ell^* < r^* \leq \overline{x}$, and the equilibrium is unique.*

[23] In classic electoral models, strict quasi-concavity requires both log-concave voter distributions and concave policy utility. In our setting, party preferences over election winner ideology are piecewise linear and can be convex due to a kink at $\mathcal{M} = 0$. However, once a candidate crosses $\mathcal{M} = 0$, further convergence increases candidate extremism. Voters near the veto player dislike this, and electoral gains from convergence thus slow dramatically. This ensures strictly quasiconcave preferences over candidate ideology.

Existence follows from a fixed-point argument on parties' best responses, which are single-valued and continuous given the continuity and strict quasiconcavity of party objectives (Lemma 5). The ordering $\ell^* < r^*$ follows from standard arguments with policy motivated parties and electoral uncertainty. Uniqueness follows because the electoral game between the policy-motivated parties is constant-sum in expected policy, so equilibria are interchangeable, while party objectives are strictly quasiconcave. Equilibrium features partial convergence, reflecting standard incentives under median voter uncertainty (Duggan, 2014). Finally, the ordering $\ell^* < r^*$ implies party L's win probability is $F(\iota_{\ell^*,r^*})$.

We focus on interior, differentiable equilibria where $-\overline{x} < \ell^* < r^* < \overline{x}$ and $\ell^* \neq 0 \neq r^*$, which are characterized by parties' first-order conditions:

$$0 = \frac{\partial V_L(\ell, r)}{\partial \ell} = \frac{\partial F(\iota_{\ell,r})}{\partial \iota_{\ell,r}} \cdot \frac{\partial \iota_{\ell,r}}{\partial \ell} \cdot (\mu_r - \mu_\ell) - \frac{\partial \mu_\ell}{\partial \ell} \cdot F\big(\iota_{\ell,r}\big), \text{ and} \tag{7}$$

$$0 = -\frac{\partial V_R(\ell, r)}{\partial r} = \frac{\partial F(\iota_{\ell,r})}{\partial \iota_{\ell,r}} \cdot \frac{\partial \iota_{\ell,r}}{\partial r} \cdot (\mu_r - \mu_\ell) - \frac{\partial \mu_r}{\partial r} \cdot \Big(1 - F\big(\iota_{\ell,r}\big)\Big). \tag{8}$$

These conditions balance electoral gains against policy costs. The first term captures the electoral benefit of convergence: the marginal increase in win probability, scaled by the policy stakes (the difference in expected policy between winning and losing). The second term captures the policy cost of convergence: less favorable expected policy if elected, weighted by win probability. Parties' marginal convergence incentives thus depend on two marginal effects: how candidates affect expected policy conditional on winning ($\frac{\partial \mu_\ell}{\partial \ell}$ and $\frac{\partial \mu_r}{\partial r}$), and how candidates shift the indifferent voter location ($\frac{\partial \iota_{\ell,r}}{\partial \ell}$ and $\frac{\partial \iota_{\ell,r}}{\partial r}$). Each effect contains a symmetric proximity component and a potentially asymmetric extremism component, depending on candidate positions relative to $\mathcal{M}$.

If candidates are on opposite sides of $\mathcal{M}$, further convergence by either party results in candidate moderation. As a result, convergence has symmetric effects on the indifferent voter location. If $\rho_{\mathcal{L}} \neq \rho_{\mathcal{R}}$, the policy effects of convergence are asymmetric: the weak-extremist party has stronger policy incentives to converge than the strong-extremist party, because candidate moderation constrains the latter party's powerful aligned partisan extremist (see Lemma 3).

If candidates are on the same side of $\mathcal{M}$, further convergence by one party reduces its candidate's extremism but convergence by the other party increases its candidate's extremism. Consequently, both parties either benefit from the indirect effect of converging further or not and, due to linearity, they have symmetric policy incentives to converge. However, convergence has asymmetric electoral effects, because the indifferent voter location is more sensitive to convergence by the party that reduces candidate extremism.

Calvert-Wittman Benchmark. We first characterize a benchmark with $\rho_E = 0$, in which partisan extremists hold no proposal rights. The electoral game is then strategically equivalent to the Calvert-Wittman model with linear loss utilities (Wittman, 1983; Calvert, 1985). If additionally $\rho_e = 1$, then the policymaking environment coincides with that model. This benchmark clarifies the role of extremist proposal rights in the analysis that follows. The indirect channel is absent, so players evaluate candidates solely on proximity. Convergence by either party has symmetric effects on policy outcomes and the indifferent voter location, so they have symmetric convergence incentives. Remark 2 summarizes the key properties: equal win probabilities, candidates equidistant from the median m of the voter distribution F, and divergence determined solely by the density $f(m)$. None of these objects depend on ρ_e or $\rho_{\mathcal{M}}$.

Remark 2. *If $\rho_E = 0$, then in any interior equilibrium: (i) party L's win probability is $P_{CW} = \frac{1}{2}$, (ii) the indifferent voter location is $\iota_{CW} = m = F^{-1}(\frac{1}{2})$, (iii) candidate divergence is $r_{CW} - \ell_{CW} = \frac{1}{f(m)}$, and (iv) the candidates are $\ell_{CW} = m - \frac{1}{2f(m)}$ and $r_{CW} = m + \frac{1}{2f(m)}$.*

General Analysis. With extremist proposal rights ($\rho_E > 0$), players consider both proximity and the candidate's impact on extremist proposals. This creates potentially asymmetric convergence incentives. We show such asymmetries generate systematic partisan advantages. The nature of these advantages depends on whether equilibrium candidates are located on opposite sides of the veto player or on the same side.

Combining the first-order conditions yields the equilibrium indifferent voter location:

$$\iota^* = F^{-1}\left(\frac{\frac{\partial \mu_r}{\partial r}\frac{\partial \iota_{\ell,r}}{\partial \ell}}{\frac{\partial \mu_r}{\partial r}\frac{\partial \iota_{\ell,r}}{\partial \ell} + \frac{\partial \mu_\ell}{\partial \ell}\frac{\partial \iota_{\ell,r}}{\partial r}}\right). \tag{9}$$

This location shifts toward a party's ideal point—reducing its win probability—when that party's candidate has stronger policymaking effects or weaker electoral effects, or when the opposing candidate has weaker policymaking effects or stronger electoral effects. The magnitude of such shifts depends on the voter distribution F. Equation (9) combined with Lemma 4 pins down equilibrium candidate positions. These positions relative to $\mathcal{M} = 0$ distinguish two qualitatively different configurations.

Definition 1. The equilibrium features (i) *no crossover* if $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, and (ii) *crossover* if $-\overline{x} < \ell^* < r^* < 0$ or $0 < \ell^* < r^* < \overline{x}$.

Which configuration arises is determined by the constituency. For any interior equilibrium, explicit quantile conditions on the voter distribution F determine whether it features no crossover, crossover, or one of two transition configurations with a candidate exactly at the veto player (Appendix B). In broad strokes, sufficiently left-leaning constituencies are L-strongholds featuring crossover, roughly centrist constituencies feature partisan balancing without crossover, and sufficiently right-leaning constituencies are R-strongholds.[24]

Definition 2. A constituency is *centrist* if the equilibrium features no crossover, and *partisan-leaning* (toward L or R) if it features crossover.

No-Crossover. In the no-crossover configuration, asymmetric extremist proposal rights generate asymmetric convergence incentives. The weak-extremist party has stronger policy incentives to moderate and is therefore favored to win. Proposition 2 characterizes no-crossover equilibrium.

Proposition 2. *If there is no crossover in equilibrium, then:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$,
b. *the indifferent voter location is* $\iota^*_{\ell,r} = \check{x}_{nc} = F^{-1}\left(\frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}\right)$,
c. *candidate divergence is* $r^* - \ell^* = 2\delta(\rho_{\mathfrak{L}} - \rho_{\mathfrak{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathfrak{L}})(1-2\delta\rho_{\mathfrak{R}})}{1-\delta\rho_E}$, *and*
d. *candidates are* $\ell^* = (1-2\delta\rho_{\mathfrak{L}})\left(\check{x}_{nc} - \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}\right)$ *and* $r^* = (1-2\delta\rho_{\mathfrak{R}})\left(\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathfrak{L}}}{1-\delta\rho_E}\right)$.

Proposition 2 shows the weak-extremist party is favored to win. If $\rho_{\mathfrak{R}} > \rho_{\mathfrak{L}}$, then $P^* > \frac{1}{2}$, so party L is advantaged. This advantage increases with the imbalance in extremist proposal rights, $|\rho_{\mathfrak{L}} - \rho_{\mathfrak{R}}|$. The indifferent voter location shifts toward the strong-extremist party, reflecting that party's weaker incentives to converge.

The weak-extremist party's advantage arises because the indirect channel creates asymmetric policy incentives. Although parties have equal electoral incentives to converge further, their policy incentives are unequal. The weak-extremist party benefits from reducing candidate extremism and constraining the powerful opposing extremist. For the strong-extremist party, in contrast, reduced candidate extremism constrains its own powerful ally, which counters the electoral gain of convergence. Facing this tradeoff, the strong-extremist party is less inclined to converge, while the weak-extremist party is more inclined to do so and thus wins more often. This imbalance is party-driven.

Candidate locations depend on the voter and proposal rights distributions. When the indifferent voter location is left of the veto player ($\iota^*_{\ell,r} = \check{x}_{nc} < 0$), party L's candidate is closer

[24] Formally, indexing constituencies by rightward shifts of the voter distribution, the equilibrium configuration passes monotonically from left crossover, through no crossover, to right crossover (Remark A.2 in Appendix B). Between these regions lie transition bands in which the constituency-misaligned party locates its candidate exactly at the veto player, reflecting the kink in party policy payoffs at $\mathcal{M} = 0$ under linear loss. Higher total extremist proposal rights ρ_E shrink both crossover regions (Corollary A.4.1).

to the indifferent voter location than party R's candidate but farther from $\mathcal{M}$. The pattern is reversed when $\check{x}_{nc} > 0$. In each case, the constituency-aligned party offers proximity, while the other party compensates with more moderation.

Notably, electoral advantage may be distinct from ideological proximity to voters. The weak-extremist party's candidate can be favored despite being further from the realized voter v more than half the time. Consider a constituency in which m leans slightly toward the strong-extremist party: that party positions closer to the median m, yet a voter at m still prefers the weak-extremist party candidate. This voter, who is close to the indifferent voter location, values candidate proximity *and* moderation, and the weak-extremist candidate's greater moderation outweighs its worse proximity.

If the extremists hold equal proposal rights ($\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$), each party's gain from constraining the opposing extremist exactly offsets its loss from constraining the aligned extremist. As a result, policy incentives are symmetric ($\frac{\partial \mu_\ell}{\partial \ell} = \frac{\partial \mu_r}{\partial r}$), eliminating the source of partisan advantage. Electoral incentives remain symmetric ($\frac{\partial \iota_{\ell,r}}{\partial \ell} = \frac{\partial \iota_{\ell,r}}{\partial r}$). The election is thus a toss-up, with $P^* = \frac{1}{2}$. Compared to the Calvert-Wittman benchmark, total extremist rights ρ_E strengthen both parties' electoral incentives to converge. Therefore, candidate divergence is reduced by a factor $(1-\delta\rho_E)$. Corollary 2.1 summarizes these results, expressing equilibrium quantities in terms of the Calvert-Wittman benchmark (Remark 2).

Corollary 2.1. *If there is no crossover in equilibrium, $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, and the corresponding Calvert-Wittman candidates are interior, then: (i) party L's win probability is $P^* = \frac{1}{2}$, (ii) the indifferent voter location is $\iota_{BE} = m = F^{-1}(\frac{1}{2})$, (iii) candidate divergence is $r_{BE} - \ell_{BE} = (1-\delta\rho_E)\cdot(r_{CW} - \ell_{CW})$, and (iv) candidates are $\ell_{BE} = (1-\delta\rho_E)\cdot\ell_{CW}$ and $r_{BE} = (1-\delta\rho_E)\cdot r_{CW}$.*

Crossover. When candidates position on the same side of $\mathcal{M} = 0$, the indifferent voter location responds asymmetrically to convergence by each party. The constituency-aligned party faces stronger electoral incentives to converge and is therefore favored to win. Proposition 3 characterizes crossover equilibrium.

Proposition 3. *If there is crossover in equilibrium such that $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, then:*

a. *party L's win probability is $P^* = \frac{1}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter location is $\iota^*_{\ell,r} = \check{x}_{l\,c} = F^{-1}\left(\frac{1}{2(1-\delta\rho_E)}\right)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}_{l\,c})}$, and*
d. *candidates are $\ell^* = \check{x}_{l\,c} - \frac{1}{2f(\check{x}_{l\,c})} \cdot \frac{1-2\delta\rho_E}{1-\delta\rho_E}$ and $r^* = \check{x}_{l\,c} + \frac{1}{2f(\check{x}_{l\,c})} \cdot \frac{1}{1-\delta\rho_E}$.*

Proposition 3 shows the constituency-aligned party is favored. If both candidates are left of $\mathcal{M}$ (so $\check{x}_{l\,c} < 0$), party L wins with probability $P^* = \frac{1}{2(1-\delta\rho_E)} > \frac{1}{2}$. This advantage increases with total extremist proposal rights ρ_E. Thus, party strongholds are more pronounced when extremists hold substantial procedural power.

The mechanism differs fundamentally from the no-crossover case. The asymmetry is voter-driven instead of party-driven. Convergence by the constituency-aligned party moves its candidate toward $\mathcal{M}$, reducing candidate extremism. Convergence by the misaligned party moves its candidate away from $\mathcal{M}$, increasing extremism. Therefore, voters near the indifferent location reward convergence by the aligned party more, giving that party stronger electoral incentives to converge. Meanwhile, parties have equivalent policy incentives to converge: although further convergence has opposite effects on partisan extremist proposals, the parties also have opposite preferences for extremism.

Proposition 3 provides an institutional mechanism for party strongholds. The result does not require intrinsic partisan attachments, voter loyalty, or other non-policy considerations. Even when the voter evaluates candidates purely on policy grounds, strongholds emerge endogenously from the interaction between collective policymaking and constituency characteristics.

Unlike the no-crossover case, in crossover equilibria electoral success and ideological proximity coincide: the electorally favored party's candidate is typically closer to the realized voter v. This follows because the favored candidate is closer to the indifferent voter location—and thus closer to most realizations of v—while also being more likely to win.

The effect of extremist proposal rights on candidate divergence differs from the no-crossover case. In crossover equilibria, higher ρ_E increases the constituency-aligned party's electoral advantage, pulling the indifferent voter location further into the tail of F. Under mild conditions on the voter distribution, this reduces $f(\check{x}_{l\,c})$ and thereby increases candidate divergence.[25] Thus, stronger extremist proposal rights increase divergence in partisan-leaning constituencies by strengthening the favored party's advantage while they reduce divergence in centrist constituencies by amplifying the electoral reward for candidate moderation. This contrast suggests that empowering extremists with proposal rights may have heterogeneous effects on candidate polarization depending on constituency characteristics.

Implications for Midterm Loss

We now apply the framework to midterm loss. We analyze legislative elections in a single district but vary a key distinction between presidential and midterm election years: whether the party holding the presidency is known (Alesina and Rosenthal, 1996).

We capture presidential power through partisan extremist proposal rights. The president's

[25]A sufficient condition is that F is symmetric about its median m.

party receives proposal rights ρ_{pres}, while the opposing party receives ρ_{opp}, with $\rho_{\text{pres}} > \rho_{\text{opp}}$.[26] In a midterm year, the president's party is known, so the baseline analysis applies directly. In a presidential election year, the voter and parties share a common prior $\lambda \in (0, 1)$ that party $\mathcal{L}$ wins the presidency. They thus face uncertainty over which party will hold greater proposal rights when the district's representative enters the collective body.[27]

If the district lacks a clear partisan lean (no-crossover equilibria), outcomes depend on the election cycle. In a midterm year, the president's party is the strong-extremist party and is therefore less inclined to converge, since a more centrist candidate constrains its co-partisan president. The president's party consequently wins less often (Figures 2C and 2D; Proposition 2). In a presidential election year, parties' convergence incentives are shaped by their beliefs about which party will win the presidency. If party $\mathcal{L}$ is favored to win the presidency, it is less inclined to converge—doing so would constrain a likely co-partisan president—and thus less likely to win (Figures 2A and 2B).[28] Crucially, the asymmetry in party win probability is smaller in presidential election years than at the midterms: uncertainty over which party will hold the presidency attenuates the asymmetry in their incentives to converge.

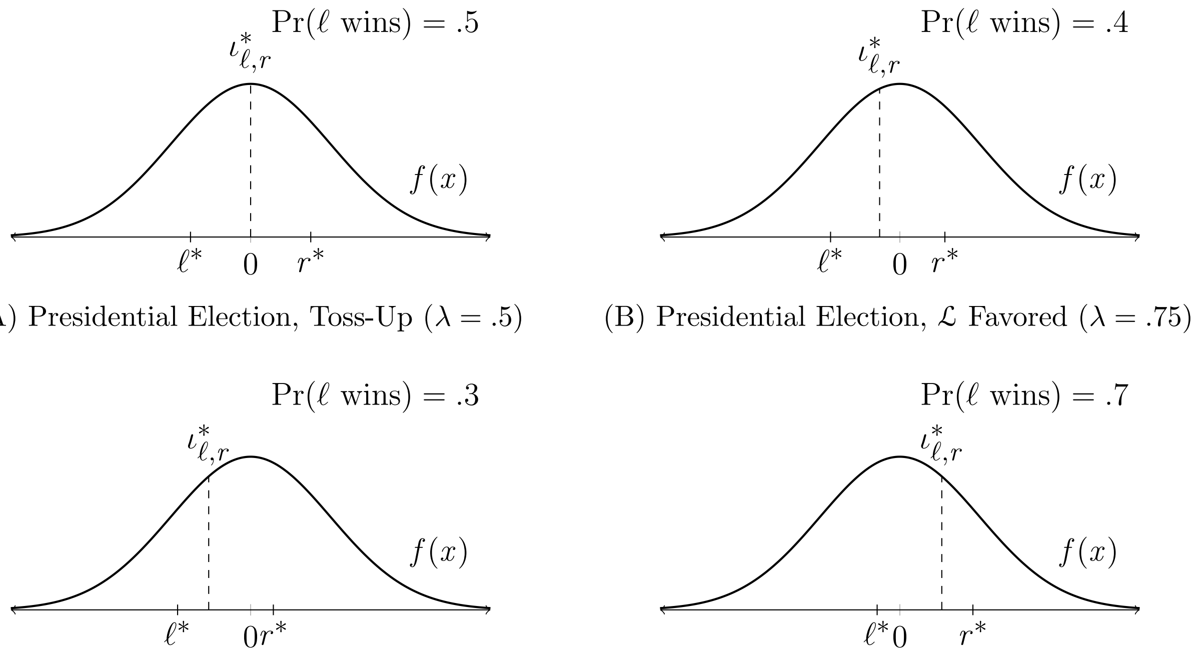
In partisan-leaning districts (crossover equilibria), regardless of which party holds the presidency, the constituency-aligned party is favored to win (Figure 3). Beliefs about the presidency affect only expectations of *relative* extremist proposal rights, which do not influence electoral outcomes in such districts (Proposition 3). Candidates are therefore identical across presidential and midterm elections.

[26]We fix the winner's and veto player's proposal rights, ρ_e and $\rho_{\mathcal{M}}$, and the voter distribution within the district across elections. We maintain Assumptions 1–2a.

[27]Players are risk-neutral, so they take expectations over policy lotteries under each possible distribution of proposal rights. Here, only relative extremist proposal rights are uncertain. The indifferent voter location is still given by Lemma 4, and λ affects electoral outcomes only through parties' calculations.

[28]Erikson (2016) shows that a party's predicted probability of winning the presidency negatively affects its performance in concurrent House elections (*anticipatory balancing*). We highlight a party-driven mechanism.

Centrist Legislative District



(A) Presidential Election, Toss-Up ($\lambda = .5$)

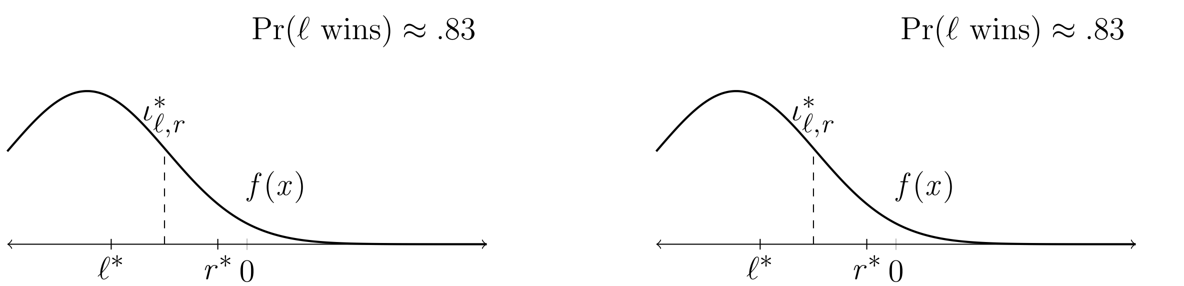
(B) Presidential Election, $\mathcal{L}$ Favored ($\lambda = .75$)

(C) Midterm Election, $\mathcal{L}$ President ($\lambda = 1$)

(D) Midterm Election, $\mathcal{R}$ President ($\lambda = 0$)

Figure 2. Equilibrium candidates (ℓ^*, r^*) and indifferent voter $\iota^*_{\ell,r}$ in a centrist district during presidential and midterm elections. Parameters: $\mathcal{M} = 0, \overline{X} = 1, c = 2, \delta = .8, \rho_e = .05, \rho_{\mathcal{M}} = .45, \rho_{\text{pres}} = .4, \rho_{\text{opp}} = .1$, and $v \sim \mathcal{N}(0, .5)$.

Left-Leaning Legislative District



(A) Presidential Election (any λ)

(B) Midterm Election

Figure 3. Equilibrium candidates (ℓ^*, r^*) and indifferent voter $\iota^*_{\ell,r}$ in left-leaning district during presidential and midterm elections. Parameters: $\mathcal{M} = 0, \overline{X} = 1, c = 2, \delta = .8, \rho_e = .05, \rho_{\mathcal{M}} = .45, \rho_{\text{pres}} = .4, \rho_{\text{opp}} = .1$, and $v \sim \mathcal{N}(-1, .5)$.

Our analysis addresses parties' probability of winning a given single district. Thus, it does not directly capture typical measures of midterm loss such as seat losses or vote share across districts (Erikson, 1988). Additionally, it sets aside dynamic linkages by instead analyzing a counterfactual comparison between otherwise identical open-seat elections. Yet, it does yield two empirical implications that can help unpack aggregate patterns.[29] First, midterm losses should concentrate in districts without a clear partisan lean. There, the president's party is strictly less likely to win during a midterm year than during a presidential election year. In partisan-leaning districts, its probability of winning is constant across elections. Second, in those districts, a more uncertain presidential election increases the difference between parties' win probabilities across the two types of election years. Intuitively, a predictable presidential election facilitates anticipatory balancing in the presidential year.

Party Preferences over Proposal Rights

Having characterized electoral equilibria, we turn to a key institutional design question: why do majority parties in legislatures concentrate proposal rights among their partisans when doing so creates an electoral disadvantage in competitive districts?

We analyze parties' preferences over shifting proposal rights from the veto player $\mathcal{M}$ to extremist $\mathcal{R}$; a reallocation that strengthens party R's legislative allies.[30] Such a shift affects party welfare through two channels.

The first is a *policymaking channel*, holding candidate positions fixed. Shifting proposal rights to extremist $\mathcal{R}$ has two effects: (i) $\mathcal{R}$ proposes more often, directly benefiting party R, and (ii) the acceptance set expands, enabling more extreme proposals from both extremists. Under Assumption 2, the direct effect dominates: party R's expected payoff increases while party L's decreases.

The second is an *electoral channel*, reflecting parties' candidate adjustments in response to changed proposal rights. The sign of this channel depends on equilibrium candidate positions. In crossover equilibria where both candidates are left of $\mathcal{M}$, the effect is positive as both candidates shift right. If both are right of $\mathcal{M}$, the effect is negative as both shift left. In no-crossover equilibria, the sign depends on the indifferent voter location and density f.[31]

Despite these competing forces, the net effect is clear: parties prefer to empower aligned extremists over centrists.

[29] Formal definitions, derivations, and conditions are in Appendix C.

[30] Appendix D provides comparative statics for other shifts in ρ and F.

[31] Specifically, the electoral channel benefits party R if $\check{x}_{nc} < \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{R}})(1-2\delta\rho_{\mathcal{L}})}{2(1-\delta\rho_E)^2}$, and benefits party L otherwise.

Remark 3. *In any differentiable interior equilibrium, increasing $\rho_{\mathcal{R}}$ at $\rho_{\mathcal{M}}$'s expense strictly increases party R's ex-ante expected payoff while strictly decreasing party L's.*

This result helps explain observed patterns in legislative organization. Majority parties consistently seek agenda control through committee assignments and procedural rules (Cox and McCubbins, 2005), even though majority status correlates with weaker electoral performance (Feigenbaum et al., 2017). While previous models study how parties allocate rights to shape policymaking (Diermeier and Vlaicu, 2011; Diermeier et al., 2015, 2016), we incorporate electoral effects. Our analysis shows that policymaking benefits from concentrated proposal rights can outweigh electoral costs—and in partisan-leaning districts, there are no electoral costs at all.

Our analysis also speaks to intra-party incentives for empowering extremists. The electoral consequences of concentrating proposal rights vary across districts: the party's win probability decreases in centrist districts but increases in its aligned partisan-leaning ones. Legislators from partisan-leaning districts thus favor empowering extremists—such as selecting a more extreme Speaker—for both policy and electoral reasons. As centrist districts become rarer, due to voter sorting or gerrymandering, parties are increasingly composed of such legislators. Changes in district composition therefore shape incentives for legislative party organization through two channels: altering the ideological makeup of party legislators and altering these legislators' electoral returns to empowering extremists.

Extensions

The baseline analysis highlighted two core mechanisms: candidates' policymaking effects determine parties' policy costs of convergence, while their effects on the indifferent voter location determine parties' electoral benefits of convergence. Asymmetric policy incentives produce partisan balancing in centrist constituencies. Asymmetric electoral incentives produce party strongholds in partisan-leaning constituencies.

We now extend the framework to probe these mechanisms and the robustness of our core results.[32] Our first two extensions allow the election's outcome to directly affect institutional rights. First, through proposal rights, by letting the distribution of these rights depend on the winner's party. Then, through veto rights, by letting the winner become the veto player. Lastly, to distinguish party-driven from voter-driven results, we vary voter sophistication about policymaking.

[32] Appendix E provides proofs, more details, and additional extensions.

Party-Dependent Extremist Proposal Rights

A single election determines which candidate wins office, but can also affect the balance of power between parties in policymaking. Since the mid-1990s, Congress has been characterized by intense competition for majority control (Lee, 2016). Under these conditions, a single district election can plausibly determine which party holds a legislative majority, and therefore controls committee chairs and other positions of institutional power.[33] The extent to which a district election affects parties' relative institutional power, however, varies across electoral and institutional contexts. When one party has a large and durable legislative majority[34], a single seat has little direct impact on parties' institutional power. We now study how such impacts of the election on parties' relative institutional power affect electoral competition.

We relax the assumption of a fixed institutional environment by allowing the distribution of (extremist) proposal rights to depend on the election winner. Let total extremist proposal rights be $\rho_E = \underline{\rho}_{\mathcal{L}} + \underline{\rho}_{\mathcal{R}} + \phi$, where $\underline{\rho}_{\mathcal{L}}$ and $\underline{\rho}_{\mathcal{R}}$ are fixed extremist rights and $\phi \geq 0$ represents variable rights allocated to the winning party's aligned extremist. If ℓ wins, extremist $\mathcal{L}$ receives proposal rights $\underline{\rho}_{\mathcal{L}} + \phi$. If r wins, extremist $\mathcal{R}$ receives $\underline{\rho}_{\mathcal{R}} + \phi$. The parameter ϕ thus captures the extent to which the election outcome affects extremists' relative proposal rights.

This setup modifies both channels of our analysis. The indifferent voter location in equilibrium is $\iota^{\phi}_{\ell,r} = \frac{\rho_e}{\rho_e+\phi} \cdot \iota_{\ell,r}$, where $\iota_{\ell,r}$ denotes the baseline indifferent voter location. For $\phi > 0$, the factor $\frac{\rho_e}{\rho_e+\phi} < 1$ reflects reduced responsiveness to candidate positions. The voter now cares about which extremist receives variable rights ϕ: a voter left of 0 prefers extremist $\mathcal{L}$, while a voter right of 0 prefers extremist $\mathcal{R}$. Consequently, voter preferences over candidates depend on candidate positions *and* which candidate's victory allocates ϕ to the voter's preferred extremist. As ϕ increases, the latter consideration receives greater weight, making the indifferent voter location less responsive to candidate positions.

Parties' policy incentives also change. Shifting a candidate's position still affects policy directly and indirectly, but the distribution of proposal rights *conditional on winning* now differs between parties. As a result, the effect of shifting a candidate e on expected policy if elected (μ^{ϕ}_e) depends both on candidate location relative to 0 *and* their party:

$$\frac{\partial \mu^{\phi}_e}{\partial e} = \frac{\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (1-2\delta(\underline{\rho}_{\mathcal{R}} + \phi \cdot \mathbb{I}\{e=r\})) & \text{if } e \in (-\overline{x}, 0), \\ (1-2\delta(\underline{\rho}_{\mathcal{L}} + \phi \cdot \mathbb{I}\{e=\ell\})) & \text{if } e \in (0, \overline{x}). \end{cases} \tag{10}$$

When candidates are located on each party's own side of 0, parties' marginal policy incentives to converge depend only on fixed extremist rights $\underline{\rho}_{\mathcal{R}}$ and $\underline{\rho}_{\mathcal{L}}$, as variable rights ϕ

[33] Even if an election does not directly determine majority status, narrow majorities imply that every seat won meaningfully affects the (relative) agenda power of the majority and minority parties.

[34] Throughout the 1960s and 70s, the Democratic party held sizable congressional majorities.

symmetrically affect the parties. When candidates are positioned on the same side, parties' policy incentives are now asymmetric, unlike in the baseline. This asymmetry stems from variable rights ϕ and the opposite effects of convergence on extremists: if $-\overline{x} < \ell < r < 0$, party R benefits more from converging as it enables extremist $\mathcal{R}$, who controls variable rights ϕ if r wins, while convergence by party L constrains extremist $\mathcal{L}$, who controls ϕ if ℓ wins.

Despite these modifications, the core results about partisan electoral advantages from the main analysis are qualitatively unchanged.

Proposition 4. *Suppose* $0 \leq \phi < \rho_E$.

(i) If there is no crossover in equilibrium, then party L's win probability is $P^* = \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}$.

(ii) If there is crossover in equilibrium such that $-\overline{x} < \ell^* < r^* < 0$, *then party L's win probability is* $P^* = \frac{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)}{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)+(1-2\delta\underline{\rho}_{\mathcal{R}})(1-2\delta\rho_E)} > \frac{1}{2}$.

Proposition 4 shows that in centrist constituencies, the weak-extremist party—based on fixed extremist proposal rights—is electorally advantaged. Party win probabilities depend only on (relative) fixed extremist proposal rights, as variable rights ϕ symmetrically impact parties' convergence incentives in centrist districts. This implies our baseline midterm-loss prediction for these districts carries over for any $\phi \in [0, \rho_E)$.

Proposition 4 also highlights that in partisan-leaning districts, the constituency-aligned party remains favored to win. As in the baseline, the indifferent voter location is more responsive to constituency-aligned party convergence—which constrains extremists— than constituency-misaligned party convergence, which enables them. This voter-based asymmetry outweighs the new asymmetry in party policy convergence incentives due to variable rights ϕ.

This extension also allows us to vary the strength of the linkage between election outcomes and institutional power captured by ϕ, such as the intensity of competition for majority control. This allows us to shed light on an empirical puzzle (Lee, 2016; Merrill et al., 2024): why do congressional races in competitive, centrist districts continue to exhibit substantial candidate divergence despite intense competition for majority control?

We consider an illustrative example of a constituency centered at the veto player ($m = 0$) with balanced fixed extremist rights ($\underline{\rho}_{\mathcal{L}} = \underline{\rho}_{\mathcal{R}}$). The effect of an increase in ϕ on candidate divergence holding ρ_E fixed, so that fixed rights decrease along $\underline{\rho}_{\mathcal{L}}(\phi) = \underline{\rho}_{\mathcal{R}}(\phi) = \frac{\rho_E - \phi}{2}$, decomposes into three forces:

$$\frac{\partial[r^* - \ell^*]}{\partial\phi} = \underbrace{-\frac{2}{\rho_e} \cdot \frac{(1-\delta)c}{1-\delta(\rho_E - \phi)}}_{\text{higher stakes } (-)} + \underbrace{\frac{1}{\rho_e} \cdot \frac{(1-\delta\rho_E)}{f(0)}}_{\text{lower voter sensitivity } (+)} + \underbrace{\frac{2\phi}{\rho_e} \cdot \frac{\delta(1-\delta)c}{(1-\delta(\rho_E - \phi))^2}}_{\text{stronger allied extremist } (+)} . \quad (11)$$

The first force encourages convergence: greater ϕ raises the electoral stakes since victory conveys institutional power to the party. The second and third forces discourage convergence. Voters are less responsive to candidate positions when elections also determine extremist power.[35] Moreover, moderation is less attractive to each party, because its aligned extremist holds greater proposal rights upon victory.

The overall effect is ambiguous. Our framework thus provides an explanation for why candidate divergence can persist in centrist districts during periods of intense majority competition, complementing explanations based on intrinsic voter attachment to parties and abstention due to alienation (Merrill et al., 2024).

Election for Veto Player

In the baseline, we fix the veto player at $\mathcal{M} = 0$ and analyze elections for a proposer who participates in bargaining. We now consider elections where the winner becomes the veto player, such as an executive with veto power or a pivotal legislator. The winner still affects other officeholders' strategic calculations through the veto player's continuation value, but now also through the additional channel of shifting the veto player's location.

Suppose the collective body consists only of the election winner e and extremists $\mathcal{L}$ and $\mathcal{R}$, with the winner serving as veto player. We assume $\rho_E < \frac{1}{2}$ so that, like the baseline, proximity concerns dominate extremism concerns. We restrict attention to parameter values such that both extremists are constrained regardless of the winner, both acceptance sets are subsets of $(-\overline{X}, \overline{X})$ and their intersection contains both candidates. If elected, candidate e's acceptance set is $A(e) = [\underline{y}(e), \overline{y}(e)]$ where $\underline{y}(e) = e - \frac{(1-\delta)c}{1-\delta\rho_E}$ and $\overline{y}(e) = e + \frac{(1-\delta)c}{1-\delta\rho_E}$. The acceptance set is now centered on the winner's ideal point e rather than 0.

A key difference from the baseline is how the winner's ideal point affects extremist proposals. In the baseline, moderation (shifting toward $\mathcal{M} = 0$) simultaneously constrains both partisan extremists. Here, shifting e rightward enables extremist $\mathcal{R}$'s proposals while constraining extremist $\mathcal{L}$'s, so both bounds of the acceptance set shift in the same direction.

The indifferent voter location is:

$$\iota^v_{\ell,r} = \frac{1}{2(1-\rho_E)} \left(\ell(1-2\rho_{\mathcal{R}}) + r(1-2\rho_{\mathcal{L}})\right). \tag{12}$$

In such an equilibrium, party L's win probability is $P^* = \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$.[36] If $\rho_{\mathcal{R}} > \rho_{\mathcal{L}}$, then $P^* < \frac{1}{2}$, so party R is favored. This reverses the baseline result, as the strong-extremist party now

[35] Krasa and Polborn (2018) identify a similar force: when voters prioritize national policymaking, parties are incentivized to nominate more extreme district candidates.

[36] The equilibrium indifferent voter location is $\check{x}^v = F^{-1}\left(\frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}\right)$, and candidates are $\ell^* = \check{x}^v - \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)}$ and $r^* = \check{x}^v + \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$.

has the electoral advantage.

The reversal stems from a different channel generating asymmetric incentives to converge. In the baseline, policy incentives are asymmetric—the weak-extremist party benefits more from moderation—while electoral incentives are symmetric. Here, policy incentives are symmetric. Shifting the veto player's position has offsetting effects on the two extremists, so convergence has equivalent policymaking costs for both parties. Electoral incentives, however, are asymmetric. The indifferent voter location is more responsive to convergence by the strong-extremist party, since it shifts the acceptance set away from its powerful ally, constraining proposals voters dislike. When the weak-extremist party converges, it shifts the acceptance set away from its ally, but this ally holds few proposal rights, so voters care less.

Elections that determine the veto player are relatively uncommon. Most legislative elections determine who participates in bargaining, not directly who holds veto power. Unlike partisan balancing, this advantage is voter-driven: with proximity voters, equilibrium reverts to Calvert-Wittman with equal win probabilities (see Appendix Corollary A.14.1). The baseline result (weak-extremist party advantage in competitive constituencies) may predominate empirically.

This extension highlights, however, that certain elections favor parties holding institutional power. Executive elections where the winner holds significant veto power could favor the party aligned with the stronger partisan extremist. This logic suggests considering whether presidential elections show different patterns than legislative elections, in particular how congressional composition relates to presidential electoral performance.[37]

Voters with Proximity Motives

Our baseline assumes voters fully anticipate how candidates affect collective policymaking. We now consider voters who evaluate candidates on proximity alone, without accounting for indirect effects on extremist proposals.[38] This allows us to identify which baseline results are party-driven versus voter-driven. With pure proximity voters, the indifferent voter location simplifies to the midpoint between candidates: $\iota^0_{\ell,r} = \frac{\ell+r}{2}$.

In constituencies without a strong partisan lean (no crossover), win probabilities are unchanged from the baseline: $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. The weak-extremist party retains its advantage because partisan balancing is party-driven—the asymmetry stems from parties' policy incentives, not voters' anticipation of policymaking consequences. Candidates diverge more

[37] Appendix E.4.2 shows that the strong-extremist party advantage can also arise in a supermajoritarian policymaking setting with two veto pivots.

[38] Appendix E.3 analyzes a model where voters place weight $\alpha \in [0, 1]$ on policymaking consequences and $(1 - \alpha)$ on proximity.

than in the baseline, by a factor of $\frac{1}{1-\delta\rho_E}$, reflecting reduced electoral rewards for moderation when voters focus only on candidate proximity.[39]

In partisan-leaning constituencies (crossover), the stronghold pattern disappears. Parties win with equal probability regardless of constituency lean, $P^* = \frac{1}{2}$. Strongholds thus only emerge in our model if voters recognize that convergence by the misaligned party increases policy extremism. When voters evaluate candidates on proximity alone, this asymmetry in electoral incentives vanishes. Of course, observed strongholds likely reflect multiple mechanisms—including intrinsic partisan attachments and local advantages—but our analysis identifies a distinct, policymaking-based channel.

This distinction has empirical implications. Partisan balancing should appear broadly across competitive constituencies. The stronghold pattern should be stronger where electorates better anticipate how candidates affect policymaking.

Conclusion

We develop a theoretical framework to study how the prospect of collective policymaking shapes electoral competition. Because the winner participates in bargaining rather than setting policy unilaterally, elections affect both the winner's proposals and those by other officeholders. Moderate winners constrain extremists, while extreme winners enable them. This indirect channel shapes how voters and parties evaluate candidates, generating electoral patterns that vary with constituency characteristics and policymaking institutions.

Our analysis unifies two empirical patterns often studied separately. Parties holding institutional power can suffer electoral disadvantages, including midterm loss, because they are reluctant to nominate moderates who would constrain powerful co-partisans. Meanwhile, party strongholds can arise from the interaction of constituency characteristics and policymaking institutions even without intrinsic partisan attachments. These mechanisms complement existing voter-driven mechanisms.

Our framework provides a lens for interpreting the effects of changes in institutional power. Reallocating proposal rights, through procedural rules or control of key institutional positions, can alter both policy outcomes and electoral prospects. Our analysis suggests policy-motivated majority parties benefit from concentrating proposal rights among partisan officeholders. Doing so enhances their policy influence, even if it negatively affects their electoral performance in centrist constituencies. This helps explain why majority parties

[39]Appendix Corollary A.15.1 shows that parties have opposing preferences over voter attentiveness to policymaking. In no-crossover equilibria, the party favored by the constituency lean prefers proximity-focused voters, while the disadvantaged party prefers attentive voters.

may seek to consolidate institutional power (Cox and McCubbins, 2005) despite its electoral downsides.

In extensions, we examine how these patterns depend on specific features of the electoral and policymaking environment. First, we study elections that also affect relative party power in policymaking, showing our core results on electoral (dis)advantages are robust. This extension also sheds light on why congressional candidates in competitive districts remain polarized during periods of intense competition for majority control (Lee, 2016). While higher stakes increase party incentives to converge, countervailing forces sustain polarization: voters are less responsive to local candidates, and—conditional on winning—moderate candidates constrain a party's powerful allies in government. Second, when the election determines who holds veto power in policymaking, we find the partisan advantage can reverse. Lastly, we show the partisan balancing pattern is party-driven and persists even when voters focus only on candidate proximity, whereas the stronghold pattern requires voters to anticipate how candidates affect extremist proposals.

Our results may help interpret empirical patterns in voter behavior by parsing how policymaking institutions impact strategic behaviors of voters *and* parties, shaping electoral outcomes. These institutions influence voters' preferences over candidates (Kedar, 2005; Duch et al., 2010), creating voting patterns often treated as separate phenomena requiring distinct assumptions (Tomz and Van Houweling, 2008). We explain phenomena like vote discounting (Adams et al., 2005) and varying responsiveness to candidate positioning (Montagnes and Rogowski, 2015) through voters' expectations about policymaking. Our specific mechanisms—e.g., extremist proposal rights affect voters' taste for moderation—also microfound observed voter heuristics (Fortunato et al., 2021).

Our analysis suggests avenues for future research. The model yields predictions about where partisan balancing should occur and how stronghold effects vary with voter sophistication. These predictions could be evaluated using variation in institutional contexts across states, countries, or time periods. The framework can also inform analyses of how redistricting shapes electoral patterns by changing the distribution of constituency types. Incorporating dynamics, incumbency advantages, and campaign spending would enrich the analysis while building on the institutional foundations established here.

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Appendix

Contents

A Proofs for Main Analysis

A.1 Policymaking Equilibrium

Let $\rho_E = \rho_{\mathcal{L}} + \rho_{\mathcal{R}}$. Define $\overline{x} = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)}$ and $\overline{x}(e) = \begin{cases} \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else.} \end{cases}$

Lemma 1 (Cardona and Ponsati (2011))**.** *For each $e \in X$, the equilibrium acceptance set is $A(e) = [-\overline{x}(e), \overline{x}(e)]$ and the unique policy lottery assigns:*

a. *probability $\rho_{\mathcal{M}}$ to 0 (the veto player's ideal point),*

b. *probability $\rho_{\mathcal{L}}$ to $-\overline{x}(e)$ (the leftmost policy in the acceptance set),*

c. *probability $\rho_{\mathcal{R}}$ to $\overline{x}(e)$ (the rightmost policy in the acceptance set), and*

d. *probability ρ_e to $\min\{\overline{x}, \max\{-\overline{x}, e\}\}$ (the election winner's proposal).*

PROOF. Given officeholder e, existence of a stationary subgame perfect equilibrium in the policymaking stage follows from Banks and Duggan (2000), and uniqueness from Cardona and Ponsati (2011). For characterization, Banks and Duggan (2000) implies $\mathcal{M}$'s acceptance set is an interval of the form $A(e) = [-y(e), y(e)]$, since $u_{\mathcal{M}}$ is symmetric about 0. When recognized, $\mathcal{M}$ proposes 0, $\mathcal{L}$ proposes $-y(e)$, $\mathcal{R}$ proposes $y(e)$, and e proposes the nearest policy to e in $A(e)$. To characterize $y(e)$, there are two cases. First, if $e \in A(e)$, then $\mathcal{M}$'s indifference condition is $c - |y(e)| = \delta(c - \rho_E|y(e)| - \rho_e|e|)$, which yields $y(e) = \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E}$. Thus, e must satisfy $c - |e| \geq \delta(c - \rho_E|y(e)| - \rho_e|e|)$, which holds if and only if $|e| \leq \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)} = \overline{x}$. Second, the preceding implies that $e \notin A(e)$ is equivalent to $e \notin [-\overline{x}, \overline{x}]$. Moreover, $\mathcal{M}$'s indifference condition is $c - |y(e)| = \delta[c - (\rho_E + \rho_e)|y(e)|]$, so $y(e) = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e)} = \overline{x}$.

Combining these two cases, we have $y(e) = \begin{cases} \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E} & \text{if } e \in [-\overline{x}, \overline{x}] \\ \overline{x} & \text{else.} \end{cases}$ The characterization of the acceptance set and proposals in the unique equilibrium yields the result. □

A.2 Preferences over Officeholder Ideology

Lemma A.1. *Under Assumptions 1 and 2, for all $i \in \mathbb{R}$, $\mathcal{U}_i(e)$ is: (i) constant over $e \leq -\overline{x}$, (ii) strictly increasing over $e \in (-\overline{x}, \min\{i, \overline{x}\})$, (iii) strictly decreasing over $e \in (\max\{i, -\overline{x}\}, \overline{x})$, and (iv) constant over $e \geq \overline{x}$.*

PROOF. For (i), all $e \leq -\overline{x}$ induce the same policy lottery, so $\mathcal{U}_i$ is constant. An analogous argument establishes (iv). For (ii), $\mathcal{U}_i$ is continuous and piecewise linear in e, so it suffices to sign its derivative wherever it exists on $(-\overline{x}, \min\{i, \overline{x}\})$. There, $e < i$ implies $\frac{\partial u_i(x_e(e))}{\partial e} = 1$, and $\frac{\partial u_i(0)}{\partial e} = 0$. Since $|u_i(x) - u_i(y)| \leq |x - y|$ for all x, y and $\left|\frac{\partial \overline{x}(e)}{\partial e}\right| = \frac{\delta\rho_e}{1-\delta\rho_E}$ for $e \neq 0$, we have $\left|\frac{\partial u_i(-\overline{x}(e))}{\partial e}\right| \leq \frac{\delta\rho_e}{1-\delta\rho_E}$ and $\left|\frac{\partial u_i(\overline{x}(e))}{\partial e}\right| \leq \frac{\delta\rho_e}{1-\delta\rho_E}$, regardless of i's position relative to the acceptance set. Thus, we have

$$\frac{\partial \mathcal{U}_i(e)}{\partial e}\Big|_{e\in(-\overline{x},\min\{i,\overline{x}\})} \geq \rho_e - (\rho_{\mathcal{L}} + \rho_{\mathcal{R}}) \cdot \frac{\delta\rho_e}{1-\delta\rho_E} > 0,$$

where the strict inequality follows from Assumption 2. Part (iii) follows analogously. $\square$

Lemma 2. *For player i: $\mathcal{U}_i$ is piecewise linear, constant over $e \leq -\overline{x}$ and $e \geq \overline{x}$, and single-peaked. If $i \in (-\overline{x}, \overline{x})\backslash\{0\}$, then $\mathcal{U}_i$ decreases weakly slower from its unique maximizer i toward $\mathcal{M} = 0$ than away from $\mathcal{M}$ and, in a neighborhood of i, strictly slower if $\rho_E > 0$. If $i \notin (-\overline{x}, \overline{x})$, then $\mathcal{U}_i$ is maximized by any e on its side of $(-\overline{x}, \overline{x})$ and strictly decreases as e shifts away over $(-\overline{x}, \overline{x})$.*

PROOF. Lemma A.1 implies each part except for the asymmetry of $\mathcal{U}_i$ around $i \in (-\overline{x}, \overline{x})\backslash\{0\}$. Consider $i \in (-\overline{x}, 0)$. Since $x_e(e) = e \in A(e)$, we have $\overline{x}(e) \geq |e|$ for all $e \in [-\overline{x}, \overline{x}]$, with strict inequality for $e \in (-\overline{x}, \overline{x})$. Thus, on $e \in (-\overline{x}, i)$ we have $-\overline{x}(e) \leq e < i < 0 < \overline{x}(e)$, so $\frac{\partial u_i(-\overline{x}(e))}{\partial e} = \frac{\partial u_i(\overline{x}(e))}{\partial e} = \frac{\delta\rho_e}{1-\delta\rho_E}$ and $\frac{\partial \mathcal{U}_i(e)}{\partial e} = \rho_e + \frac{\delta\rho_e\rho_E}{1-\delta\rho_E}$. On $e \in (i, 0)$, we have $i < e$ and $0 < \overline{x}(e)$, but i may lie inside or below $[-\overline{x}(e), \overline{x}(e)]$. Wherever differentiable, $\frac{\partial \mathcal{U}_i(e)}{\partial e} = -\rho_e + \frac{\delta\rho_e\rho_E}{1-\delta\rho_E}$ if $i > -\overline{x}(e)$, and $\frac{\partial \mathcal{U}_i(e)}{\partial e} = -\rho_e + \frac{\delta\rho_e(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})}{1-\delta\rho_E}$ if $i < -\overline{x}(e)$. Both are strictly negative by Assumption 2. Therefore $\mathcal{U}_i$ decreases at rate $\rho_e + \frac{\delta\rho_e\rho_E}{1-\delta\rho_E}$ as e moves away from i, and at rate $\rho_e - \frac{\delta\rho_e\rho_E}{1-\delta\rho_E}$ or $\rho_e - \frac{\delta\rho_e(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})}{1-\delta\rho_E}$ as e moves toward $\mathcal{M} = 0$. The respective gaps, $\frac{2\delta\rho_e\rho_E}{1-\delta\rho_E}$ and $\frac{2\delta\rho_e\rho_{\mathcal{R}}}{1-\delta\rho_E}$, are nonnegative, so the asymmetry is weak throughout. Finally, since $\overline{x}(i) > |i|$ and $\overline{x}(\cdot)$ is continuous, $i > -\overline{x}(e)$ for e in a neighborhood of i, where the gap is $\frac{2\delta\rho_e\rho_E}{1-\delta\rho_E} > 0$ for $\rho_E > 0$. The case $i \in (0, \overline{x})$ is symmetric. $\square$

Lemma 3. *For each party $P \in \{L, R\}$, we have $\mathcal{U}_P(e) = u_P(\mu_e)$. If $\rho_{\mathcal{L}} > \rho_{\mathcal{R}}$, then*

$$\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} < -\rho_e < \frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}. \quad (4)$$

If $\rho_{\mathcal{L}} < \rho_{\mathcal{R}}$, the inequalities are reversed. If $\rho_{\mathcal{L}} = \rho_{\mathcal{R}}$, they are equalities.

PROOF. For any election winner e, party ideal points are outside $\mathcal{M}$'s acceptance set: $L < -\overline{x}(e) < \overline{x}(e) < R$ for all e. Thus, for $P \in \{L, R\}$, we have $\mathcal{U}_P(e) = \rho_e \cdot (-|P - x_e(e)|) + \rho_{\mathcal{L}} \cdot (-|P + \overline{x}(e)|) + \rho_{\mathcal{R}} \cdot (-|P - \overline{x}(e)|) + \rho_{\mathcal{M}} \cdot (-|P - 0|) = -|P - (\rho_e \cdot x_e(e) + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \overline{x}(e))| = u_P(\mu_e)$.

For the second part, we have $\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(-\overline{x},0)} = -\rho_e - \frac{\delta\rho_e(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})}{1-\delta\rho_E} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(-\overline{x},0)}$ and $\frac{\partial \mathcal{U}_L(e)}{\partial e}\Big|_{e\in(0,\overline{x})} = -\rho_e + \frac{\delta\rho_e(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})}{1-\delta\rho_E} = -\frac{\partial \mathcal{U}_R(e)}{\partial e}\Big|_{e\in(0,\overline{x})}$. The result follows. $\square$

For a candidate pair (ℓ, r), let $\Delta(\ell, r; i) = \mathcal{U}_i(\ell) - \mathcal{U}_i(r)$ denote player i's expected utility of electing candidate ℓ over candidate r. Then,

$$\Delta(\ell, r; i) = \rho_{\mathcal{L}}\big(u_i(-\overline{x}(\ell)) - u_i(-\overline{x}(r))\big) + \rho_e\big(u_i(x_e(\ell)) - u_i(x_e(r))\big) + \rho_{\mathcal{R}}\big(u_i(\overline{x}(\ell)) - u_i(\overline{x}(r))\big).$$

Lemma 4. *For candidate pair $-\overline{x} \leq \ell < r \leq \overline{x}$, the voter is indifferent if and only if v is equal to:*

$$\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}\left(\frac{\ell+r}{2} - \delta\rho_E\Big(\ell\cdot\mathbb{I}\{\ell>0\} + r\cdot\mathbb{I}\{r<0\}\Big)\right), \tag{5}$$

which satisfies $\iota_{\ell,r} \in \big(\max\{\ell, -\overline{x}(r)\}, \min\{r, \overline{x}(\ell)\}\big)$. The voter chooses ℓ if $v < \iota_{\ell,r}$ and chooses r otherwise.

PROOF. Let $-\overline{x} < \ell < r < \overline{x}$. The proof has three parts. Part 1 shows a unique indifferent voter $\iota_{\ell,r}$ satisfying $\iota_{\ell,r} \in (\ell, r)$. Part 2 shows $\iota_{\ell,r} \in (-\overline{x}(r), \overline{x}(\ell))$. Part 3 characterizes $\iota_{\ell,r}$.

Part 1. Lemma A.1 implies $\Delta(\ell, r; i) > 0$ for all $i \leq \ell$ and $\Delta(\ell, r; i) < 0$ for all $i \geq r$. Note $\mathcal{U}_i(e)$ is continuous in i given any e, which implies $\Delta(\ell, r; i)$ is continuous in i. We show $\Delta(\ell, r; i)$ strictly decreases over $i \in (\ell, r)$. Specifically, for $i \in (\max\{-\overline{x}(r), \ell\}, \min\{\overline{x}(\ell), r\})$ we have $\frac{\partial\Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[(\rho_{\mathcal{L}} + \rho_{\mathcal{R}})(\overline{x}(r) - \overline{x}(\ell)) + \rho_e(\ell + r - 2i)\Big] = -2\rho_e < 0$. Next, for $i \in (\ell, -\overline{x}(r))$ we have $\frac{\partial\Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[\rho_{\mathcal{L}}(-2i - \overline{x}(r) - \overline{x}(\ell)) + \rho_{\mathcal{R}}(\overline{x}(r) - \overline{x}(\ell)) + \rho_e(\ell + r - 2i)\Big] = -2(\rho_e + \rho_{\mathcal{L}}) < 0$. And for $i \in (\overline{x}(\ell), r)$ we have $\frac{\partial\Delta(\ell,r;i)}{\partial i} = \frac{\partial}{\partial i}\Big[\rho_{\mathcal{L}}(\overline{x}(r) - \overline{x}(\ell)) + \rho_{\mathcal{R}}(\overline{x}(r) + \overline{x}(\ell) - 2i) + \rho_e(\ell + r - 2i)\Big] = -2(\rho_e + \rho_{\mathcal{R}}) < 0$. Altogether, this implies $\Delta(\ell, r; i) = 0$ for a unique $i = \iota_{\ell,r} \in (\ell, r)$.

Part 2. We show $\iota_{\ell,r} < \overline{x}(\ell)$. An analogous argument shows $\iota_{\ell,r} > -\overline{x}(r)$. If $r \leq \overline{x}(\ell)$, then by Part 1 we have $\iota_{\ell,r} < \overline{x}(\ell)$. Thus, suppose $r > \overline{x}(\ell)$. First, Lemma A.1 implies $\Delta(\ell, r; \ell) > 0$. Second, we show $\Delta(\ell, r; \overline{x}(\ell)) < 0$, which then implies $\iota_{\ell,r} < \overline{x}(\ell)$. To begin, we have

$$\begin{aligned}\Delta(\ell, r; \overline{x}(\ell)) &= \rho_e\Big(r + \ell - 2\overline{x}(\ell)\Big) + \rho_E\left(\frac{\delta\rho_e\cdot(r - |\ell|)}{1-\delta\rho_E}\right)\\ &= \frac{\rho_e}{1-\delta\rho_E}\Big(r + (1 - 2\delta(\rho_E + \rho_e))\cdot\ell\cdot\mathbb{I}\{\ell>0\} + (1 + 2\delta\rho_e)\cdot\ell\cdot\mathbb{I}\{\ell<0\} - 2(1-\delta)c\Big).\end{aligned}$$

There are two cases. For Case 1, consider $\ell \geq 0$. Then, we have $r + (1 - 2\delta(\rho_E + \rho_e))\cdot\ell - 2(1-\delta)c < 2(1 - \delta(\rho_E + \rho_e))\cdot r - 2(1-\delta)c = 2(1 - \delta(\rho_E + \rho_e))\cdot(r - \overline{x}) < 0$, where the first inequality follows from Assumption 2a and the second inequality from $r < \overline{x}$. Thus, $\Delta(\ell, r; \overline{x}(\ell)) < 0$ for all $\ell \in [0, r)$. For Case 2, consider $\ell < 0$. Then, we have $r + (1+2\delta\rho_e)\cdot\ell - 2(1-\delta)c < \overline{x} + (1+2\delta\rho_e)\cdot\ell - 2(1-\delta)c = -(1-2\delta(\rho_E+\rho_e))\cdot\overline{x} + (1+2\delta\rho_e)\cdot\ell < 0$, where first inequality follows from $r < \overline{x}$ and the second inequality from Assumption 2a and

$\ell < 0$. Thus, $\Delta(\ell, r; \overline{x}(\ell)) < 0$ for all $\ell \in (-\overline{x}, \min\{r, 0\})$.

Part 3. Part 1 and 2 imply $\Delta(\ell, r; \iota_{\ell,r}) = \rho_E(\overline{x}(r) - \overline{x}(\ell)) + \rho_e(\ell + r - 2\iota_{\ell,r})$. We solve for $\iota_{\ell,r}$ using $\overline{x}(r) - \overline{x}(\ell) = \frac{\delta\rho_e(|r|-|\ell|)}{1-\delta\rho_E}$. If $-\overline{x} < \ell < r < 0$, then $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e\Big(\frac{\ell+(1-2\delta\rho_E)\cdot r}{1-\delta\rho_E} - 2\iota_{\ell,r}\Big)$, so $\Delta(\ell, r; \iota_{\ell,r}) = 0$ yields $\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}(\frac{r+\ell}{2} - \delta\rho_E \cdot r)$. If $0 < \ell < r < \overline{x}$, then $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e\Big(\frac{(1-2\delta\rho_E)\cdot\ell + r}{1-\delta\rho_E} - 2\iota_{\ell,r}\Big)$, so $\iota_{\ell,r} = \frac{1}{1-\delta\rho_E}(\frac{r+\ell}{2} - \delta\rho_E \cdot \ell)$. If $-\overline{x} < \ell < 0 < r < \overline{x}$, then $\Delta(\ell, r; \iota_{\ell,r}) = \rho_e(\frac{\ell+r}{1-\delta\rho_E} - 2\iota_{\ell,r})$, so $\iota_{\ell,r} = \frac{\ell+r}{2(1-\delta\rho_E)}$.

Finally, for boundary pairs ($\ell = -\overline{x}$ or $r = \overline{x}$), the argument applies verbatim. Lemma A.1 yields the endpoint signs in Part 1 if $-\overline{x} \le \ell < r \le \overline{x}$. Then, at most one inequality in each case of Part 2 becomes weak while the overall inequality remains strict. The closed form in Part 3 is identical. $\square$

A.3 Electoral Calculus

Notation. Define $\mu'_- \equiv \rho_e \frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}$ and $\mu'_+ \equiv \rho_e \frac{1-2\delta\rho_{\mathfrak{L}}}{1-\delta\rho_E}$. We can rewrite (3) as

$$\mu_e = \frac{(\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot (1-\delta)c}{1-\delta\rho_E} + e \cdot \Big(\mu'_- \cdot \mathbb{I}\{e \in [-\overline{x}, 0)\} + \mu'_+ \cdot \mathbb{I}\{e \in (0, \overline{x}]\}\Big). \tag{A.1}$$

Then, we have $\frac{\partial \mu_e}{\partial e} = \mu'_-$ if $e \in (-\overline{x}, 0)$ and $\frac{\partial \mu_e}{\partial e} = \mu'_+$ if $e \in (0, \overline{x})$.

For $-\overline{x} < \ell < r < \overline{x}$, define the *electoral stakes* as $\Delta_R(\ell, r) \equiv \mu_r - \mu_\ell$. Lemma 3 implies $\Delta_R(\ell, r) = \Delta(\ell, r; L) = -\Delta(\ell, r; R) > 0$. Analogous stakes appear in the extensions, where we write $\Delta_R^k(\ell, r) \equiv \mu_r^k - \mu_\ell^k$ for the relevant superscript k. We have

$$\Delta_R(\ell, r) = \begin{cases} \mu'_- \cdot (r - \ell) & \text{if } -\overline{x} < \ell < r < 0, \\ \mu'_+ \cdot r - \mu'_- \cdot \ell & \text{if } -\overline{x} < \ell \le 0 \le r < \overline{x}, \\ \mu'_+ \cdot (r - \ell) & \text{if } 0 < \ell < r < \overline{x}. \end{cases} \tag{A.2}$$

Define $\iota'_{nc} \equiv \frac{1}{2(1-\delta\rho_E)}$ and $\iota'_c \equiv \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. By Lemma 4, given $-\overline{x} < \ell < r < \overline{x}$, we have $\frac{\partial \iota_{\ell,r}}{\partial \ell} = \iota'_{nc}$ if $\ell \in (-\overline{x}, \min\{0, r\})$ and $\frac{\partial \iota_{\ell,r}}{\partial \ell} = \iota'_c$ if $\ell \in (0, \min\{r, \overline{x}\})$, and moreover, $\frac{\partial \iota_{\ell,r}}{\partial r} = \iota'_c$ if $r \in (\max\{\ell, -\overline{x}\}, 0)$ and $\frac{\partial \iota_{\ell,r}}{\partial r} = \iota'_{nc}$ if $r \in (\max\{0, \ell\}, \overline{x})$.

Lemma 5. *For each party $P \in \{L, R\}$, the continuation value from a candidate pair satisfying $\ell < r$ is:*

$$V_P(\ell, r) = F(\iota_{\ell,r}) \cdot u_P(\mu_\ell) + \big(1 - F(\iota_{\ell,r})\big) \cdot u_P(\mu_r), \tag{6}$$

which is continuous and strictly quasiconcave in P's own candidate.

PROOF. The characterization of $V_P(\ell, r)$ follows from Lemma 3 and 4. Continuity of $V_P(\ell, r)$ follows from continuity of $\iota_{\ell,r}$ and μ_e. We show that for each $r \in (-\overline{x}, \overline{x}]$, V_L is strictly quasiconcave over $\ell \in [-\overline{x}, r)$. By similar arguments, V_R is strictly quasiconcave in r. We consider two cases.

Case 1: Consider $r \in (-\overline{x}, 0]$. First, suppose there is an interior maximizer $\ell^* \in (-\overline{x}, r)$. Since V_L is differentiable w.r.t. ℓ on $(-\overline{x}, r)$, such an interior maximizer must satisfy:

$$0 = \frac{\partial V_L(\ell, r)}{\partial \ell} \iff f\big(\iota_{\ell,r}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell, r) - F\big(\iota_{\ell,r}\big) \cdot \mu'_- = 0. \tag{A.3}$$

Then, at such a solution $\ell^* \in (-\overline{x}, r)$, we have:

$$\begin{aligned}
\frac{\partial^2 V_L(\ell, r)}{\partial \ell^2}\Big|_{\ell=\ell^*} &= f'(\iota_{\ell^*,r}) \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 - 2 f(\iota_{\ell^*,r}) \cdot \iota'_{nc} \cdot \mu'_- && \text{(A.4)}\\
&= f'(\iota_{\ell^*,r}) \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 - 2\frac{f(\iota_{\ell^*,r})^2}{F\big(\iota_{\ell^*,r}\big)} \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 && \text{(A.5)}\\
&= 2 \cdot \Delta_R(\ell, r) \cdot (\iota'_{nc})^2 \cdot \left(\frac{f'(\iota_{\ell^*,r})}{2} - \frac{f(\iota_{\ell^*,r})^2}{F\big(\iota_{\ell^*,r}\big)}\right) && \text{(A.6)}\\
&< 0, && \text{(A.7)}
\end{aligned}$$

where (A.5) follows from substituting $\mu'_- = \frac{f\left(\iota_{\ell^*,r}\right)}{F\left(\iota_{\ell^*,r}\right)} \cdot \Delta_R(\ell^*, r) \cdot \iota'_{nc}$ based on (A.3), and (A.7) from $\Delta_R(\ell, r) > 0$ and log-concavity of f. Thus, any $\ell^* \in (-\overline{x}, r)$ that solves first-order condition (A.3) must be a strict local maximizer.

If no interior maximizer exists, then $\lim_{\ell \to r^-} \frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ implies $\frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ for all $\ell \in (-\overline{x}, r)$. Continuity at $\ell = -\overline{x}$ implies $V_L(\ell, r)$ is strictly quasiconcave on $[-\overline{x}, r]$ for all $r \leq 0$.

Case 2: Consider $r \in (0, \overline{x}]$. First, we note the following fact:

$$\frac{\iota'_{nc}}{\iota'_c} - \frac{\mu'_-}{\mu'_+} = \frac{1}{1 - 2\delta\rho_E} - \frac{1 - 2\delta\rho_{\mathfrak{R}}}{1 - 2\delta\rho_{\mathfrak{L}}} = \frac{4\delta\rho_{\mathfrak{R}}(1 - \delta\rho_E)}{(1 - 2\delta\rho_E)(1 - 2\delta\rho_{\mathfrak{L}})} \geq 0, \tag{A.8}$$

where the inequality follows from Assumption 2 and $\rho_{\mathfrak{R}}, \rho_{\mathfrak{L}} \geq 0$. We consider three subcases.

Subcase (i): Suppose $0 < r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. First, we show $\frac{\partial V_L(\ell,r)}{\partial \ell} < 0$ for $\ell \in (0,r)$:

$$\frac{\partial V_L(\ell,r)}{\partial \ell}\Big|_{\ell\in(0,r)} = f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot (r-\ell) - F\big(\iota_{\ell,r}\big) \cdot \mu'_+ \tag{A.9}$$

$$< f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \ell\Big) - F\big(\iota_{\ell,r}\big) \cdot \mu'_+ \tag{A.10}$$

$$= f\big(\iota_{\ell,r}\big) \cdot \iota'_c \cdot \mu'_+ \cdot \Big(-\ell + \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})} \cdot \frac{1}{\iota'_c}\Big) \tag{A.11}$$

$$< 0. \tag{A.12}$$

(A.10) follows from $r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$, while (A.12) follows from $\ell > 0$ and $\frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})} \cdot \frac{1}{\iota'_c} \geq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c} \geq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$, where the first inequality follows from $\iota_{\ell,r} > \iota_{0,r}$ for $\ell \in (0,r)$ and $\frac{F}{f}$ nondecreasing under log-concavity of f, and the second from (A.8). Second, note $\lim_{\ell\to 0^-} \frac{\partial V_L(\ell,r)}{\partial \ell} = f(\iota_{0,r}) \cdot \iota'_{nc} \cdot \mu'_+ \cdot r - F(\iota_{0,r}) \cdot \mu'_- < 0$ since we assumed $r < \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$. Thus, any interior maximizer must satisfy $\ell^* \in (-\overline{x}, 0)$. Analogous to (A.4)–(A.7), log-concavity of f implies $\frac{\partial^2 V_L(\ell,r)}{\partial \ell^2}\Big|_{\ell=\ell^*} < 0$. Thus, V_L is strictly quasiconcave on $[-\overline{x}, r]$.

Subcase (ii): Suppose $\frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} \leq r \leq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{1}{\iota'_c}$. First, we have:

$$\frac{\partial V_L(\ell,r)}{\partial \ell}\Big|_{\ell\in(-\overline{x},0)} = f(\iota_{\ell,r}) \cdot \iota'_{nc} \cdot (\mu'_+ \cdot r - \mu'_- \cdot \ell) - F(\iota_{\ell,r}) \cdot \mu'_- \tag{A.13}$$

$$\geq f(\iota_{\ell,r}) \cdot \iota'_{nc} \cdot \Big(\mu'_+ \cdot \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}} - \mu'_- \cdot \ell\Big) - F(\iota_{\ell,r}) \cdot \mu'_- \tag{A.14}$$

$$> f(\iota_{\ell,r}) \cdot \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \mu'_- - F(\iota_{\ell,r}) \cdot \mu'_- \tag{A.15}$$

$$\geq 0, \tag{A.16}$$

where (A.13) follows from differentiating and simplifying; (A.14) follows from $r \geq \frac{F(\iota_{0,r})}{f(\iota_{0,r})} \cdot \frac{\mu'_-}{\mu'_+} \cdot \frac{1}{\iota'_{nc}}$; (A.15) from $\ell < 0$ and simplifying; and (A.16) from $\iota_{0,r} > \iota_{\ell,r}$ for $\ell < 0$ and $\frac{F}{f}$

nondecreasing under log-concavity of f. Similarly, we have:

$$\frac{\partial V_L(\ell,r)}{\partial \ell}\Big|_{\ell\in(0,r)} = f(\iota_{\ell,r})\cdot\iota_c'\cdot\mu_+'\cdot(r-\ell) - F(\iota_{\ell,r})\cdot\mu_+' \tag{A.17}$$

$$\leq f(\iota_{\ell,r})\cdot\iota_c'\cdot\mu_+'\cdot\Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})}\frac{1}{\iota_c'} - \ell\Big) - F(\iota_{\ell,r})\cdot\mu_+' \tag{A.18}$$

$$< f(\iota_{\ell,r})\cdot\mu_+'\Big(\frac{F(\iota_{0,r})}{f(\iota_{0,r})} - \frac{F(\iota_{\ell,r})}{f(\iota_{\ell,r})}\Big) \tag{A.19}$$

$$\leq 0, \tag{A.20}$$

where (A.18) follows from $r \leq \frac{F(\iota_{0,r})}{f(\iota_{0,r})}\cdot\frac{1}{\iota_c'}$; (A.19) follows from $\ell > 0$ and simplifying; and (A.20) from $\iota_{0,r} < \iota_{\ell,r}$ and $\frac{F}{f}$ nondecreasing under log-concavity of f; the strict inequality (A.19) yields $\frac{\partial V_L(\ell,r)}{\partial \ell} < 0$. Thus, V_L is strictly quasiconcave over $[-\overline{x}, r]$.

Subcase (iii): Suppose $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})}\cdot\frac{1}{\iota_c'}$. Then (A.8) implies $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})}\cdot\frac{\mu_-'}{\mu_+'}\cdot\frac{1}{\iota_{nc}'}$. Therefore we must have $\frac{\partial V_L(\ell,r)}{\partial \ell} > 0$ for all $\ell \in (-\overline{x}, 0)$, by (A.13)-(A.16). Also, we have $\lim_{\ell\to 0^+}\frac{\partial V_L(\ell,r)}{\partial \ell} = f(\iota_{0,r})\cdot\iota_c'\cdot\mu_+'\cdot r - F(\iota_{0,r})\cdot\mu_+' > 0$, where the inequality follows from $r > \frac{F(\iota_{0,r})}{f(\iota_{0,r})}\cdot\frac{1}{\iota_c'}$. Lastly, since $\lim_{\ell\to r^-}\frac{\partial V_L(\ell,r)}{\partial \ell} < 0$, continuity of $\frac{\partial V_L(\ell,r)}{\partial \ell}$ on $(0,r)$ implies there must exist an $\ell^* \in (0,r)$ such that $\frac{\partial V_L(\ell,r)}{\partial \ell}\Big|_{\ell=\ell^*} = 0$. Analogous to (A.4)-(A.7), log-concavity of f implies $\frac{\partial^2 V_L(\ell,r)}{\partial \ell^2}\Big|_{\ell=\ell^*} < 0$. Thus, V_L is strictly quasiconcave on $[-\overline{x}, r]$. □

A.4 Equilibrium

Proposition 1. *An equilibrium exists. Every equilibrium satisfies $-\overline{x} \leq \ell^* < r^* \leq \overline{x}$, and the equilibrium is unique.*

PROOF. Throughout, candidate strategy spaces are $[-\overline{x}, \overline{x}]$.[40] We begin by noting two facts. First, *Fact (a)* is that for all $r \in [-\overline{x}, \overline{x}]$ and $\ell' \in [r, \overline{x}]$, party L's payoff is at most $u_L(\mu_r)$. If $\ell' = r$, then the policy lottery is induced by r regardless of the voter's tie-breaking. If $\ell' > r$, then Lemma 4 applied to the ordered pair (r, ℓ') gives payoff $F(\iota_{r,\ell'})\, u_L(\mu_r) + (1 - F(\iota_{r,\ell'}))\, u_L(\mu_{\ell'}) \leq u_L(\mu_r)$, since $\mu_{\ell'} \geq \mu_r$ and u_L is decreasing in the policy mean.

Second, *Fact (b)* is that for all $r > -\overline{x}$ and $\varepsilon > 0$ small, $V_L(r-\varepsilon, r) = u_L(\mu_r) + F(\iota_{r-\varepsilon,r})\cdot(\mu_r - \mu_{r-\varepsilon}) > u_L(\mu_r)$. This follows from full support of F and strict monotonicity of μ_e over $[-\overline{x}, \overline{x}]$ via Assumption 2. The symmetric statements hold for party R.

[40]Uniqueness is with respect to this normalization. In the unrestricted game any candidate outside $[-\overline{x}, \overline{x}]$ is payoff-equivalent to the nearest endpoint for all players.

No equilibrium has $\ell^* \geq r^*$. Suppose otherwise. If $r^* > -\overline{x}$, then Fact (a) bounds L's payoff by $u_L(\mu_{r^*})$, and Fact (b) shows the deviation $\ell = r^* - \varepsilon$ is strictly profitable. If $r^* = -\overline{x}$, then $\ell^* \geq -\overline{x} = r^*$, and the symmetric Fact (a) bounds R's payoff by $u_R(\mu_{\ell^*})$. If $\ell^* < \overline{x}$, the symmetric Fact (b) shows the deviation $r = \ell^* + \varepsilon$ is strictly profitable. If $\ell^* = \overline{x}$, the deviation $r = \overline{x}$ yields $u_R(\mu_{\overline{x}})$, which strictly exceeds R's payoff $F(\iota_{-\overline{x},\overline{x}})\, u_R(\mu_{-\overline{x}}) + (1 - F(\iota_{-\overline{x},\overline{x}}))\, u_R(\mu_{\overline{x}})$, since $\iota_{-\overline{x},\overline{x}} = 0$, $F(0) \in (0,1)$, and $\mu_{-\overline{x}} < \mu_{\overline{x}}$.

Next, we establish existence. Let $D = [-\overline{x}, \overline{x}]^2$ and define the constraint correspondences $\Gamma_L(r) = [-\overline{x}, r]$ and $\Gamma_R(\ell) = [\ell, \overline{x}]$, which are nonempty, compact, convex valued and continuous. Extend the ordered-pair payoffs to the diagonal by $V_L(r,r) = u_L(\mu_r)$ and $V_R(\ell,\ell) = u_R(\mu_\ell)$. This extension is continuous on the graphs of Γ_L and Γ_R. By Lemma 5, $V_L(\cdot, r)$ is continuous and strictly quasiconcave on $\Gamma_L(r)$. Thus, $BR_L(r) = \arg\max_{\ell \in \Gamma_L(r)} V_L(\ell, r)$ is single-valued and, by Berge's maximum theorem, continuous. A similar argument applies for BR_R. Therefore the map $\Phi = (BR_L, BR_R) : [-\overline{x}, \overline{x}]^2 \to [-\overline{x}, \overline{x}]^2$ is a continuous function mapping a nonempty, convex, and compact set into itself. Brouwer's theorem yields a fixed point (ℓ^*, r^*) with $\ell^* \leq r^*$. By Fact (b), $BR_L(r) < r$ for $r > -\overline{x}$, and $BR_R(\ell) > \ell$ for $\ell < \overline{x}$. If $r^* = -\overline{x}$, then $\ell^* = -\overline{x} < \overline{x}$, so $r^* = BR_R(\ell^*) > \ell^* = -\overline{x}$, a contradiction. Thus, $r^* > -\overline{x}$ and $\ell^* = BR_L(r^*) < r^*$.

To complete the existence argument, we show that there are no profitable deviations in the unconstrained game. By construction, ℓ^* is optimal for L on $[-\overline{x}, r^*]$. For $\ell' \in (r^*, \overline{x}]$, Fact (a) implies an upper bound on payoffs equal to $u_L(\mu_{r^*}) = \lim_{\ell \uparrow r^*} V_L(\ell, r^*) \leq V_L(\ell^*, r^*)$, where the inequality follows from optimality on $[-\overline{x}, r^*]$ and continuity. The symmetric argument applies to R. Therefore (ℓ^*, r^*) is an equilibrium.

Finally, we establish uniqueness. By Lemma 3 and linear loss, $u_L(\mu_e) = L - \mu_e$ and $u_R(\mu_e) = \mu_e - R$ for all $e \in [-\overline{x}, \overline{x}]$. Letting $\pi(\ell, r) \equiv \mathbb{E}[\mu_e]$ denote expected policy from winning candidate e under sequentially rational voting, we therefore have $V_L(\ell, r) + V_R(\ell, r) = \mathbb{E}\big[(L - \mu_e) + (\mu_e - R)\big] = L - R$. Thus, the electoral game is constant-sum between R and L, with R maximizing π and L minimizing it. Uniqueness then follows because equilibria are interchangeable in constant-sum games, and the parties' objective functions are strictly quasiconcave. First, using interchangeability, let (ℓ_1^*, r_1^*) and (ℓ_2^*, r_2^*) be equilibria. Optimality for R at (ℓ_1^*, r_1^*) and for L at (ℓ_2^*, r_2^*) yields $\pi(\ell_2^*, r_2^*) \leq \pi(\ell_1^*, r_2^*) \leq \pi(\ell_1^*, r_1^*)$, and the symmetric chain gives the reverse ordering, so all four profiles yield the same expected policy and (ℓ_1^*, r_2^*) is an equilibrium. That is, $\pi(\ell_1^*, r_2^*) = \pi(\ell_2^*, r_2^*) \leq \pi(\ell, r_2^*)$ for all ℓ, and $\pi(\ell_1^*, r_2^*) = \pi(\ell_1^*, r_1^*) \geq \pi(\ell_1^*, r)$ for all r. Second, as shown above, no equilibrium has $\ell^* \geq r^*$, so $\ell_1^*, \ell_2^* \in [-\overline{x}, r_2^*)$. Furthermore, they each maximize $V_L(\cdot, r_2^*)$ over $[-\overline{x}, \overline{x}]$, and therefore over $\Gamma_L(r_2^*)$, on which $V_L(\cdot, r_2^*)$ is continuous and strictly quasiconcave by Lemma 5. The maximizer on that set is unique, so $\ell_1^* = \ell_2^*$. Analogously, $r_1^* = r_2^*$. $\square$

Proposition 2. *If there is no crossover in equilibrium, then:*

a. *party L's win probability is* $P^* = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$,

b. *the indifferent voter location is* $\iota^*_{\ell,r} = \check{x}_{nc} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}\Big)$,

c. *candidate divergence is* $r^* - \ell^* = 2\delta(\rho_{\mathfrak{L}} - \rho_{\mathfrak{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathfrak{L}})(1-2\delta\rho_{\mathfrak{R}})}{1-\delta\rho_E}$, *and*

d. *candidates are* $\ell^* = (1-2\delta\rho_{\mathfrak{L}})\Big(\check{x}_{nc} - \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}\Big)$ *and* $r^* = (1-2\delta\rho_{\mathfrak{R}})\Big(\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathfrak{L}}}{1-\delta\rho_E}\Big)$.

PROOF. Suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ is an equilibrium. This requires

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - F\big(\iota_{\ell^*,r^*}\big) \cdot \mu'_-, \text{ and} \tag{A.21}$$

$$0 = -\frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - \Big(1 - F\big(\iota_{\ell^*,r^*}\big)\Big) \cdot \mu'_+. \tag{A.22}$$

Combining (A.21) and (A.22) yields $F\big(\iota_{\ell^*,r^*}\big) = \frac{\mu'_+}{\mu'_+ + \mu'_-} = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$, since $\mu'_+ = \frac{1-2\delta\rho_{\mathfrak{L}}}{1-\delta\rho_E}\rho_e$ and $\mu'_- = \frac{1-2\delta\rho_{\mathfrak{R}}}{1-\delta\rho_E}\rho_e$. Thus, $\iota_{\ell^*,r^*} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}\Big) = \check{x}_{nc}$. Substituting into (A.21) and simplifying yields $\ell^* = (1-2\delta\rho_{\mathfrak{L}}) \cdot (\frac{r^*}{1-2\delta\rho_{\mathfrak{R}}} - \frac{1}{f(\check{x}_{nc})})$. Combining with $\check{x}_{nc} = \frac{\ell^*+r^*}{2(1-\delta\rho_E)}$ yields $\ell^* = (1-2\delta\rho_{\mathfrak{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = (1-2\delta\rho_{\mathfrak{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}\Big)$. □

Corollary 2.1. *If there is no crossover in equilibrium,* $\rho_{\mathfrak{L}} = \rho_{\mathfrak{R}}$, *and the corresponding Calvert-Wittman candidates are interior, then: (i) party L's win probability is* $P^* = \frac{1}{2}$, *(ii) the indifferent voter location is* $\iota_{BE} = m = F^{-1}(\frac{1}{2})$, *(iii) candidate divergence is* $r_{BE} - \ell_{BE} = (1-\delta\rho_E)\cdot(r_{CW} - \ell_{CW})$, *and (iv) candidates are* $\ell_{BE} = (1-\delta\rho_E)\cdot\ell_{CW}$ *and* $r_{BE} = (1-\delta\rho_E)\cdot r_{CW}$.

PROOF. This is a special case of Proposition 2. □

Proposition 3. *If there is crossover in equilibrium such that* $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, *then:*

a. *party L's win probability is* $P^* = \frac{1}{2(1-\delta\rho_E)}$,

b. *the indifferent voter location is* $\iota^*_{\ell,r} = \check{x}_{l\,c} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)}\Big)$,

c. *candidate divergence is* $r^* - \ell^* = \frac{1}{f(\check{x}_{l\,c})}$, *and*

d. *candidates are* $\ell^* = \check{x}_{l\,c} - \frac{1}{2f(\check{x}_{l\,c})} \cdot \frac{1-2\delta\rho_E}{1-\delta\rho_E}$ *and* $r^* = \check{x}_{l\,c} + \frac{1}{2f(\check{x}_{l\,c})} \cdot \frac{1}{1-\delta\rho_E}$.

PROOF. Suppose $-\overline{x} < \ell^* < r^* < 0$ is an equilibrium. This requires

$$0 = \frac{\partial V_L(\ell, r^*)}{\partial \ell}\Big|_{\ell=\ell^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - F\big(\iota_{\ell^*,r^*}\big) \cdot \mu'_-, \text{ and} \tag{A.23}$$

$$0 = -\frac{\partial V_R(\ell^*, r)}{\partial r}\Big|_{r=r^*} = f\big(\iota_{\ell^*,r^*}\big) \cdot \iota'_{c} \cdot \Delta_R(\ell^*, r^*) - \Big(1 - F\big(\iota_{\ell^*,r^*}\big)\Big) \cdot \mu'_-. \tag{A.24}$$

Combining (A.23) and (A.24) yields $F\big(\iota_{\ell^*,r^*}\big) = \frac{\mu'_{-}\cdot\iota'_{nc}}{\mu'_{-}\cdot\iota'_{nc}+\mu'_{-}\cdot\iota'_{c}} = \frac{1}{2(1-\delta\rho_E)}$ since $\iota'_c = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$ and $\iota'_{nc} = \frac{1}{2(1-\delta\rho_E)}$. Thus, $\iota_{\ell^*,r^*} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)}\Big) = \check{x}_{l\,c}$. Substituting into (A.23) yields

$$\begin{aligned} 0 &= f(\check{x}_{l\,c})\cdot\frac{\rho_e\cdot(1-2\delta\rho_{\mathfrak{R}})}{2(1-\delta\rho_E)^2}\cdot(r^*-\ell^*) - \frac{\rho_e\cdot(1-2\delta\rho_{\mathfrak{R}})}{2(1-\delta\rho_E)^2} \\ &\propto r^* - \ell^* - \frac{1}{f(\check{x}_{l\,c})}. \end{aligned} \tag{A.25}$$

Combining (A.25) with $\iota_{\ell^*,r^*} = \frac{\ell^*+(1-2\delta\rho_E)r^*}{2(1-\delta\rho_E)} = \check{x}_{l\,c}$ yields $\ell^* = \check{x}_{l\,c} - \frac{1}{f(\check{x}_{l\,c})}\cdot\frac{1-2\delta\rho_E}{2\cdot(1-\delta\rho_E)}$ and $r^* = \check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\cdot\frac{1}{2\cdot(1-\delta\rho_E)}$. □

Features of Equilibrium Given equilibrium candidates (ℓ^*, r^*), let $\pi(\ell^*, r^*) = F(\iota_{\ell^*,r^*})\cdot\mu_{\ell^*} + (1-F(\iota_{\ell^*,r^*}))\cdot\mu_{r^*}$ denote ex-ante expected policy. Substituting for μ_{ℓ^*} and μ_{r^*} yields:

$$\begin{aligned} \pi(\ell^*, r^*) &= \rho_e\cdot[F(\iota_{\ell^*,r^*})\cdot\ell^* + (1-F(\iota_{\ell^*,r^*}))\cdot r^*] \\ &\quad + (\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})\cdot\Big(\frac{(1-\delta)c + \delta\rho_e\cdot\big(F(\iota_{\ell^*,r^*})\cdot|\ell^*| + (1-F(\iota_{\ell^*,r^*}))\cdot|r^*|\big)}{1-\delta\rho_E}\Big). \end{aligned} \tag{A.26}$$

Proposition A.1. *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, ex-ante expected policy equals $\mu_{e^*_{nc}}$, the mean of the policy lottery induced by an officeholder with ideal point* $e^*_{nc} = \begin{cases} \check{x}_{nc}\cdot(1-2\delta\rho_{\mathfrak{R}}) & \text{if } \check{x}_{nc}\geq 0, \\ \check{x}_{nc}\cdot(1-2\delta\rho_{\mathfrak{L}}) & \text{else.}\end{cases}$.

PROOF. There are two cases. Case (i): $\check{x}_{nc}\geq 0$. Then, (A.26) implies $\pi(\ell^*, r^*) = \rho_e\cdot\Big(\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathfrak{L}}}\cdot F(\iota_{\ell^*,r^*})\cdot\ell^* + (1-F(\iota_{\ell^*,r^*}))\cdot r^*\Big) + (\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})\cdot\Bigg(\frac{(1-\delta)c+\delta\rho_e\cdot\big(\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathfrak{L}}}\cdot F(\iota_{\ell^*,r^*})\cdot\ell^*+(1-F(\iota_{\ell^*,r^*}))\cdot r^*\big)}{1-\delta\rho_E}\Bigg) = \rho_e\cdot\check{x}_{nc}\cdot(1-2\delta\rho_{\mathfrak{R}}) + (\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})\cdot\overline{x}(\check{x}_{nc}\cdot(1-2\delta\rho_{\mathfrak{R}})) = \mu_{\check{x}_{nc}\cdot(1-2\delta\rho_{\mathfrak{R}})}$.

Case (ii): $\check{x}_{nc} < 0$. Then, (A.26) implies $\pi(\ell^*, r^*) = \rho_e\cdot\Big(F(\iota_{\ell^*,r^*})\cdot\ell^* + \frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}}(1-F(\iota_{\ell^*,r^*}))\cdot r^*\Big) + (\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})\cdot\Bigg(\frac{(1-\delta)c-\delta\rho_e\cdot\big(F(\iota_{\ell^*,r^*})\cdot\ell^*+\frac{1-2\delta\rho_{\mathfrak{L}}}{1-2\delta\rho_{\mathfrak{R}}}(1-F(\iota_{\ell^*,r^*}))\cdot r^*\big)}{1-\delta\rho_E}\Bigg) = \rho_e\cdot\check{x}_{nc}\cdot(1-2\delta\rho_{\mathfrak{L}}) + (\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})\cdot\overline{x}(\check{x}_{nc}\cdot(1-2\delta\rho_{\mathfrak{L}})) = \mu_{\check{x}_{nc}\cdot(1-2\delta\rho_{\mathfrak{L}})}$. □

Proposition A.2. *If $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$, ex-ante expected policy equals $\mu_{e^*_{l\,c}}$, the mean of the policy lottery induced by an officeholder with ideal point $e^*_{l\,c} = \check{x}_{l\,c}$.*

PROOF. In such an equilibrium, $\ell^* < \check{x}_{l\,c} < r^* < 0$. Thus, (A.26) implies $\pi(\ell^*, r^*) = \rho_e\cdot\Big(F(\iota_{\ell^*,r^*})\cdot\ell^* + (1-F(\iota_{\ell^*,r^*}))\cdot r^*\Big) + (\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})\cdot\Bigg(\frac{(1-\delta)c-\delta\rho_e\cdot\big(F(\iota_{\ell^*,r^*})\cdot\ell^*+(1-F(\iota_{\ell^*,r^*}))\cdot r^*\big)}{1-\delta\rho_E}\Bigg) = \rho_e\cdot\check{x}_{l\,c} + (\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})\cdot\overline{x}(\check{x}_{l\,c}) = \mu_{\check{x}_{l\,c}}$. □

B Equilibrium Configuration

We now characterize the configuration of the unique equilibrium in the baseline setting. An equilibrium is *interior* if $-\overline{x} < \ell < r < \overline{x}$. An interior equilibrium is *differentiable* if $\ell^* \neq 0 \neq r^*$.

Define the quantiles $\check{x}_{rc} \equiv F^{-1}\left(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\right)$, $\check{x}_{nc} \equiv F^{-1}\left(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right)$, and $\check{x}_{l\,c} \equiv F^{-1}\left(\frac{1}{2(1-\delta\rho_E)}\right)$.

Remark A.1. Assumption 2 implies $\check{x}_{rc} \leq \check{x}_{nc} \leq \check{x}_{l\,c}$.

Differentiable Interior Equilibria Propositions 2 and 3 characterize no-crossover and left-crossover equilibria. We now characterize right-crossover equilibria in Proposition A.3.

Proposition A.3. *If $0 < \ell^* < r^* < \overline{x}$ is an equilibrium:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter is $\check{x}_{rc} = F^{-1}\left(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\right)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}_{rc})}$, and*
d. *candidates are $\ell^* = \check{x}_{rc} - \frac{1}{2\,f(\check{x}_{rc})} \cdot \frac{1}{1-\delta\rho_E}$, $r^* = \check{x}_{rc} + \frac{1}{2\,f(\check{x}_{rc})} \cdot \frac{1-2\delta\rho_E}{1-\delta\rho_E}$.*

PROOF. Analogous to the proof of Proposition 3. □

Non-Differentiable Interior Equilibria

Claim A.1. If $-\overline{x} < \ell^* < r^* = 0$ is an equilibrium:

a. party L's win probability is $P^* \in \left[\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}, \frac{1}{2(1-\delta\rho_E)}\right]$,
b. the indifferent voter is $\iota_{\ell^*,0} \in \left[\check{x}_{nc}, \check{x}_{l\,c}\right]$, and
c. candidates are $\ell^* \in \left[-\frac{1}{f(\check{x}_{l\,c})}, -\frac{1-2\delta\rho_{\mathcal{L}}}{f(\check{x}_{nc})}\right]$ and $r^* = 0$.

PROOF. Suppose $-\overline{x} < \ell^* < r^* = 0$ is an equilibrium. For L, we must have $0 = \frac{\partial V_L(\ell,0)}{\partial \ell}\Big|_{\ell=\ell^*} = f\left(\iota_{\ell^*,0}\right) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) - F\left(\iota_{\ell^*,0}\right) \cdot \mu'_- = F\left(\iota_{\ell^*,0}\right) + f\left(\iota_{\ell^*,0}\right) \cdot \frac{\ell^*}{2(1-\delta\rho_E)}$, which implies $\ell^* = -2(1-\delta\rho_E) \cdot \frac{F(\iota_{\ell^*,0})}{f\left(\iota_{\ell^*,0}\right)}$. For R, we must have $\lim_{\hat{r}\to 0^+} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}} \leq 0 \leq \lim_{\hat{r}\to 0^-} \frac{\partial V_R(\ell^*,r)}{\partial r}\Big|_{r=\hat{r}}$. The first inequality is equivalent to $0 \geq -f\left(\iota_{\ell^*,0}\right) \cdot \iota'_{nc} \cdot \Delta_R(\ell^*, r^*) + \left(1 - F\left(\iota_{\ell^*,0}\right)\right) \cdot \mu'_+$. Substituting L's condition into R's and simplifying yields $F\left(\iota_{\ell^*,0}\right) \geq \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Similarly, R's second inequality is equivalent to $0 \leq -f\left(\iota_{\ell^*,0}\right) \cdot \iota'_c \cdot \Delta_R(\ell^*, r^*) + \left(1 - F\left(\iota_{\ell^*,0}\right)\right) \cdot \mu'_-$. Substituting L's condition into R's and simplifying yields $F\left(\iota_{\ell^*,0}\right) \leq \frac{1}{2(1-\delta\rho_E)}$. Together, these inequalities imply $F(\iota_{\ell^*,0}) \in \left[\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}, \frac{1}{2(1-\delta\rho_E)}\right]$, so $\iota_{\ell^*,0} \in [\check{x}_{nc}, \check{x}_{l\,c}]$. Next, log-concavity of f implies that $\frac{F}{f}$ is nondecreasing, so the characterization of ℓ^* yields $\ell^* \in$

$\left[-2(1-\delta\rho_E)\frac{F(\check{x}_{l\,c})}{f\left(\check{x}_{l\,c}\right)}, -2(1-\delta\rho_E)\frac{F(\check{x}_{nc})}{f\left(\check{x}_{nc}\right)}\right]$ and then using the two inequalities for R yields $\ell^* \in \left[-\frac{1}{f(\check{x}_{l\,c})}, -\frac{1-2\delta\rho_{\mathcal{L}}}{f(\check{x}_{nc})}\right]$. □

Claim A.2. If $0 = \ell^* < r^* < \overline{x}$ is an equilibrium:

a. party L's win probability is $P^* \in \left[\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}, \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\right]$,

b. the indifferent voter is $\iota_{0,r^*} \in [\check{x}_{rc}, \check{x}_{nc}]$, and

c. candidates are $\ell^* = 0$ and $r^* \in \left[\frac{1-2\delta\rho_{\mathfrak{R}}}{f(\check{x}_{nc})}, \frac{1}{f(\check{x}_{rc})}\right]$.

PROOF. Analogous to Claim A.1. □

Finally, define $G(x) \equiv x + \frac{F(x)}{f(x)}$ and $H(x) \equiv x - \frac{1-F(x)}{f(x)}$. The auxiliary functions facilitate conditions on F that distinguish which equilibrium configuration arises.

Lemma A.2. *The functions G and H are strictly increasing, and satisfy $H(x) < G(x)$ for all x.*

PROOF. Log-concavity of f implies log-concavity of F and $1-F$ (Bagnoli and Bergstrom, 2005), so F/f is nondecreasing and $(1-F)/f$ is nonincreasing. Since the identity function is strictly increasing, G and H are therefore strictly increasing. Finally, $H < G$ follows from $F/f > 0$ and $(1-F)/f > 0$. □

Lemma A.3. *In any interior equilibrium:*

a. *if $-\overline{x} < \ell^* < r^* < 0$, then $\iota_{\ell^*,r^*} = \check{x}_{l\,c}$, $\ell^* = H(\check{x}_{l\,c})$, and $r^* = G(\check{x}_{l\,c})$;*

b. *if $-\overline{x} < \ell^* < r^* = 0$, then $\iota_{\ell^*,0}$ is the unique zero of G, which is in $[\check{x}_{nc}, \check{x}_{l\,c}]$, and $\ell^* = 2(1-\delta\rho_E)\,\iota_{\ell^*,0}$;*

c. *if $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$: $\iota_{\ell^*,r^*} = \check{x}_{nc}$, $\ell^* = (1-2\delta\rho_{\mathcal{L}})\,H(\check{x}_{nc})$, and $r^* = (1-2\delta\rho_{\mathfrak{R}})\,G(\check{x}_{nc})$;*

d. *if $0 = \ell^* < r^* < \overline{x}$, then ι_{0,r^*} is the unique zero of H, which is in $[\check{x}_{rc}, \check{x}_{nc}]$, and $r^* = 2(1-\delta\rho_E)\,\iota_{0,r^*}$; and*

e. *if $0 < \ell^* < r^* < \overline{x}$, then $\iota_{\ell^*,r^*} = \check{x}_{rc}$, $\ell^* = H(\check{x}_{rc})$, and $r^* = G(\check{x}_{rc})$.*

PROOF. Parts (a), (c), and (e) restate Propositions 3, 2, and A.3 using $F(\check{x}_{l\,c}) = \frac{1}{2(1-\delta\rho_E)}$, $F(\check{x}_{nc}) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$, $F(\check{x}_{rc}) = \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$, and the implied values of $1-F$ (e.g., $1-F(\check{x}_{nc}) = \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$). For part (b), Claim A.1 gives $\ell^* = -2(1-\delta\rho_E)\cdot\frac{F(\iota_{\ell^*,0})}{f(\iota_{\ell^*,0})}$, and Lemma 4 gives $\iota_{\ell^*,0} = \frac{\ell^*}{2(1-\delta\rho_E)}$. Combining, we have $\iota_{\ell^*,0} = -\frac{F(\iota_{\ell^*,0})}{f(\iota_{\ell^*,0})}$, i.e., $G(\iota_{\ell^*,0}) = 0$. The unique zero follows from Lemma A.2, and the interval from Claim A.1. Part (d) is symmetric via Claim A.2: $r^* = 2(1-\delta\rho_E)\cdot\frac{1-F(\iota_{0,r^*})}{f(\iota_{0,r^*})}$ and $\iota_{0,r^*} = \frac{r^*}{2(1-\delta\rho_E)}$, so $H(\iota_{0,r^*}) = 0$. □

Corollary A.3.1. *Existence of an interior equilibrium of types (a)–(e) (as enumerated in Lemma A.3) implies, respectively: (a)* $G(\check{x}_{l\,c}) < 0$*; (b)* $G(\check{x}_{nc}) \leq 0 \leq G(\check{x}_{l\,c})$*; (c)* $H(\check{x}_{nc}) < 0 < G(\check{x}_{nc})$*; (d)* $H(\check{x}_{rc}) \leq 0 \leq H(\check{x}_{nc})$*; (e)* $H(\check{x}_{rc}) > 0$*.*

PROOF. For types (a), (c), and (e), the conditions restate the configuration constraints on (ℓ^*, r^*) via Lemma A.3, using $1-2\delta\rho_{\mathcal{L}} > 0$ and $1-2\delta\rho_{\mathcal{R}} > 0$ (Assumption 2). For types (b) and (d), the zero of G is in $[\check{x}_{nc}, \check{x}_{l\,c}]$, the zero of H is in $[\check{x}_{rc}, \check{x}_{nc}]$, and G and H are strictly increasing. □

Equilibrium Configuration The following results, referenced in the main text, characterize the equilibrium configuration in terms of the voter distribution.

Proposition A.4. *An interior equilibrium features: (i) crossover with* $\ell^* < r^* < 0$ *if and only if* $G(\check{x}_{l\,c}) < 0$*; (ii) no crossover if and only if* $H(\check{x}_{nc}) < 0 < G(\check{x}_{nc})$*; and (iii) crossover with* $0 < \ell^* < r^*$ *if and only if* $H(\check{x}_{rc}) > 0$*. If* $G(\check{x}_{nc}) \leq 0 \leq G(\check{x}_{l\,c})$*, then party R's candidate is* $r^* = 0$*. If* $H(\check{x}_{rc}) \leq 0 \leq H(\check{x}_{nc})$*, then party L's candidate is at* $\ell^* = 0$*.*

PROOF. Each condition is necessary by Corollary A.3.1. Next, define $a \equiv H(\check{x}_{rc})$, $b \equiv H(\check{x}_{nc})$, $c \equiv G(\check{x}_{nc})$, and $d \equiv G(\check{x}_{l\,c})$. Then, $a \leq b < c \leq d$ holds due to Remark A.1, the strict monotonicity of G and H, and $H < G$ via Lemma A.2. Conditions (a)–(e) of Corollary A.3.1 classify the position of 0 within the ordered list $a \leq b < c \leq d$, and are mutually exclusive and exhaustive. Respectively, they correspond to: $d < 0$; $c \leq 0 \leq d$; $b < 0 < c$; $a \leq 0 \leq b$; and $a > 0$. By Proposition 1, the equilibrium exists and is unique. Moreover, since it is interior, it is of exactly one of the five types in Lemma A.3, and by Corollary A.3.1 its type's condition holds. Thus, the equilibrium features type k if and only if condition k holds. The transition-region cases restate types (b) and (d) of Lemma A.3. □

Corollary A.4.1. *Increasing* ρ_E *shrinks the set of voter distributions for which the equilibrium features crossover. That is, the cutpoint* $G(\check{x}_{l\,c})$ *is strictly increasing in* ρ_E*, and* $H(\check{x}_{rc})$ *is strictly decreasing.*

PROOF. Note that $\check{x}_{l\,c} = F^{-1}\big(\frac{1}{2(1-\delta\rho_E)}\big)$ is strictly increasing in ρ_E and G is strictly increasing. Similarly, note that $\check{x}_{rc} = F^{-1}\big(\frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}\big)$ is strictly decreasing in ρ_E and H is strictly increasing. Thus, as ρ_E increases, the crossover conditions $G(\check{x}_{l\,c}) < 0$ and $H(\check{x}_{rc}) > 0$ from Proposition A.4 each hold for a smaller set of voter distributions. □

Remark A.2. Fix F and consider the translated family $F_t(x) = F(x-t)$ for $t \in \mathbb{R}$ such that the equilibrium is interior. Let $t_1 = -G(\check{x}_{l\,c})$, $t_2 = -G(\check{x}_{nc})$, $t_3 = -H(\check{x}_{nc})$, and $t_4 = -H(\check{x}_{rc})$, all evaluated under F. Then, we have $t_1 \leq t_2 < t_3 \leq t_4$, with $t_1 < t_2$ if and only if $\rho_{\mathcal{L}} > 0$

and $t_3 < t_4$ if and only if $\rho_{\mathcal{R}} > 0$. The equilibrium features left crossover if and only if $t < t_1$, no crossover if and only if $t \in (t_2, t_3)$, and right crossover if and only if $t > t_4$. For $t \in [t_1, t_2]$ party R's candidate is at 0. Similarly, for $t \in [t_3, t_4]$ party L's candidate is at 0.

PROOF. Under F_t, each quantile shifts to $\check{x}_{\cdot} + t$, while F_t/f_t and $(1 - F_t)/f_t$ evaluated at the shifted quantile are unchanged, so $G_t(\check{x}_{\cdot} + t) = G(\check{x}_{\cdot}) + t$ and $H_t(\check{x}_{\cdot} + t) = H(\check{x}_{\cdot}) + t$. Applying Proposition A.4 under F_t yields the stated classification. The ordering of thresholds follows from $\check{x}_{rc} \le \check{x}_{nc} \le \check{x}_{l\,c}$ via Remark A.1, strict monotonicity of G and H, and $H < G$. Finally, $F(\check{x}_{l\,c}) - F(\check{x}_{nc}) = \frac{\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}$ and $F(\check{x}_{nc}) - F(\check{x}_{rc}) = \frac{\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}$, so $\check{x}_{nc} < \check{x}_{l\,c}$ (i.e., $t_1 < t_2$) if and only if $\rho_{\mathcal{L}} > 0$, and $\check{x}_{rc} < \check{x}_{nc}$ (i.e., $t_3 < t_4$) if and only if $\rho_{\mathcal{R}} > 0$. □

C Proofs for Midterm Loss Application

We compare equilibria across two electoral environments: a *presidential-year*, in which the identity of the president is uncertain, and a *midterm*, in which the president's party is known.

Electoral Environments. Suppose the president's co-partisan extremist receives the larger share of proposal rights ρ_{pres}, while the opposition extremist receives $\rho_{\text{opp}} < \rho_{\text{pres}}$. An $\mathcal{L}$ presidency induces $\rho_{\mathcal{L}} = \rho_{\text{pres}}$ and $\rho_{\mathcal{R}} = \rho_{\text{opp}}$, and vice versa under an $\mathcal{R}$ presidency. Crucially, $\rho_E = \rho_{\text{pres}} + \rho_{\text{opp}}$ is invariant to the president's party.

In the *presidential-year* environment, voters and parties share a common prior $\lambda \in (0, 1)$ that $\mathcal{L}$ will hold the presidency, yielding expected proposal rights

$$\rho^{pe}(\lambda) \equiv \lambda \cdot \rho^{\mathcal{L}} + (1 - \lambda) \cdot \rho^{\mathcal{R}} = \big(\rho_e,\ \rho_{\mathcal{M}},\ \lambda\rho_{\text{pres}} + (1 - \lambda)\rho_{\text{opp}},\ (1 - \lambda)\rho_{\text{pres}} + \lambda\rho_{\text{opp}}\big), \tag{A.27}$$

where $\rho^{\mathcal{L}} = (\rho_e, \rho_{\mathcal{M}}, \rho_{\text{pres}}, \rho_{\text{opp}})$ and $\rho^{\mathcal{R}} = (\rho_e, \rho_{\mathcal{M}}, \rho_{\text{opp}}, \rho_{\text{pres}})$. In the *midterm* environment under a party $\mathcal{P} \in \{\mathcal{L}, \mathcal{R}\}$ president, the proposal-right vector is simply $\rho^{\mathcal{P}}$.

We fix voter distribution F across environments, with voter ideal points drawn independently across election cycles and independently of the presidential race. Let $P_L(\rho)$ denote party L's equilibrium win probability in the district election given expected proposal rights ρ.

Lemma A.4. *Equilibrium in the presidential-year district election is equivalent to equilibrium in the baseline setting with $\rho = \rho^{pe}(\lambda)$.*

PROOF. The result follows from two features of $\rho^{pe}(\lambda)$. First, the indifferent voter characterization in Lemma 4 depends on extremist proposal rights only through total extremism ρ_E, and $\rho_E^{pe}(\lambda) = \rho_E^{\mathcal{L}} = \rho_E^{\mathcal{R}} = \rho_{\text{pres}} + \rho_{\text{opp}}$ for all $\lambda \in [0, 1]$. Second, since parties have linear loss utility, their expected payoff from officeholder e depends only on the mean of the policy lottery. By linearity of μ_e in $(\rho_{\mathcal{L}}, \rho_{\mathcal{R}})$ for fixed ρ_E, we have $\lambda\mu_e(\rho^{\mathcal{L}})+(1-\lambda)\mu_e(\rho^{\mathcal{R}}) = \mu_e(\rho^{pe}(\lambda))$. $\square$

Lemma A.4 implies the equilibrium in the presidential-year district election—if interior and differentiable—is characterized by Propositions 2–3 evaluated at $\rho = \rho^{pe}(\lambda)$.

Remark A.3. The crossover conditions $G(\check{x}_{l\,c}) < 0$ and $H(\check{x}_{rc}) > 0$ from Proposition A.4 depend on extremist proposal rights only through ρ_E, which is constant across environments. The no-crossover cutoff $\check{x}_{nc}(\rho_{\mathcal{L}}) \equiv F^{-1}\big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\big)$ is strictly decreasing in $\rho_{\mathcal{L}}$, and extremist $\mathcal{L}$'s (expected) proposal rights range over $[\rho_{\text{opp}}, \rho_{\text{pres}}]$ across environments. Since G and H are strictly increasing by Lemma A.2, condition

$$G\big(\check{x}_{nc}(\rho_{\text{pres}})\big) > 0 \quad \text{and} \quad H\big(\check{x}_{nc}(\rho_{\text{opp}})\big) < 0 \tag{A.28}$$

ensures no crossover in every environment. Propositions A.6–A.7 impose (A.28).

Midterm Loss. We analyze how a party's district election win probability differs between a midterm election under a co-partisan president and the presidential-year baseline. Party L's *net midterm loss probability* conditional on an $\mathcal{L}$ presidency is:

$$\psi_L(\lambda) \equiv P_L\big(\rho^{pe}(\lambda)\big) - P_L\big(\rho^{\mathcal{L}}\big). \tag{A.29}$$

If $\psi_L(\lambda) > 0$, then party L is more likely to win the election in a presidential year than in a midterm under a co-partisan $\mathcal{L}$ president.[41] Define $\psi_R(\lambda) \equiv \big(1-P_L(\rho^{pe}(\lambda))\big)-\big(1-P_L(\rho^{\mathcal{R}})\big) = P_L(\rho^{\mathcal{R}}) - P_L(\rho^{pe}(\lambda))$ analogously.

Proposition A.5. *In a district featuring crossover equilibrium, $\psi_L(\lambda) = 0$ for all $\lambda \in (0, 1)$.*

PROOF. If $-\overline{x} < \ell^* < r^* < 0$, Proposition 3 implies $P_L(\rho) = F(\check{x}_{l\,c}) = \frac{1}{2(1-\delta\rho_E)}$, which depends on extremist proposal rights only through ρ_E. Since ρ_E is identical under $\rho^{pe}(\lambda)$ and $\rho^{\mathcal{L}}$, we have $P_L(\rho^{pe}(\lambda)) = P_L(\rho^{\mathcal{L}})$. Analogously, $\psi_L(\lambda) = 0$ if $0 < \ell^* < r^* < \overline{x}$. $\square$

Proposition A.6. *In a district satisfying (A.28), $\psi_L(\lambda) > 0$ for all $\lambda \in (0, 1)$. Moreover, $\psi_L(\lambda)$ is strictly decreasing in λ.*

PROOF. In a no-crossover equilibrium, Proposition 2 implies $P_L(\rho) = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Under $\rho^{pe}(\lambda)$, we have $\rho_{\mathcal{L}} = \lambda\rho_{\text{pres}} + (1-\lambda)\rho_{\text{opp}}$. Under $\rho^{\mathcal{L}}$, we have $\rho_{\mathcal{L}} = \rho_{\text{pres}}$. Thus,

$$\psi_L(\lambda) = \frac{1 - 2\delta\big(\lambda\rho_{\text{pres}} + (1-\lambda)\rho_{\text{opp}}\big)}{2(1-\delta\rho_E)} - \frac{1 - 2\delta\rho_{\text{pres}}}{2(1-\delta\rho_E)} = \frac{\delta(\rho_{\text{pres}} - \rho_{\text{opp}})}{1-\delta\rho_E} \cdot (1-\lambda) > 0,$$

[41] Equivalently, under independent voter draws across cycles, $\psi_L(\lambda) = \Pr(L \text{ wins in presidential year}) \cdot \Pr(L \text{ loses in midterm}) - \Pr(L \text{ loses in presidential year}) \cdot \Pr(L \text{ wins in midterm}) = P_L(\rho^{pe}(\lambda)) - P_L(\rho^{\mathcal{L}})$.

and $\frac{\partial \psi_L(\lambda)}{\partial \lambda} = -\frac{\delta(\rho_{\text{pres}} - \rho_{\text{opp}})}{1-\delta\rho_E} < 0.$ $\square$

Ex-Ante Expected Midterm Loss. Weighting each party's midterm loss by the prior probability that a co-partisan holds the presidency yields the *ex-ante net midterm loss probability* $\psi(\lambda) \equiv \lambda\, \psi_L(\lambda) + (1-\lambda)\, \psi_R(\lambda)$.

Proposition A.7. *In a district satisfying (A.28), $\psi(\lambda) > 0$ for all $\lambda \in (0,1)$, and $\psi(\lambda)$ is maximized at $\lambda = \frac{1}{2}$, strictly increasing for $\lambda < \frac{1}{2}$, and strictly decreasing for $\lambda > \frac{1}{2}$.*

PROOF. By symmetry of the conditional proposal rights, $\psi_R(\lambda) = P_L(\rho^{\mathcal{R}}) - P_L(\rho^{pe}(\lambda)) = \frac{\delta(\rho_{\text{pres}} - \rho_{\text{opp}})}{1-\delta\rho_E} \cdot \lambda$. Then, substituting yields $\psi(\lambda) = \frac{2\delta(\rho_{\text{pres}} - \rho_{\text{opp}})}{1-\delta\rho_E} \cdot \lambda(1-\lambda) > 0$, which is strictly concave in λ and maximized at $\lambda = \frac{1}{2}$. $\square$

D Comparative Statics

We study comparative statics of proposal power shifts on ex-ante expected policy. Throughout, we define the effect of increasing ρ_i at the expense of ρ_j as the derivative of equilibrium expected policy along a transfer of proposal rights from j to i, i.e., $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_i - \rho_j)} \equiv \frac{d}{d\tau}\, \pi\big(\ell^*, r^*; \ldots, \rho_i + \tau, \ldots, \rho_j - \tau, \ldots\big)\big|_{\tau=0}$, where the equilibrium (ℓ^*, r^*) varies with the proposal rights. We first provide a detailed proof for one shift, following the example in the Party Preferences over Proposal Rights section in the main text.

Proposition A.8. *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathcal{R}} - \rho_{\mathcal{M}})} > 0$.*

PROOF. From Proposition A.1, we have $\frac{\partial \pi(\ell^*, r^*)}{\partial(\rho_{\mathcal{R}} - \rho_{\mathcal{M}})} = \frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}} - \frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{M}}} = \frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}}$. Differentiating and rearranging yields:

$$\frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}} = \underbrace{\overline{x}(e^*_{nc}) + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial \rho_{\mathcal{R}}}\Big|_{e=e^*_{nc}}}_{\text{policymaking channel } (+)} + \underbrace{\left(\rho_e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial e}\Big|_{e=e^*_{nc}}\right) \cdot \frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}}}_{\text{electoral channel } (+/-)}.$$

The policymaking channel captures the effects of shifting proposal power from $\mathcal{M}$ to $\mathcal{R}$, holding fixed candidates. The first term, $\overline{x}(e^*_{nc}) > 0$, captures the direct effect. The second term, $(\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial \rho_{\mathcal{R}}}\Big|_{e=e^*_{nc}} \lesseqgtr 0$, captures the indirect effects through enabling extremists. The sign is positive if $\rho_{\mathcal{R}} \geq \rho_{\mathcal{L}}$ and negative otherwise. The total policymaking channel is $\frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E} \cdot \overline{x}(e^*_{nc}) > 0$. The direct effect dominates the indirect effects due to Assumption 2.

The electoral channel consists of two multiplicative terms. The first term, $\rho_e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{\partial \overline{x}(e)}{\partial e}\Big|_{e=e^*_{nc}} = \frac{\rho_e}{1-\delta\rho_E} \cdot \big(1 - 2\delta(\mathbb{I}\{\check{x}_{nc} > 0\} \cdot \rho_{\mathcal{R}} + \mathbb{I}\{\check{x}_{nc} < 0\} \cdot \rho_{\mathcal{L}})\big) > 0$, captures how shifts in the win-probability weighted election winner mean ideology e^*_{nc} affect policymaking outcomes. The second term, $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} \lesseqgtr 0$, captures how shifting proposal rights from $\mathcal{M}$ to $\mathcal{R}$ affects the win-probability weighted election winner mean ideology e^*_{nc}. The sign of the electoral channel depends on the second term, $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}}$. If $\check{x}_{nc} < 0$, then $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} = (1 - 2\delta\rho_{\mathcal{L}}) \cdot \frac{\partial \check{x}_{nc}}{\partial \rho_{\mathcal{R}}} > 0$, which follows from $\frac{\partial \check{x}_{nc}}{\partial \rho_{\mathcal{R}}} = \frac{1}{f(\check{x}_{nc})} \cdot \frac{\delta(1-2\delta\rho_{\mathcal{L}})}{2(1-\delta\rho_E)^2} > 0$. If $\check{x}_{nc} \geq 0$, then $\frac{\partial e^*_{nc}}{\partial \rho_{\mathcal{R}}} = (1-2\delta\rho_{\mathcal{R}}) \cdot \frac{\partial \check{x}_{nc}}{\partial \rho_{\mathcal{R}}} - 2\delta\check{x}_{nc} = 2\delta\big(-\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}\big)$. Thus, the sign of the electoral channel is positive if and only if $\check{x}_{nc} \leq \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}$ and negative otherwise.

Lastly, we show the total effect is strictly positive. If $\check{x}_{nc} \leq \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}$, both channels are positive. So suppose $\check{x}_{nc} > \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}$. Then, we have:

$$\begin{aligned}\frac{\partial \mu_{e^*_{nc}}}{\partial \rho_{\mathcal{R}}} &= \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E}\overline{x}(e^*_{nc}) + 2\delta\rho_e \cdot \frac{1-2\delta\rho_{\mathcal{L}}}{1-\delta\rho_E} \cdot \Big(-\check{x}_{nc} + \frac{1}{2f(\check{x}_{nc})} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{2(1-\delta\rho_E)^2}\Big) \\ &= \frac{1-2\delta\rho_{\mathcal{L}}}{(1-\delta\rho_E)^2}\bigg((1-\delta)c + (1-2\delta\rho_{\mathcal{L}})\delta\rho_e\Big(-\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)\bigg) > 0,\end{aligned}$$

where the inequality follows as $\ell^* < 0$ implies $\check{x}_{nc} < \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}$. □

Proposition A.9. *If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then (a) $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})} > 0$; (b) $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_e-\rho_{\mathcal{M}})} > 0$ if and only if $\check{x}_{nc} > 0$; (c) $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_e)} > 0$ if $\check{x}_{nc} < 0$.*

PROOF. *Part (a):* From Proposition A.8, we have $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{M}})} > 0$ and $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{L}}-\rho_{\mathcal{M}})} < 0$. Thus, $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})} = \frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{M}})} - \frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{L}}-\rho_{\mathcal{M}})} > 0$.

Part (b): Taking the derivative, we have $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_e-\rho_{\mathcal{M}})} = \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E} \cdot \check{x}_{nc}$. Thus, $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_e-\rho_{\mathcal{M}})} > 0$ if $\check{x}_{nc} > 0$ and $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_e-\rho_{\mathcal{M}})} < 0$ if $\check{x}_{nc} < 0$.

Part (c): Proposition A.8 and part (b) imply $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_e)} = \frac{1-2\delta\rho_{\mathcal{L}}}{(1-\delta\rho_E)^2}\bigg((1-\delta)c + (1-2\delta\rho_{\mathcal{L}})\delta\rho_e\Big(-\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)\bigg) - \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\check{x}_{nc}$. Thus, $\check{x}_{nc} < 0$ implies $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_e)} > 0$. □

Proposition A.10. *If $-\overline{x} < \ell^* < r^* < 0$, then (a) $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{M}})} > 0$; (b) $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{M}}-\rho_{\mathcal{L}})} > 0$; (c) $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{M}}-\rho_e)} > 0$; (d) $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})} > 0$; (e) $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_e)} > 0$; (f) $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{L}}-\rho_e)} < 0$.*

PROOF. *Part (a):* $\frac{\partial \pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{M}})} = \frac{\partial}{\partial \rho_{\mathcal{R}}}\Big[\rho_e \cdot \check{x}_{l\,c} + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}})\frac{(1-\delta)c - \delta\rho_e\check{x}_{l\,c}}{1-\delta\rho_E}\Big] = \frac{1-2\delta\rho_{\mathcal{L}}}{(1-\delta\rho_E)^2}\bigg((1-\delta)c + \delta\rho_e\Big(-\check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}}\Big)\bigg) > 0$. The inequality follows from $\check{x}_{l\,c} < 0$.

Part (b): $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{M}}-\rho_{\mathcal{L}})} = -\frac{\partial}{\partial\rho_{\mathcal{L}}}\left[\rho_e\cdot\check{x}_{l\,c}+(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})\frac{(1-\delta)c-\delta\rho_e\check{x}_{l\,c}}{1-\delta\rho_E}\right] = \frac{1-2\delta\rho_{\mathcal{R}}}{(1-\delta\rho_E)^2}\left((1-\delta)c-\delta\rho_e\left(\check{x}_{l\,c}+\frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\right)\right) > 0$. The inequality follows since $r^* < 0$ implies $\check{x}_{l\,c} + \frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)} < 0$.

Part (c): $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{M}}-\rho_e)} = -\frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\check{x}_{l\,c} > 0$, where the inequality again follows from $\check{x}_{l\,c} < 0$.

Part (d): From parts (a) and (b), it follows that $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{L}})} = \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{M}})} + \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{M}}-\rho_{\mathcal{L}})} > 0$.

Part (e): From parts (a) and (c), it follows that $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_e)} = \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{R}}-\rho_{\mathcal{M}})} + \frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{M}}-\rho_e)} > 0$.

Part (f): From part (b) and (c), we have $\frac{\partial\pi(\ell^*,r^*)}{\partial(\rho_{\mathcal{L}}-\rho_e)} = \frac{1-2\delta\rho_{\mathcal{R}}}{(1-\delta\rho_E)^2}\Big(-(1-\delta)c+\delta\rho_e\big(\check{x}_{l\,c}+\frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\big)\Big) - \frac{1-2\delta\rho_{\mathcal{R}}}{1-\delta\rho_E}\check{x}_{l\,c} = \frac{1-2\delta\rho_{\mathcal{R}}}{(1-\delta\rho_E)^2}\Big(-(1-\delta)c-(1-\delta(\rho_E+\rho_e))\check{x}_{l\,c}+\delta\rho_e\frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)}\Big)$. By Proposition 3, $\ell^* = \check{x}_{l\,c} - \frac{1}{2f(\check{x}_{l\,c})}\cdot\frac{1-2\delta\rho_E}{1-\delta\rho_E}$, so $\ell^* > -\overline{x}$ implies $\check{x}_{l\,c} > -\overline{x} + \frac{1}{2f(\check{x}_{l\,c})}\cdot\frac{1-2\delta\rho_E}{1-\delta\rho_E}$. Since $\check{x}_{l\,c}$ enters the bracketed term with coefficient $-(1-\delta(\rho_E+\rho_e)) < 0$, substituting this bound for $\check{x}_{l\,c}$, holding $f(\check{x}_{l\,c})$ at its equilibrium value, and using $(1-\delta(\rho_E+\rho_e))\,\overline{x} = (1-\delta)c$ yields

$$
\begin{aligned}
-(1-\delta)c-(1-\delta(\rho_E+\rho_e))\check{x}_{l\,c}+\delta\rho_e\frac{1}{f(\check{x}_{l\,c})}\frac{1}{2(1-\delta\rho_E)} &< \frac{1}{2f(\check{x}_{l\,c})}\cdot\frac{\delta\rho_e-(1-\delta(\rho_E+\rho_e))(1-2\delta\rho_E)}{1-\delta\rho_E} \\
&= -\frac{1-2\delta(\rho_E+\rho_e)}{2f(\check{x}_{l\,c})} < 0,
\end{aligned}
$$

where the final inequality follows from Assumption 2a. □

Proposition A.11. *If $-\overline{x} < \ell^* < r^* < \overline{x}$, a marginal rightward shift of the voter distribution increases $\pi(\ell^*, r^*)$.*

PROOF. Consider the translated family $F_t(x) = F(x-t)$ and let (ℓ_t^*, r_t^*) denote the unique equilibrium under F_t, restricted to t for which it is interior. By Remark A.2, as t increases the equilibrium configuration passes through the five types of Lemma A.3, each prevailing on an interval of t. We show π is strictly increasing in t within each configuration. Since the representations in Lemma A.3 coincide at the configuration boundaries, π is continuous in t, and the conclusion follows.

First, we consider differentiable configurations. Each equilibrium quantile shifts one-for-one with t, with $\check{x}.$ mapping to $\check{x}.+t$, and $f_0(\check{x}.) = f_t(\check{x}.+t)$. Therefore it suffices to sign $\frac{\partial\pi}{\partial\check{x}.}$. If $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$, then $\frac{\partial\pi(\ell^*,r^*)}{\partial\check{x}_{nc}} = \frac{\rho_e}{1-\delta\rho_E}\cdot(1-2\delta\rho_{\mathcal{R}})\cdot(1-2\delta\rho_{\mathcal{L}}) > 0$. If $-\overline{x} < \ell^* < r^* < 0$, then $\frac{\partial\pi(\ell^*,r^*)}{\partial\check{x}_{l\,c}} = \frac{\rho_e\cdot(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E} > 0$, and symmetrically $\frac{\partial\pi(\ell^*,r^*)}{\partial\check{x}_{rc}} > 0$ if $0 < \ell^* < r^* < \overline{x}$.

Second, we consider the non-differentiable case with $r_t^* = 0$. The case with $\ell_t^* = 0$ is symmetric. By Lemma A.3, the equilibrium indifferent voter ι_t is the unique zero of $G_t(x) = G(x-t)+t$, so $\iota_t = t+G^{-1}(-t)$ and $F_t(\iota_t) = F\big(G^{-1}(-t)\big)$, with $\ell_t^* = 2(1-\delta\rho_E)\,\iota_t < 0$.

Consider $t' > t$ with both equilibria in this configuration. Since G is strictly increasing by Lemma A.2 and F has full support, $F_{t'}(\iota_{t'}) < F_t(\iota_t)$. Moreover, $G(b) - G(a) \geq b - a$ for $b > a$ since F/f is nondecreasing, so $G^{-1}(-t) - G^{-1}(-t') \leq t' - t$, and therefore $\iota_{t'} \geq \iota_t$ and $\ell^*_{t'} \geq \ell^*_t$. Denoting $\pi_t = \mu_0 + F_t(\iota_t)\big(\mu_{\ell^*_t} - \mu_0\big)$, where μ_0 does not depend on t and $\mu_{\ell^*_t} - \mu_0 < 0$ since μ_e is strictly increasing, we have

$$\pi_{t'} - \pi_t \;=\; \underbrace{\big(F_{t'}(\iota_{t'}) - F_t(\iota_t)\big)}_{<0} \cdot \underbrace{\big(\mu_{\ell^*_{t'}} - \mu_0\big)}_{<0} \;+\; \underbrace{F_t(\iota_t)}_{>0} \cdot \underbrace{\big(\mu_{\ell^*_{t'}} - \mu_{\ell^*_t}\big)}_{\geq 0} \;>\; 0.$$

□

E Extensions

E.1 Party-Dependent Extremist Proposal Rights

Fix ρ_e, $\rho_{\mathcal{M}}$, and let total extremist rights be $\rho_E = \underline{\rho}_{\mathcal{L}} + \underline{\rho}_{\mathcal{R}} + \phi$. Since ρ_e and $\rho_{\mathcal{M}}$ are fixed, so is ρ_E. Accordingly, comparative statics in ϕ exchange fixed extremist rights for variable ones one-for-one, along paths satisfying $\underline{\rho}_{\mathcal{L}}(\phi) + \underline{\rho}_{\mathcal{R}}(\phi) = \rho_E - \phi$. Essentially, ϕ captures how much extremists' proposal rights depend on the winner's party. In particular, suppose that if ℓ wins, we have $\rho_{\mathcal{L}} = \underline{\rho}_{\mathcal{L}} + \phi$ and $\rho_{\mathcal{R}} = \underline{\rho}_{\mathcal{R}}$, while if r wins, we have $\rho_{\mathcal{L}} = \underline{\rho}_{\mathcal{L}}$ and $\rho_{\mathcal{R}} = \underline{\rho}_{\mathcal{R}} + \phi$. We maintain Assumptions 1 and 2a, along with $\phi \in [0, \frac{1}{2\delta} - \underline{\rho}_{\mathcal{L}} - \underline{\rho}_{\mathcal{R}} - \rho_e)$, which restates Assumption 2a given $\rho_E = \underline{\rho}_{\mathcal{L}} + \underline{\rho}_{\mathcal{R}} + \phi$. Finally, note that given an officeholder e and proposal rights ρ, equilibrium policymaking is identical to the baseline.

Voter Calculus. The key difference is a shift in the weights of the policy lottery. In a slight abuse of notation, let $\mathcal{U}_i^{\phi}(e) = \rho_e \cdot u_i(x_e(e)) + (\underline{\rho}_{\mathcal{L}} + \phi \cdot \mathbb{I}\{e = \ell\}) \cdot u_i(-\overline{x}(e)) + (\underline{\rho}_{\mathcal{R}} + \phi \cdot \mathbb{I}\{e = r\}) \cdot u_i(\overline{x}(e)) + \rho_{\mathcal{M}} \cdot u_i(0)$, and define $\Delta^{\phi}(\ell, r; i) = \mathcal{U}_i^{\phi}(\ell) - \mathcal{U}_i^{\phi}(r)$. It can be verified, following the proof of Lemma 4, that there is a unique indifferent voter. For the equilibrium candidate pairs below, this voter satisfies $\iota^{\phi}_{\ell,r} \in (\max\{\ell, -\overline{x}(r)\}, \min\{r, \overline{x}(\ell)\})$. Solving for $\iota^{\phi}_{\ell,r}$ yields

$$\iota^{\phi}_{\ell,r} = \frac{\rho_e}{\rho_e + \phi} \cdot \frac{1}{1 - \delta\rho_E}\left(\frac{\ell + r}{2} - \delta\rho_E\Big(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\Big)\right).$$

Note $\iota^{\phi}_{\ell,r} = \frac{\rho_e}{\rho_e + \phi} \cdot \iota_{\ell,r}$, where $\iota_{\ell,r}$ is the baseline indifferent voter. Since $\frac{\rho_e}{\rho_e+\phi} < 1$ for $\phi > 0$, the indifferent voter is less responsive to candidate positions, since voters' preferences over candidates are now partially also affected by their relative preference over extremists.

Party Calculus. Let $\mu_e^\phi = \rho_e \cdot e + (\underline{\rho}_{\mathcal{R}} - \underline{\rho}_{\mathcal{L}} - \phi \cdot (\mathbb{I}\{e=\ell\} - \mathbb{I}\{e=r\})) \cdot \overline{x}(e)$. Then,

$$\frac{\partial \mu_\ell^\phi}{\partial \ell} = \frac{\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (1-2\delta\underline{\rho}_{\mathcal{R}}) & \text{if } \ell < 0 \\ (1-2\delta(\underline{\rho}_{\mathcal{L}}+\phi)) & \text{if } \ell > 0, \end{cases} \quad \frac{\partial \mu_r^\phi}{\partial r} = \frac{\rho_e}{1-\delta\rho_E} \cdot \begin{cases} (1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)) & \text{if } r < 0 \\ (1-2\delta\underline{\rho}_{\mathcal{L}}) & \text{if } r > 0. \end{cases}$$

Lastly, let $\Delta_R^\phi(\ell, r) \equiv \mu_r^\phi - \mu_\ell^\phi$ denote the electoral stakes.

Proposition A.12 characterizes no-crossover equilibria and establishes Proposition 4(i).

Proposition A.12. *In any equilibrium such that $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}$,*

b. *the indifferent voter is $\iota_{\ell^*,r^*}^\phi = \check{x}_{nc}^\phi = F^{-1}\Big(\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}\Big)$,*

c. *candidate divergence is $r^*-\ell^* = \frac{\rho_e+\phi}{\rho_e}\cdot\frac{1-\delta\rho_E}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\Big(2\delta(\underline{\rho}_{\mathcal{L}}-\underline{\rho}_{\mathcal{R}})\cdot\check{x}_{nc}^\phi+\frac{1}{f(\check{x}_{nc}^\phi)}\cdot\frac{(1-2\delta\underline{\rho}_{\mathcal{L}})(1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\Big)- \frac{2\phi}{\rho_e}\cdot\frac{(1-\delta)c}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}$, and*

d. *candidates are $\ell^* = \frac{\rho_e+\phi}{\rho_e}\cdot\frac{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{L}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\Big(\check{x}_{nc}^\phi - \frac{1}{2f(\check{x}_{nc}^\phi)}\cdot\frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\Big) + \frac{\phi}{\rho_e}\cdot\frac{(1-\delta)c}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}$ and $r^* = \frac{\rho_e+\phi}{\rho_e}\cdot\frac{(1-\delta\rho_E)(1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\Big(\check{x}_{nc}^\phi + \frac{1}{2f(\check{x}_{nc}^\phi)}\cdot\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\Big) - \frac{\phi}{\rho_e}\cdot\frac{(1-\delta)c}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}$.*

PROOF. Fix $\phi \in [0, \frac{1}{2\delta} - \underline{\rho}_{\mathcal{L}} - \underline{\rho}_{\mathcal{R}} - \rho_e)$. Suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$. The FOCs are:

$$0 = f\big(\iota_{\ell^*,r^*}^\phi\big)\cdot\Delta_R^\phi(\ell^*, r^*)\cdot\frac{\partial\iota_{\ell,r^*}^\phi}{\partial\ell}\Big|_{\ell=\ell^*} - F\big(\iota_{\ell^*,r^*}^\phi\big)\cdot\frac{\partial\mu_\ell^\phi}{\partial\ell}\Big|_{\ell=\ell^*},$$
$$0 = f\big(\iota_{\ell^*,r^*}^\phi\big)\cdot\Delta_R^\phi(\ell^*, r^*)\cdot\frac{\partial\iota_{\ell^*,r}^\phi}{\partial r}\Big|_{r=r^*} - \Big(1-F\big(\iota_{\ell^*,r^*}^\phi\big)\Big)\cdot\frac{\partial\mu_r^\phi}{\partial r}\Big|_{r=r^*}.$$

Moreover, we have $\frac{\partial\iota_{\ell,r}^\phi}{\partial\ell} = \frac{\partial\iota_{\ell,r}^\phi}{\partial r} = \frac{\rho_e}{\rho_e+\phi}\frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial\mu_\ell^\phi}{\partial\ell}\Big|_{\ell=\ell^*} = \frac{\rho_e\cdot(1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta\rho_E}$ and $\frac{\partial\mu_r^\phi}{\partial r}\Big|_{r=r^*} = \frac{\rho_e\cdot(1-2\delta\underline{\rho}_{\mathcal{L}})}{1-\delta\rho_E}$. Combining FOCs yields $F(\iota_{\ell,r}^\phi) = \frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}$, so $\iota_{\ell,r}^\phi = F^{-1}\Big(\frac{1-2\delta\underline{\rho}_{\mathcal{L}}}{2(1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}}))}\Big) = \check{x}_{nc}^\phi$. Next, substituting μ_e^ϕ and $\overline{x}(e)$, the electoral stakes for $\ell^* < 0 < r^*$ are

$$\Delta_R^\phi(\ell^*, r^*) = \mu_{r^*}^\phi - \mu_{\ell^*}^\phi = \frac{\rho_e\big((1-2\delta\underline{\rho}_{\mathcal{L}})\,r^* - (1-2\delta\underline{\rho}_{\mathcal{R}})\,\ell^*\big) + 2\phi(1-\delta)c}{1-\delta\rho_E}.$$

Substituting into L's FOC and simplifying yields

$$r^* = \frac{1-2\delta\underline{\rho}_{\mathcal{R}}}{1-2\delta\underline{\rho}_{\mathcal{L}}}\cdot\ell^* + \frac{\rho_e+\phi}{\rho_e}\cdot\frac{(1-2\delta\underline{\rho}_{\mathcal{R}})(1-\delta\rho_E)}{1-\delta(\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})}\cdot\frac{1}{f(\check{x}_{nc}^\phi)} - \frac{2\phi}{\rho_e}\cdot\frac{(1-\delta)c}{1-2\delta\underline{\rho}_{\mathcal{L}}}.$$

Combining with $\check{x}_{nc}^{\phi} = \frac{\rho_e}{\rho_e+\phi} \cdot \frac{1}{1-\delta\rho_E} \cdot \frac{\ell^*+r^*}{2}$ yields ℓ^* and r^*. $\square$

Example: Divergence with Balanced Extremists. Suppose the voter distribution F has median $m = 0$ and extremists have equal fixed proposal power, $\underline{\rho}_{\mathcal{L}} = \underline{\rho}_{\mathcal{R}}$. Then Proposition A.12 implies $\check{x}_{nc}^{\phi} = F^{-1}(\frac{1}{2}) = 0$, and $r^* - \ell^* = \frac{\rho_e+\phi}{\rho_e} \cdot \frac{(1-\delta\rho_E)}{f(0)} - \frac{2\phi}{\rho_e} \cdot \frac{(1-\delta)c}{1-\delta(\rho_E-\phi)}$. Fix ρ_e and $\rho_{\mathcal{M}}$, so that ρ_E is fixed, and maintain $\underline{\rho}_{\mathcal{L}} = \underline{\rho}_{\mathcal{R}}$. Then, differentiating with respect to ϕ shifts fixed rights along $\underline{\rho}_{\mathcal{L}}(\phi) = \underline{\rho}_{\mathcal{R}}(\phi) = \frac{\rho_E-\phi}{2}$, so we have:

$$\frac{\partial[r^*-\ell^*]}{\partial\phi} = \underbrace{-\frac{2}{\rho_e} \cdot \frac{(1-\delta)c}{1-\delta(\rho_E-\phi)}}_{\text{election stakes channel } (-)} + \underbrace{\frac{1}{\rho_e} \cdot \frac{(1-\delta\rho_E)}{f(0)}}_{\text{voter channel } (+)} + \underbrace{\frac{2\phi}{\rho_e} \cdot \frac{\delta(1-\delta)c}{(1-\delta(\rho_E-\phi))^2}}_{\text{extremist stronger if winning channel } (+)} \lesseqgtr 0.$$

Increasing ϕ incentivizes convergence by raising the stakes of the election. It also incentivizes divergence because voters are less sensitive to candidates and both parties, conditional on winning, want to constrain extremists less due to their aligned extremist holding more proposal power. The overall effect of increasing ϕ may be positive or negative.

Proposition A.13 characterizes crossover equilibria and establishes Proposition 4(ii).

Proposition A.13. *Suppose $\phi < \rho_E$, so that $\underline{\rho}_{\mathcal{L}} + \underline{\rho}_{\mathcal{R}} > 0$. In any equilibrium such that $-\overline{x} < \ell^* < r^* < 0$, party L's win probability is*

$$P^* = \frac{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)}{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)+(1-2\delta\underline{\rho}_{\mathcal{R}})(1-2\delta\rho_E)} > \frac{1}{2}.$$

PROOF. Suppose $-\overline{x} < \ell^* < r^* < 0$. The FOCs are analogous to those in the proof of Proposition A.12, but now with $\frac{\partial\mu_\ell^\phi}{\partial\ell}\big|_{\ell=\ell^*} = \frac{\rho_e(1-2\delta\underline{\rho}_{\mathcal{R}})}{1-\delta\rho_E}$, $\frac{\partial\mu_r^\phi}{\partial r}\big|_{r=r^*} = \frac{\rho_e(1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi))}{1-\delta\rho_E}$, $\frac{\partial\iota_{\ell,r}^\phi}{\partial\ell} = \frac{\rho_e}{\rho_e+\phi} \cdot \frac{1}{2(1-\delta\rho_E)}$, and $\frac{\partial\iota_{\ell,r}^\phi}{\partial r} = \frac{\rho_e}{\rho_e+\phi} \cdot \frac{1-2\delta\rho_E}{2(1-\delta\rho_E)}$. Combining the FOCs yields

$$F\left(\iota_{\ell^*,r^*}^\phi\right) = \frac{\frac{\partial\mu_r^\phi}{\partial r} \cdot \frac{\partial\iota_{\ell,r}^\phi}{\partial\ell}}{\frac{\partial\mu_r^\phi}{\partial r} \cdot \frac{\partial\iota_{\ell,r}^\phi}{\partial\ell} + \frac{\partial\mu_\ell^\phi}{\partial\ell} \cdot \frac{\partial\iota_{\ell,r}^\phi}{\partial r}} = \frac{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)}{1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi)+(1-2\delta\underline{\rho}_{\mathcal{R}})(1-2\delta\rho_E)}.$$

Finally, $P^* > \frac{1}{2}$ if and only if $1-2\delta(\underline{\rho}_{\mathcal{R}}+\phi) > (1-2\delta\underline{\rho}_{\mathcal{R}})(1-2\delta\rho_E)$, which rearranges to $2\delta\underline{\rho}_{\mathcal{R}}\,\phi < (\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})(1-2\delta\underline{\rho}_{\mathcal{R}})$. If $\underline{\rho}_{\mathcal{R}} = 0$, this holds since $\underline{\rho}_{\mathcal{L}} > 0$. If $\underline{\rho}_{\mathcal{R}} > 0$, then Assumption 2a implies $\phi < \frac{1}{2\delta} - \underline{\rho}_{\mathcal{R}}$, so $2\delta\underline{\rho}_{\mathcal{R}}\,\phi < \underline{\rho}_{\mathcal{R}}(1-2\delta\underline{\rho}_{\mathcal{R}}) \leq (\underline{\rho}_{\mathcal{L}}+\underline{\rho}_{\mathcal{R}})(1-2\delta\underline{\rho}_{\mathcal{R}})$. $\square$

E.2 Election for Veto Player

Suppose the collective body consists only of election winner e and extremists $\mathcal{L}$ and $\mathcal{R}$. We assume $\rho_E < \frac{1}{2}$ and focus on the case in which candidates constrain both extremists in equilibrium policymaking.

Policymaking. To characterize policymaking, let $\underline{y}(e) = e - \frac{(1-\delta)c}{1-\delta\rho_E}$ and $\overline{y}(e) = e + \frac{(1-\delta)c}{1-\delta\rho_E}$. If $-\overline{X} < \underline{y}(e)$ and $\overline{y}(e) < \overline{X}$, then e's acceptance set is $A(e) = [\underline{y}(e), \overline{y}(e)]$. Let $\mathcal{U}_i^v(e) = \rho_e \cdot u_i(e) + \rho_{\mathcal{L}} \cdot u_i(\underline{y}(e)) + \rho_{\mathcal{R}} \cdot u_i(\overline{y}(e))$, and $\Delta^v(\ell, r; i) = \mathcal{U}_i^v(\ell) - \mathcal{U}_i^v(r)$, and $\mu_e^v = \rho_e \cdot e + \rho_{\mathcal{L}} \cdot \underline{y}(e) + \rho_{\mathcal{R}} \cdot \overline{y}(e) = e + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) \cdot \frac{(1-\delta)c}{1-\delta\rho_E}$.

Lemma A.5. *If $-\overline{X} < \underline{y}(r) < \ell < r < \overline{y}(\ell) < \overline{X}$, then there is a unique indifferent voter $\iota_{\ell,r}^v = \frac{1}{2(1-\rho_E)}\big(\ell \cdot (1 - 2\rho_{\mathcal{R}}) + r \cdot (1 - 2\rho_{\mathcal{L}})\big)$, which satisfies $\iota_{\ell,r}^v \in (\ell, r)$.*

PROOF. It can be verified that $\rho_E < \frac{1}{2}$ implies $\Delta^v(\ell, r; i) > 0$ for all $i \leq \ell$ and $\Delta^v(\ell, r; i) < 0$ for all $i \geq r$. For $i \in (\ell, r)$, we have $\Delta^v(\ell, r; i) = \rho_e(\ell + r - 2i) + (\rho_{\mathcal{R}} - \rho_{\mathcal{L}})(r - \ell)$, so $\frac{\partial \Delta^v(\ell,r;i)}{\partial i} = -2\rho_e < 0$. Thus, there is a unique indifferent voter $\iota_{\ell,r}^v \in (\ell, r)$, characterized by $\Delta^v(\ell, r; i) = 0$. □

Proposition A.14. *In any equilibrium such that $-\overline{X} < \underline{y}(r^*) < \ell^* < r^* < \overline{y}(\ell^*) < \overline{X}$:*

a. *L's equilibrium win probability is $P^* = \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$,*
b. *the indifferent voter is $\iota_{\ell^*,r^*}^v = \check{x}^v = F^{-1}\Big(\frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}\Big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}^v)}$, and*
d. *candidates are $\ell^* = \check{x}^v - \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)}$ and $r^* = \check{x}^v + \frac{1}{f(\check{x}^v)} \cdot \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$.*

PROOF. Suppose $-\overline{X} < \underline{y}(r^*) < \ell^* < r^* < \overline{y}(\ell^*) < \overline{X}$. The FOCs are:

$$0 = f\big(\iota_{\ell^*,r^*}^v\big) \cdot \Delta_R^v(\ell^*, r^*) \cdot \frac{\partial \iota_{\ell,r^*}^v}{\partial \ell}\Big|_{\ell=\ell^*} - F\big(\iota_{\ell^*,r^*}^v\big) \cdot \frac{\partial \mu_\ell^v}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\big(\iota_{\ell^*,r^*}^v\big) \cdot \Delta_R^v(\ell^*, r^*) \cdot \frac{\partial \iota_{\ell^*,r}^v}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota_{\ell^*,r^*}^v\big)\Big) \cdot \frac{\partial \mu_r^v}{\partial r}\Big|_{r=r^*},$$

where $\frac{\partial \mu_\ell^v}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\partial \mu_r^v}{\partial r}\big|_{r=r^*} = 1$, $\frac{\partial \iota_{\ell,r^*}^v}{\partial \ell}\big|_{\ell=\ell^*} = \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$, and $\frac{\partial \iota_{\ell^*,r}^v}{\partial r}\big|_{r=r^*} = \frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\big(\iota_{\ell^*,r^*}^v\big) = \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)}$. Thus, we must have $\iota_{\ell^*,r^*}^v = \check{x}^v$. Combining with the FOCs yields the candidate locations ℓ^* and r^*. □

The candidate profile characterized above satisfies the maintained fully constrained interior inequalities if: (i) $\frac{1}{f(\check{x}^v)} < \frac{(1-\delta)c}{1-\delta\rho_E}$, (ii) $\overline{X} > \check{x}^v + \frac{1}{f(\check{x}^v)} \frac{1-2\rho_{\mathcal{R}}}{2(1-\rho_E)} + \frac{(1-\delta)c}{1-\delta\rho_E}$, and (iii) $-\overline{X} < \check{x}^v - \frac{1}{f(\check{x}^v)} \frac{1-2\rho_{\mathcal{L}}}{2(1-\rho_E)} - \frac{(1-\delta)c}{1-\delta\rho_E}$.

Corollary A.14.1. *With pure proximity voters, L's win probability in the veto player election is $P^* = \frac{1}{2}$.*

PROOF. In this case, the indifferent voter is $\iota^0_{\ell,r} = \frac{\ell+r}{2}$. The marginal policy cost $\frac{\partial \mu^v_\ell}{\partial \ell} = \frac{\partial \mu^v_r}{\partial r} = 1$ and electoral benefit $\frac{\partial \iota^0_{\ell,r}}{\partial \ell} = \frac{\partial \iota^0_{\ell,r}}{\partial r} = \frac{1}{2}$ of converging are both symmetric. Thus, $P^* = \frac{1}{2}$. $\square$

E.3 Proximity Voters

Suppose the voter evaluates candidates based on a weighted average between full sophistication and proximity concerns. Let $\alpha \in [0,1]$ parametrize voters' weight on sophistication and $1-\alpha$ the weight on proximity. Denote a voter i's ex-ante utility of electing candidate ℓ over candidate r as $\Delta^\alpha(\ell, r; i) \equiv \alpha \cdot \Delta(\ell, r; i) + (1-\alpha) \cdot (u_i(\ell) - u_i(r))$. For $\alpha = 1$, we recover the baseline model. For $\alpha = 0$, we are in the pure proximity voting case described in the main text. Solving for the indifferent voter yields:

$$\iota^\alpha_{\ell,r} = \frac{1}{1-\delta\rho_E}\Big(\frac{\ell+r}{2} \cdot \frac{\alpha\rho_e + (1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e + (1-\alpha)} - \frac{\alpha\rho_e \cdot \delta\rho_E}{\alpha\rho_e + (1-\alpha)}\big(\ell \cdot \mathbb{I}\{\ell > 0\} + r \cdot \mathbb{I}\{r < 0\}\big)\Big).$$

Proposition A.15. *In any equilibrium such that $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter is $\iota^\alpha_{\ell^*,r^*} = \check{x}_{nc} = F^{-1}\Big(\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{\alpha\rho_e+(1-\alpha)}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\Big(2\delta(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\Big)$,*
d. *and candidates are $\ell^* = \frac{(1-2\delta\rho_{\mathcal{L}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = \frac{(1-2\delta\rho_{\mathcal{R}})\cdot(\alpha\rho_e+(1-\alpha))}{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})}\frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}\Big)$.*

PROOF. Fix $\alpha \in [0,1]$ and suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$ in equilibrium. The FOCs are:

$$0 = f\big(\iota^\alpha_{\ell^*,r^*}\big) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial \iota^\alpha_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\big(\iota^\alpha_{\ell^*,r^*}\big) \cdot \mu'_-$$
$$0 = f\big(\iota^\alpha_{\ell^*,r^*}\big) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial \iota^\alpha_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota^\alpha_{\ell^*,r^*}\big)\Big) \cdot \mu'_+.$$

Since there is no crossover, we have $\frac{\partial \iota^\alpha_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} = \frac{\partial \iota^\alpha_{\ell^*,r}}{\partial r}\Big|_{r=r^*} = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$. Combining the FOCs yields $F\big(\iota^\alpha_{\ell^*,r^*}\big) = \frac{\mu'_+}{\mu'_+ + \mu'_-} = \frac{1-2\delta\rho_{\mathcal{L}}}{2(1-\delta\rho_E)}$. Thus, $\iota^\alpha_{\ell^*,r^*} = \check{x}_{nc}$. Substituting

$\check{x}_{nc}$ into L's FOC and simplifying yields:

$$r^* = \ell^* \cdot \frac{1-2\delta\rho_{\mathcal{R}}}{1-2\delta\rho_{\mathcal{L}}} + \frac{1-2\delta\rho_{\mathcal{R}}}{f(\check{x}_{nc})} \cdot \frac{\alpha\rho_e + (1-\alpha)}{\alpha\rho_e + (1-\alpha)(1-\delta\rho_E)}.$$

Solving the system of two equations yields ℓ^* and r^*. $\square$

In particular, the equilibrium indifferent-voter location is independent of α, although proximity weighting does change the candidates supporting that location.

Corollary A.15.1. *Suppose $\alpha \in (0,1)$, $\rho_E > 0$, $\check{x}_{nc} \neq 0$, and $-\overline{x} < \ell^* < 0 < r^* < \overline{x}$. The party on the same side of 0 as $\check{x}_{nc}$ strictly prefers to decrease α, while the other party strictly prefers to increase α.*

PROOF. Ex-ante expected policy is $\pi^\alpha(\ell^*, r^*) = F(\check{x}_{nc})\cdot(\mu_{\ell^*}-\mu_{r^*})+\mu_{r^*} = \frac{1}{1-\delta\rho_E}\Big((1-\delta)c(\rho_{\mathcal{R}}-\rho_{\mathcal{L}}) + \rho_e \cdot \frac{\ell^*+r^*}{2} \cdot \frac{(1-2\delta\rho_{\mathcal{L}})(1-2\delta\rho_{\mathcal{R}})}{1-\delta\rho_E}\Big)$. Thus, we have $\frac{\partial \pi^\alpha(\ell^*,r^*)}{\partial\alpha} = \check{x}_{nc} \cdot \Big(\frac{\rho_e\cdot(1-2\delta\,\rho_{\mathcal{L}})\cdot(1-2\delta\,\rho_{\mathcal{R}})}{1-\delta\rho_E}\Big) \cdot \Big(-\frac{\delta\,\rho_e\,\rho_E}{(\alpha\rho_e+(1-\alpha)(1-\delta\rho_E))^2}\Big)$, so $\frac{\partial\pi^\alpha(\ell^*,r^*)}{\partial\alpha} \propto -\check{x}_{nc}$. Therefore $\check{x}_{nc} > 0$ implies $\pi^\alpha(\ell^*, r^*)$ strictly decreases in α, and vice versa. $\square$

Proposition A.16. *In any equilibrium such that $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$:*

a. *party L's win probability is $P^* = \frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$,*
b. *the indifferent voter is $\iota^\alpha_{\ell^*,r^*} = \check{x}^\alpha_{l\,c} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}\Big)$,*
c. *candidate divergence is $r^* - \ell^* = \frac{1}{f(\check{x}^\alpha_{l\,c})}$, and*
d. *candidates are $\ell^* = \check{x}^\alpha_{l\,c} - \frac{1}{f(\check{x}^\alpha_{l\,c})} \cdot \frac{(1-2\delta\rho_E)\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e+(1-\alpha))}$ and $r^* = \check{x}^\alpha_{l\,c} + \frac{1}{f(\check{x}^\alpha_{l\,c})} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{2(1-\delta\rho_E)(\alpha\rho_e+(1-\alpha))}$.*

PROOF. Fix $\alpha \in [0,1]$ and suppose $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$ in equilibrium. The FOCs are:

$$0 = f\big(\iota^\alpha_{\ell^*,r^*}\big) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial \iota^\alpha_{\ell,r^*}}{\partial\ell}\Big|_{\ell=\ell^*} - F\big(\iota^\alpha_{\ell^*,r^*}\big) \cdot \mu'_-$$
$$0 = f\big(\iota^\alpha_{\ell^*,r^*}\big) \cdot \Delta_R(\ell^*, r^*) \cdot \frac{\partial \iota^\alpha_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota^\alpha_{\ell^*,r^*}\big)\Big) \cdot \mu'_-.$$

Combining these FOCs yields $F\big(\iota^\alpha_{\ell^*,r^*}\big) = \frac{\frac{\partial \iota^\alpha_{\ell,r^*}}{\partial\ell}\big|_{\ell=\ell^*}}{\frac{\partial \iota^\alpha_{\ell,r^*}}{\partial\ell}\big|_{\ell=\ell^*} + \frac{\partial \iota^\alpha_{\ell^*,r}}{\partial r}\big|_{r=r^*}} = \frac{1}{2(1-\delta\rho_E)}\cdot\frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}$.
Let $\check{x}^\alpha_{l\,c} = F^{-1}\Big(\frac{1}{2(1-\delta\rho_E)} \cdot \frac{\alpha\rho_e+(1-\alpha)(1-\delta\rho_E)}{\alpha\rho_e+(1-\alpha)}\Big)$. In equilibrium, $\check{x}^\alpha_{l\,c} = \iota^\alpha_{\ell^*,r^*}$, which implies

$$r^* = \check{x}^\alpha_{l\,c} \cdot \frac{2(1-\delta\rho_E)\cdot(\alpha\rho_e + (1-\alpha))}{\alpha\rho_e \cdot (1-2\delta\rho_E) + (1-\alpha)\cdot(1-\delta\rho_E)} - \ell^* \cdot \frac{\alpha\rho_e + (1-\alpha)\cdot(1-\delta\rho_E)}{\alpha\rho_e \cdot (1-2\delta\rho_E) + (1-\alpha)\cdot(1-\delta\rho_E)}.$$

Moreover, FOCs imply $r^* = \frac{1}{f(\check{x}^{\alpha}_{l\,c})} + \ell^*$. Solving the system of equations yields ℓ^*, r^*. $\square$

Corollary A.16.1. *Suppose $\alpha \in (0,1)$, $\rho_E > 0$, and $-\overline{x} < \ell^* < r^* < 0 < \overline{x}$. Party R has a strict preference for increasing α while L has a strict preference for decreasing α.*

PROOF. The ex-ante expected policy is: $\pi^{\alpha}(\ell^*, r^*) = F(\check{x}^{\alpha}_{l\,c})(\mu_{\ell^*} - \mu_{r^*}) + \mu_{r^*} = \frac{1}{1-\delta\rho_E}\Big((1-\delta)c \cdot (\rho_{\mathcal{R}} - \rho_{\mathcal{L}}) + \rho_e\, \check{x}^{\alpha}_{l\,c}(1 - 2\delta\rho_{\mathcal{R}})\Big) = \mu_{\check{x}^{\alpha}_{l\,c}}$. Therefore $\frac{\partial \pi^{\alpha}(\ell^*, r^*)}{\partial \alpha} = \frac{\partial \mu_{\check{x}^{\alpha}_{l\,c}}}{\partial \alpha} \propto \frac{\partial \check{x}^{\alpha}_{l\,c}}{\partial \alpha} > 0$. $\square$

E.4 Additional Extensions

E.4.1 Voters Overestimate Election Winner's Proposal Rights

Voters may overestimate the influence of their election winner in policymaking. Here, we consider one possibility: parties know the true distribution of proposal rights ρ, while the voter believes that it is $\rho^{\epsilon} = (\rho_e + \epsilon, \rho_{\mathcal{M}} - \epsilon, \rho_{\mathcal{L}}, \rho_{\mathcal{R}})$. Assume $\epsilon \in (0, \frac{1}{2\delta} - \rho_e - \rho_E)$ and $\delta > \frac{c - \overline{X}}{c - (\rho_E + \rho_e + \epsilon)\overline{X}}$. These are analogous to Assumptions 2a and 1 under the perceived rights ρ^{ϵ}. They ensure that, under the voter's perceived model, the indifferent voter is a centrist. Lemma 4 implies there is a unique indifferent voter $\iota^{\epsilon}_{\ell,r}$, which is at the same location as the baseline setting: $\iota^{\epsilon}_{\ell,r} = \iota_{\ell,r}$. As a result, party incentives to converge are identical to the baseline, so equilibrium is identical. Thus, our results are robust to voters overestimating the institutional power their local representative will hold, as long as they correctly perceive extremist proposal rights.

E.4.2 Election with Supermajority Policymaking

Suppose there are two fixed veto pivots located symmetrically around 0, $v_L = -\nu$ and $v_R = \nu$ with $\nu > 0$, each with proposal power $\rho_{v_L} = \rho_{v_R} = \frac{1 - \rho_e - \rho_{\mathcal{L}} - \rho_{\mathcal{R}}}{2}$. A proposal passes if and only if both pivots accept. We keep Assumptions 1 and 2a, and assume $c \geq \frac{(3-\delta)\nu}{1-\delta}$, which ensures both pivots can pass their ideal points in policymaking (verified below). We restrict attention to candidates between the pivots and accordingly characterize policymaking for $e \in [-\nu, \nu]$.

Policymaking. Let $A^s(e)$ denote the equilibrium acceptance set given e, which is the intersection of the two pivots' acceptance sets. Given linear loss utility, v_R's indifference condition pins down the lower bound of $A^s(e)$ and v_L's pins down the upper bound. Define

$$\underline{x}^s(e) = \frac{-(1-\delta)c + \nu\left(1 - \delta + 2\delta\rho_{\mathcal{R}}(1 + \delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}}))\right) + \delta\rho_e(1 - 2\delta\rho_{\mathcal{R}})\, e}{1 - \delta\rho_E},$$

$$\overline{x}^s(e) = \frac{(1-\delta)c - \nu\left(1 - \delta + 2\delta\rho_{\mathcal{L}}(1 - \delta(\rho_{\mathcal{L}} - \rho_{\mathcal{R}}))\right) + \delta\rho_e(1 - 2\delta\rho_{\mathcal{L}})\, e}{1 - \delta\rho_E}.$$

Lemma A.6. *If $e \in [-\nu, \nu]$, then $A^s(e) = [\underline{x}^s(e), \overline{x}^s(e)]$, with $\underline{x}^s(e) \le -\nu$ and $\overline{x}^s(e) \ge \nu$.*

PROOF. Write $A^s(e) = A^s_{v_L}(e) \cap A^s_{v_R}(e)$, where $A^s_{v_P}(e) = [\underline{a}^s_{v_P}(e), \overline{a}^s_{v_P}(e)]$ is pivot v_P's acceptance set. Since $v_L < v_R$ and both pivots evaluate the same continuation lottery, $\underline{a}^s_{v_L}(e) < \underline{a}^s_{v_R}(e)$ and $\overline{a}^s_{v_L}(e) < \overline{a}^s_{v_R}(e)$, so $A^s(e) = [\underline{a}^s_{v_R}(e), \overline{a}^s_{v_L}(e)]$. Conjecture the ordering $\underline{a}^s_{v_R}(e) \le -\nu \le e \le \nu \le \overline{a}^s_{v_L}(e)$. Then, if recognized, e, v_L, and v_R propose their ideal points, $\mathcal{L}$ proposes $\underline{a}^s_{v_R}(e)$, and $\mathcal{R}$ proposes $\overline{a}^s_{v_L}(e)$, all of which pass. The pivots' indifference conditions,

$$u_{v_R}\big(\underline{a}^s_{v_R}(e)\big) + (1-\delta)c = \delta\Big(\rho_e u_{v_R}(e) + \rho_{\mathcal{L}} u_{v_R}\big(\underline{a}^s_{v_R}(e)\big) + \rho_{\mathcal{R}} u_{v_R}\big(\overline{a}^s_{v_L}(e)\big) + \tfrac{\rho_{\mathcal{M}}}{2} u_{v_R}(-\nu) + \tfrac{\rho_{\mathcal{M}}}{2} u_{v_R}(\nu)\Big),$$
$$u_{v_L}\big(\overline{a}^s_{v_L}(e)\big) + (1-\delta)c = \delta\Big(\rho_e u_{v_L}(e) + \rho_{\mathcal{L}} u_{v_L}\big(\underline{a}^s_{v_R}(e)\big) + \rho_{\mathcal{R}} u_{v_L}\big(\overline{a}^s_{v_L}(e)\big) + \tfrac{\rho_{\mathcal{M}}}{2} u_{v_L}(-\nu) + \tfrac{\rho_{\mathcal{M}}}{2} u_{v_L}(\nu)\Big),$$

are linear under the conjectured ordering:

$$(1-\delta\rho_{\mathcal{L}})\,\underline{a}^s_{v_R}(e) + \delta\rho_{\mathcal{R}}\,\overline{a}^s_{v_L}(e) = -(1-\delta)c + \delta\rho_e e + \nu(1-\delta+2\delta\rho_{\mathcal{R}}),$$
$$\delta\rho_{\mathcal{L}}\,\underline{a}^s_{v_R}(e) + (1-\delta\rho_{\mathcal{R}})\,\overline{a}^s_{v_L}(e) = (1-\delta)c + \delta\rho_e e - \nu(1-\delta+2\delta\rho_{\mathcal{L}}).$$

This system has determinant $1-\delta\rho_E > 0$ and unique solution $\big(\underline{x}^s(e), \overline{x}^s(e)\big)$. It remains to verify the conjectured behavior. Since $\overline{x}^s(e)$ is increasing in e, it is minimized on $[-\nu,\nu]$ at $e=-\nu$, where $\overline{x}^s(-\nu) \ge \nu$ reduces to $(1-\delta)c \ge \nu\big(2-\delta+\delta(\rho_{\mathcal{L}}-\rho_{\mathcal{R}})+\delta\rho_e-2\delta^2\rho_{\mathcal{L}}(\rho_{\mathcal{L}}-\rho_{\mathcal{R}}+\rho_e)\big)$. The right side is less than $(3-\delta)\nu$ under Assumption 2a, so $c \ge \frac{(3-\delta)\nu}{1-\delta}$ suffices. Symmetrically, $\underline{x}^s(e) \le \underline{x}^s(\nu) \le -\nu$. Thus, $\pm\nu, e \in A^s(e)$. Moreover, $c \ge \frac{(3-\delta)\nu}{1-\delta}$ implies $A^s(e) \subset (-\overline{x}, \overline{x})$, so Assumption 1 places the extremists' ideal points outside $A^s(e)$, and each optimally proposes the nearest bound. Uniqueness of the policymaking equilibrium follows as in Lemma 1 (Cardona and Ponsati, 2011). □

A key difference from the main model is that shifting an officeholder between the pivots, $e \in [-\nu,\nu]$, moves *both* bounds of $A^s(e)$ in the same direction, rather than in opposite directions.

Voter Calculus. If officeholder $e \in (-\nu,\nu)$, then player i's continuation value is $\mathcal{U}^s_i(e) = \rho_e u_i(e) + \rho_{\mathcal{L}} u_i(\underline{x}^s(e)) + \rho_{\mathcal{R}} u_i(\overline{x}^s(e)) + \frac{\rho_{\mathcal{M}}}{2} u_i(-\nu) + \frac{\rho_{\mathcal{M}}}{2} u_i(\nu)$. Let $\Delta^s(\ell, r; i) = \mathcal{U}^s_i(\ell) - \mathcal{U}^s_i(r)$.

Lemma A.7. *If $-\nu < \ell < r < \nu$, then there is a unique indifferent voter $\iota^s_{\ell,r} = \frac{1}{2(1-\delta\rho_E)}\Big(\ell(1-2\delta\rho_{\mathcal{R}}) + r(1-2\delta\rho_{\mathcal{L}})\Big)$, which satisfies $\iota^s_{\ell,r} \in (\ell, r)$.*

PROOF. The pivots' proposals do not vary with e, so they cancel in Δ^s. For $i \le \ell$, we have $u_i(\ell) - u_i(r) = r - \ell$. Furthermore, since u_i is 1-Lipschitz, Lemma A.6 implies $u_i\big(\underline{x}^s(\ell)\big) -$

$u_i\big(\underline{x}^s(r)\big) \geq -\frac{\delta\rho_e(1-2\delta\rho_{\mathfrak{R}})}{1-\delta\rho_E}(r-\ell)$ and $u_i\big(\overline{x}^s(\ell)\big) - u_i\big(\overline{x}^s(r)\big) \geq -\frac{\delta\rho_e(1-2\delta\rho_{\mathfrak{L}})}{1-\delta\rho_E}(r-\ell)$. Therefore $\Delta^s(\ell,r;i) \geq (r-\ell)\,\rho_e\Big(1-\frac{\delta\rho_E}{1-\delta\rho_E}\Big) > 0$ by Assumption 2. Symmetrically, $\Delta^s(\ell,r;i) < 0$ for $i \geq r$. For $i \in (\ell,r)$, since $\underline{x}^s(\cdot) \leq -\nu < i < \nu \leq \overline{x}^s(\cdot)$,

$$\Delta^s(\ell,r;i) = (r-\ell)\,\frac{\delta\rho_e\,(\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})}{1-\delta\rho_E} + \rho_e(\ell+r-2i),$$

which is strictly decreasing in i. Solving $\Delta^s(\ell,r;i)=0$ for i yields the result. $\square$

Party Calculus. Given $e \in (-\nu,\nu)$, the mean of the equilibrium policy lottery is $\mu^s_e = \rho_e\cdot e + \rho_{\mathfrak{L}}\cdot\underline{x}^s(e) + \rho_{\mathfrak{R}}\cdot\overline{x}^s(e)$ since the pivots' proposals cancel. Substituting for $\underline{x}^s(e)$ and $\overline{x}^s(e)$ and simplifying yields $\mu^s_e = \frac{1-4\delta^2\rho_{\mathfrak{L}}\rho_{\mathfrak{R}}}{1-\delta\rho_E}\rho_e\cdot e + (\rho_{\mathfrak{R}}-\rho_{\mathfrak{L}})\Big(\frac{(1-\delta)\,c-\nu\,(1-\delta(1-4\delta\rho_{\mathfrak{L}}\rho_{\mathfrak{R}}))}{1-\delta\rho_E}\Big)$.

Proposition A.17. *In any equilibrium such that $-\nu < \ell^* < r^* < \nu$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$,*
b. *the indifferent voter is $\iota^s_{\ell^*,r^*} = \check{x}_\nu = F^{-1}\big(\frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}\big)$,*
c. *candidate divergence is $r^*-\ell^* = \frac{1}{f(\check{x}_\nu)}$,*
d. *and candidates are $\ell^* = \check{x}_\nu - \frac{1}{f(\check{x}_\nu)}\frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$ and $r^* = \check{x}_\nu + \frac{1}{f(\check{x}_\nu)}\frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$.*

PROOF. Suppose $-\nu < \ell^* < r^* < \nu$. The FOCs take the baseline form, with $\frac{\partial\iota^s_{\ell,r}}{\partial\ell} = \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$ and $\frac{\partial\iota^s_{\ell,r}}{\partial r} = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$ (Lemma A.7), and $\frac{\partial\mu^s_\ell}{\partial\ell}\big|_{\ell=\ell^*} = \frac{\partial\mu^s_r}{\partial r}\big|_{r=r^*} = \frac{1-4\delta^2\rho_{\mathfrak{L}}\rho_{\mathfrak{R}}}{1-\delta\rho_E}\rho_e$. Combining the FOCs and using the equality of the μ^s slopes yields $F\big(\iota^s_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}$, so $\iota^s_{\ell^*,r^*} = \check{x}_\nu$. Since μ^s_e is linear over $(-\nu,\nu)$, the electoral stakes are $\Delta^s_R(\ell^*,r^*) = \frac{\partial\mu^s_e}{\partial e}\cdot(r^*-\ell^*)$. Substituting into L's FOC yields $r^*-\ell^* = \frac{1}{f(\check{x}_\nu)}$, and combining with $\iota^s_{\ell^*,r^*} = \check{x}_\nu$ yields the candidate locations. $\square$

Since $P^* > \frac{1}{2}$ if and only if $\rho_{\mathfrak{L}} > \rho_{\mathfrak{R}}$, the party with the *stronger* extremist is electorally advantaged. This matches the election for a veto player above, and differs from the baseline.

E.4.3 Party-Dependent Election Winner Proposal Rights

The election winner's institutional rights may be contingent on their party. For instance, candidates from one party might be more skilled or effective legislators if elected, and therefore be more likely to propose. Here, we consider a setting where (i) if ℓ wins, the distribution of proposal rights is $\rho = (\rho_e,\rho_{\mathfrak{M}},\rho_{\mathfrak{L}},\rho_{\mathfrak{R}})$, and (ii) if r wins, the distribution is $\rho^\beta = (\rho_e-\beta,\rho_{\mathfrak{M}}+\beta,\rho_{\mathfrak{L}},\rho_{\mathfrak{R}})$, where $\beta \in (0,\rho_e)$. We maintain Assumptions 1 & 2a and focus on no-crossover equilibria.

Policymaking. If ℓ wins, equilibrium policymaking is identical to the baseline. If r wins, policymaking is analogous but with ρ^β instead of ρ. Define $\overline{x}^\beta = \frac{(1-\delta)c}{1-\delta(\rho_E+\rho_e-\beta)}$ and $\overline{x}^\beta(r) = \begin{cases} \frac{(1-\delta)c+\delta(\rho_e-\beta)|r|}{1-\delta\rho_E} & \text{if } r \in [-\overline{x}^\beta, \overline{x}^\beta], \\ \overline{x}^\beta & \text{else} \end{cases}$. If r wins, the acceptance set is $A(r) = [-\overline{x}^\beta(r), \overline{x}^\beta(r)]$.

Voter Calculus. A player i's continuation value from ℓ as officeholder is $\mathcal{U}_i(\ell)$ while r as officeholder yields $\mathcal{U}_i^\beta(r) = (\rho_e - \beta)u_i(x_r(r)) + \rho_{\mathcal{L}}u_i(-\overline{x}^\beta(r)) + \rho_{\mathcal{R}}u_i(\overline{x}^\beta(r)) + (\rho_{\mathcal{M}} + \beta)u_i(0)$. Let $\Delta^\beta(\ell, r; i) = \mathcal{U}_i(\ell) - \mathcal{U}_i^\beta(r)$. For interior candidates, $-\overline{x} < \ell < r < \overline{x}^\beta$, we have:

$$\Delta^\beta(\ell, r; i) = \rho_{\mathcal{L}}\big(-|i + \overline{x}(\ell)| + |i + \overline{x}^\beta(r)|\big) + \rho_e\big(-|i - \ell| + |i - r|\big)$$
$$+ \rho_{\mathcal{R}}\big(-|i - \overline{x}(\ell)| + |i - \overline{x}^\beta(r)|\big) - \beta\big(-|i| + |i - r|\big).$$

Lemma A.8. *If $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$, then there is a unique indifferent voter:*

$$\iota_{\ell,r}^\beta = \frac{1}{2(1-\delta\rho_E)} \cdot \begin{cases} (\frac{\rho_e}{\rho_e-\beta} \cdot \ell + r) & \text{if } r \in [-\frac{\rho_e}{\rho_e-\beta} \cdot \ell, \overline{x}^\beta) \\ (\ell + \frac{\rho_e-\beta}{\rho_e} \cdot r) & \text{if } r \in (0, -\frac{\rho_e}{\rho_e-\beta} \cdot \ell), \end{cases}$$

which satisfies $\iota_{\ell,r}^\beta \in (\max\{\ell, -\overline{x}^\beta(r)\}, \min\{r, \overline{x}(\ell)\})$.

PROOF. Consider $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$. The proof is similar to the proof of Lemma 4: Part 1 shows $\iota_{\ell,r}^\beta \in (\max\{\ell, -\overline{x}^\beta(r)\}, \min\{r, \overline{x}(\ell)\})$ and Part 2 characterizes it.

Part 1: We show $\Delta^\beta(\ell, r; \ell) > 0$ and $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) > 0$, which imply $\Delta^\beta(\ell, r; i) > 0$ for all $i \leq \max\{\ell, -\overline{x}^\beta(r)\}$. An analogous proof shows $\Delta^\beta(\ell, r; i) < 0$ for all $i \geq \min\{r, \overline{x}(\ell)\}$.

First, $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$ implies $\Delta^\beta(\ell, r; \ell) = \rho_{\mathcal{L}}(-\ell - \overline{x}(\ell) + |\ell + \overline{x}^\beta(r)|) + \rho_{\mathcal{R}}(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell$. If $\ell \geq -\overline{x}^\beta(r)$, then $\Delta^\beta(\ell, r; \ell) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell$, so substituting and simplifying yields $\Delta^\beta(\ell, r; \ell) = \frac{1}{1-\delta\rho_E}\big((\rho_e - \beta)r - (1 - 2\delta\rho_E)\rho_e\ell\big) > 0$. Otherwise $\ell < -\overline{x}^\beta(r)$, which yields $\Delta^\beta(\ell, r; \ell) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + (\rho_e - \beta)r - \rho_e\ell - 2\rho_{\mathcal{L}}(\ell + \overline{x}^\beta(r)) > 0$ by the preceding case and $\ell < -\overline{x}^\beta(r)$. Thus, $\Delta^\beta(\ell, r; \ell) > 0$.

Second, $-\overline{x} < \ell < 0 < r < \overline{x}^\beta$ implies $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) = \rho_{\mathcal{L}}(-|\overline{x}^\beta(r) - \overline{x}(\ell)|) + \rho_{\mathcal{R}}(\overline{x}^\beta(r) - \overline{x}(\ell)) + \rho_e(\overline{x}^\beta(r) - |\overline{x}^\beta(r) + \ell|) + (\rho_e - \beta)r$. If $\ell \geq -\overline{x}^\beta(r)$, then it is straightforward to verify $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) > 0$. If instead $\ell < -\overline{x}^\beta(r)$, we have $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) = \rho_E(\overline{x}^\beta(r) - \overline{x}(\ell)) + 2\rho_e\overline{x}^\beta(r) + \rho_e\ell + (\rho_e - \beta)r$. Substituting and simplifying yields $\Delta^\beta(\ell, r; -\overline{x}^\beta(r)) = \frac{1}{1-\delta\rho_E}\big(\rho_e(\ell + 2(1-\delta)c) + (1 + 2\delta\rho_e)(\rho_e - \beta)r\big) > 0$ by Assumption 2a.

Part 2: Note $\Delta^\beta(\ell, r; i)$ is continuous and strictly decreasing over $i \in (\ell, r)$ since, wherever it is differentiable on (ℓ, r), it satisfies $\frac{\partial\Delta^\beta(\ell,r;i)}{\partial i} \leq -2\rho_e + 2\beta < 0$. Thus, a unique $\iota_{\ell,r}^\beta$ solves $\Delta^\beta(\ell, r; i) = 0$, characterized by $(\rho_e - \beta \cdot \mathbb{I}\{i > 0\}) \cdot i = \frac{1}{2(1-\delta\rho_E)}(\rho_e\ell + (\rho_e - \beta)r)$. $\square$

Let $\mu_r^\beta = (\rho_e - \beta)r + (\rho_{\mathfrak{R}} - \rho_{\mathfrak{L}}) \cdot \overline{x}^\beta(r)$ be the mean of the policy lottery induced by r.

Proposition A.18. *Suppose $\check{x}_{nc} \neq 0$. In any equilibrium such that $-\overline{x} < \ell^* < 0 < r^* < \overline{x}^\beta$:*

a. *party L's win probability is $P^* = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$;*

b. i. *if $\check{x}_{nc} > 0$, then candidates are $\ell^* = \frac{\rho_e-\beta}{\rho_e}(1-2\delta\rho_{\mathfrak{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = (1-2\delta\rho_{\mathfrak{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}\Big)$;*

   ii. *if $\check{x}_{nc} < 0$, then candidates are $\ell^* = (1-2\delta\rho_{\mathfrak{L}})\Big(\check{x}_{nc} - \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathfrak{R}}}{2(1-\delta\rho_E)}\Big)$ and $r^* = \frac{\rho_e}{\rho_e-\beta}(1-2\delta\rho_{\mathfrak{R}})\Big(\check{x}_{nc} + \frac{1}{f(\check{x}_{nc})} \cdot \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}\Big)$.*

PROOF. Fix $\beta \in [0, \rho_e)$. Suppose $-\overline{x} < \ell^* < 0 < r^* < \overline{x}^\beta$ is an equilibrium. The FOCs are:

$$0 = f\big(\iota^\beta_{\ell^*,r^*}\big) \cdot \Delta^\beta_R(\ell^*, r^*) \cdot \frac{\partial \iota^\beta_{\ell,r^*}}{\partial \ell}\Big|_{\ell=\ell^*} - F\big(\iota^\beta_{\ell^*,r^*}\big) \cdot \frac{\partial \mu_\ell}{\partial \ell}\Big|_{\ell=\ell^*},$$
$$0 = f\big(\iota^\beta_{\ell^*,r^*}\big) \cdot \Delta^\beta_R(\ell^*, r^*) \cdot \frac{\partial \iota^\beta_{\ell^*,r}}{\partial r}\Big|_{r=r^*} - \Big(1 - F\big(\iota^\beta_{\ell^*,r^*}\big)\Big) \cdot \frac{\partial \mu^\beta_r}{\partial r}\Big|_{r=r^*}.$$

We have $\frac{\partial \mu_\ell}{\partial \ell}\big|_{\ell=\ell^*} = \mu'_-$ and $\frac{\partial \mu^\beta_r}{\partial r}\big|_{r=r^*} = \frac{\rho_e-\beta}{\rho_e}\mu'_+$. There are two cases.

Case (i): If $r^* \in (-\frac{\rho_e}{\rho_e-\beta} \cdot \ell^*, \overline{x}^\beta)$, then $\frac{\partial \iota^\beta_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{\rho_e}{\rho_e-\beta}\frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial \iota^\beta_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{1}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^\beta_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$, so $\iota^\beta_{\ell^*,r^*} = \check{x}_{nc}$. Moreover, the FOCs imply $r^* = \frac{\rho_e}{\rho_e-\beta}\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathfrak{L}}}\ell^* + (1-2\delta\rho_{\mathfrak{R}})\frac{1}{f(\check{x}_{nc})}$. Finally, combining with $\check{x}_{nc} = \frac{1}{2(1-\delta\rho_E)} \cdot (r^* + \frac{\rho_e}{\rho_e-\beta}\ell^*)$ yields ℓ^* and r^* for $\check{x}_{nc} > 0$.

Case (ii): If $r^* \in (0, -\frac{\rho_e}{\rho_e-\beta} \cdot \ell^*)$, then $\frac{\partial \iota^\beta_{\ell,r^*}}{\partial \ell}\big|_{\ell=\ell^*} = \frac{1}{2(1-\delta\rho_E)}$ and $\frac{\partial \iota^\beta_{\ell^*,r}}{\partial r}\big|_{r=r^*} = \frac{\rho_e-\beta}{\rho_e}\frac{1}{2(1-\delta\rho_E)}$. Combining the FOCs, substituting and simplifying yields $F\big(\iota^\beta_{\ell^*,r^*}\big) = \frac{1-2\delta\rho_{\mathfrak{L}}}{2(1-\delta\rho_E)}$, so $\iota^\beta_{\ell^*,r^*} = \check{x}_{nc}$. Moreover, the FOCs imply $r^* = \frac{\rho_e}{\rho_e-\beta}\Big(\frac{1-2\delta\rho_{\mathfrak{R}}}{1-2\delta\rho_{\mathfrak{L}}}\ell^* + (1-2\delta\rho_{\mathfrak{R}})\frac{1}{f(\check{x}_{nc})}\Big)$. Finally, combining with $\check{x}_{nc} = \frac{1}{2(1-\delta\rho_E)} \cdot (\frac{\rho_e-\beta}{\rho_e}r^* + \ell^*)$ yields ℓ^* and r^* for $\check{x}_{nc} < 0$. □

The contingent rights β alter convergence incentives through both the party policy channel and the voter channel. Notably, Proposition A.18 shows parties' win probabilities in centrist districts are identical to the baseline, since the two effects cancel out, but candidate locations shift systematically. If the district has a slight right lean ($\check{x}_{nc} > 0$), then party L's candidate is more moderate than in the baseline while party R's is unchanged. If it has a slight left lean ($\check{x}_{nc} < 0$), then party R's candidate is more extreme while party L's is unchanged.

F Weak Veto Player

Suppose Assumptions 1 and 2 hold, but Assumption 2a does not. Substantively, this captures an election for a major office (ρ_e high), or into a policymaking system where the main veto player is unlikely to propose ($\rho_{\mathcal{M}}$ low). We focus on the case in which $r \geq |\ell|$.

Now, if r is sufficiently more extreme than ℓ, the indifferent voter may not be a centrist, as $\iota_{\ell,r} > \overline{x}(\ell)$.

Lemma A.9. *If $-\overline{x} < \ell < r < \overline{x}$ and $|\ell| \leq r$, then the indifferent voter is*

$$\iota^{wv}_{\ell,r} = \begin{cases} \frac{\rho_e}{\rho_e+\rho_{\mathcal{R}}}\frac{1}{2(1-\delta\rho_E)}\Big(r + \ell\big(1-2\delta(\rho_{\mathcal{L}} \cdot \mathbb{I}\{\ell>0\} + \rho_{\mathcal{R}} \cdot \mathbb{I}\{\ell<0\})\big)\Big) + \frac{\rho_{\mathcal{R}}}{\rho_e+\rho_{\mathcal{R}}}\frac{(1-\delta)c}{1-\delta\rho_E} & \text{if } r \in (\overline{r}(\ell), \overline{x}), \\ \iota_{\ell,r} & \text{otherwise,} \end{cases}$$

where $\overline{r}(\ell) = 2(1-\delta)c - (1+2\delta\rho_e) \cdot \ell \cdot \mathbb{I}\{\ell<0\} - (1-2\delta(\rho_E+\rho_e)) \cdot \ell \cdot \mathbb{I}\{\ell>0\}$.

PROOF. Existence and uniqueness follow as in Part 1 of the proof of Lemma 4. Lemma A.1, which uses only Assumptions 1 and 2, implies $\Delta(\ell, r; i) > 0$ for $i \leq \ell$ and $\Delta(\ell, r; i) < 0$ for $i \geq r$, and $\Delta(\ell, r; i)$ is continuous and strictly decreasing in i on (ℓ, r). Since $r \geq |\ell|$ implies $\overline{x}(r) \geq \overline{x}(\ell) \geq |\ell|$, we have $-\overline{x}(r) \leq \ell$, so the indifferent voter can exit the acceptance-set interval only on the right. From the computation of $\Delta(\ell, r; \overline{x}(\ell))$ in Part 2 of that proof, which does not use Assumption 2a, we have $\Delta(\ell, r; \overline{x}(\ell)) = \frac{\rho_e}{1-\delta\rho_E}\big(r - \overline{r}(\ell)\big)$, so $\iota^{wv}_{\ell,r} > \overline{x}(\ell)$ if and only if $r > \overline{r}(\ell)$. If $r \leq \overline{r}(\ell)$, the indifferent voter is in $(-\overline{x}(r), \overline{x}(\ell)]$ and Part 3 of that proof yields $\iota^{wv}_{\ell,r} = \iota_{\ell,r}$. If instead $r > \overline{r}(\ell)$, then $\iota^{wv}_{\ell,r} \in (\overline{x}(\ell), r)$, where

$$\Delta(\ell, r; i) = \rho_{\mathcal{L}}\big(\overline{x}(r) - \overline{x}(\ell)\big) + \rho_{\mathcal{R}}\big(\overline{x}(r) + \overline{x}(\ell) - 2i\big) + \rho_e(\ell + r - 2i).$$

Solving $\Delta(\ell, r; i) = 0$ and substituting $\overline{x}(e) = \frac{(1-\delta)c+\delta\rho_e|e|}{1-\delta\rho_E}$ yields the displayed expression. □

Crucially, now shifting ℓ more extreme has opposing effects on the indifferent voter. Conditional on ℓ winning, $\mathcal{R}$'s proposal $\overline{x}(\ell)$ shifts closer to $\iota_{\ell,r}$, while $\mathcal{L}$'s proposal shifts away. In contrast, marginal changes to r have the same impact as the baseline.