Lobbying and Policy Extremism in Repeated Elections [*]

Peter Bils[†]   John Duggan[‡]   Gleason Judd[§]

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Abstract

We study a model of repeated elections that features privately informed politicians and ideologically extreme lobby groups. We establish existence of a class of perfect Bayesian equilibria. If office incentives are high, then all equilibria feature strong parties: liberal politicians all choose the same policy, as do all conservative politicians. When the effectiveness of money approaches zero, these equilibrium policies converge to the median, providing a dynamic version of the median voter theorem. When the effectiveness of money becomes large, however, the most polarized strongly partisan equilibria become arbitrarily extremist, and thus highly effective lobbying creates the possibility of arbitrarily extreme policy outcomes. In case the effectiveness of money is not large, lobbying incentives can push politicians to choose more moderate policies than they otherwise would, and an increase in the effectiveness of money can increase the welfare of the median voter.

Keywords: accountability, adverse selection, interest group, lobbying, median voter, repeated elections

JEL Codes: C72, C73, D72, D78

1   Introduction

Democratic theorists have long celebrated elections as a mechanism to discipline officeholders, while also highlighting the dangers posed by organized groups with particular interests, or "factions."[1] Elections enable voters to remove incumbents who enact unpopular policy, but officeholders face considerable pressure from policy-motivated interest groups. Lobbying is a prominent way that groups influence policy, and a correspondingly large literature in political economy studies its impact on policy outcomes (Grossman and Helpman, 1994; Kang, 2015).[2] Although lobbying can distort policy if the preferences of powerful interest groups diverge from those of the public at large, the threat of losing re-election encourages officeholders to choose policies that are sufficiently popular with voters, and the electoral consequences of policy choices have the potential to feed back into lobby group efforts. In this article, we propose a dynamic model to study how lobbying by ideological interest groups interacts with electoral accountability to shape policy in democratic systems.

Existing theoretical work typically considers these forces separately. On one hand, elections have been shown to moderate policy through several channels, including electoral competition (Black, 1948; Downs, 1957) and dynamic policy responsiveness by incumbents (Banks and Duggan, 2008; Duggan and Forand, 2021). On the other hand, a large literature suggests that lobbying distorts political motives by shifting policy choices away from the median voter (Grossman and Helpman, 1996; Fox and Rothenberg, 2011). Nevertheless, lobby groups evidently have reason to consider the electoral prospects of their policy initiatives: if a group advocates a policy choice that risks an electoral loss, then it must compensate the officeholder for those electoral costs; moreover, if the incumbent loses re-election, then the new officeholder may be less receptive to the group's lobbying. In particular, the new officeholder may be lobbied by groups from the opposite ideological extreme, which increases the policy stakes and highlights the endogeneity of lobbying incentives due to repeated interaction over time. Overall, analyzing the effect of interest groups on policymaking in democracies involves highly complex, dynamic incentives and, consequently, difficult analytical challenges.

We provide a formal analysis of elections over time that allows us to trace causal mechanisms that are crucial for understanding the role and effects of money in electoral politics, and we find that the interaction of accountability and lobbying has a stark effect on ideological cohesion within parties. If money is highly effective or office incentives are large, then all politicians from the same side of the ideological spectrum choose the same policy, a phenomenon we refer to as "strong parties." Furthermore, we highlight conditions under which a dynamic version of the median voter theorem holds, so that the electoral mechanism nullifies the adverse effects of lobbying: if office incentives are high, then equilibrium policies converge to the median as the effectiveness of money becomes small. Conversely, we also provide conditions such that the centrifugal effect of lobbying offsets the centripetal effect of elections and produces arbitrarily extremist policy outcomes:

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[1] See Madison (1787, 1788).

[2] See also De Figueiredo and Silverman (2006); Baumgartner et al. (2009), and Richter et al. (2009).

when the effectiveness of money becomes large, the most polarized equilibria become arbitrarily extremist. Another implication of our results is that lobbyist expenditures are non-monotonic in the effectiveness of money, and they are highest when money is moderately effective.

The operation of dynamic incentives in the model is subtle, and our analysis reveals counter-intuitive possibilities when the effectiveness of money is not large. As the effectiveness of money increases, it becomes less costly for a lobby group to pull policy outcomes in the direction of its preference, and the direct effect on policy outcomes is that they become more extreme. This applies when a politician is initially lobbied to a policy in the win set and is pulled to a more extreme policy in the win set, and when a politician is initially lobbied to a losing policy and is pulled to a more extreme losing policy. However, there is also an indirect effect through the endogeneity of lobbying incentives: when an incumbent is replaced by a challenger, there is a chance that the opposing lobby group pulls the new officeholder to worse policies, making both groups more willing to compromise. The indirect effect can dominate: we give a numerical example in which the effectiveness of money grows, and greater equilibrium compromise *raises* the welfare of the median voter.

As well, it is intuitive that within an equilibrium, lobby groups will pull politicians to policies that are more extreme than they would otherwise choose. Insofar as policy payoffs are concerned, this insight is correct. However, lobby groups have an extra incentive to compromise that office-holders lack: if an incumbent is replaced by a challenger, then in addition to future policy payoffs, lobby groups must anticipate future payments to politicians, which are of course endogenous. We give an example in which the moderating influence of continuation payments offsets the extremal influence of policy incentives, and some politician types who would choose losing policies are lobbied to winning ones. For such a politician type, the default choice in the absence of a lobby offer is to shirk, by choosing her ideal policy and foregoing re-election, but the active lobby group pushes the politician to compromise and pays to compensate the politician, in order to avoid the risk of future payments to politicians.

In our model, policymaking occurs repeatedly over an infinite time horizon. In each period, a lobby group makes an offer to an incumbent politician; the incumbent either accepts this offer and chooses the proposed policy, or she rejects the offer and chooses policy independently; and there is an election between the incumbent and a challenger. Voters observe the policy choice of the incumbent, but not the ideology of either politician. We analyze stationary equilibria, in which the incumbent always chooses the same policy if re-elected, so that voters face the choice between a known incumbent and a relatively unknown challenger. The incumbent politician may face a trade off between the short term gains from choosing her ideal policy and the long term gains of compromising her choice, choosing a more moderate policy in order to gain re-election. A lobby group, anticipating voter choices and politician incentives, can make an offer to the incumbent politician that consists of a desired policy and a transfer to the politician. We model this transfer as a monetary payment from the lobby group to the politician, but it may more generally represent resources that are desirable to the politician, such as (unmodeled) campaign contributions, drafting of model legislation, or promises of future revolving-door opportunities.

These strategic incentives determine a centrally located "win set," which consists of policy choices that are sufficient for re-election: if the incumbent chooses a policy in the win set, then she is re-elected, and otherwise, she is replaced by the challenger. Given the win set, in the absence of an offer from a lobby group, the incumbent optimally chooses a winning policy if she is moderate, while more extreme politicians may choose their ideal points, foregoing re-election. This policy choice in lieu of a lobby offer is a "default" policy that serves as a reversion point in the lobbying stage. Anticipating the default policy choice, one of two lobby groups makes an offer to the incumbent that consists of a proposed policy and a monetary payment to the politician. We assume that liberal politicians are lobbied by the left interest group, while conservative politicians are lobbied by the right group; and for simplicity, we assume that the contract, if accepted by the politician, is binding. The proposed policy maximizes the payoff of the lobby group, subject to the reservation payoff of the officeholder, and the monetary payment compensates the politician for moving policy from her default choice.

We establish existence of a simple lobbying equilibrium, in which politicians and voters use stationary strategies and such that choices are optimal at every point in the game, and we provide results on strongly partisan equilibria, on policy extremism as the effectiveness of money becomes large, and convergence to the median as the effectiveness of money approaches zero. The existence proof relies on a fixed point approach, but it is novel in that we do not impose, ex ante, any structure on equilibria: rather than anticipating the form of equilibria in advance (and build that into the existence proof), the argument takes place in a space of "continuation distributions," which are consistent with a large class of strategy profiles. This strengthens our characterization results, and it suggests a technique that may be of general use in the analysis of complex, dynamic models. The dynamic median voter theorem connects our game-theoretic analysis to the social choice theory literature, and it illustrates the attraction of the median, even in the presence of lobby groups with incentives to pull policy outcomes to the extremes of the policy space. Finally, our results on the effectiveness of money establish that lobbying can precipitate extreme policies as the role of money in elections grows large, but that in some cases, lobbying can have a positive influence on voter welfare. This suggests that caution should be taken in setting restrictions on political contributions, and it points to the importance of the further study of the linkages between money and policy.

The analysis of lobbying in this paper contributes to the literature on electoral accountability and builds on the repeated elections framework of Duggan (2000),[3] in which lobby groups are not modeled and the incumbent politician chooses policy independently in each period. As in our paper, voters observe policy choices but do not observe the preferences of politicians, so that elections are subject to pure adverse selection. Early studies of electoral accountability include Barro (1973), who studies an electoral model in which there is one type of politician and voters observe policy choices, and Ferejohn (1986), who analyzes a pure moral hazard setting, where policy choices are not perfectly observable. In the pure adverse selection context, closer to our work, Bernhardt et al. (2004) study the effect of term limits; Bernhardt et al. (2009) add partisanship to the model by

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[3]See Duggan and Martinelli (2017) and Ashworth (2012) for recent overviews.

assuming that challengers can be drawn from different pools, depending on their partisan affiliation; and Bernhardt et al. (2011) add valence to the model, so that a politician's type is composed of two components, valence (which is observed) and her ideal point (which is not). The pure adverse selection model is extended to the multidimensional setting by Banks and Duggan (2008), who establish a dynamic median voter theorem in one dimension. Our median voter result reinforces that of the latter paper by showing that convergence to the median policy obtains even when lobby groups have incentives to pull policy away from the median.[4]

Much of the previous literature on lobbying and elections studies models in which donations from interest groups increase a politician's probability of winning the election (Austen-Smith, 1987; Baron, 1989). There is also a prominent literature studying lobbying as an instrument to buy votes (Groseclose and Snyder, 1996; Banks, 2000; Dal Bó, 2007; Dekel et al., 2009), as well as a large formal literature in which lobbying provides information to politicians (Potters and Van Winden, 1990, 1992; Austen-Smith and Wright, 1992, 1994).[5] We focus on lobbying as a means for interest groups to directly influence policy content via quid pro quo transfers.[6] While quid pro quo exchanges are against the law, there is substantial evidence that politicians are able to maneuver around these restrictions in practice (De Figueiredo and Garrett, 2004).[7] Austen-Smith and Wright (1994) examine how two lobbies may attempt to influence the same politicians to try and offset one other. We abstract from this possibility by assuming that only one lobby group is active at a given time, and that it only attempts to influence politicians on the same side of the median. In our context, however, this does not appear to be an onerous assumption, as it captures the empirical regularity that interest groups tend to lobby ideological allies.[8]

Another branch of the literature uses the common agency approach to study lobbying. Grossman and Helpman (1996) analyze a static model of campaign finance, in which interest groups contribute to political parties to gain influence or serve electoral motives, and Grossman and Helpman (1994) use the common agency framework to explore how special interests affect trade policy. Martimort and Semenov (2007) consider how officeholder decisions are affected, in an election-free setting, by contributions from competing lobbying firms that have opposing ideological preferences. However, these papers do not consider the dynamic incentives inherent in the repeated elections framework.

Closest to the analysis of this paper is Snyder and Ting (2008), who also study a model of repeated elections with lobbying, but the papers differ in many important ways. First, Snyder and Ting (2008) assume that politicians are purely office motivated, so that they do not face a trade off between policy and re-election. An implication of their assumption is that if two policies fail to gain re-election, then a politician is indifferent between them. This leads to uninteresting stationary

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[4]We also generalize Banks and Duggan (2008) by allowing partisanship, as in Bernhardt et al. (2009).

[5]See also Schnakenberg and Turner (2021) and Schnakenberg (2017) for recent work in this vein. See Grossman and Helpman (2002) and Wright (2002) for extensive overviews of the preceding literature.

[6]See an extension in Acemoglu et al. (2013) for recent work that also models lobbying in this fashion.

[7]See Grossman and Helpman (2002) for a more complete discussion.

[8]See, e.g., Bauer et al. (1964), Hojnacki and Kimball (1998), Kollman (1997), Milbrath (1976), and Carpenter et al. (2004).

subgame perfect equilibria in which the incumbent politician always chooses the ideal point of the interest group and voters always remove the incumbent in favor of a challenger. Such equilibria cannot occur in our model, because politicians care about policy. Second, and more subtly, Snyder and Ting (2008) assume that politicians differ only in their innate benefit from holding office, and that this type is revealed to voters after a politician's first term of office. Thus, once a first-term incumbent is re-elected, the politician no longer has an incentive to signal her type. Third, and perhaps most importantly, Snyder and Ting (2008) assume that lobby groups are short-lived, and that in each period there is a single lobby group with ideal policy drawn independently over time. In contrast, we analyze the influence of two competing lobby groups that persist over time and that care about the future consequences of policy decisions, not just pertaining to the incumbent's re-election chances, but anticipating the ideology of future challengers and the effect of lobbying on future policies.

In Section 2, we describe the model of repeated elections with lobbying. The simple lobbying equilibrium concept is defined in Section 3, and in Section 4, we establish equilibrium existence and provide a characterization in terms of cutpoints in the space of policies. Section 5 presents results on electoral incentives and strong partisanship. In Section 6, we examine the consequences for policy outcomes when the effectiveness of money becomes large. Section 7 concludes, and proofs are contained in the appendix.

2 Repeated Elections with Lobbying

We analyze repeated elections as a game among the following players: ideological lobby groups who can exchange money for policy, politicians who have policymaking power, and voters who decide between the incumbent politician and a challenger in majority-rule elections. Voters and politicians belong to a continuum $N = [\underline{\theta}, \overline{\theta}]$ of citizen types, and each type θ is associated with an ideal point $x(\theta)$ belonging to the policy space $X = [0, 1]$. We assume $0 \leq \underline{\theta} < \overline{\theta}$, where the first inequality is a useful normalization. There are two lobby groups, L and R, who for simplicity have the policy preferences of the most extreme citizen types: we equate L with $\underline{\theta}$ and R with $\overline{\theta}$, and we assume that the ideal point of group L is the left-most policy, $x(\underline{\theta}) = 0$, and the ideal point of group R is the right-most policy, $x(\overline{\theta}) = 1$.[9] The distribution of citizen types is given by a density $f(\theta)$ on N, and we assume the median, denoted θ_m, is unique. We assume that citizen types are private information, so a politician's type is not directly observable by voters, but we assume types are observable by lobby groups.[10] Of course, it may be that types can be inferred by voters from observed behavior in equilibrium.

A citizen's preferences over policy are represented by a utility function that is indexed by her

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[9]Throughout the paper we discuss these players as ideological interest groups, however, they could be interpreted more generally. One such possibility is to consider them as party leaders, who spend resources to influence policy choices of party members.

[10]Interest groups likely have better information about policymaker preferences due to frequent interactions with politicians and their staffs. Additionally, firms often hire former officeholders and staffers who have well-developed relationships with policymakers (see, e.g., Blanes i Vidal, Draca and Fons-Rosen, 2012).

type: let $u_\theta(x)$ denote the utility of a type θ citizen from policy x. Since we equate lobby groups with the extreme types, $u_L(x) = u_{\underline{\theta}}(x)$ is the utility of group L from policy x and $u_R(x) = u_{\overline{\theta}}(x)$ is the utility of group R. We assume that the utility function $u_\theta(x)$ is differentiable and strictly concave with unique maximizer $x(\theta)$. Moreover, to facilitate the analysis, we impose the following general functional form restriction:

$$u_\theta(x) = \theta v(x) - c(x) + k_\theta, \tag{1}$$

where $v' > 0$ and $v'' \leq 0$, and $c' \geq 0$ and $c'' > 0$, and k_θ is a term that is constant in policy but can depend on type; in particular, v is concave and c is strictly convex. Then the ideal point $x(\theta)$ of citizen θ is the unique solution to the first order condition $\theta v'(x) = c'(x)$, and it follows from the implicit function theorem that $x(\theta)$ is differentiable and strictly increasing; in other words, citizen types are ranked in terms of policy preferences, with higher types corresponding to higher ideal points. Therefore, the ideal point $x_m = x(\theta_m)$ is the median of the voters' ideal points. Without loss of generality, we assume the median voter weakly prefers policy $x = 1$ to $x = 0$, i.e., $u_m(1) \geq u_m(0)$, so that the right lobby group is weakly more moderate than the left, in terms of voter preferences.[11] In addition to policy utility, a politician receives a benefit of $\beta \geq 0$ in each period she holds office.

Along with the choice of policy by the politicians, we allow for the possibility of monetary transfers from lobby groups to politicians. Utility from monetary payments is quasi-linear, e.g., the utility of lobby group $G \in \{L, R\}$ from policy y and monetary payment m to the incumbent is $u_G(y) - m$. Similarly, the utility to a type θ politician from entering the contract (y, m) with the group is $u_\theta(y) + \gamma m + \beta$, with the difference that the impact of the monetary payment is multiplied by the parameter $\gamma \geq 0$. Note that when $\gamma = 0$, lobbying plays no role in the model, and we obtain the model of Duggan (2000) and the one-dimensional version of Banks and Duggan (2008), which do not permit lobbying, as a special case.

Our model conceptualizes lobbying as the expenditure of resources by two extreme interest groups to influence the content of enacted policy, rather than seeking access or providing information. These assumptions are stark, but they facilitate the analysis of the tradeoff between electoral motivations and lobbyist influence when these considerations are potentially at odds with one another. Additionally, lobbies have been shown to influence the content of policies in various issue areas (e.g., Richter et al. (2009), Bombardini and Trebbi (2011), and Kang (2015)).[12] Although quid pro quo lobbying is illegal, the transfer m can represent interactions between politicians and lobbyists more generally, such as (unmodeled) campaign contributions, provision of model legislation, or charitable donations.[13] Given our interest in understanding the impact of special interests

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[11]A special class of models are those that are symmetric around the median, in which case $x_m = 1/2$ and $u_m(0) = u_m(1)$. Our general formulation allows for asymmetries, and we assume $u_m(1) \geq u_m(0)$ merely to simplify the characterization by eliminating cases that are, by reflection across the median, already covered in the analysis.

[12]Powell (2014) emphasizes that "the influence of contributions is most likely to occur earlier in the legislative process, where less visible actions are taken to kill bills quietly or to negotiate the details of legislation that can matter so greatly to donors." (pp.75–76)

[13]See Grossman and Helpman (2002) and Großer et al. (2013) for more discussion of this issue; and Bertrand et al.

when lobbying influences policy content, quid pro quo transfers appear to be a reasonable way to model more covert forms of influence that occur out of the public eye and involve implicit transfers of favors for policy.

The parameter γ measures the effectiveness of money and will be a central focus of the subsequent analysis. In particular, we view γ as an institutional parameter that summarizes restrictions on lobbying expenditures, including expenditures to cover travel and personal expenses. More generally, γ represents constraints on any expenditures that can (explicitly or implicitly) be linked to policy choices; under this interpretation, γ incorporates regulation of campaign advertising by outside groups, limits on donations by corporations, unions, and other organizations, and rules requiring disclosure of funding sources. We will be interested in the effect of γ due to variation in restrictions on political expenditures, e.g., allowances in the US system for 527 committees or political nonprofit 501(c) groups. Alternatively, we can write the type θ politician's utility from contract (y, m) as $u_\theta(y) + m$ and the group's utility as $\gamma u_G(y) - m$. In this case, γ is interpreted as the "stakes" of the game for the lobby, and a higher γ corresponds to the lobbies having a stronger interest in the policy outcome.[14]

Along with their citizen types, politicians are distinguished by their party affiliation and preference for holding office. We assume there are two parties, where $\pi \in \{0, 1\}$ denotes the party affiliation of a politician. Here, $\pi = 0$ indicates that the politician belongs to the liberal party, and $\pi = 1$ indicates membership in the conservative party. Party affiliation will be used in equilibrium by voters to make inferences about the policy preferences of untried challengers. Let $h^\pi(\theta)$ denote the density of citizen types within the pool of candidates in party $\pi = 0, 1$. We maintain the weak assumption that there is an open set around θ_m that is contained in the support of both challenger distributions, i.e., there is an open set $\tilde{N} \subseteq N$ such that $\theta_m \in \tilde{N}$ and for almost every $\theta \in \tilde{N}$, we have $h^\pi(\theta) > 0$. For example, given $\epsilon > 0$, we could allow h^0 to be uniform on $[\underline{\theta}, \theta_m + \epsilon]$ and h^1 to be uniform on $[\theta_m - \epsilon, \overline{\theta}]$. Our existence and characterization results, Propositions 1 and 2, do not require this assumption, however, and they hold with completely general densities.

Each period $t = 0, 1, \ldots$ begins with a politician θ_t, the *incumbent,* who has some partisan affiliation π_t and who holds a political office. If the incumbent is strictly to the left of the median, i.e., $\theta_t < \theta_m$, then lobby group L is the *active group*; and if the incumbent is weakly to the right of the median, i.e., $\theta_t \geq \theta_m$, then R is the active group. In general, we write $G(\theta)$ for the active group given an incumbent with type θ. Selecting $G(\theta_m) = R$ is without loss of generality because $\theta_t = \theta_m$ is a zero probability event.

The timing of moves in the first term of office for the politician is as follows:

(1) The active group G_t offers a binding contract (y_t, m_t), where $y_t \in X$ is a policy to be chosen

(2020) on influence via charitable donations. Also, Powell (2014) notes that "there are a range of behaviors in which the legislator, consciously or unconsciously, prioritizes the interests of donors over those of constituents. Influence occurs when a legislator acts to favor donors in a way he or she would not have absent contributions." (p.83)

[14]The model is changed only nominally if we rescale monetary payments by the factor $\frac{1}{\gamma}$, which amounts to a change of units of measure. Then a type θ politician's utility from (y, m) becomes $u_\theta(y) + m + \beta$, and the utility of group G is $u_G(y) - \frac{1}{\gamma} m$. A scalar transformation of the group's utility then yields $\gamma u_G(y) - m$.

henceforth by the politician and m_t is a monetary payment to be made in period t if the officeholder enters the contract; this offer is not observed by voters.

(2) The officeholder decides whether to accept or reject the offer, $a_t \in \{0, 1\}$, where $a_t = 1$ indicates acceptance and $a_t = 0$ rejection; this acceptance decision is not observed by voters.

(3) The officeholder chooses policy x_t, and this is observed by voters; if the officeholder accepts the offer, $a_t = 1$, then she is committed to policy $x_t = y_t$, and otherwise, x_t is unrestricted.

(4) A candidate θ'_t, the *challenger,* is drawn from the density function $h^{1-\pi_t}(\theta)$ for the opposition party to challenge the incumbent; the challenger's type θ'_t is observed by lobby groups but not by voters, and the partisan affiliation of θ'_t is $1 - \pi_t$, and this is observed by lobby groups and voters.

(5) Each voter casts a ballot in a majority-rule election between the incumbent and challenger, with the winner taking office at the beginning of period $t + 1$.

In step (5), we assume that the incumbent wins in case of an exact tie. If period t is not the first term of office for the politician, then either the politician has entered a contract (y_s, m_s) with the active lobby group in some previous period $s < t$, in which case she is committed to $x_t = y_s$ and we set $a_t = 1$, or the politician has opted not to engage with the active group, in which case the current period consists of steps (3)–(5). The model is summarized diagrammatically in Figure 1, which depicts the sequence of play and accumulation of payoffs for the type θ politician and active group.

Before we proceed to discuss information and payoffs in greater detail, three remarks are in order. First, our specification of utilities is a general one that captures the canonical model with *quadratic utility,*[15] in which θ is identified with the ideal point of a citizen and utility is defined as $u_\theta(x) = -(x - \theta)^2$. To see this, we expand this expression as $-x^2 + 2x\theta - \theta^2$, and we then set

$$v(x) = 2x, \ c(x) = x^2, \ k_\theta = -\theta^2.$$

Second, we model the lobby group's offer as a binding contract in which the payment m_t is made once, and thereafter the politician is committed to y_t. Because of the stationary nature of equilibria analyzed in the sequel, we could as well have modeled interaction between lobbyists and politicians as a series of short-term contracts that hold only for a single period. The current specification, by virtue of reducing interaction to a single exchange, can be viewed as an analytically tractable way of modeling such short-term contracts. Third, we have assumed that an officeholder is lobbied by the proximate interest group, i.e., liberal politicians deal with group L and conservative politicians deal with group R, consistent with the empirical regularity that interest groups tend to lobby ideological allies.

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[15]The functional form in (1) is used by Duggan and Martinelli (2017), who also introduce an exponential specification as a special case. Specifically, the *exponential utility* functional form is $u_\theta(x) = xe^\theta - e^x + k_\theta$.

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Figure 1: Flow chart of the model
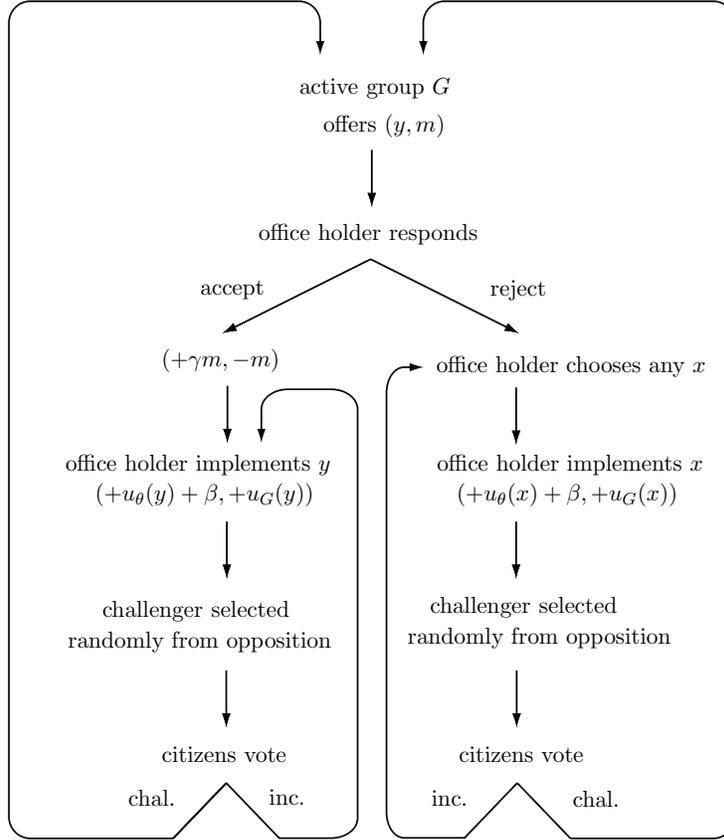
As described above, we assume that a politician's type is private information and not directly observable by voters. Voters do observe the policy choices of the incumbent politician, and thus elections are characterized by pure adverse selection. In contrast, lobby groups have more information, due to greater experience or political connections, and we assume that the active lobby group does observe the type of a first-term politician before making an offer. We assume that lobbying takes place "behind closed doors," so that voters do not observe the offer made by the active group or the acceptance decision of the politician.

All players discount the streams of utility by the common factor $\delta \in [0, 1)$. Given a sequence of offers $(y_1, m_1), (y_2, m_2), \ldots$, a sequence of acceptance decisions $a_1, a_2, \ldots$, and a sequence $x_1, x_2, \ldots$ of policy choices, the discounted sum of per period payoffs of a type θ citizen is given by the expression

$$\sum_{t=1}^{\infty} \delta^{t-1} \left[a_t u_\theta(y_t) + (1 - a_t) u_\theta(x_t) + I_t(a_t \gamma m_t + \beta) \right],$$

where $I_t \in \{0, 1\}$ is an indicator variable that takes the value one if the citizen holds office in period

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t and zero otherwise. In the above, note that the office benefit accrues to the citizen only if she holds office ($I_t = 1$), and she receives the monetary payment only if she holds office and accepts the lobby group's offer ($I_t a_t = 1$). The discounted sum of per period payoffs of lobby group $G \in \{L, R\}$ is

$$\sum_{t=1}^{\infty} \delta^{t-1} \left[a_t u_G(y_t) + (1 - a_t) u_G(x_t) - I_t a_t m_t \right],$$

where now $I_t = 1$ indicates that the lobby group is active in period t, and $I_t = 0$ indicates it is inactive.

3 Simple Lobbying Equilibrium

The analysis focuses on a selection of perfect Bayesian equilibrium (PBE) of the model of repeated elections with lobbying. It is known that in repeated games, many outcomes can be supported by strategies in which players condition on histories in complex ways, and in the repeated elections framework, Duggan (2014a) shows that arbitrary paths of policies can be supported by perfect Bayesian equilibria when citizens are sufficiently patient. The complexity of these strategies is implausible in models of elections and voting, however, and we therefore consider strategies that can be described by means of simple behavioral rules. We study equilibria that are stationary, in the sense that the active lobby group's offer depends only on the citizen type and party affiliation of the current politician; the acceptance decision of a politician depends only on the offer by the active group, and her policy choice in lieu of acceptance is independent of the prior history; and each citizen's vote in an election depends only on the policy choice and partisan affiliation of the incumbent in the preceding period, and the updating of voter beliefs depends only on the current prior and the policy choice in the current period.

After formulating stationary strategies and beliefs, we then define our equilibrium concept by imposing the final assumption that voters use retrospective voting strategies with an intuitive form: for each type θ, the type θ voter re-elects an incumbent from party π if and only if the politician's policy choice in the preceding period is greater than or equal to the continuation value of a challenger. This, in turn, implies that the median voter type is a representative voter, i.e., the incumbent is re-elected if and only if she offers the median voter an expected discounted payoff from re-election at least equal to the continuation value of a challenger. If the lobby group's offer is rejected, an officeholder whose ideal point is acceptable to the median voter simply chooses that policy, whereas other politicians are faced with a trade off: compromise by choosing the best policy acceptable to the median, or shirk by choosing her own ideal point. Finally, the active lobby group makes the most advantageous offer possible, subject to the constraint that the politician receives utility at least equal to the payoff of "going it alone."

Formally, given an incumbent belonging to party π, a strategy of lobby group L is described by mappings $\lambda_L^\pi \colon [\underline{\theta}, \theta_m) \to X$ and $\mu_L^\pi \colon [\underline{\theta}, \theta_m) \to \mathbb{R}$, where $(\lambda_L^\pi(\theta), \mu_L^\pi(\theta))$ is the offer made by the group; that is, L offers to transfer monetary amount $\mu_L^\pi(\theta)$ to the politician in exchange for

the commitment to choose policy $\lambda_L^\pi(\theta)$ thereafter. Similarly, a strategy for group R consists of mappings $\lambda_R^\pi\colon [\theta_m, \overline{\theta}] \to X$ and $\mu_R^\pi\colon [\theta_m, \overline{\theta}] \to \mathbb{R}$, with the same interpretation. A strategy of a type θ politician with partisan affiliation π is represented by mappings $\alpha_\theta^\pi\colon X \times \mathbb{R} \to \{0, 1\}$ and $\xi_\theta^\pi \in X$, where $\alpha_\theta^\pi(y, m) = 1$ if and only if the politician accepts the offer (y, m) from the active group, and ξ_θ^π is the default policy chosen by the politician if she rejects the offer. The voting strategy of a type θ citizen is represented by a mapping $\nu_\theta^\pi\colon X \to \{0, 1\}$, where $\nu_\theta^\pi(x) = 1$ if and only if the type θ citizen votes to re-elect an incumbent from party π who chooses x in the preceding period. In addition to strategies that specify the actions of all players after all histories, we must specify a belief system for the voters because they do not observe the types of the incumbent and challenger. This a set of mappings $\kappa_\theta\colon X \times \{0, 1\} \to \Delta(N)$, where $\Delta(N)$ is the set of probability distributions over citizen types, and $\kappa_\theta(x, \pi)$ represents the type θ voter's beliefs about the type of an incumbent from party π following policy choice x in the preceding period. Given that the incumbent is from party π, the voters' beliefs about the challenger's type are of course represented by the prior density $h^{1-\pi}(\cdot)$.[16,17]

A strategy profile $\sigma = (\lambda, \mu, \alpha, \xi, \nu)$ is *sequentially rational* given belief system κ if the following conditions are satisfied in every period: (i) for every type θ and party π, neither active lobby group can profitably deviate from $(\lambda_G^\pi(\theta), \mu_G^\pi(\theta))$ to a different offer, (ii) for every type θ, party π, and lobby offer (y, m), $\alpha_\theta^\pi(y, m)$ is an optimal response for the politician, (iii) for every type θ and party π, conditional on rejecting the lobby group's offer, a politician cannot profitably deviate from ξ_θ^π to another policy choice, and (iv) for all policy choices x, each type θ voter votes for a candidate who provides the highest expected discounted payoff conditional on their information. Condition (iv) is equivalent to the assumption of "sincere voting," but it does not assume voter myopia: each voter calculates the expected payoffs from the incumbent and challenger in a sophisticated way, and then she chooses between them optimally.[18] Beliefs κ are *consistent* with σ if for all $x \in X$ and all $\pi \in \{0, 1\}$, $\kappa(\cdot|x, \pi)$ is derived via Bayes rule on the public path of play determined by σ; if citizens observe a policy that occurs off the public path of play under σ, then consistency places no restrictions on beliefs other than stationarity.[19,20] An assessment $\Psi = (\sigma, \kappa)$ is a *stationary perfect*

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[16]In what follows, the calculation of expected payoffs assumes that strategies are jointly measurable: $\lambda_G^\pi(\theta)$, $\mu_G^\pi(\theta)$, and ξ_θ^π are measurable in θ, and $\alpha_\theta^\pi(x, m)$ and $\nu_\theta^\pi(x)$ are measurable in (θ, x). Henceforth, for ease of exposition, technical measurability issues will be suppressed.

[17]To avoid problems in the application of Bayes rule, we restrict attention to strategy profiles such that for each policy x, the set of types θ such that $x \in \{\xi_\theta, \lambda_{G(\theta)}^0(\theta), \lambda_{G(\theta)}^1(\theta)\}$ is a (possibly empty) interval, so it is either a singleton or has positive measure.

[18]Condition (iv) is in the spirit of eliminating undominated strategies in voting subgames. In the current context, however, each voter is massless and cannot affect the outcome of an election; a consequence is that voting for the worst of two candidates is not, technically, dominated. Nevertheless, such behavior appears implausible, and it is precluded by condition (iv).

[19]Formally, a t-period sequence $((x_1, \pi_1), \ldots, (x_t, \pi_t))$ of policy choices and election-winning parties is *on the public path of play* if for each $k < t$ with $\pi_k = \pi_{k+1}$, there is a politician type θ with $h^{\pi_k}(\theta) > 0$ such that the politician chooses $x_k = x_{k+1}$.

[20]Specifically, our formulation of beliefs incorporates a strong form of stationarity: after any two histories, if an incumbent from party π chooses policy x, then voters update beliefs in the same way, to $\kappa(\cdot|x, \pi)$. In particular, if a politician deviates by choosing a policy off the public path of play, then this does not affect the updating of voter beliefs about future officeholders.

Bayesian equilibrium if σ is sequentially rational given κ and κ is consistent with σ.

Next, we define several technical concepts that play key roles in the analysis, and we specialize stationary PBE further to impose intuitive restrictions on voting and policy choices. Given an assessment $\Psi = (\sigma, \kappa)$, each voter type θ can calculate the expected discounted payoff, conditional on some policy choice x, from re-electing an incumbent belonging to party π; we denote this by $V_\theta^{I,\pi}(x|\Psi)$. Similarly, let $V_\theta^{C,\pi}(\Psi)$ denote the continuation value from electing the challenger for a citizen type θ, given that the incumbent belongs to π. Note that stationarity implies that $V_\theta^{I,\pi}(x|\Psi)$ and $V_\theta^{C,\pi}(\Psi)$ are constant across time periods.

In any period, the above continuation values are generated by two probability distributions, depending on the winner of the election at the end of the period. First, if the incumbent is from party π, chooses x, and is reelected, then the assessment Ψ determines a probability measure $\mathscr{P}^{I,\pi,x}$ over sequences $\{(\pi_t, \theta_t, G_t, y_t, m_t, a_t, x_t)\}$ of 7-tuples, where: θ_t is the incumbent's type, π_t is her partisanship, (y_t, m_t) is the offer of the active group G_t, a_t is the response of the incumbent, and x_t is the policy outcome t periods hence. For each t, let $\mathscr{P}_{X,t}^{I,\pi,x}$ denote the marginal on policy outcomes after t periods, and define the normalized discounted sum

$$P_X^{I,\pi,x} = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} \mathscr{P}_{X,t}^{I,\pi,x},$$

which is the *incumbent continuation distribution* following policy x by an incumbent belonging to π. Then we can write the incumbent continuation value as

$$V_\theta^{I,\pi}(x|\Psi) = \frac{\mathbb{E}_{P_X^{I,\pi,x}}[u_\theta(z)]}{1-\delta},$$

where z represents a policy outcome, and the expectation is taken with respect to the distribution $P_X^{I,\pi,x}$ over policy outcomes. Second, if the incumbent from party π is removed from office, then Ψ determines a probability measure $\mathscr{P}^{C,\pi}$ over sequences $\{(\pi_t, \theta_t, G_t, y_t, m_t, a_t, x_t)\}$. Let $\mathscr{P}_{X,t}^{C,\pi}$ denote the marginal on policy outcomes t periods hence, and define

$$P_X^{C,\pi} = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} \mathscr{P}_{X,t}^{C,\pi},$$

which is the *challenger continuation distribution*.[21] Then we can write the challenger continuation value as

$$V_\theta^{C,\pi}(\Psi) = \frac{\mathbb{E}_{P_X^{C,\pi}}[u_\theta(z)]}{1-\delta}.$$

Importantly, these distributions are independent of citizen type: all citizens view the incumbent as the same lottery, in effect, and similarly for the challenger.

Furthermore, the functional form in (1) implies that the median voter type θ_m is decisive in

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[21] This concept is used in Banks and Duggan (2008) in their analysis of repeated elections in multiple dimensions.

majority voting between lotteries.[22] Specifically, a majority of voters strictly prefer the challenger to the incumbent if and only if this is the median voter's preference, i.e., $V_m^{C,\pi}(\Psi) > V_m^{I,\pi}(x|\Psi)$. Conversely, at least half of all voters weakly prefer the incumbent if and only if $V_m^{I,\pi}(x|\Psi) \geq V_m^{C,\pi}(\Psi)$. Thus, in a stationary PBE, an incumbent is re-elected following the policy choice x only if $V_m^{I,\pi}(x|\Psi) \geq V_m^{C,\pi}(\Psi)$, in which case, by stationarity, they will be reelected after every history at which they choose x. Define the *win set,* denoted $W^\pi(\Psi)$, as the set of policy choices x such that incumbents from party π are re-elected if they choose $x \in W^\pi(\Psi)$, and note that by stationarity, if an incumbent chooses $x \in W^\pi(\Psi)$ in her first period of office and is reelected, then she continually chooses x while in office will be continually reelected. Thus, for any policy choice $x \in W^\pi(\Psi)$ on the public path of play, we have $V_m^{I,\pi}(x|\Psi) = \frac{u_m(x)}{1-\delta}$, and for such policies, we must have

$$\frac{u_m(x)}{1-\delta} \;\geq\; V_m^{C,\pi}(\Psi).$$

In words, if an incumbent is re-elected after the choice of policy x on the public path of play, then that policy must provide the median voter a payoff at least equal to the value of a challenger. We consider a selection of equilibria such that the above inequality is necessary and sufficient for re-election, consistent with a "what have you done for me lately" mindset on the part of voters.

To describe the equilibrium incentives of voters and politicians, it is useful to define the *dynamic policy utility* of the type θ citizen from policy choice x by an incumbent belonging to party π as

$$U_\theta^\pi(x|\Psi) \;=\; \begin{cases} \frac{u_\theta(x)}{1-\delta} & \text{if } x \in W^\pi(\Psi), \\ u_\theta(x) + \delta V_\theta^{C,\pi}(\Psi) & \text{else,} \end{cases}$$

which represents the citizen's discounted expected payoff when choice of policy in the win set is necessary and sufficient for re-election. Thus, if a policy in the win set is chosen, then it will continue to be chosen in every period thereafter; but if a policy outside the win set is chosen, then it will be in place for just one period, after which a challenger will take office. Similarly, to analyze the optimization problem of a politician, we define the *dynamic office rents* from policy choice x by an incumbent belonging to π as

$$B^\pi(x|\Psi) \;=\; \begin{cases} \frac{\beta}{1-\delta} & \text{if } x \in W^\pi(\Psi), \\ \beta & \text{else.} \end{cases}$$

In a stationary equilibrium, if a policy in the win set is chosen, then it will continue to be chosen by the politician, who receives the office benefit β in each period; and if it does not belong to the win set, then the incumbent holds office and receives the office benefit for just one period.

To formulate the optimization problem facing a group G in equilibrium, we must also consider the payments it makes in the event that the incumbent is removed from office. To that end, in any period with an incumbent from party π, let $\mathscr{P}_{G,t}^{C,\pi}$ denote the marginal of $\mathscr{P}^{C,\pi}$ on monetary

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[22]See Duggan (2014b) for details of this claim.

payments made t periods hence by group G in case the incumbent is removed. Let

$$M_G^{C,\pi}(\Psi) \;=\; \sum_{t=1}^{\infty} \delta^{t-1} \mathbb{E}_{\mathscr{P}_{G,t}^{C,\pi}}[m]$$

be the *continuation payment* of group G, which represents the expected discounted monetary payments of G following removal of an incumbent from party π. Then the *dynamic payment* of group G following policy choice x is

$$M_G^{\pi}(x|\Psi) \;=\; \begin{cases} \delta M_G^{C,\pi}(\Psi) & \text{if } x \notin W^{\pi}(\Psi), \\ 0 & \text{else,} \end{cases}$$

reflecting the fact that if the incumbent chooses a policy outside the win set, then in addition to the group's dynamic policy utility from a challenger, it expects to enter into costly contracts with future politicians.

Finally, we can now define the concept of *simple lobbying equilibrium,* in which we specialize stationary PBE by strengthening sequential rationality as follows: (i) for every type θ and each party π with active group G, the offer $(\lambda_G^{\pi}(\theta), \mu_G^{\pi}(\theta))$ solves

$$\max_{(y,m)} U_G^{\pi}(y|\Psi) - M_G^{\pi}(y|\Psi) - m$$
$$\text{s.t. } U_\theta^{\pi}(y|\Psi) + \gamma m + B^{\pi}(y|\Psi) \;\geq\; U_\theta^{\pi}(\xi_\theta^{\pi}|\Psi) + B^{\pi}(\xi_\theta^{\pi}|\Psi)$$

for the active group G; (ii) for every type θ, each party π, and each offer (y,m), we have $\alpha_\theta^{\pi}(y,m) = 1$ if and only if

$$U_\theta^{\pi}(y|\Psi) + \gamma m + B^{\pi}(y|\Psi) \;\geq\; U_\theta^{\pi}(\xi_\theta^{\pi}|\Psi) + B^{\pi}(\xi_\theta^{\pi}|\Psi);$$

(iii) for every type θ and each party π, the default policy ξ_θ^{π} solves

$$\max_x U_\theta^{\pi}(x|\Psi) + B^{\pi}(x|\Psi);$$

and (iv) for each party π, the win set is

$$W^{\pi}(\Psi) \;=\; \left\{ x \in X \;\middle|\; \frac{u_m(x)}{1-\delta} \;\geq\; V_m^{C,\pi}(\Psi) \right\}.$$

That is, in a simple lobbying equilibrium, the active group offers a contract that maximizes its dynamic utility, subject to the participation constraint of the politician. Here, the group's monetary expenditure in the current period is the transfer m to the incumbent, and in case the contract specifies a policy choice y outside the win set, the group also accounts for future payments. The politician accepts the offer if and only if the utility from the contract is at least equal to the utility of declining and choosing the default policy, ξ_θ^{π}, which maximizes the dynamic utility of the politician. Finally, the incumbent is re-elected if and only if the median voter weakly prefers her policy choice

to an untried challenger.

In equilibrium, the active group may offer policies that lead to re-election of the incumbent or to removal of the incumbent. Let

$$\mathcal{E}^\pi(\Psi) \;=\; \{\theta \mid \lambda^\pi_{G(\theta)}(\theta) \in W^\pi(\Psi)\}$$

denote the set of politician types from party π such that the active group offers a winning policy, so that the incumbent wins election. Because it plays an important role in the proof of equilibrium existence, we observe that the challenger continuation values $V^{C,\pi}_\theta(\Psi)$ are determined by a system of two recursive equations,

$$V^{C,\pi}_\theta(\Psi) \;=\; \int_{\theta' \in \mathcal{E}^{1-\pi}(\Psi)} \frac{u_\theta(\lambda^{1-\pi}_{G(\theta')}(\theta'))}{1-\delta} h^{1-\pi}(\theta')d\theta' \tag{2}$$
$$+ \int_{\theta' \notin \mathcal{E}^{1-\pi}(\Psi)} \big[u_\theta(\lambda^{1-\pi}_{G(\theta')}(\theta')) + \delta V^{C,1-\pi}_\theta(\Psi)\big] h^{1-\pi}(\theta')d\theta',$$

for $\pi \in \{0,1\}$. In words, if an incumbent from party π is replaced by a challenger, then if the challenger's type θ' is such that the active group $G(\theta')$ offers a winning policy, then that politician remains in office forever and chooses the policy agreed to; and if θ' is such that the active group offers a losing policy, then that policy is in place for just one period, after which another challenger takes office. An implication of the contraction mapping theorem is that the continuation values $V^{C,\pi}_\theta(\Psi)$ are the unique solution to the system of equations in (2).

We end this section with several comments on simple lobbying equilibrium. First, we assume the active group always makes an offer, but the group can offer the politician's default with no payment, $(y,m) = (\xi^\pi_\theta, 0)$, effectively choosing to forego lobbying, so our assumption is without loss of generality. Second, the politician is assumed to always accept the group's offer when she weakly prefers it to the default, whereas it may seem that acceptance is necessitated by PBE only if this preference is strict. In fact, the restriction is essentially without loss of generality. When the optimal contract (y,m) is distinct from the default $(\xi^\pi_\theta, 0)$, the active group receives positive rents from the exchange, but then it must be that the constraint holds with equality, i.e.,

$$U^\pi_\theta(y|\Psi) + \gamma m + B^\pi(y|\Psi) \;=\; U^\pi_\theta(\xi^\pi_\theta|\Psi) + B^\pi(\xi^\pi_\theta|\Psi)$$

and the politician accepts, for otherwise the group could increase its transfer to the politician by a small amount $\epsilon > 0$. The politician then strictly prefers the offer to the default and accepts. But then for small enough $\epsilon > 0$, the contract $(y, m + \epsilon)$ is strictly better for the group than (y,m), contradicting optimality of the latter contract. Third, we have observed that in a stationary equilibrium, every policy in the win set must be at least as good for the median voter as a challenger, and our equilibrium concept is maximally permissive, in the sense that we impose equality in the definition of the win set. Similar to the preceding comment, we assume the politician is re-elected when the median voter is indifferent between the incumbent and challenger, but this is without

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loss of generality (it is necessitated by existence of an optimal policy) in all but the extreme case in which the win set is a singleton consisting of the median policy. Finally, we note that simple lobbying equilibrium is a selection of stationary PBE in which the decisions of players are described by relatively simple behavioral rules, but players are not prevented from deviating to more complex strategies: given a simple lobbying equilibrium, no player can increase her payoff by deviating to any other different strategy, stationary or otherwise.

4   Equilibrium Existence and Characterization

In this section, we establish existence of simple lobbying equilibrium and a partitional characterization of equilibrium in terms of cutoff policies. To begin, we establish in Proposition 1 that there is at least one simple lobbying equilibrium. Note that Propositions 1 and 2 do not rely on the assumption that an open set around θ_m is contained in the support of both challenger densities.

Proposition 1. *A simple lobbying equilibrium exists.*

The proof, which is provided in the appendix, consists of a fixed point argument. A novel aspect is that the fixed point belongs to the set of pairs $(\mathbf{P}, \mathbf{M})$, where $\mathbf{P} = (P_X^{C,0}, P_X^{C,1})$ represents two challenger continuation distributions, and $\mathbf{M} = (M_L^{C,0}, M_L^{C,1}, M_R^{C,0}, M_R^{C,1})$ represents the expected discounted payments made by a lobby group if a challenger is elected. For example, $P_X^{C,0}$ summarizes the distribution over future policies when an incumbent from party $\pi = 0$ is replaced by a challenger, and $M_L^{C,0}$ summarizes the expected discounted payments of lobby group L when an incumbent from party $\pi = 0$ is replaced by a challenger. The pair $(\mathbf{P}, \mathbf{M})$ is a sufficient statistic to compute equilibrium payoffs of all citizen types, and thus from such a pair, we can deduce the implied win set, optimal default policy choices of each politician, and the optimal offers by lobby groups. These optimal choices then imply an "updated" pair, denoted $(\tilde{\mathbf{P}}, \tilde{\mathbf{M}})$, which may or may not be the same as the initial one. A fixed point is a pair $(\mathbf{P}^*, \mathbf{M}^*)$ that is mapped to itself, in this sense, so that optimal voting and policy choices given $(\mathbf{P}^*, \mathbf{M}^*)$ in fact generate the same distributions and give us a simple lobbying equilibrium.

A byproduct of the proof is a characterization of equilibria in terms of two 6-tuples of cutoff policies, $(\underline{c}^\pi, \underline{e}^\pi, \underline{w}^\pi, \overline{w}^\pi, \overline{e}^\pi, \overline{c}^\pi)$, such that

$$\underline{c}^\pi \;\leq\; \underline{w}^\pi \;\leq\; x_m \;\leq \overline{w}^\pi \;\leq\; \overline{c}^\pi \quad \text{and} \quad \underline{e}^\pi \;\leq\; x_m \;\leq\; \overline{e}^\pi \tag{3}$$

for $\pi \in \{0, 1\}$. These determine the win set, default policy choices, and policies offered by lobby groups in the following way. First, the *win set* is the closed interval $W^\pi(\Psi) = [\underline{w}^\pi, \overline{w}^\pi]$, where by condition (iv) in the definition of simple lobby equilibrium, we have

$$\frac{u_m(\underline{w}^\pi)}{1 - \delta} \;=\; V_m^{C,\pi}(\Psi) \quad \text{and} \quad \frac{u_m(\overline{w}^\pi)}{1 - \delta} \;\geq\; V_m^{C,\pi}(\Psi), \tag{4}$$

with the latter holding with equality unless $\overline{w}^{\pi} = 1$.[23] Second, the default policy choice of a type θ politician is to choose the closest policy in the win set to her ideal point $x(\theta)$ if that ideal point is in the *compromise set,* $C^{\pi}(\Psi) = [\underline{c}^{\pi}, \overline{c}^{\pi}]$, so that, in particular, if $x(\theta) \in W^{\pi}(\Psi)$, then the politician chooses her ideal point by default and is re-elected thereafter; and if, e.g., $\overline{w}^{\pi} < x(\theta) \leq \overline{c}^{\pi}$, then she chooses the endpoint $\overline{w}^{\pi}$ by default and is subsequently re-elected. And if the politician's ideal point belongs to the *shirk set,* $S^{\pi}(\Psi) = [0, \underline{c}^{\pi}) \cup (\overline{c}^{\pi}, 1]$, then in the absence of a lobby offer, she shirks by choosing her ideal point and is removed from office. Third, the *election set* of ideal points of politician types that are offered a winning policy is the closed interval $E^{\pi}(\Psi) = [\underline{e}^{\pi}, \overline{e}^{\pi}]$. For $x(\theta) \in E^{\pi}(\Psi)$, the active group's offer maximizes joint surplus for the politician and group, subject to the win set constraint, i.e., $\lambda_G^{\pi}(\theta)$ solves

$$\max_{y \in W^{\pi}(\Psi)} u_{\theta}(y) + \gamma u_G(y).$$

And if the ideal point of the politician is outside the interval, i.e., $x(\theta) \notin E^{\pi}(\Psi)$, then the group's offer maximizes the unconstrained joint surplus, i.e., $\lambda_G^{\pi}(\theta)$ solves

$$\max_{y \in X} u_{\theta}(y) + \gamma u_G(y),$$

and furthermore the policy offered lies outside the win set, i.e., $\lambda_G^{\pi}(\theta) \notin W^{\pi}(\Psi)$.

We say a simple lobbying equilibrium possessing the above structure has the *partitional form.* Politicians with sufficiently moderate ideal points $x(\theta) \in [\underline{e}^{\pi}, \overline{e}^{\pi}]$ are lobbied to winning policies, while those who are more ideologically extreme are lobbied to losing policies. Note that because lobby group offers maximize the joint surplus $u_{\theta}(y) + \gamma u_G(y)$, the policy choice of a politician in the win set is pulled strictly in the direction of the lobby group's ideal point, except perhaps the politician type with ideal point equal to $\overline{w}^{\pi}$. As well, a politician with ideal policy outside the election set is pulled strictly in the direction of the active lobby group. For example, a politician whose type $\theta \geq \theta_m$ satisfies $x(\theta) < \overline{w}^{\pi}$ or $x(\theta) > \overline{e}^{\pi}$ is lobbied by group R, and thus her policy choice is pulled to the right as a consequence of lobbying. The only types for which this extremization effect does not hold strictly are those in the interval $[\overline{w}^{\pi}, \overline{e}^{\pi}]$. For example, if $\overline{w}^{\pi} \leq x(\theta) \leq \overline{c}^{\pi}$, then the politician would compromise at the endpoint $\overline{w}^{\pi}$ in the absence of lobbying, and she is lobbied to this same policy. Interestingly, it is also possible that $\overline{c}^{\pi} \leq x(\theta) \leq \overline{e}^{\pi}$, in which case the politician would shirk by default, but the group lobbies the politician to compromise her policy choice. In fact, on the basis of policy outcomes alone, the lobby group has greater incentives to shirk (indeed, the type $\overline{\theta}$ politician would choose a losing policy by default and be replaced by a challenger), but if future policies are the result of costly lobbying, the group may push a politician to compromise, thereby avoiding future payments by the lobby group. We provide an example of this moderating effect in the next section.

The partitional structure of equilibrium is depicted in Figure 2, where we illustrate the policy

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[23]This claim uses the simplifying assumption that $u_m(1) \geq u_m(0)$.

choices of right-leaning politicians in three cases: (i) $\overline{e}^\pi < \overline{w}^\pi < \overline{c}^\pi$, (ii) $\overline{w}^\pi < \overline{e}^\pi < \overline{c}^\pi$, and (iii) $\overline{w}^\pi < \overline{c}^\pi < \overline{e}^\pi$. Here, arrows represent policy choices of a small sample of politician types, with the arrow emanating from the politician's ideal point and pointing to their her choice. The heavy black intervals indicate the intervals of politicians that are lobbied to the endpoint $\overline{w}^\pi$ of the win set. In case (i), all politician types are lobbied to strictly more extreme policies than their default choices; in case (ii), the choices of politicians in the interval $[\overline{w}^\pi, \overline{e}^\pi]$ are preserved by lobbying; while in case (iii), some politician types are lobbied to more moderate policies, so that the lobby group avoids future payments that would be incurred if a challenger were elected.

Figure 2: Partitional form of simple lobbying equilibrium
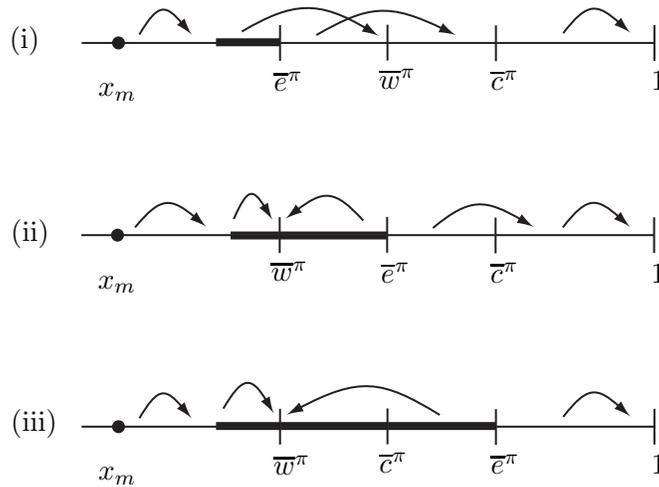
The next characterization result establishes that all simple lobbying equilibria have the partitional form.

Proposition 2. *Every simple lobbying equilibrium has the partitional form.*

The partitional form arises from the ordering of policy utility by type. First, moderate politicians can win re-election simply by choosing their ideal policy, but are lobbied to choose more extreme policies in equilibrium. If the win set is not too narrow, then politicians closest to θ_m are lobbied to a policy inside the win set, which maximizes joint surplus. Moving outwards in either direction, however, there is eventually a politician type for which the joint surplus maximizing policy hits the edge of the win set. This type's ideal point is the inner bound of the set of types choosing the edge of the win set. In Figure 2, the left-hand endpoint of the bold, black interval depicts this type for right-leaning politicians.

For types more extreme than this threshold, but moderate enough to have ideal points in the win set, the lobby group faces a dilemma: (i) lobby the officeholder out of the win set to the joint surplus maximizing policy, sacrificing re-election; or (ii) lobby them to the edge of the win set, thus obtaining a worse policy outcome today, but retaining the officeholder forever. The cut-point $\overline{e}^\pi$ corresponds to the politician ideal point at which group R is indifferent between the two options,

and $\underline{e}^\pi$ is analogous for L. Roughly, whether these cut-points fall outside the win set depends on how strongly the active group wants to lock-in future policy and how strongly politicians value re-election. The incentive to comprise decreases as politician types become more extreme, and the cut-point $\bar{c}^\pi$ corresponds to the politician ideal point to the right of the median that, in the absence of a lobby offer, is indifferent between compromising and shirking, and analogously for $\underline{c}^\pi$. Based on policy payoffs alone, the lobby groups are more inclined to shirk than less extreme politicians, but $\bar{e}^\pi$ and $\underline{e}^\pi$ may fall outside the compromise set if removal of the politician leads to high future payments by the active lobby group.

Example 1. Figure 3 gives numerical equilibrium computations for four parameterizations of the model. Here, we fix $N = [0, 2]$, $\theta_m = 1$, $\beta = .1$, and $\delta = .2$. To generate shirking in equilibrium, we use a variant of quadratic utility, namely, $u_\theta(x) = \theta x - x^2 - (\theta/2)^2$. It follows that the space of ideal points is $[0, 1]$ and $x_m = 0.5$.

The figure depicts the equilibrium policy strategy as a function of the politician's type, along with the win set and compromise set, for two values of γ and two challenger distributions. The two panels in the left-hand column set $\gamma = .05$, and the panels in the right-hand column increase the effectiveness of lobbying to $\gamma = .2$. The top row represents the model with a challenger density that is uniform on $[0, 2]$ and independent of the party in power, and the bottom row represents the model with a challenger density that depends on the party in power: if a politician from the right party holds office, then the challenger is drawn from the uniform distribution on $[0, 1]$, and if the incumbent belongs to the left party, then the challenger is drawn from the uniform distribution on $[1, 2]$. The gray dashed line indicates the right-hand endpoint of the win set, and the gray dotted line indicates the ideal point of the most extreme politician type who compromises. The gap between the two lines measures the amount of compromise generated in equilibrium, and the size of the win set indicates the median voter's expected payoff from a challenger, with larger win sets corresponding to lower welfare for the median voter.

Figure 3 illustrates the partitional form of equilibria and suggests two intuitions. First, in comparing the non-partisan challenger density (uniform on $[0, 2]$) with the partisan density (which draws from the side of the spectrum opposite the incumbent), the win set is larger in the non-partisan case. This is to be expected: the two densities produce the same distribution of ideological extremism (measured in terms of distance from the median), but the partisan challenger density creates a greater "threat" for a lobby group, and so it increases the incentives of lobby groups to compromise. This leads to more moderate policy choices, and to a higher ex ante expected payoff to the median voter. Second, the median voter is worse off when lobbying is more effective. This effect is small in the figure, but it is also intuitive: when lobby groups become more effective, they pull policy choices of officeholders toward the extremes of the policy space. We conjecture that the first effect holds widely when comparing the partisan and non-partisan densities, irrespective of the office benefit or discount factor. However, we show in Section 6 that the intuitive comparative static on γ can be overturned by second-order equilibrium effects. □

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Figure 3: Four parameterizations
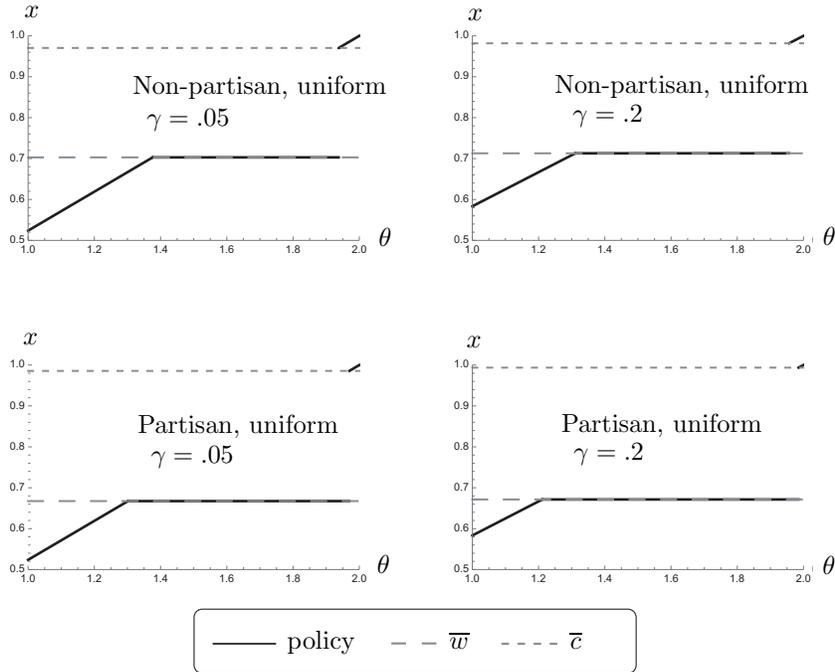
5 Effects of Electoral Incentives on Partisanship

Having established existence and provided a characterization of simple lobbying equilibria, it is of interest to consider the effect of electoral incentives on policy outcomes in the presence of lobby groups. To this end, we show that parties become strong when office incentives are high, in the sense that all liberal politicians choose the same policy, all conservative politicians choose the same policy, and these policies are sufficient for re-election of the incumbent. Moreover, the gap between these policy outcomes goes to zero, and all politicians are lobbied to policies close to the median as office incentives increase.

This issue of policy responsiveness is a central question in the literature on electoral accountability, and it is known that a positive result holds in the repeated elections model without lobbying: Banks and Duggan (2008) show that when $\delta > 0$ and the office benefit β is sufficiently large, all politician types choose the median policy. In the presence of lobbying, a form of this responsiveness result holds, but lobbying creates a wedge between policy choices and the median voter's preferences. In fact, assuming $\gamma > 0$, no politician type will choose policies at (or close to) the median ideal point x_m in a simple lobbying equilibrium, unless the win set is a singleton consisting of just the median voter's ideal point. This follows since the median politician type is lobbied to the policy $\lambda_G^\pi(\theta)$, which maximizes the sum $u_G(y) + \frac{1}{\gamma}u_m(y)$ and thus solves the first order condition

$$u_G'(y) + \frac{1}{\gamma}u_m'(y) = 0.$$

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Of course, $u'_m(x_m) = 0$ and $u'_G(x_m) \neq 0$, so it follows that the median politician is pulled strictly in the direction of the lobby group's ideal point. That is, simple lobbying equilibria exhibit a "flight from the center." There exist equilibria with this property even when office benefit is arbitrarily large (and re-election incentives are strongest), and we will see in the next section that the wedge between policy choices and the median ideal point grows when the effectiveness of lobbying becomes large.

Next, we we fix the effectiveness of money and examine the structure of equilibria when office incentives are high, i.e., $\delta\beta$ is large. A simple lobbying equilibrium is *strongly partisan* if: (i) there is a single win set $W = [\underline{w}, \overline{w}]$ with $\underline{w} < \overline{w}$ that is independent of the incumbent's party; (ii) for all politician types $\theta < \theta_m$ in the support of h^0 or h^1, lobby group L offers the policy $\lambda_L^\pi(\theta) = \underline{w}$; and (iii) for all politician types $\theta \geq \theta_m$ in the support of h^0 or h^1, lobby group R offers the policy $\lambda_R^\pi(\theta) = \overline{w}$. In such an equilibrium, all liberal politicians choose the same policy, as do all conservative politicians, and an incumbent's policy choices always ensure re-election; see Figure 4. The width of the win set, $\overline{w} - \underline{w}$, then measures the extent of polarization. Let $x_G(\theta)$ maximize the joint surplus $u_G(x) + \frac{1}{\gamma} u_\theta(x)$ of the lobby group G and the type θ politician, with weight $\frac{1}{\gamma}$ on the politician. Note that in a strongly partisan equilibrium, it necessarily follows that

$$x_L(\theta_m) \leq \underline{w} \quad \text{and} \quad \overline{w} \leq x_R(\theta_m), \tag{5}$$

for lobby group R must offer the median politician type the policy $\overline{w}$, and by Proposition 2, this offer is either $x_R(\theta_m)$ or, if $\overline{w} < x_R(\theta_m)$, then it is the endpoint $\overline{w}$; with similar observations holding for group L.

Figure 4: Strongly partisan equilibrium
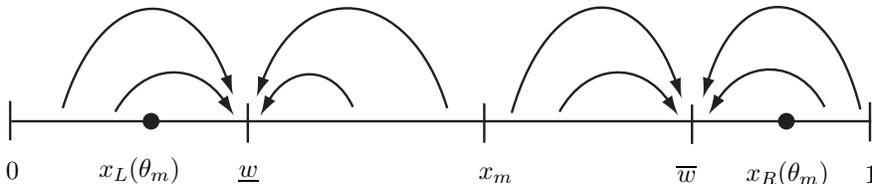
The next result establishes that if office incentives are high, then all simple lobbying equilibria are strongly partisan, and the extent of polarization is limited by the ability of the lobby groups to move the median politician type from her ideal point: in fact, the inequalities in (5), along with the median indifference, $u_m(\underline{w}) = u_m(\overline{w})$, are necessary and sufficient for existence of a strongly partisan equilibrium with win set $[\underline{w}, \overline{w}]$. Since $\gamma > 0$, we have

$$x_L(\theta_m) < x_m < x_R(\theta_m),$$

and thus when $\delta\beta$ is sufficiently large, there are equilibria in which each type pools on one of two policies, those policies being on either side of (and distinct from) the median.

Proposition 3. *Fix $\gamma > 0$. When $\delta\beta$ is sufficiently large: every simple lobbying equilibrium is strongly partisan; furthermore, the most polarized strongly partisan equilibrium is such that one of the inequalities in (5) holds with equality; and finally, there is a strongly partisan equilibrium with win set $[\underline{w}, \overline{w}]$ if and only if $u_m(\underline{w}) = u_m(\overline{w})$ and the inequalities in (5) hold.*

Proposition 3 considers equilibria when, holding all other parameters constant, the office incentive $\delta\beta$ is sufficiently large. The precise sufficient condition we use in the proof is

$$\delta\beta \;\; > \;\; \max\Big\{2(u_\theta(x(\theta)) - u_\theta(x_m)), 2\gamma(u_G(x_G) - u_G(x_m)) \mid \theta \in [\underline{\theta}, \overline{\theta}], G \in \{L, R\}\Big\},$$

which describes a region of the parameter space such as that pictured in Figure 5, below. Intuitively, for both the politician and active group, we make $\delta\beta$ large relative to the difference in policy utility from their ideal policy versus the median's ideal. An implication of Proposition 3 is that when office benefit is large, there is an indeterminacy of equilibria in the model. If $\underline{w}$ and $\overline{w}$ are such that $\underline{w} < x_m < \overline{w}$, satisfy (5), and the median voter is indifferent between them, then the win set $[\underline{w}, \overline{w}]$ can be supported when $\delta\beta$ is sufficiently large. But if we choose $\underline{w}'$ and $\overline{w}'$ with $\underline{w} < \underline{w}' < x_m < \overline{w}' < \overline{w}$ to make the median voter indifferent, then we can also support the smaller win set $[\underline{w}', \overline{w}']$. By this reasoning, we can support a continuum of win sets. However, the proposition also imposes an upper bound on this indeterminacy, as (5) implies that the win set must be contained in the interval $[x_L(\theta_m), x_R(\theta_m)]$.

Next, we present an example of a strongly partisan equilibrium in which the lobby groups push some politician types to moderate their policies, even though they would shirk by default.

Example 2. For simplicity, we consider a symmetric version of the model in which the median ideal policy is $x_m = \frac{1}{2}$; the challenger density, h, is independent of the party in power; h is symmetric about the median θ_m; and utilities are quadratic. Fix $\gamma > 0$, and let $\overline{w} = \frac{1}{1+\gamma}(\frac{1}{2}) + \frac{\gamma}{1+\gamma}(1) = \frac{1+2\gamma}{2(1+\gamma)}$ be equal to the surplus maximizing policy for the median politician and lobby group R. Similarly, let $\underline{w} = \frac{1}{1+\gamma}(\frac{1}{2}) + \frac{\gamma}{1+\gamma}(0) = \frac{1}{2(1+\gamma)}$ be the surplus maximizing policy for the median and group L. By (5), it follows that given γ, the largest win set that can possibly be supported in a strongly partisan equilibrium is $[\underline{w}, \overline{w}]$, and we specify Ψ such that each politician type is lobbied to the endpoint of the win set $[\underline{w}, \overline{w}]$ closest to the group, and each type is compensated for that choice. By Proposition 3, there is such an equilibrium if $\delta\beta$ is sufficiently large, but we assume that

$$0 \;\; < \;\; \delta \;\; < \;\; \overline{\delta} \;\; \equiv \;\; \frac{2u_{\overline{\theta}}(\overline{w})}{u_{\overline{\theta}}(\underline{w}) + u_{\overline{\theta}}(\overline{\theta})} \;\; < \;\; 1,$$

and, initially, that $\beta = 0$. Consider the default policy choices of the politicians. Clearly, if $x(\theta) \in [\underline{w}, \overline{w}]$, then the default choice of the politician is the ideal policy, $\xi_\theta = x(\theta)$. Let $\overline{\theta}_w$ be the politician type with ideal point equal to $\overline{w}$, i.e., $x(\overline{\theta}_w) = \overline{w}$. Because politician types $\theta \in [\theta_m, \overline{\theta}_w]$ are lobbied

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to $\overline{w}$, we specify that lobby group R transfers $u_\theta(x(\theta)) - u_\theta(\overline{w}) = -u_\theta(\overline{w})$ to each such type. Let

$$\overline{m} = -\frac{1}{\gamma} \int_{\theta_m}^{\overline{\theta}_w} u_\theta(\overline{w}) h(\theta) d\theta > 0$$

be the ex ante transfers to these politicians from the group, giving us a positive lower bound on the transfers made by each group.

We claim that the type $\overline{\theta}$ politician shirks by default. Indeed, the politician's payoff from shirking is

$$u_{\overline{\theta}}(1) + \frac{\delta[\frac{1}{2} u_{\overline{\theta}}(\underline{w}) + \frac{1}{2} u_{\overline{\theta}}(\overline{w})]}{1 - \delta} = \frac{\delta[u_{\overline{\theta}}(\underline{w}) + u_{\overline{\theta}}(\overline{w})]}{2(1 - \delta)},$$

where the continuation value reflects the fact that the challenger is lobbied by the two groups with equal probability and is pushed to $\underline{w}$ or $\overline{w}$, depending on whether the active group is L or R. The politician's payoff from compromise is $\frac{u_{\overline{\theta}}(\overline{w})}{1-\delta}$. Then the optimal default choice of the politician is to shirk if

$$\frac{\delta[u_{\overline{\theta}}(\underline{w}) + u_{\overline{\theta}}(\overline{w})]}{2(1 - \delta)} - \frac{u_{\overline{\theta}}(\overline{w})}{1 - \delta} > 0,$$

which holds by $\delta < \overline{\delta}$. As we increase β, the type $\overline{\theta}$ politician's incentive to shirk decreases, and the politician is exactly indifferent between shirking and compromising when the extra office benefit from compromise, namely $\frac{\delta\beta}{1-\delta}$, equals the policy benefit from shirking. That is, when office benefit equals $\overline{\beta}$ defined by

$$\overline{\beta} = \frac{1}{\delta} \left[\frac{\delta[u_{\overline{\theta}}(\underline{w}) + u_{\overline{\theta}}(\overline{w})]}{2(1 - \delta)} - \frac{u_{\overline{\theta}}(\overline{w})}{1 - \delta} \right].$$

For the example, we will choose $\beta < \overline{\beta}$ close to $\overline{\beta}$ and $\delta < \overline{\delta}$ close to $\overline{\delta}$. By the above arguments, it follows that the default policy choice of the type $\overline{\theta}$ politician is to shirk. Note that the lower bound $\overline{m} > 0$ is independent of these parameters.

Now, if group R lobbies the type $\overline{\theta}$ incumbent to shirk by choosing her default policy $x(\overline{\theta}) = 1$, then no payment to the politician is required. Yet, the group's discounted payoff is less than or equal to

$$\frac{\delta[u_{\overline{\theta}}(\underline{w}) + u_{\overline{\theta}}(\overline{w})]}{2(1 - \delta)} - \delta\overline{m}$$

because the group expects to make future payments of at least $\overline{m}$ after the incumbent is replaced by a challenger. On the other hand, if lobby group R pushes the politician to compromise at $\overline{w}$, its discounted payoff is

$$\frac{u_{\overline{\theta}}(\overline{w})}{1 - \delta} - \frac{1}{\gamma} \left[\frac{\delta[u_{\overline{\theta}}(\underline{w}) + u_{\overline{\theta}}(\overline{w})]}{2(1 - \delta)} - \frac{u_{\overline{\theta}}(\overline{w}) + \delta\beta}{1 - \delta} \right],$$

where the term in brackets is the payment to compensate the politician. Then the lobby group

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pushes the politician to compromise if

$$\frac{u_{\bar{\theta}}(\overline{w})}{1-\delta} - \frac{1}{\gamma}\left[\frac{\delta[u_{\bar{\theta}}(\underline{w}) + u_{\bar{\theta}}(\overline{w})]}{2(1-\delta)} - \frac{u_{\bar{\theta}}(\overline{w}) + \delta\beta}{1-\delta}\right] > \frac{\delta[u_{\bar{\theta}}(\underline{w}) + u_{\bar{\theta}}(\overline{w})]}{2(1-\delta)} - \delta\overline{m},$$

or equivalently, if

$$(1+\gamma)\left[u_{\bar{\theta}}(\overline{w}) - \frac{\delta}{2}\left(u_{\bar{\theta}}(\underline{w}) + u_{\bar{\theta}}(\overline{w})\right)\right] + \delta\beta > -\gamma(1-\delta)\delta\overline{m}. \tag{6}$$

The right-hand side of (6) is strictly negative, and the left-hand side equals zero when evaluated at $\overline{\delta}$ and $\overline{\beta}$. Thus, we can choose $\beta < \overline{\beta}$ close to $\overline{\beta}$ and $\delta < \overline{\delta}$ close to $\overline{\delta}$ so that (6) holds.

The above argument shows that given $\underline{w}$ and $\overline{w}$, and given the default choices of politicians, lobby group R's optimal offer is indeed to push the type $\overline{\theta}$ politician to compromise at $\overline{w}$. Because types $\theta \in (\overline{\theta}_w, 1)$ have greater incentives to compromise, it is also optimal to push them to $\overline{w}$. A symmetric argument establishes that lobby group L's optimal offers are to lobby types $\theta < \theta_m$ to $\underline{w}$, and we can assign voter beliefs at out of equilibrium histories as in the proof of existence. Thus, we have specified a simple lobbying equilibrium in which the lobby groups induce compromise by some politician types who would shirk by default, in order to avoid costly lobbying in future periods. □

A close reading of the proof of Proposition 3 reveals that if $\delta\beta$ is sufficiently large given a particular value of the effectiveness of money, γ, then the result holds when γ decreases to zero. Note that when γ goes to zero, thereby approximating the model with no lobbying, we have $x_G(\theta_m) \to x_m$ for both lobby groups, and we find that in every simple lobbying equilibrium, all politician types choose policies arbitrarily close to the median ideal policy. The next result confirms that when the office incentive is large, electoral incentives generate policy outcomes close to the median voter's ideal point as as the effectiveness of money becomes small. For the one-dimensional case, this responsiveness result shows that the dynamic median voter theorem of Banks and Duggan (2008) is robust to the introduction of limited lobbying.[24]

Proposition 4. *If $\delta\beta$ is sufficiently large, then as $\gamma \to 0$, every simple lobbying equilibrium is strongly partisan, and the win set $[\underline{w}, \overline{w}]$ in the most polarized strongly partisan equilibrium converges to the median ideal policy, i.e., $\underline{w} \to x_m$ and $\overline{w} \to x_m$.*

The dynamic median voter theorem fixes $\delta\beta$ at a large value, and it delivers a positive result when the effectiveness of money is small. It is also of interest to understand the effect of money in the polar situation, where money has a large impact, and thus γ is high. We turn to this question in the next section.

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[24]The framework of this paper also allows for partisan challenger pools, which is not allowed by Banks and Duggan (2008), so in this respect we strengthen the dynamic median voter theorem.

Figure 5: Sufficient conditions for strongly partisan equilibria
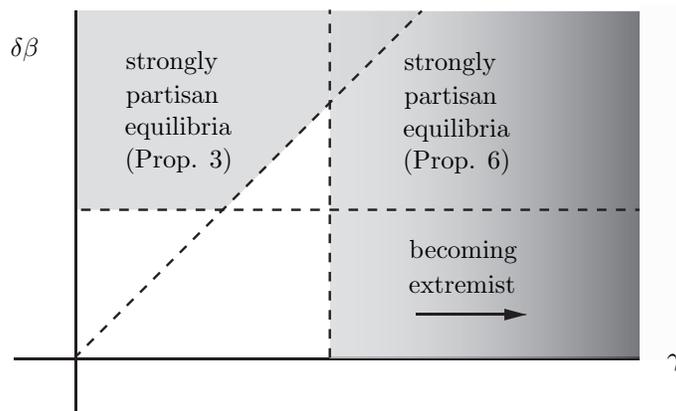
6 Effect of Money on Extremism

We now characterize policy outcomes as the effectiveness of money becomes large. As mentioned earlier, there are different channels through which this effect could be realized in reality: one is through the lifting of restrictions on expenditures by lobby groups for personal expenses and travel of politicians; another is the relaxation of restrictions on provision of services such as model legislation; and yet another is the easing of restrictions on campaign contributions, which can then be used by a politician to secure future gains from office. Although we do not model the role of campaign contributions explicitly, the analysis of the parameter γ, which summarizes the effectiveness of lobby contributions, can provide insight into aspects of general quid pro quo exchange, in which the value of contributions to the politician (or alternatively, the deadweight loss due to lobbying frictions) varies with the electoral or technological environment.

We begin by noting that the monetary transfer to a politician is non-monotonic in the effectiveness of money; in particular, lobby group spending goes to zero when γ is very small or very large, and it is thus highest for intermediate levels of γ.[25] If γ is large, then lobby groups can influence policy at low cost; intuitively, when restrictions on lobby expenditures are loose, each dollar goes further, and politicians can achieve large policy changes at low cost. Conversely, if the effectiveness of money is low, then lobby groups find it costly to move policy; substantively, the "bang for the buck" in shaping policy diminishes if there are tight restrictions on how groups can use their money. This result, which is formalized next, suggests that the empirical relationship between lobbying restrictions and observed expenditures may be more nuanced than is popularly believed.

Proposition 5. *If $\delta\beta$ is sufficiently large, then for all $\pi \in \{0, 1\}$, all θ in the support of h^π, and*

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[25]See Acemoglu et al. (2013) for a similar result in which the relationship between influence and transfers is non-monotonic.

all simple lobbying equilibria, we have

$$\lim_{\gamma \to 0} \mu^\pi_{G(\theta)}(\theta) \;=\; \lim_{\gamma \to \infty} \mu^\pi_{G(\theta)}(\theta) \;=\; 0,$$

and in particular, equilibrium lobbying expenditures are non-monotonic in γ.

The intuition for the result that $\lim_{\gamma \to 0} \mu^\pi_{G(\theta)}(\theta) = 0$ is easy, but it masks some subtleties in the proof. It relies on Proposition 3 for the fact that when office incentives are high, all equilibria are strongly partisan, and moreover, the default policy of every politician type outside the win set is to compromise. And it relies on Proposition 4 for the fact that as γ goes to zero, the win set shrinks to the median ideal policy, x_m. For example, given any $\theta > \theta_m$, when γ is small enough, the politician's ideal policy $x(\theta)$ exceeds $\overline{w}$; her default policy is $\overline{w}$; and R lobbies the politician to her default policy at no monetary cost. Thus, the crux of the proof is to show that payments to the median politician go to zero. By Proposition 4, the median politician is lobbied to $\overline{w}$, and this converges to x_m. Then the amount lobby group R is willing to pay for $\overline{w}$ instead of x_m goes to zero, and since the group can obtain x_m by default at no cost, the result follows.

The remainder of the analysis focuses on the case in which money is very effective, i.e., γ becomes large. In this case, the surplus-maximizing policies converge to the lobby groups' ideal points, i.e., $x_L(\theta_m) \to 0$ and $x_R(\theta_m) \to 1$. We say a sequence of simple lobbying equilibria *becomes extremist* if lobby group R offers all conservative politician types policies arbitrarily close to one along the sequence, and lobby group L offers all liberal politician types policies close to zero: formally, for each $\pi \in \{0, 1\}$, we have $\lambda^\pi_R(\theta_m) \to 1$ and $\lambda^\pi_L(\theta_m) \to 0$. The policies offered may be winning or losing, but an immediate implication is that the median voter's continuation value of a challenger cannot, in the limit, exceed the payoff from the worst policy for the median, i.e.,

$$\limsup \, (1 - \delta) V^{C,\pi}_m(\Psi) \;\leq\; u_m(0),$$

which implies that either $\overline{w}^\pi \to 1$ or $\underline{w}^\pi \to 0$ (or both). The main result of this section establishes that if the effectiveness of money is sufficiently high, then regardless of office incentives, every simple lobbying equilibrium is strongly partisan, and the most polarized strongly partisan equilibria become extremist, driving policies to the extremes of the policy space.

Proposition 6. *Assume the median voter is indifferent between the lobby groups, i.e., $u_m(0) = u_m(1)$, and fix $\epsilon > 0$. If γ is sufficiently large, then for all $\delta \geq \epsilon$ and for all β, every simple lobbying equilibrium is strongly partisan; furthermore, the most polarized strongly partisan equilibria become extremist as $\gamma \to \infty$.*

Proposition 6 is depicted in Figure 5.[26] The result imposes a small amount of symmetry in the model, in that the median voter is indifferent between the lobby groups. In the complementary

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[26]The proposition establishes a threshold on γ that is uniform in $\delta \geq \epsilon$ and in β, while Figure 5 indicates that equilibria are strongly partisan for all $\delta\beta > 0$. The result does hold for β close to zero, but we caution that the uniform threshold requires a positive (arbitrarily small) lower bound on δ.

case that $u_m(0) < u_m(1)$, money continues to have a polarizing—if less stark—effect on policy outcomes. In the general case, where lobby group R may be advantaged relative to lobby group L vis a vis the median voter, it may be that when the effectiveness of money is high, some simple lobbying equilibria are not strongly partisan and policies offered by group L do not converge to the worst policy, $x = 0$, for the median voter. Nevertheless, if such equilibria exist as γ becomes large, then the median voter's continuation value of a challenger can, in the limit, be no greater than the utility from the right-most extreme policy. Formally, we say a sequence of equilibria *becomes weakly extremist* if

$$\limsup (1 - \delta) V_m^{C,\pi}(\Psi) \quad \leq \quad u_m(1)$$

for each party $\pi \in \{0, 1\}$. Next, we establish that any exceptional equilibria—ones that are excluded by the minimal symmetry in Proposition 6—must become weakly extremist.

Proposition 7. *Assume $u_m(0) < u_m(1)$, and fix $\epsilon > 0$. Let $\gamma \to \infty$, and let δ and β vary arbitrarily with γ subject to $\delta \geq \epsilon$. Every selection of strongly partisan equilibria become weakly extremist, and if non-strongly partisan simple lobbying equilibria exist for arbitrarily high γ, then these equilibria also become weakly extremist.*

Propositions 6 and 7 establish the potentially polarizing effect of money in elections. As money becomes more effective, lobby groups offer the most extreme winning policies, i.e., equilibria become strongly partisan. Moreover, there exist equilibria supporting arbitrarily extreme policies, with liberal politicians choosing policies close to zero and conservative politicians choosing policies close to one. In such cases, Proposition 3 establishes that there may be multiple strongly partisan equilibria, and not all equilibria will exhibit extremism to this extent. It is possible to show, however, that as money becomes effective, limits of simple lobbying equilibria correspond to equilibria of the model in which office benefit is zero, there are no lobby groups, and there are just two extreme politician types. We define the *dichotomous model* as in Section 2, but now we assume: $\beta = 0$; there are no lobby groups, so that politician choices are given by the default policies $\xi_{\bar{\theta}}$ and $\xi_{\underline{\theta}}$; and given an incumbent from party π, the challenger's type is $\underline{\theta}$ with probability $H^\pi(\theta_m)$ and $\bar{\theta}$ with complementary probability.[27] The statement of the next result considers convergent sequences of win sets, without loss of generality, and it relies on Proposition 6, which implies that equilibria are strongly partisan when γ is high, so that these limits are independent of the incumbent's party.

Proposition 8. *Assume the median voter is indifferent between the lobby groups. Fix δ and β, and let $\gamma \to \infty$. For every selection of simple lobbying equilibria with convergent win sets, i.e., $\underline{w}^\pi \to \underline{w}$ and $\overline{w}^\pi \to \tilde{w}$, there is a strongly partisan equilibrium of the dichotomous model with win set $[\underline{w}, \tilde{w}]$.*

An implication is that equilibria of the dichotomous model provide bounds on simple lobbying equilibria as γ becomes large. In particular, the least polarized equilibrium of the dichotomous

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[27]When the challenger distribution is independent of the party in power, so that $H^1 = H^0$, the dichotomous model is a special case of Banks and Duggan (2008).

model serves as a lower bound on polarization in the original model, and in this equilibrium, at least one extreme type must be indifferent between compromise and shirking. Then the win set $[\underline{w}, \tilde{w}]$ must satisfy

$$\frac{u_{\bar{\theta}}(\tilde{w})}{1-\delta} \;=\; u_{\bar{\theta}}(1) + \delta\left[\frac{H^{\pi}(\theta_m)u_{\bar{\theta}}(\underline{w}) + (1-H^{\pi}(\theta_m))u_{\bar{\theta}}(\tilde{w})}{1-\delta}\right] \tag{7}$$

for some party π, or it must satisfy the corresponding equality for type $\underline{\theta}$. This equality cannot be satisfied when polarization is small, i.e., $\underline{w}$ and $\tilde{w}$ are close to x_m, and it follows that even the least polarized equilibria of the original model exhibit non-trivial polarization when money is effective. We conclude that when γ is large, all equilibria are strongly partisan, and moreover, the policies delivered by liberal and conservative politicians are bounded away from the median ideal point.

Condition (7) becomes more transparent when the challenger is drawn from the side of the spectrum opposite the incumbent, so that $H^1(\theta_m) = 1 - H^0(\theta_m) = 1$. For $\pi = 1$, it reduces to

$$u_{\bar{\theta}}(1) - u_{\bar{\theta}}(\tilde{w}) \;=\; \delta(u_{\bar{\theta}}(1) - u_{\bar{\theta}}(\underline{w})),$$

which shows that the restrictiveness of the bound interacts with the discount factor. When citizens are impatient, so that δ is close to zero, the above bound implies that $u_{\bar{\theta}}(1) - u_{\bar{\theta}}(\tilde{w})$ is close to zero, which implies that $\tilde{w}$ is close to one. Thus, if voters are impatient, then when γ becomes large, all equilibria are strongly partisan, and *all equilibria* become arbitrarily extremist. In case voters are patient, the bound from condition (7) becomes less restrictive; in this case, there exist arbitrarily extremist equilibria by Proposition 7, but equilibria with win sets close to the median ideal policy can also be supported.

Our results on the effectiveness of money do not state a general comparative static, such as the one suggested by Figure 3. As discussed there, when γ decreases, the direct effect is to cause lobby groups to moderate their policy offers, which increases the equilibrium payoff of the median voter from a challenger. However, there is a subtle, indirect effect of equilibrium incentives. Conditional on lobbying a politician to a winning policy, or conditional on lobbying a politician who shirks by default, an increase in γ does indeed lead the lobby group to make more extreme offers—but this direct effect implies that if an incumbent is lobbied outside the win set, then the policies of the challenger who replaces her will be more extreme. This means that the opposing lobby poses a more significant "threat," which can cause a lobby group to prefer the marginal winning policy to the option of shirking. This has the countervailing effect of moderating policy and increasing the equilibrium expected payoff of the median voter.

Example 3. To demonstrate that the intuitively obvious comparative static does not hold generally, we computed equilibria in an environment conducive to the indirect effect. We set $N = [0,2]$, $\theta_m = 1$, $\beta = 0$, and $\delta = .2$. The indirect effect is small if citizens are too risk averse (in that case, the opposing group will lobby politicians to losing policies with very small probability), so we use the same specification of utility, $u_{\theta}(x) = \theta x - x^2 - (\theta/2)^2$, as in Example 1. We use

a challenger distribution that depends on the party of the incumbent, which accentuates the risk created by the opposing group, and indeed, we use a single-peaked density that piles mass on challenger types that are lobbied to losing policies. For our purposes, it suffices to use defined on $[1, 2]$ as

$$
h^1(\theta) \;=\; \begin{cases} 2 & \text{if } 1.94 \leq \theta \leq 1.98, \\ .9583 & \text{else,} \end{cases}
$$

and symmetrically about 1 for h^0. Figure 6 illustrates the equilibrium relationship between the effectiveness of money (on the horizontal axis) and the right-hand endpoint of the win set, as we vary γ from 0 to .2. Initially, the win set becomes smaller as γ increases, reflecting the fact that the median voter's expected payoff from a challenger *increases* as money becomes more effective.

Figure 6: Non-monotonicity of median voter welfare in γ
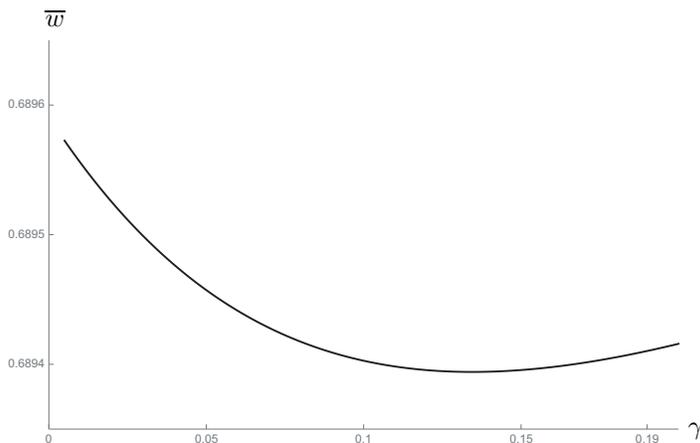
At work is the indirect effect described above. As γ continues to increase, the direct effect dominates, a feature borne out in our numerical experiments and consistent with the results of this section. Importantly, this non-monotonicity of voter welfare with respect to the effectiveness of money would not arise in a simpler, two-period model. In such a model, policies in the second period would be a trivial function of the lobby groups' influence, as there are no incentives for re-election, and an increase in γ would unambiguously lower the median voter's expected payoff from a challenger. The indirect effect could arise in a model with three or more periods, and it is possible that the median's payoff from electing a challenger to office in early periods could increase with γ, but the mechanical influence of money in the last period would exaggerate the direct effect of money in later periods. This could propagate to earlier periods, raising the possibility that the non-monotonicity was simply an artifact of the finite horizon. Our example shows that the non-monotonicity in voter welfare can arise in the more natural infinite-horizon setting, where there is no last-period distortion, and the non-monotonicity is reflected in the continuation value of a challenger evaluated in any period of the game. □

7 Conclusion

We have proposed a model of lobbying in the context of repeated elections that allows us to analyze the complex dynamic incentives faced by lobby groups, politicians, and voters. The presence of lobbying implies, under parameters of interest, that political outcomes are characterized by strong parties, in the sense that all liberal politicians deliver one policy and all conservative politicians deliver another. The centripetal effects of office incentives, found in prior work without lobbying, are robust to the introduction of extreme lobby groups: when office incentives are high and the effectiveness of money is low, equilibrium policies are close to the median. However, money has a centrifugal effect: fixing office incentives, there exist arbitrarily extreme equilibria as money becomes more effective, and a wedge is introduced between liberal and conservative policy choices that bounds equilibrium policies away from the median ideal point. Our analysis cautions against simple conclusions drawn from imprecise intuitions or political principles: the relaxation of restrictions on political contributions, which may be interpreted as an increase in the parameter γ, can lead to extreme policies, but lobbying can have a positive influence on voter welfare through the indirect effect of money on the incentives of groups to compromise. Thus, prescriptions for lobbying or campaign regulations should be subject to careful analysis.

Our findings also have implications for turnover. We find that incumbents always win re-election when money is very effective. Notably, this is not because groups "buy the election." Instead, highly effective lobbying creates cohesive parties with all members enacting the same policy, which always wins re-election. In contrast, if lobbying is not highly effective, then there can be turnover because extremist politicians do not cater to voters. In this way, introducing very effective interest groups into the electoral environment can reduce turnover even if those groups are extreme.

We have provided a baseline model of lobbying in which the mechanism at work is that of policy concession in exchange for sidepayments to politicians, but many avenues for future research remain open. Explicitly modeling electoral campaigns in the dynamic setting and tracing the effect of money through campaign financing may provide deeper insights into how money interacts with electoral accountability. We conjecture that the analysis would be largely unaffected, and that the effect of money could indeed be amplified in such a model. In the current framework, lobby groups must take the win set as given, and some politicians may be lobbied to relatively centrist policies because this "win set constraint" is binding. When lobby groups can contribute to political campaigns, however, it may be that they can affect the perceptions of voters and electoral outcomes, effectively enlarging the set of policies leading to re-election, relaxing the win set constraint, and creating the scope for more extreme policies. Additionally, campaign contributions could come from small ideological donors, as in Bouton et al. (2018).

We have assumed that lobbyists perfectly observe the incumbent's type. Although lobbyists are certainly better informed than voters, they likely have some uncertainty about politician preferences. In that setting, the lobbyist would be motivated to propose a menu of policies and transfers to partially screen politicians. Interestingly, the strongly partisan equilibria of our paper are robust to this type of uncertainty when γ is sufficiently high. That is, given a strongly partisan equi-

librium in our model, if γ is high enough, then there would be an equilibrium of the model with private information in which each lobby group offered the closest endpoint of the win set, along with a payment that compensated the most reluctant politician type. We conjecture that a form of Propositions 6 and 7 would carry over, but we leave that question open.

Finally, future work could endogenize who lobbies whom. Here, interest groups are restricted to lobbying officeholders on their own side of the spectrum. Although this setup has empirical justification, existing work highlights incentives to lobby politicians on the opposite side of the spectrum (e.g., Felli and Merlo, 2006, 2007).

A   Appendix

A.1   Existence of Equilibrium: Proof of Proposition 1

The existence proof consists of a fixed point argument, a byproduct of which is a characterization of optimal lobby offers and default policies that give rise to the partitional form; this is used to prove Proposition 2 in the next subsection. The fixed point argument is novel, in that it takes place in the product space of continuation distributions and group monetary payments. Before setting up the approach, we first note that the possible equilibrium monetary payments of a lobby group can be bounded: the lowest payment is zero (because politicians will never pay to have a policy different from their default), and the highest payment is

$$\overline{m} \;=\; \frac{1}{1-\delta} \max\left\{ u_{\overline{\theta}}(1) - u_{\overline{\theta}}(0), u_{\underline{\theta}}(0) - u_{\underline{\theta}}(1) \right\}$$

(because the group has the option of offering a payment of zero). Thus, we can take $\frac{\overline{m}}{1-\delta}$ as an upper bound on the discounted sum of payments by any group, which allows us to restrict payments to the compact interval $K = [0, \overline{m}]$.

We endow the space $\Delta(X)$ of Borel probability measures on X with the topology of weak convergence, making it compact. We denote a pair of continuation distributions by $\mathbf{P} = \left(P_X^{C,0}, P_X^{C,1} \right)$, where $P_X^{C,\pi}$ represents the continuation distribution following removal of an incumbent from party π, and we denote a quadruple of payments by $\mathbf{M} = (M_L^{C,0}, M_L^{C,1}, M_R^{C,0}, M_R^{C,1})$, where $M_G^{C,\pi}$ denotes the expected discounted payments made by group G, conditional on removing an incumbent from party π. We endow $\Delta(X)^2$, K^4, and $\Delta(X)^2 \times K^4$ with their product topologies. Mathematically, at this point, we consider an arbitrary pair $(\mathbf{P}, \mathbf{M}) \in \Delta(X)^2 \times K^4$. The arguments of this subsection construct a particular mapping, $\phi \colon \Delta(X)^2 \times K^4 \to \Delta(X)^2 \times K^4$, from the set of pairs $(\mathbf{P}, \mathbf{M})$ into itself. The construction takes place in a number of steps, and along the way we take note of continuity properties that will be critical for the existence proof.

Challenger continuation values: Given $\mathbf{P}$, we infer challenger continuation values as

$$V_\theta^{C,\pi}(\mathbf{P}) \;=\; \frac{\mathbb{E}_{P_X^{C,\pi}}[u_\theta(x)]}{1-\delta}, \tag{8}$$

where the expectation over policies x is with respect to the challenger continuation distribution $P_X^{C,\pi}$. Note that because u_θ is bounded and continuous, the continuation values $V_\theta^{C,\pi}(\mathbf{P})$ vary continuously as a function of $\mathbf{P}$. In fact, because the $u_\theta(x)$ is jointly continuous in x and θ, a version of Lebesgue's dominated convergence theorem implies that the continuation value $V_\theta^{C,\pi}(\mathbf{P})$ is jointly continuous as a function of θ and $\mathbf{P}$.

Win set: These continuation values determine a win set via the policies acceptable to the median voter, as in

$$W^\pi(\mathbf{P}) \;=\; \left\{ x \in X \mid \frac{u_m(x)}{1-\delta} \;\geq\; V_m^{C,\pi}(\mathbf{P}) \right\}.$$

Note that the median type's ideal policy belongs to the win set, i.e., $x_m \in W^\pi(\mathbf{P})$, and we can write this non-empty interval as $W^\pi(\mathbf{P}) = [\underline{w}^\pi(\mathbf{P}), \overline{w}^\pi(\mathbf{P})]$. Let $\overline{\theta}_w$ be the unique citizen type with ideal point equal to the greater endpoint, i.e., $x(\overline{\theta}_w) = \overline{w}^\pi(\mathbf{P})$, and let $\underline{\theta}_w$ be such that $x(\underline{\theta}_w) = \underline{w}^\pi(\mathbf{P})$. We will write these as $\overline{\theta}_w^\pi(\mathbf{P})$ and $\underline{\theta}_w^\pi(\mathbf{P})$ to make dependence on π and $\mathbf{P}$ explicit. By continuity of $V_m^{C,\pi}(\mathbf{P})$, along with strict concavity of u_m, the endpoints of this interval vary continuously as a function of $\mathbf{P}$, as do the cutoff types. In particular, $W^\pi(\mathbf{P})$, viewed as a function of $\mathbf{P}$, is a continuous correspondence.

Dynamic payoffs: We adapt the above notation for dynamic payoffs as follows: the dynamic policy utility for type θ is

$$
U_\theta^\pi(x|\mathbf{P}) \;\;=\;\; \begin{cases} \frac{u_\theta(x)}{1-\delta} & \text{if } x \in W^\pi(\mathbf{P}), \\ u_\theta(x) + \delta V_\theta^{C,\pi}(\mathbf{P}) & \text{else,} \end{cases}
$$

and dynamic office rents are

$$
B^\pi(x|\mathbf{P}) \;\;=\;\; \begin{cases} \frac{\beta}{1-\delta} & \text{if } x \in W^\pi(\mathbf{P}), \\ \beta & \text{else.} \end{cases}
$$

Thus, a policy x belongs to the win set $W^\pi(\mathbf{P})$ if and only if the dynamic payoff from x is at least equal to the continuation value of a challenger for the median voter. Note that these functions are not generally continuous; however, $U_\theta^\pi(x|\mathbf{P})$ is jointly continuous on $(x, \theta, \mathbf{P})$ triples such that $x \in W^\pi(\mathbf{P})$ and is jointly continuous on triples such that $x \notin W^\pi(\mathbf{P})$; and $B^\pi(x, \mathbf{P})$ is constant on pairs $(x, \mathbf{P})$ with $x \notin W^\pi(\mathbf{P})$ and jointly continuous on pairs with $x \in W^\pi(\mathbf{P})$. This joint continuity property will be important below, when we establish continuity of optimal lobby offers. We infer dynamic payments for group G as

$$
M_G^\pi(x|\mathbf{P}, \mathbf{M}) \;\;=\;\; \begin{cases} \delta M_G^{C,\pi} & \text{if } x \notin W^\pi(\mathbf{P}) \\ 0 & \text{else.} \end{cases}
$$

Thus, if a politician is lobbied by group G, then the expected discounted payments from the lobby are $m + M_G^\pi(x|\mathbf{P}, \mathbf{M})$, where m is the current offer, and $M_G^\pi(x|\mathbf{P}, \mathbf{M})$ represents the expected flow of future payments. Note that the dynamic payment $M_G^\pi(x|\mathbf{P}, \mathbf{M})$ is constant on triples $(x, \mathbf{P}, \mathbf{M})$ with $x \in W^\pi(\mathbf{P})$, is jointly continuous on triples with $x \notin W^\pi(\mathbf{P})$, and is constant in x when $x \notin W^\pi(\mathbf{P})$.

Default policies: These quantities determine default policy choices $\xi_\theta^\pi(\mathbf{P})$ for each politician type from each party as the solution to the following optimization problem:

$$
\max_x U_\theta^\pi(x|\mathbf{P}) + B^\pi(x|\mathbf{P}).
$$

Of course, the optimal policy choice of a politician type with $x(\theta) \in W^\pi(\mathbf{P})$, or equivalently, $\underline{\theta}_w^\pi(\mathbf{P}) \leq \theta \leq \overline{\theta}_w^\pi(\mathbf{P})$, is simply her ideal point. Otherwise, if $x(\theta) > \overline{w}^\pi(\mathbf{P})$, then the politician must choose between compromising at the greater endpoint of the win set or choosing her ideal

point and foregoing re-election. Note that the politician weakly prefers to compromise if and only if

$$\frac{u_\theta(\overline{w}^\pi(\mathbf{P})) + \beta}{1 - \delta} \geq u_\theta(x(\theta)) + \beta + \delta V_\theta^{C,\pi}(\mathbf{P}), \tag{9}$$

where the left-hand side of the inequality represents the type θ politician's discounted payoff from compromising at the nearest winning policy, and the right-hand side is the expected discounted payoff from shirking.

We claim that equality holds in (9) for at most one type $\overline{\theta}_c > \overline{\theta}_w^\pi(\mathbf{P})$, that it holds strictly for θ between $\overline{\theta}_w^\pi(\mathbf{P})$ and $\overline{\theta}_c$, and that the reverse inequality holds strictly for $\theta > \overline{\theta}_c$. We write the equality as

$$\frac{u_\theta(\overline{w}^\pi(\mathbf{P})) + \beta}{1 - \delta} - [u_\theta(x(\theta)) + \beta + \delta V_\theta^{C,\pi}(\mathbf{P})] = 0. \tag{10}$$

The first derivative of the left-hand side above with respect to θ is

$$\frac{v(\overline{w}^\pi(\mathbf{P}))}{1 - \delta} - v(x(\theta)) - \frac{\delta \mathbb{E}_{P_X^{C,\pi}}[v(x)]}{1 - \delta},$$

where we use the envelope theorem to neglect the indirect effect on $u_\theta(x(\theta))$ through the ideal point. Note that $v(x(\theta))$ is strictly increasing, so that the left-hand side of (10) is strictly concave in θ. Furthermore, (9) holds strictly for type $\overline{\theta}_w^\pi(\mathbf{P})$. Since the left-hand side of the inequality is strictly concave, it can then hold with equality for at most one type greater than $\overline{\theta}_w^\pi(\mathbf{P})$, and the claim follows. We write $\overline{\theta}_c^\pi(\mathbf{P})$ to make the dependence of this cutoff on $\mathbf{P}$ explicit; since (10) is continuous in $\mathbf{P}$, it follows that $\overline{\theta}_c^\pi(\mathbf{P})$ is also a continuous function of $\mathbf{P}$. Let $\overline{c}^\pi(\mathbf{P}) = x(\overline{\theta}_c^\pi(\mathbf{P}))$ be the ideal point of the type $\theta > \overline{\theta}_w^\pi(\mathbf{P})$ that is just indifferent between compromising and shirking. A similar analysis for types $\theta < \underline{\theta}_w^\pi(\mathbf{P})$ yields a cutoff $\underline{\theta}_c^\pi(\mathbf{P})$ that determines the willingness to compromise of such types, and we let $\underline{c}^\pi(\mathbf{P})$ be the ideal point of the type $\theta < \underline{\theta}_w^\pi(\mathbf{P})$ that is indifferent between compromise and shirking.

This gives us a partition of the policy space and a characterization of the politicians' optimal policy choices, according to whether a politician's ideal point is winning, or whether her ideal point is not winning but she optimally chooses to compromise her choice in order to gain re-election, or whether her ideal point is losing and she optimally shirks, choosing her ideal point and forgoing re-election. In particular, we define the partition

$$\begin{aligned}
W^\pi(\mathbf{P}) &= [\underline{w}^\pi(\mathbf{P}), \overline{w}^\pi(\mathbf{P})] \\
C^\pi(\mathbf{P}) &= [\underline{c}^\pi(\mathbf{P}), \underline{w}^\pi(\mathbf{P})) \cup (\overline{w}^\pi(\mathbf{P}), \overline{c}^\pi(\mathbf{P})] \\
S^\pi(\mathbf{P}) &= [0, \underline{c}^\pi(\mathbf{P})) \cup (\overline{c}^\pi(\mathbf{P}), 1]
\end{aligned}$$

consisting of the sets of ideal points of winners, compromisers, and shirkers, and we specify the

politician's default policy choices as

$$
\xi_\theta^\pi(\mathbf{P}) \;=\; \begin{cases}
x(\theta) & \text{if } x(\theta) \in [0, \underline{c}^\pi(\mathbf{P})) \\
\underline{w}^\pi(\mathbf{P}) & \text{if } x(\theta) \in [\underline{c}^\pi(\mathbf{P}), \underline{w}^\pi(\mathbf{P})), \\
x(\theta) & \text{if } x(\theta) \in W^\pi(\mathbf{P}) \\
\overline{w}^\pi(\mathbf{P}) & \text{if } x(\theta) \in (\overline{w}^\pi(\mathbf{P}), \overline{c}^\pi(\mathbf{P})] \\
x(\theta) & \text{if } x(\theta) \in (\overline{c}^\pi(\mathbf{P}), 1].
\end{cases}
$$

Thus, winners and shirkers optimally choose their ideal points, with winners being re-elected and shirkers being removed from office, whereas compromisers choose the winning policy closest to their ideal point.

Finally, we claim that the maximized value of the politician's objective function,

$$
\max_x U_\theta^\pi(x|\mathbf{P}) \;=\; U_\theta^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}) + B^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}),
$$

is jointly continuous as a function of θ and $\mathbf{P}$. Indeed, we can write this as

$$
\max\left\{ \max_{x \in W^\pi(\mathbf{P})} \frac{u_\theta(x) + \beta}{1 - \delta}, \; \sup_{x \notin W^\pi(\mathbf{P})} u_\theta(x) + V_\theta^{C,\pi}(\mathbf{P}) \right\},
$$

decomposing the politician's global maximization problem into two smaller ones. Because $W^\pi(\mathbf{P})$ is a continuous correspondence, the theorem of the maximum implies that the maximized value of each smaller problem is jointly continuous in θ and $\mathbf{P}$, and this continuity is preserved by the maximum operation, as claimed.

Lobby offers, part 1: Next, we examine the constrained optimization problem of the active lobby, translated to the current context with arbitrary continuation distributions. When $\gamma = 0$, the problem is trivial, and the active lobby group simply offers the officeholder their default policy with no payment. For the remainder of the analysis of lobby offers, we assume $\gamma > 0$. For every type θ and each party π with active group G, the optimal offer $(\lambda_G^\pi(\theta|\mathbf{P}, \mathbf{M}), \mu_G^\pi(\theta|\mathbf{P}, \mathbf{M}))$ solves

$$
\max_{(y,m)} U_G^\pi(y|\mathbf{P}) - m - M_G^\pi(y|\mathbf{P}, \mathbf{M})
$$

$$
\text{s.t. } U_\theta^\pi(y|\mathbf{P}) + \gamma m + B^\pi(y|\mathbf{P}) \;\geq\; U_\theta^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}) + B^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}),
$$

where the inequality is the *participation constraint* of the politician. This will be binding at a solution, so we can convert the constrained optimization problem into an unconstrained one by substituting the constraint into the objective function to obtain

$$
\max_y U_G^\pi(y|\mathbf{P}) - M_G^\pi(y|\mathbf{P}, \mathbf{M}) + \frac{1}{\gamma}\left[U_\theta^\pi(y|\mathbf{P}) + B^\pi(y|\mathbf{P}) - U_\theta^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}) - B^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}) \right].
$$

Because the politician's payoff from the default policy is continuous, as observed above, it follows that the objective function of the lobby group is jointly continuous when restricted to triples $(y, \theta, \mathbf{P})$

such that $y \in W^\pi(\mathbf{P})$, and similarly, it is jointly continuous when restricted to triples $(y, \theta, \mathbf{P})$ such that $y \notin W^\pi(\mathbf{P})$.

The optimal offer of the lobby group depends on the default policy choice of the politician and thus on the location of the politician's ideal point. There are three cases corresponding to the location of the politician's ideal point:

1. $x(\theta) \in W^\pi(\mathbf{P})$

2. $x(\theta) \in C^\pi(\mathbf{P})$

3. $x(\theta) \in S^\pi(\mathbf{P})$.

We examine the lobby group's optimization problem in each case. In each case, we examine the optimal offer of the group in the win set, say y', and the optimal offer (if any) outside the win set, say y'', with the global optimum being the preferred of the two offers. Because the analysis for group L is symmetric, we focus on the optimal offer of group R in what follows, and we therefore consider politician types $\theta \geq \theta_m$.

Case 1: Assume $x(\theta) \in W^\pi(\mathbf{P})$. The lobby group can buy policy $y' \in W^\pi(\mathbf{P})$. In this case, the default policy of the politician is $x(\theta)$, and the dynamic rents from y' and $x(\theta)$ are the same. Thus, y' solves

$$\max_{y \in W^\pi(\mathbf{P})} U_R^\pi(y|\mathbf{P}) - M_R^\pi(y|\mathbf{P}, \mathbf{M}) + \frac{1}{\gamma} U_\theta^\pi(y|\mathbf{P}) - \frac{1}{\gamma} U_\theta^\pi(x(\theta)|\mathbf{P}),$$

where the second and fourth terms are constant. In terms of stage utilities, after normalizing by $1 - \delta$ and dropping the constant terms, the policy y' solves

$$\max_{y \in W^\pi(\mathbf{P})} u_{\bar{\theta}}(y) + \frac{1}{\gamma} u_\theta(y). \tag{11}$$

The lobby group can also buy $y'' \notin W^\pi(\mathbf{P})$. In this case, the dynamic rents from $x(\theta)$ are $\frac{\beta}{1-\delta}$, while the dynamic rents from y'' are β. Then y'' solves

$$\max_{y \in X \setminus W^\pi(\mathbf{P})} U_R^\pi(y|\mathbf{P}) - M_R^\pi(y|\mathbf{P}, \mathbf{M}) + \frac{1}{\gamma} \left(U_\theta^\pi(y|\mathbf{P}) - U_\theta^\pi(x(\theta)|\mathbf{P}) - \frac{\delta\beta}{1-\delta} \right).$$

In terms of stage utilities, the group's problem is

$$\max_{y \in X \setminus W^\pi(\mathbf{P})} u_{\bar{\theta}}(y) + \delta V_{\bar{\theta}}^{C,\pi}(\mathbf{P}) - M_R^\pi(y|\mathbf{P}, \mathbf{M}) + \frac{1}{\gamma} \left(u_\theta(y) + \delta V_\theta^{C,\pi}(\mathbf{P}) - \frac{u_\theta(x(\theta))}{1-\delta} - \frac{\delta\beta}{1-\delta} \right).$$

Note that $M_R^\pi(y|\mathbf{P}, \mathbf{M})$ is constant in $y \notin W^\pi(\mathbf{P})$, so the objective function is, up to a constant term, equivalent to (11).

Case 2: Assume $x(\theta) \in C^\pi(\mathbf{P})$. The lobby group can buy policy $y' \in W^\pi(\mathbf{P})$. In this case, the default policy of the politician is $\overline{w}^\pi(\mathbf{P})$, and the dynamic rents from both y' and the default are

$\frac{\beta}{1-\delta}$. Thus, y' solves

$$\max_{y \in W^\pi(\mathbf{P})} U_R^\pi(y|\mathbf{P}) - M_R^\pi(y|\mathbf{P},\mathbf{M}) + \frac{1}{\gamma}\left(U_\theta^\pi(y|\mathbf{P}) - U_\theta^\pi(\overline{w}^\pi(\mathbf{P})|\mathbf{P})\right).$$

In terms of stage utilities, after normalizing by $1 - \delta$ and dropping constant terms, the policy y' again solves (11), as in Case 1. The lobby group can also buy $y'' \notin W^\pi(\mathbf{P})$. In this case, the dynamic rents from the default are $\frac{\beta}{1-\delta}$, while the dynamic rents from y'' are β. Then y'' solves

$$\max_{y \in X \backslash W^\pi(\mathbf{P})} U_R^\pi(y|\mathbf{P}) - M_R^\pi(y|\mathbf{P},\mathbf{M}) + \frac{1}{\gamma}\left(U_\theta^\pi(y|\mathbf{P}) - U_\theta^\pi(\overline{w}^\pi(\mathbf{P})|\mathbf{P}) - \frac{\delta\beta}{1-\delta}\right).$$

In terms of stage utilities, this is

$$\max_{y \in X \backslash W^\pi(\mathbf{P})} u_{\overline{\theta}}(y) + \delta V_{\overline{\theta}}^{C,\pi}(\mathbf{P}) - M_R^\pi(y|\mathbf{P},\mathbf{M}) + \frac{1}{\gamma}\left(u_\theta(y) + \delta V_\theta^{C,\pi}(\mathbf{P}) - \frac{u_\theta(\overline{w}^\pi(\mathbf{P}))}{1-\delta} - \frac{\delta\beta}{1-\delta}\right),$$

and again, up to a constant, this is the same objective function as in (11).

Case 3: Assume $x(\theta) \in S^\pi(\mathbf{P})$. The lobby group can buy policy $y' \in W^\pi(\mathbf{P})$. In this case, the default policy of the politician is $x(\theta)$, the dynamic rents from y' are $\frac{\beta}{1-\delta}$, and the dynamic rents from the default are β. Thus, y' solves

$$\max_{y \in W^\pi(\mathbf{P})} U_R^\pi(y|\mathbf{P}) - M_R^\pi(y|\mathbf{P},\mathbf{M}) + \frac{1}{\gamma}\left(U_\theta^\pi(y|\mathbf{P}) + \frac{\delta\beta}{1-\delta} - U_\theta^\pi(x(\theta)|\mathbf{P})\right).$$

In terms of stage utilities, after normalizing by $1 - \delta$ and dropping constant terms, the policy y' once again solves (11). The lobby group can also buy policy $y'' \notin W^\pi(\mathbf{P})$. Then the dynamic rents from y'' and the default are both β, and y'' solves

$$\max_{y \in X \backslash W^\pi(\mathbf{P})} U_R^\pi(y|\mathbf{P}) - M_R^\pi(y|\mathbf{P},\mathbf{M}) + \frac{1}{\gamma}\left(U_\theta^\pi(y|\mathbf{P}) - U_\theta^\pi(x(\theta)|\mathbf{P})\right).$$

In terms of stage utilities, this is

$$\max_{y \in X \backslash W^\pi(\mathbf{P})} u_{\overline{\theta}}(y) + \delta V_{\overline{\theta}}^{C,\pi}(\mathbf{P}) - M_R^\pi(y|\mathbf{P},\mathbf{M}) + \frac{1}{\gamma}\left(u_\theta(y) + \delta V_\theta^{C,\pi}(\mathbf{P}) - u_\theta(x(\theta)) + \delta V_\theta^{C,\pi}(\mathbf{P})\right),$$

and once again, this objective function coincides with (11), up to a constant.

Interim conclusions: We conclude that every policy $y' \in W^\pi(\mathbf{P})$ that is optimal for lobby group R subject to being in the win set solves

$$\max_{y \in W^\pi(\mathbf{P})} u_{\overline{\theta}}(y) + \frac{1}{\gamma}u_\theta(y), \tag{12}$$

and every policy $y'' \notin W^\pi(\mathbf{P})$ that is optimal subject to not being in the win set maximizes the

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same objective function,

$$\max_{y \in X \setminus W^\pi(\mathbf{P})} u_{\overline{\theta}}(y) + \frac{1}{\gamma} u_\theta(y). \tag{13}$$

That is, the optimal policy in each region maximizes the *joint surplus* function, $u_{\overline{\theta}}(y) + \frac{1}{\gamma} u_\theta(y)$, where the weight on the politician's utility decreases with the effectiveness of money in elections. Using the functional form of stage utilities, we can identify a *composite voter type,* namely, $\theta' = \frac{\theta + \gamma\overline{\theta}}{1+\gamma}$, such that optimal policies maximize the stage utility $u_{\theta'}$ of this composite type. Indeed, note that

$$u_{\overline{\theta}}(y) + \frac{1}{\gamma} u_\theta(y) \;=\; \left(\overline{\theta} + \frac{1}{\gamma}\right) v(y) - \left(1 + \frac{1}{\gamma}\right) c(y) + k_{\overline{\theta}} + \frac{k_\theta}{\gamma},$$

which is a positive affine transformation of $u_{\theta'}$.

The objective function of the group is strictly concave, so there is exactly one policy that is optimal subject to the constraint that the policy is winning; henceforth, to bring out the dependence on the politician's type and party and on the given probability distributions, we denote this by $y_w^\pi(\theta|\mathbf{P})$. Similarly, there is at most one policy that is optimal subject to the constraint that the policy is losing; we denote this by $y_\ell^\pi(\theta|\mathbf{P})$. Because the set $X \setminus W^\pi(\mathbf{P})$ is not compact, it is possible that there is no optimal losing policy for the group, but our analysis below shows that this complication is moot: if the group's optimal policy does not solve (12), then (13) has a solution, and that will be the optimal policy choice for the group. In fact, if (13) fails to have a solution, then the supremum of group payoffs over $X \setminus W^\pi(\mathbf{P})$ is approached by policies converging to $\overline{w}^\pi(\mathbf{P})$, and in this case, we adopt the convention that the optimal losing policy is $y_\ell^\pi(\mathbf{P}) = \overline{w}^\pi(\mathbf{P})$. Because this does not affect the analysis, we set aside this distinction in the remainder of the argument.

Recall that the restrictions of $U_R^\pi(x|\mathbf{P})$ and $U_\theta^\pi(x|\mathbf{P})$ to triples $(x, \theta, \mathbf{P})$ such that $x \in W^\pi(\mathbf{P})$ are each jointly continuous in θ and $\mathbf{P}$. Because $y_w^\pi(\theta|\mathbf{P})$ is uniquely defined and $W^\pi(\mathbf{P})$ is a continuous correspondence, the theorem of the maximum therefore implies that $y_w^\pi(\theta|\mathbf{P})$ is a jointly continuous function of θ and $\mathbf{P}$. Likewise, the lobby group's objective function is jointly continuous on the triples with $x \notin W^\pi(\mathbf{P})$, and $y_\ell^\pi(\theta|\mathbf{P})$ is jointly continuous in θ and $\mathbf{P}$. Of course, symmetric conclusions hold for lobby group L. Let

$$\mathcal{E}^\pi(\mathbf{P}, \mathbf{M}) \;=\; \{\theta \mid \lambda_{G(\theta)}^\pi(\theta|\mathbf{P}, \mathbf{M}) \in W^\pi(\mathbf{P})\}$$

denote the set of politician types from party π such that the active group offers a winning policy given the challenger distributions $\mathbf{P}$ and continuation payments $\mathbf{M}$, so that the incumbent wins election.

The question that remains is which is better for the lobby group—offering the optimal winning policy $y_w^\pi(\theta|\mathbf{P})$ or the optimal losing policy $y_\ell^\pi(\theta|\mathbf{P})$—and this depends on the constant term.

To win or not to win: Is it better to lobby for a policy that will ensure re-election of the politician, or to lobby for a policy that is best in the short run? That depends on the constant terms in the above analysis. It is clearly optimal to lobby for a winning policy if the constraint

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$y \in W^\pi(\mathbf{P})$ is not restrictive at the optimal winning policy $y_w^\pi(\theta|\mathbf{P})$, i.e., the policy $y_w^\pi(\theta|\mathbf{P})$ is the unconstrained maximizer of $u_{\overline{\theta}}(y) + \frac{1}{\gamma}u_\theta(x)$. Indeed, this is so because the lobby group can obtain the maximizer of joint surplus without having to compensate the politician for the loss of office benefit. Let $\hat{\theta}_w^\pi(\mathbf{P})$ be the highest politician type such that the constraint that $y_w^\pi(\theta|\mathbf{P})$ belongs to the win set is not restrictive. That is, $\hat{\theta}_w^\pi(\mathbf{P})$ is the unique type θ such that the maximizer of the joint surplus function $u_{\overline{\theta}}(y) + \frac{1}{\gamma}u_{\hat{\theta}_w^\pi(\mathbf{P})}(y)$ is exactly the right-hand endpoint of the win set, $\overline{w}^\pi(\mathbf{P})$, i.e., it solves

$$x\left(\frac{\theta + \gamma\overline{\theta}}{1 + \gamma}\right) \quad = \quad \overline{w}^\pi(\mathbf{P}),$$

and if there is no solution, because $x\left(\frac{\theta_m + \gamma\overline{\theta}}{1+\gamma}\right) > \overline{w}^\pi(\mathbf{P})$, set $\hat{\theta}_w^\pi(\mathbf{P}) = 0$. Of course, for higher politician types $\theta > \hat{\theta}_w^\pi(\mathbf{P})$, the policy that maximizes surplus subject to being in the win set is just $\overline{w}^\pi(\mathbf{P})$. Note that it is uniquely optimal for the lobby group to offer the winning policy $y_w^\pi(\theta|\mathbf{P}) < \overline{w}^\pi(\mathbf{P})$ to every lower type $\theta < \hat{\theta}_w^\pi(\mathbf{P})$. Thus, we define the optimal policy offer

$$\lambda_R^\pi(\theta|\mathbf{P}, \mathbf{M}) \quad = \quad y_w^\pi(\theta|\mathbf{P})$$

for all θ with $\theta_m < \theta \le \hat{\theta}_w^\pi(\mathbf{P})$, with transfer $\mu_R^\pi(\theta|\mathbf{P}, \mathbf{M})$ determined by the politician's default policy choice through the participation constraint, i.e.,

$$\mu_R^\pi(\theta|\mathbf{P}, \mathbf{M}) \quad = \quad \frac{1}{\gamma}\left[U_\theta^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}) + B^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}) - U_\theta^\pi(y_w^\pi(\theta|\mathbf{P})|\mathbf{P}) - B^\pi(y_w^\pi(\theta|\mathbf{P})|\mathbf{P})\right],$$

so that the optimal offer gives the politician her reservation value of choosing the default policy, $\xi_\theta^\pi(\mathbf{P})$.

The optimal offer to types $\theta > \hat{\theta}_w^\pi(\mathbf{P})$ is more complex. In what follows, we define

$$\Phi_R^\pi(\theta|\mathbf{P}, \mathbf{M}) \quad = \quad U_R^\pi(y_w^\pi(\theta|\mathbf{P})|\mathbf{P}) + \frac{1}{\gamma}\left[U_\theta^\pi(y_w^\pi(\theta|\mathbf{P})|\mathbf{P}) + \frac{\beta}{1-\delta}\right] \tag{14}$$
$$-U_R^\pi(y_\ell^\pi(\theta|\mathbf{P})|\mathbf{P}) + \delta M_R^{C,\pi} - \frac{1}{\gamma}\left[U_\theta^\pi(y_\ell^\pi(\theta|\mathbf{P})|\mathbf{P}) + \beta\right]$$

to be the lobby group's payoff from the optimal winning policy minus the payoff from the optimal losing policy, where we simplify the expression by canceling terms for default utility and rents. Note that lobby group R strictly prefers to lobby the type $\hat{\theta}_w^\pi(\mathbf{P})$ politician to a winning policy, so that $\Phi_R^\pi(\hat{\theta}_w^\pi(\mathbf{P})|\mathbf{P}) > 0$. And since the maximizer of joint surplus $u_{\overline{\theta}}(y) + \frac{1}{\gamma}u_\theta(y)$ is greater than $\overline{w}^\pi(\mathbf{P})$ for all $\theta > \hat{\theta}_w^\pi(\mathbf{P})$, the optimal policy subject to belonging to the win set for such types is the right-hand endpoint of the win set, i.e., $y_w^\pi(\theta|\mathbf{P}) = \overline{w}^\pi(\mathbf{P})$, as noted above. Furthermore, recall that the restrictions of $U_R^\pi(x|\mathbf{P})$, $U_\theta^\pi(x|\mathbf{P})$, and $M_R^\pi(x|\mathbf{P}, \mathbf{M})$ to quadruples $(x, \theta, \mathbf{P}, \mathbf{M})$ such that $x \in W^\pi(\mathbf{P})$ are jointly continuous in $(x, \theta, \mathbf{P}, \mathbf{M})$, as are the restrictions to quadruples with $x \notin W^\pi(\mathbf{P})$. Because $y_w^\pi(\theta|\mathbf{P})$ and $y_\ell^\pi(\theta|\mathbf{P})$ are jointly continuous in θ and $\mathbf{P}$, we conclude that $\Phi_R^\pi(\theta|\mathbf{P}, \mathbf{M})$ is jointly continuous in θ, $\mathbf{P}$, and $\mathbf{M}$.

Rewriting (14) in terms of stage utilities, the lobby group's payoff from the optimal winning policy $y_w^\pi(\theta|\mathbf{P})$ minus the payoff from the optimal losing policy $y_\ell^\pi(\theta|\mathbf{P})$ is

$$\Phi_R^\pi(\theta|\mathbf{P},\mathbf{M}) = \frac{1}{1-\delta}\left[u_{\bar{\theta}}(y_w^\pi(\theta|\mathbf{P})) + \frac{1}{\gamma}(u_\theta(y_w^\pi(\theta|\mathbf{P})) + \beta)\right] \tag{15}$$
$$-\left(u_{\bar{\theta}}(y_\ell^\pi(\theta|\mathbf{P})) + \delta V_{\bar{\theta}}^{C,\pi}(\mathbf{P})\right) + \delta M_R^{C,\pi} - \frac{1}{\gamma}\left(u_\theta(y_\ell^\pi(\theta|\mathbf{P})) + \delta V_\theta^{C,\pi}(\mathbf{P}) + \beta\right).$$

As mentioned above, when $y_w^\pi(\theta|\mathbf{P}) < \overline{w}^\pi(\mathbf{P})$, it is optimal for the lobby group to offer this winning policy. For $\theta > \hat{\theta}_w^\pi(\mathbf{P})$, we have $y_w^\pi(\theta|\mathbf{P}) = \overline{w}^\pi(\mathbf{P})$, and it follows that

$$\frac{d}{d\theta}\Phi_R^\pi(\theta|\mathbf{P},\mathbf{M}) = \frac{1}{1-\delta}\left[\frac{\partial}{\partial\theta}\frac{u_\theta(\overline{w}^\pi(\mathbf{P}))}{\gamma}\right] - \frac{\partial}{\partial\theta}\left(u_{\bar{\theta}}(y_\ell^\pi(\theta\mid\mathbf{P})) + \frac{1}{\gamma}u_\theta(y_\ell^\pi(\theta\mid\mathbf{P}))\right) - \frac{\delta}{\gamma}\frac{\partial}{\partial\theta}V_\theta^{C,\pi}(\mathbf{P})$$
$$= \frac{1}{1-\delta}\left[\frac{v(\overline{w}^\pi(\mathbf{P}))}{\gamma}\right] - \frac{1}{\gamma}v(y_\ell^\pi(\theta\mid\mathbf{P})) - \frac{\delta}{\gamma}\left[\frac{\mathbb{E}_{P_X^{C,\pi}}[v(x)]}{1-\delta}\right],$$

where we use the envelope theorem to suppress the indirect effect on the joint surplus $u_{\bar{\theta}}(y_\ell^\pi(\theta|\mathbf{P}) - \frac{1}{\gamma}u_\theta(y_\ell^\pi(\theta|\mathbf{P}))$ through the optimal losing policy. Note that $y_\ell^\pi(\theta|\mathbf{P})$ is strictly increasing in θ, so the above expression for $\frac{d}{d\theta}\Phi_R^\pi(\theta|\mathbf{P},\mathbf{M})$ is strictly decreasing in θ, and thus $\Phi_R^\pi(\theta|\mathbf{P},\mathbf{M})$ is strictly concave in $\theta > \hat{\theta}_w^\pi(\mathbf{P})$.

Lobby offers, part 2: Recall that lobby R strictly prefers to offer the optimal winning policy for the type $\hat{\theta}_w^\pi(\mathbf{P})$ politician, so that $\Phi_R^\pi(\hat{\theta}_w^\pi(\mathbf{P})|\mathbf{P},\mathbf{M}) > 0$. By strict concavity of $\Phi_R^\pi(\theta|\mathbf{P},\mathbf{M})$ in $\theta > \hat{\theta}_w^\pi(\mathbf{P})$, it follows that indifference between the optimal winning and optimal losing policies, i.e., $\Phi_R^\pi(\theta|\mathbf{P},\mathbf{M}) = 0$, holds for at most one type $\bar{\theta}_e$, that $\Phi_R^\pi(\theta|\mathbf{P},\mathbf{M}) > 0$ holds for all θ between $\hat{\theta}_w^\pi(\mathbf{P})$ and $\bar{\theta}_e$, and that $\Phi_R^\pi(\theta|\mathbf{P},\mathbf{M}) < 0$ for all $\theta > \bar{\theta}_e$. We henceforth write $\bar{\theta}_e^\pi(\mathbf{P},\mathbf{M})$ to make dependence of this cutoff on $\mathbf{P}$ and $\mathbf{M}$ explicit. If there is no solution to $\Phi_R^\pi(\theta|\mathbf{P},\mathbf{M}) = 0$, we set $\bar{\theta}_e^\pi(\mathbf{P},\mathbf{M}) = \bar{\theta}$. Importantly, since $\bar{\theta}_e^\pi(\mathbf{P},\mathbf{M})$ is uniquely defined and $\Phi_R^\pi(\theta|\mathbf{P},\mathbf{M})$ is jointly continuous in θ, $\mathbf{P}$, and $\mathbf{M}$, it follows that $\bar{\theta}_e^\pi(\mathbf{P},\mathbf{M})$ is also a continuous function of $\mathbf{P}$ and $\mathbf{M}$. A similar analysis holds for types $\theta < \theta_m$, yielding cutoff $\underline{\theta}_e^\pi(\mathbf{P},\mathbf{M})$ that is also jointly continuous.

With this background, we specify the optimal policy offer of lobby group R as

$$\lambda_R^\pi(\theta|\mathbf{P},\mathbf{M}) = \begin{cases} y_w^\pi(\theta|\mathbf{P}) & \text{if } \theta \leq \hat{\theta}_w^\pi(\mathbf{P}), \\ \overline{w}^\pi(\mathbf{P}) & \text{if } \hat{\theta}_w^\pi(\mathbf{P}) < \theta \leq \bar{\theta}_e^\pi(\mathbf{P},\mathbf{M}), \\ y_\ell^\pi(\theta|\mathbf{P}) & \text{if } \bar{\theta}_e^\pi(\mathbf{P},\mathbf{M}) < \theta. \end{cases}$$

In the above definition, there is some redundancy, as the first and second cases could be collapsed into one. When the politician is not too extreme, the lobby group offers the optimal winning policy, thereby removing the need to compensate the politician for lost office benefit at not too great a cost in terms of policy. When the politician is more extreme, the policy offer maximizes the joint surplus function $u_{\bar{\theta}}(y) + \frac{1}{\gamma}u_\theta(y)$, pulling policy from the ideal point of the politician toward the ideal policy of the group. Note that by continuity of the optimal winning policy $y_w^\pi(\theta|\mathbf{P})$ and the optimal losing policy $y_\ell^\pi(\theta|\mathbf{P})$, the optimal policy outcome $\lambda_R^\pi(\theta|\mathbf{P},\mathbf{M})$ is jointly continuous in $(\theta,\mathbf{P},\mathbf{M})$ at

all $\theta \neq \overline{\theta}_e^\pi(\mathbf{P}, \mathbf{M})$.

Monetary transfers are determined by the politician's participation constraint: in case $\theta \leq \hat{\theta}_w^\pi(\mathbf{P})$, we specify

$$\mu_R^\pi(\theta|\mathbf{P}, \mathbf{M}) = \frac{1}{\gamma}\left[U_\theta^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}) + B^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}) - U_\theta^\pi(y_w^\pi(\theta|\mathbf{P})|\mathbf{P}) - B^\pi(y_w^\pi(\theta|\mathbf{P})|\mathbf{P})\right],$$

in case $\theta_w^\pi(\mathbf{P}) < \theta \leq \overline{\theta}_e^\pi(\mathbf{P}, \mathbf{M})$, we specify

$$\mu_R^\pi(\theta|\mathbf{P}, \mathbf{M}) = \frac{1}{\gamma}\left[U_\theta^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}) + B^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}) - U_\theta^\pi(\overline{w}^\pi(\mathbf{P})|\mathbf{P}) - B^\pi(\overline{w}^\pi(\mathbf{P})|\mathbf{P})\right],$$

and in case $\overline{\theta}_e^\pi(\mathbf{P}, \mathbf{M}) < \theta$, we specify

$$\mu_R^\pi(\theta|\mathbf{P}, \mathbf{M}) = \frac{1}{\gamma}\left[U_\theta^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}) + B^\pi(\xi_\theta^\pi(\mathbf{P})|\mathbf{P}) - U_\theta^\pi(y_\ell^\pi(\theta|\mathbf{P})|\mathbf{P}) - B^\pi(y_\ell^\pi(\theta|\mathbf{P})|\mathbf{P})\right].$$

Again, $\mu_R^\pi(\theta|\mathbf{P}, \mathbf{M})$ is jointly continuous in $(\theta, \mathbf{P}, \mathbf{M})$ at all $\theta \neq \overline{\theta}_e^\pi(\mathbf{P}, \mathbf{M})$, by continuity of the optimal winning and losing policies, continuity of the politician's payoff from the default policy, and continuity of $\overline{\theta}_e^\pi(\mathbf{P}, \mathbf{M})$.

Although the preceding analysis has focused on lobby group R, we use analogous arguments to deduce an optimal policy offer $\lambda_L^\pi(\theta|\mathbf{P}, \mathbf{M})$ and monetary transfer $\mu_L^\pi(\theta|\mathbf{P}, \mathbf{M})$ for group L to politician types $\theta < \theta_m$. These will have similar continuity properties, which we exploit in the fixed point argument below.

Updating continuation distributions and payments: We now update the continuation distributions, $\mathbf{P} = (P_X^{C,0}, P_X^{C,1})$ and monetary payments $\mathbf{M} = (M_L^{C,0}, M_L^{C,1}, M_R^{C,0}, M_R^{C,1})$ fixed at the beginning of the argument. Technically, we specify two probability measures $\tilde{\mathbf{P}} = (\tilde{P}_X^{C,0}, \tilde{P}_X^{C,1})$ and four payments $\tilde{\mathbf{M}} = (\tilde{M}_L^{C,0}, \tilde{M}_L^{C,1}, \tilde{M}_R^{C,0}, \tilde{M}_R^{C,1})$, where for every measurable set $Z \subseteq X$, $\tilde{P}_X^{C,\pi}(Z)$ represents the probability (discounted appropriately over time) of a policy in the set Z, conditional on replacing an incumbent from party π with an untried challenger; and $\tilde{M}_G^{C,\pi}$ is the expected, discounted sum of payments by group G, conditional on removing an incumbent from party π. To construct $\tilde{\mathbf{P}}$ and $\tilde{\mathbf{M}}$, we use the above analysis to update $\mathbf{P}$ and $\mathbf{M}$ in the period immediately after a challenger is elected, and we then use the original $\mathbf{P}$ and $\mathbf{M}$ to evaluate future policy choices and monetary payments if the challenger is removed from office after her first term.

We begin by specifying $\tilde{P}_X^{C,\pi}$. If an incumbent from party π is removed from office, then she is replaced by a challenger from party $1 - \pi$. Define the measures $Q_w^\pi(\cdot|\mathbf{P}, \mathbf{M})$ and $Q_\ell^\pi(\cdot|\mathbf{P}, \mathbf{M})$ on X

so that for all measurable $Z \subseteq X$,

$$Q_w^\pi(Z|\mathbf{P}, \mathbf{M}) = \int_{\substack{\theta \,:\, \theta < \theta_m, \\ \lambda_L^{1-\pi}(\theta|\mathbf{P}, \mathbf{M}) \,\in\, Z \cap W^{1-\pi}(\mathbf{P})}} h^{1-\pi}(\theta) d\theta \quad + \quad \int_{\substack{\theta \,:\, \theta \geq \theta_m, \\ \lambda_R^{1-\pi}(\theta|\mathbf{P}, \mathbf{M}) \,\in\, Z \cap W^{1-\pi}(\mathbf{P})}} h^{1-\pi}(\theta) d\theta$$

$$Q_\ell^\pi(Z|\mathbf{P}, \mathbf{M}) = \int_{\substack{\theta \,:\, \theta < \theta_m, \\ \lambda_L^{1-\pi}(\theta|\mathbf{P}, \mathbf{M}) \,\in\, Z \setminus W^{1-\pi}(\mathbf{P})}} h^{1-\pi}(\theta) d\theta \quad + \quad \int_{\substack{\theta \,:\, \theta \geq \theta_m, \\ \lambda_R^{1-\pi}(\theta|\mathbf{P}, \mathbf{M}) \,\in\, Z \setminus W^{1-\pi}(\mathbf{P})}} h^{1-\pi}(\theta) d\theta,$$

where $Q_w^\pi(Z|\mathbf{P}, \mathbf{M})$ represents probability mass on winning policies, and $Q_\ell^\pi(Z|\mathbf{P}, \mathbf{M})$ will be used to assign probability mass to losing policies. Define the updated challenger continuation distribution after removal of an incumbent from party π as follows: for all measurable $Z \subseteq X$, we have

$$\tilde{P}_X^{C,\pi}(Z) = Q_w^\pi(Z|\mathbf{P}, \mathbf{M}) + (1 - \delta)Q_\ell^\pi(Z|\mathbf{P}, \mathbf{M}) + \delta Q_\ell^\pi(X|\mathbf{P}, \mathbf{M})P_X^{C,1-\pi}(Z), \quad (16)$$

and note that $\tilde{P}_X^{C,\pi}(X) = 1$, so that $\tilde{P}_X^{C,\pi}$ is indeed a probability measure. In words, we allocate probability mass to Z for each type that is lobbied to a winning policy in the set, because those politicians stay in office and choose that policy thereafter. We also allocate probability to Z for losing types that choose a policy in the set, but now discounted, because such a politician holds office for only one term, and subsequently an amount of probability is allocated via the initial challenger continuation distribution $P_X^{C,1-\pi}$.

Next, we specify $\tilde{M}_R^{C,\pi}$ as follows:

$$\tilde{M}_R^{C,\pi} = \int_{\substack{\theta \,:\, \theta < \theta_m, \\ \lambda_L^{1-\pi}(\theta|\mathbf{P}, \mathbf{M}) \,\in\, X \setminus W^{1-\pi}(\mathbf{P})}} \delta M_R^{C,\pi} h^{1-\pi}(\theta) d\theta \quad + \quad \int_{\substack{\theta \,:\, \theta \geq \theta_m, \\ \lambda_R^{1-\pi}(\theta|\mathbf{P}, \mathbf{M}) \,\in\, W^{1-\pi}(\mathbf{P})}} \mu_R^{1-\pi}(\theta|\mathbf{P}, \mathbf{M}) h^{1-\pi}(\theta) d\theta$$

$$+ \quad \int_{\substack{\theta \,:\, \theta \geq \theta_m, \\ \lambda_R^{1-\pi}(\theta|\mathbf{P}, \mathbf{M}) \,\in\, X \setminus W^{1-\pi}(\mathbf{P})}} \left(\mu_R^{1-\pi}(\theta|\mathbf{P}, \mathbf{M}) + \delta M_R^{C,\pi} \right) h^{1-\pi}(\theta) d\theta \quad (17)$$

When an incumbent from party π is removed from office, she is replaced by a challenger drawn from party $1 - \pi$. In case the newly elected politician is type $\theta < \theta_m$, she is lobbied by group L, and if L offers a winning policy, then the incumbent remains in office thereafter, and the payments of lobby group R are zero. If L offers a losing policy, then group R's payments are zero in the first term of the new officeholder, and after that, they are given by the discounted stream M_R^π of payments of a replacement drawn from party π. This is represented by the integral in the first term above. In case the newly elected politician is type $\theta \geq \theta_m$, she is lobbied by group R, and this is represented by the second and third terms. If R offers a winning policy, then the group makes a one-time payment of $\mu_R^{1-\pi}(\theta|\mathbf{P}, \mathbf{M})$ to the incumbent from party $1 - \pi$. Finally, if R offers a losing policy, then the

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group makes a one-time payment of $\mu_R^{1-\pi}(\theta|\mathbf{P}, \mathbf{M})$, followed by the discounted stream $\delta M_R^{C,\pi}$ of payments to a replacement drawn from party π.

We construct $\tilde{M}_L^{C,\pi}$ analogously, and for the remainder of the proof, we let $\tilde{\mathbf{P}} = (\tilde{P}_X^{C,0}, \tilde{P}_X^{C,1})$ and $\tilde{\mathbf{M}} = (\tilde{M}_L^{C,0}, \tilde{M}_L^{C,1}, \tilde{M}_R^{C,0}, \tilde{M}_R^{C,1})$ denote the updated continuation distributions and monetary payments of the lobby groups.

Fixed point argument: The above analysis determines a mapping $\phi \colon \Delta(X)^2 \times K^4 \to \Delta(X)^2 \times K^4$ as follows: for each $(\mathbf{P}, \mathbf{M}) \in \Delta(X)^2 \times K^4$, we define

$$\phi(\mathbf{P}, \mathbf{M}) \quad = \quad (\tilde{\mathbf{P}}, \tilde{\mathbf{M}})$$

to consist of the updated continuation distributions and group payments, revised to account for optimal lobbying, policy choice, and voting, given $(\mathbf{P}, \mathbf{M})$, as described in (16) and (17). The next step is to use the mapping ϕ to obtain a fixed point, i.e., a pair $(\mathbf{P}^*, \mathbf{M}^*)$ such that $\phi(\mathbf{P}^*, \mathbf{M}^*) = (\mathbf{P}^*, \mathbf{M}^*)$, which will yield a simple lobbying equilibrium. The mathematical technology used in the proof is Schauder's fixed point theorem, which requires the following: $\Delta(X)^2 \times K^4$ is a non-empty, convex, compact subset of a locally Hausdorff linear space, and the mapping ϕ is continuous. The first half of this requirement is delivered by well-known properties of the set of Borel probability measures on a compact subset of finite-dimensional Euclidean space, endowed with the topology of weak convergence. The crux of the proof is, therefore, to demonstrate that the mapping ϕ is continuous.

To this end, we consider a convergent sequence $\{(\mathbf{P}^m, \mathbf{M}^m)\}$ of pairs with $\mathbf{P}^m = (P_X^{C,0,m}, P_X^{C,1,m}) \in \Delta(X)^2$ and $\mathbf{M}^m = (M_L^{C,0,m}, M_L^{C,1,m}, M_R^{C,0,m}, M_R^{C,1,m}) \in K^4$ for each m. Let $(\mathbf{P}, \mathbf{M})$ denote the limit of this sequence, with $\mathbf{P} = (P_X^{C,0}, P_X^{C,1})$ and $\mathbf{M} = (M_L^{C,0}, M_L^{C,1}, M_R^{C,0}, M_R^{C,1})$. In particular, we have $P_X^{C,\pi,m} \to P_X^{C,\pi}$ weakly for each $\pi = 0, 1$ and $M_G^{C,\pi,m} \to M_G^{C,\pi}$ in the usual Euclidean topology for each $\pi = 0, 1$ and each $G = L, R$. We must show that $\phi(\mathbf{P}^m, \mathbf{M}^m) \to \phi(\mathbf{P}, \mathbf{M})$. We write the values of ϕ along the sequence as

$$\phi(\mathbf{P}^m, \mathbf{M}^m) \quad = \quad (\tilde{\mathbf{P}}^m, \tilde{\mathbf{M}}^m) \quad = \quad ((\tilde{P}_X^{C,0,m}, \tilde{P}_X^{C,1,m}), (\tilde{M}_L^{C,0,m}, \tilde{M}_L^{C,1,m}, \tilde{M}_R^{C,0,m}, \tilde{M}_R^{C,1,m}))$$

and we write the value at $(\mathbf{P}, \mathbf{M})$ as

$$\phi(\mathbf{P}, \mathbf{M}) \quad = \quad (\tilde{\mathbf{P}}, \tilde{\mathbf{M}}) \quad = \quad ((\tilde{P}_X^{C,0}, \tilde{P}_X^{C,1}), (\tilde{M}_L^{C,0}, \tilde{M}_L^{C,1}, \tilde{M}_R^{C,0}, \tilde{M}_R^{C,1}))$$

Then we must show that $\tilde{P}_X^{C,\pi,m} \to \tilde{P}_X^{C,\pi}$ weakly for each $\pi = 0, 1$, and that $\tilde{M}_G^{C,\pi,m} \to \tilde{M}_G^{C,\pi}$ for each $\pi = 0, 1$ and each $G = L, R$. Because the arguments for the remaining cases are analogous, we will show that $\tilde{P}_X^{C,0,m} \to \tilde{P}_X^{C,0}$ weakly and that $\tilde{M}_R^{C,0,m} \to \tilde{M}_R^{C,0}$.

To prove $\tilde{P}_X^{C,0,m} \to \tilde{P}_X^{C,0}$ weakly, it suffices to consider any closed set $Z \subseteq X$ and to show that

$$\limsup \tilde{P}_X^{C,0,m}(Z) \quad \leq \quad \tilde{P}_X^{C,0}(Z).$$

Using (16), since $P_X^{C,1,m} \to P_X^{C,1}$ weakly, it follows that $\limsup P_X^{C,1,m}(Z) \leq P_X^{C,1}(Z)$. Thus, it

suffices to show that $Q_w^0(Z|\mathbf{P}^m,\mathbf{M}^m) \to Q_w^0(Z|\mathbf{P},\mathbf{M})$ and $Q_\ell^0(Z|\mathbf{P}^m,\mathbf{M}^m) \to Q_\ell^0(Z|\mathbf{P},\mathbf{M})$. To this end, define the indicator functions $I_w\colon N \to \mathbb{R}$ and $I_w^m\colon N \to \mathbb{R}$ by

$$
I_w(\theta) \;=\; \left\{
\begin{array}{ll}
1 & \text{if } \theta < \theta_m \text{ and } \lambda_L^1(\theta|\mathbf{P},\mathbf{M}) \in Z \cap W^1(\mathbf{P}) \\
1 & \text{if } \theta \ge \theta_m \text{ and } \lambda_R^1(\theta|\mathbf{P},\mathbf{M}) \in Z \cap W^1(\mathbf{P}), \\
0 & \text{else,}
\end{array}
\right.
$$

and

$$
I_w^m(\theta) \;=\; \left\{
\begin{array}{ll}
1 & \text{if } \theta < \theta_m \text{ and } \lambda_L^1(\theta|\mathbf{P}^m,\mathbf{M}^m) \in Z \cap W^1(\mathbf{P}^m) \\
1 & \text{if } \theta \ge \theta_m \text{ and } \lambda_R^1(\theta|\mathbf{P}^m,\mathbf{M}^m) \in Z \cap W^1(\mathbf{P}^m), \\
0 & \text{else,}
\end{array}
\right.
$$

$m = 1,2,\ldots$. Then we can write

$$
Q_w^0(Z|\mathbf{P},\mathbf{M}) \;=\; \int_N I_w(\theta) h^1(\theta) d\theta \quad \text{and} \quad Q_w^0(Z|\mathbf{P}^m,\mathbf{M}^m) \;=\; \int_N I_w^m(\theta) h^1(\theta) d\theta.
$$

Now, consider any $\theta \in N$ such that $\theta \ne \overline{\theta}_e^1(\mathbf{P})$, so that the lobby group R has a strict preference to lobby the type θ politician to a winning policy, $\lambda_R^1(\theta|\mathbf{P},\mathbf{M}) \in W^1(\mathbf{P})$, or to a losing policy $\lambda_R^1(\theta|\mathbf{P},\mathbf{M}) \notin W^1(\mathbf{P})$. In either case, continuity of the lobby group's optimal offer implies $\lambda_R^1(\theta|\mathbf{P}^m,\mathbf{M}^m) \to \lambda_R^1(\theta|\mathbf{P},\mathbf{M})$. Thus, if $\lambda_R^1(\theta|\mathbf{P},\mathbf{M})$ is winning, then $\lambda_R^1(\theta|\mathbf{P}^m,\mathbf{M}^m)$ is winning for high enough m; and, if $\lambda_R^1(\theta|\mathbf{P},\mathbf{M}) \notin W^1(\mathbf{P})$, then $\lambda_R^1(\theta|\mathbf{P}^m,\mathbf{M}^m) \notin W^1(\mathbf{P}^m)$ for sufficiently high m. Likewise, for all $\theta \ne \underline{\theta}_e^1(\mathbf{P},\mathbf{M})$, $\lambda_L^1(\theta|\mathbf{P}^m,\mathbf{M}^m)$ is winning for sufficiently high m if $\lambda_L^1(\theta|\mathbf{P},\mathbf{M})$ is winning, and $\lambda_L^1(\theta|\mathbf{P}^m,\mathbf{M}^m)$ is losing for sufficiently high m if $\lambda_L^1(\theta|\mathbf{P},\mathbf{M})$ is losing. We have established that the functions I_w^m converge pointwise almost everywhere to the function I_w. By Lebesgue's dominated convergence theorem, we conclude that the integrals converge, and thus $Q_w^0(Z|\mathbf{P}^m,\mathbf{M}^m) \to Q_w^0(Z|\mathbf{P},\mathbf{M})$. An analogous argument, defining I_ℓ and I_ℓ^m using the set of losing policies, $X \setminus W^1(\mathbf{P})$ and $X \setminus W^1(\mathbf{P}^m)$ respectively, establishes that $Q_\ell^0(Z|\mathbf{P}^m,\mathbf{M}^m) \to Q_\ell^0(Z|\mathbf{P},\mathbf{M})$. We conclude that $\tilde{P}_X^{C,0,m} \to \tilde{P}_X^{C,0}$, and an analogous argument yields $\tilde{\mathbf{P}}^m \to \tilde{\mathbf{P}}$.

To prove $\tilde{M}_R^{C,0,m} \to \tilde{M}_R^{C,0}$, we define the functions $I_{R,\ell}\colon N \to \mathbb{R}$, $I_{R,w}\colon N \to \mathbb{R}$, $I_{R,\ell}^m\colon N \to \mathbb{R}$,

and $I_{R,w}^m \colon N \to \mathbb{R}$ by

$$
I_{R,\ell}(\theta) = \begin{cases} \delta M_R^{C,0} & \text{if } \theta < \theta_m \text{ and } \lambda_L^1(\theta|\mathbf{P},\mathbf{M}) \in X \setminus W^1(\mathbf{P}) \\ \mu_R^1(\theta|\mathbf{P},\mathbf{M}) + \delta M_R^{C,0} & \text{if } \theta \geq \theta_m \text{ and } \lambda_R^1(\theta|\mathbf{P},\mathbf{M}) \in X \setminus W^1(\mathbf{P}) \\ 0 & \text{else,} \end{cases}
$$

$$
I_{R,w}(\theta) = \begin{cases} \mu_R^1(\theta|\mathbf{P},\mathbf{M}) & \text{if } \theta \geq \theta_m \text{ and } \lambda_R^1 I(\theta|\mathbf{P},\mathbf{M}) \in W^1(\mathbf{P}) \\ 0 & \text{else,} \end{cases}
$$

$$
I_{R,\ell}^m(\theta) = \begin{cases} \delta M_R^{C,0,m} & \text{if } \theta < \theta_m \text{ and } \lambda_L^1(\theta|\mathbf{P}^m,\mathbf{M}^m) \in X \setminus W^1(\mathbf{P}^m) \\ \mu_R^1(\theta|\mathbf{P}^m,\mathbf{M}^m) + \delta M_R^{C,0,m} & \text{if } \theta \geq \theta_m \text{ and } \lambda_R^1(\theta|\mathbf{P}^m,\mathbf{M}^m) \in X \setminus W^1(\mathbf{P}^m) \\ 0 & \text{else,} \end{cases}
$$

$$
I_{R,w}^m(\theta) = \begin{cases} \mu_R^1(\theta|\mathbf{P}^m,\mathbf{M}^m) & \text{if } \theta \geq \theta_m \text{ and } \lambda_R^1 I(\theta|\mathbf{P}^m,\mathbf{M}^m) \in W^1(\mathbf{P}^m) \\ 0 & \text{else.} \end{cases}
$$

Then we can write

$$
\tilde{M}_R^{C,0} = \int_N I_{R,\ell}(\theta) h^1(\theta) d\theta + \int_N I_{R,w}(\theta) h^1(\theta) d\theta
$$

and

$$
\tilde{M}_R^{C,0,m} = \int_N I_{R,\ell}^m(\theta) h^1(\theta) d\theta + \int_N I_{R,w}^m(\theta) h^1(\theta) d\theta.
$$

Now consider any $\theta \notin \{\overline{\theta}_e^1(\mathbf{P},\mathbf{M}), \theta_e^1(\mathbf{P},\mathbf{M})\}$, so that the active lobby group has a strict preference to lobby the type θ politician to a winning policy or to a losing policy. Then we have $\lambda_R^1(\theta|\mathbf{P}^m,\mathbf{M}^m) \to \lambda_R^1(\theta|\mathbf{P},\mathbf{M})$ if $\theta \geq \theta_m$, and we have $\lambda_L^1(\theta|\mathbf{P}^m,\mathbf{M}^m) \to \lambda_L^1(\theta|\mathbf{P},\mathbf{M})$ if $\theta < \theta_m$. As well, $M_R^{C,0,m} \to M_R^{C,0}$, and in case $\theta \geq \theta_m$, by continuity of $\mu_R^1(\theta|\mathbf{P},\mathbf{M})$ at $\theta \neq \overline{\theta}_e^1(\mathbf{P},\mathbf{M})$, we have $\mu_R^1(\theta|\mathbf{P}^m,\mathbf{M}^m) \to \mu_R^1(\theta|\mathbf{P},\mathbf{M})$. By Lebesgue's dominated convergence theorem, we conclude that

$$
\int_N I_{R,\ell}^m(\theta) h^1(\theta) d\theta \to \int_N I_{R,\ell}(\theta) h^1(\theta) d\theta \quad \text{and} \quad \int_N I_{R,w}^m(\theta) h^1(\theta) d\theta \to \int_N I_{R,w}(\theta) h^1(\theta) d\theta.
$$

Thus, we have $\tilde{M}_R^{C,0,m} \to \tilde{M}_R^{C,0}$, and by analogous arguments, $\tilde{\mathbf{M}}^m \to \tilde{\mathbf{M}}$.

Therefore, we have proved that ϕ is a continuous mapping from $\Delta(X)^2 \times K^4$ into itself, and Schauder's fixed point theorem implies the existence of a fixed point $(\mathbf{P}^*, \mathbf{M}^*)$, i.e., a pair such that $\phi(\mathbf{P}^*, \mathbf{M}^*) = (\mathbf{P}^*, \mathbf{M}^*)$.

Existence of equilibrium: Given the fixed point of the mapping ϕ selected above, we define the assessment $\Psi = (\sigma, \kappa)$ so that the strategy profile $\sigma = (\lambda, \mu, \alpha, \xi, \nu)$ is such that all citizens use the optimal strategies derived above given $(\mathbf{P}^*, \mathbf{M}^*)$, and κ is derived from Bayes rule when possible. That is, for every type θ and party π with active group G, we have $\lambda_G^\pi(\theta) = \lambda_G^\pi(\theta|\mathbf{P}^*, \mathbf{M}^*)$

and $\mu_G^\pi(\theta) = \mu_G^\pi(\theta|\mathbf{P}^*, \mathbf{M}^*)$; each politician type θ from each party π accepts (y, m) if and only if

$$U_\theta^\pi(y|\mathbf{P}^*) + \gamma m + B^\pi(y|\mathbf{P}^*) \quad \geq \quad U_\theta^\pi(\xi_\theta^\pi(\mathbf{P}^*)|\mathbf{P}^*) + B^\pi(\xi_\theta^\pi(\mathbf{P}^*)|\mathbf{P}^*);$$

for each type θ and party π, the politician's default choice is $\xi_\theta^\pi = \xi_\theta^\pi(\mathbf{P}^*)$; and the win set is $W^\pi(\Psi) = W^\pi(\mathbf{P}^*)$. Note that $\mathcal{E}^\pi(\Psi) = \mathcal{E}^\pi(\mathbf{P}^*, \mathbf{M}^*)$, by construction. The belief system κ_θ is pinned down by Bayes rule unless the policy chosen by the incumbent is off the public path of play. Below, we describe the updating of voter beliefs off the path of play, and we specify voting strategies that generate the win set $W^\pi(\Psi)$.

Now, let $O^\pi(\mathbf{P}^*, \mathbf{M}^*)$ be the set of off-path policy choices, i.e., it consists of each policy x such that there is no type θ with $h^\pi(\theta) > 0$ such that $\lambda_{G(\theta)}^\pi(\theta) = x$. We partition into the set $O_w^\pi(\mathbf{P}^*, \mathbf{M}^*)$ of off-path choices that are winning and the set $O_\ell^\pi(\mathbf{P}^*, \mathbf{M}^*)$ of off-path choices that are losing. Following an off-path policy choice $x \in O_w^\pi(\mathbf{P}^*, \mathbf{M}^*)$ in the win set, we specify voter beliefs so that the incumbent is the median citizen type with probability one, and that the incumbent rejected the lobby offer, so that the politician is expected to choose the median policy x_m if re-elected. For losing off-path policies, we consider two cases.

In case there are politician types that are not lobbied to winning policies, i.e., $\mathcal{E}^\pi(\Psi) \subsetneq [\underline{\theta}, \overline{\theta}]$, we specify beliefs following an off-path policy choice $x \in O_\ell^\pi(\mathbf{P}^*, \mathbf{M}^*)$ so that voters place probability one on the incumbent being a type θ' such that $\theta' \notin \mathcal{E}^\pi(\Psi)$. We then specify that each type θ citizen votes to re-elect the incumbent if and only if the incumbent provides an expected discounted payoff that is at least as high as the challenger, i.e.,

$$\nu_\theta^\pi(x) \quad = \quad \begin{cases} 1 & \text{if } x \notin O(\mathbf{P}^*, \mathbf{M}^*) \text{ and } u_\theta(x) \geq (1-\delta)V_\theta^{C,\pi}(\mathbf{P}^*), \\ 1 & \text{if } x \in O_w(\mathbf{P}^*, \mathbf{M}^*) \text{ and } u_\theta(x_m) \geq (1-\delta)V_\theta^{C,\pi}(\mathbf{P}^*), \\ 1 & \text{if } x \in O_\ell(\mathbf{P}^*, \mathbf{M}^*) \text{ and } u_\theta(x(\theta')) \geq (1-\delta)V_\theta^{C,\pi}(\mathbf{P}^*), \\ 0 & \text{else.} \end{cases}$$

Thus, if $x \in W^\pi(\mathbf{P}^*)$ is winning, then regardless of whether the policy is on or off path, this specification indeed requires the voter to vote for a candidate that delivers the highest expected discounted utility, and since the median voter type is pivotal, the incumbent is re-elected. And if $x \notin W^\pi(\Psi)$, then again voters votes for the candidate delivering the highest payoff, and the incumbent is rejected in favor of the challenger.

However, it may be that all politician types are lobbied to winning policies, i.e., $\mathcal{E}^\pi(\Psi) = [\underline{\theta}, \overline{\theta}]$. In this case, all politician types are expected to choose either $\underline{w}^\pi(\mathbf{P}^*)$ or $\overline{w}^\pi(\mathbf{P}^*)$. We then specify beliefs $\kappa_\theta(x, \pi)$ so that the type $\theta \leq \theta_m$ voter places probability one on the incumbent being a type θ' that is conservative, i.e., $\theta_m \leq \theta'$, and so that the type $\theta > \theta_m$ voter places probability one on the incumbent being liberal, i.e., $\theta' < \theta_m$. We again specify that each type θ citizen votes to re-elect the incumbent if and only if the incumbent provides an expected discounted payoff that is at least

as high as the challenger, i.e., for $\theta \leq \theta_m$,

$$\nu_\theta^\pi(x) = \begin{cases} 1 & \text{if } x \notin O(\mathbf{P}^*, \mathbf{M}^*) \text{ and } u_\theta(x) \geq (1-\delta)V_\theta^{C,\pi}(\mathbf{P}^*), \\ 1 & \text{if } x \in O_w(\mathbf{P}^*, \mathbf{M}^*) \text{ and } u_\theta(x_m) \geq (1-\delta)V_\theta^{C,\pi}(\mathbf{P}^*), \\ 0 & \text{else,} \end{cases}$$

and similarly for $\theta > \theta_m$. Note that if x is off-path and does not belong to the win set, i.e., $x \in O_\ell(\mathbf{P}^*, \mathbf{M}^*)$, then the voter votes for the challenger, who offers an expected discounted payoff at least as high as the payoff from re-electing the incumbent, the latter being $\frac{u_\theta(\overline{w}^\pi(\mathbf{P}^*))}{1-\delta}$ for $\theta \leq \theta_m$. If $x \in W^\pi(\mathbf{P}^*)$, then because the median voter type is pivotal, the incumbent is re-elected. And if $x \notin W^\pi(\mathbf{P}^*)$, then the median voter is again pivotal—in fact, if x is off the public path of play, then every voter rejects the incumbent—and the challenger is elected.

To check that Ψ constitutes a simple lobbying equilibrium, it suffices to show that the induced continuation values in (8) are in fact the challenger continuation values determined by Ψ. Indeed, let $\mathbf{P}^* = (P_X^{C,0,*}, P_X^{C,1,*})$, and note that $P_X^{C,\pi,*}$ satisfies

$$P_X^{C,\pi,*} = Q_w^\pi(\cdot|\mathbf{P}^*, \mathbf{M}^*) + (1-\delta)Q_\ell^\pi(\cdot|\mathbf{P}^*, \mathbf{M}^*) + \delta Q_\ell^\pi(X|\mathbf{P}^*, \mathbf{M}^*)P_X^{C,1-\pi,*} \tag{18}$$

for $\pi \in \{0,1\}$. To confirm that $V_\theta^{C,\pi}(\mathbf{P}^*) = \frac{\mathbb{E}_{P_X^{C,\pi,*}}[u_\theta(x)]}{1-\delta}$, we integrate $\frac{u_\theta(x)}{1-\delta}$ with respect to $P_X^{C,\pi,*}$, which yields

$$\begin{aligned} \frac{\mathbb{E}_{P_X^{C,\pi,*}}[u_\theta(x)]}{1-\delta} &= \int_{\theta' \in \mathcal{E}^{1-\pi}(\Psi)} \frac{u_\theta(\lambda_{G(\theta')}^{1-\pi}(\theta'))}{1-\delta} h^{1-\pi}(\theta')d\theta' \\ &\quad + \int_{\theta' \notin \mathcal{E}^{1-\pi}(\Psi)} \left[u_\theta(\lambda_{G(\theta')}^{1-\pi}(\theta')) + \delta \frac{\mathbb{E}_{P_X^{C,1-\pi,*}}[u_\theta(x)]}{1-\delta} \right] h^{1-\pi}(\theta')d\theta', \end{aligned}$$

where we use a change of variables to integrate with respect to the density $h^{1-\pi}(\theta')$, in place of the distributions $Q_w^\pi(\cdot|\mathbf{P}^*, \mathbf{M}^*)$ and $Q_\ell^\pi(\cdot|\mathbf{P}^*, \mathbf{M}^*)$. Since $\mathcal{E}^\pi(\mathbf{P}^*, \mathbf{M}^*) = \mathcal{E}^\pi(\Psi)$, it follows that $V_\theta^{C,0}(\mathbf{P}^*)$ satisfies the recursive system in (2) that uniquely identifies the challenger continuation values determined by the assessment Ψ, and we conclude that $V_\theta^{C,\pi}(\Psi) = V_\theta^{C,\pi}(\mathbf{P}^*)$ for each $\pi \in \{0,1\}$, as required.

A.2 Partitional Form: Proof of Proposition 2

Let $\Psi = (\sigma, \mu)$ be any simple lobbying equilibrium, and let $P_X^{C,\pi}$ be the challenger continuation distributions given an incumbent from party $\pi = 0, 1$, and let $M_G^{C,\pi} = M_G^{C,\pi}(\Psi)$ be the continuation payment of group $G = L, R$ when an incumbent from party $\pi = 0, 1$ is removed. The existence proof in the preceding subsection shows that the induced win set is an interval $[\underline{w}^\pi, \overline{w}^\pi]$; that optimal default policies are determined by compromising cutoffs $\underline{c}^\pi$ and $\overline{c}^\pi$ such that a politician whose ideal point belongs to the interval $[\underline{c}^\pi, \overline{c}^\pi]$ chooses the winning policy closest to her ideal point; and that optimal lobby offers are determined by cutoff types $\underline{\theta}_e^\pi$ and $\overline{\theta}_e^\pi$ such that group R offers

a winning policy to any type $\theta \in [\theta_m, \overline{\theta}_e^\pi]$ and a losing policy to types $\theta > \overline{\theta}_e^\pi$, and symmetrically for group L. Letting $\overline{e}^\pi = x(\overline{\theta}_e^\pi)$ and $\underline{e}^\pi = x(\underline{\theta}_e^\pi)$, we see that the equilibrium is pinned down by 6-tuples $(\underline{c}^\pi, \underline{e}^\pi, \underline{w}^\pi, \overline{w}^\pi, \overline{e}^\pi, \overline{c}^\pi)$, $\pi \in \{0, 1\}$, satisfying (3). We conclude that the equilibrium has the partitional form

A.3 Partisanship and the Dynamic Median Voter Theorem: Proof of Propositions 3 and 4

We prove Proposition 3 in four steps and complete the proof of Proposition 4 in a fifth. Let Θ_L^π consist of types $\theta < \theta_m$ belonging to the support of h^π, and let Θ_R^π consist of types $\theta \geq \theta_m$ belonging to the support of h^π.

Step 1: We first show that when $\delta\beta$ is high, each lobby group G offers only winning policies, i.e., for all $\pi \in \{0, 1\}$ and all $\theta \in \Theta_G^\pi$, we have $\lambda_G^\pi(\theta) \in W^\pi(\Psi)$. Note that in a simple lobbying equilibrium, the median ideal point is always winning: $x_m \in W^\pi(\Psi)$. Thus, in the absence of a lobby offer, a politician's optimal payoff from compromise can be no worse than the payoff from choosing the median ideal point, and so the net benefit of compromise for a type θ politician from party π is at least equal to

$$\frac{u_\theta(x_m) + \beta}{1 - \delta} - [u_\theta(x(\theta)) + \beta + \delta V_\theta^{C,\pi}(\Psi)]$$
$$\geq \left[\frac{u_\theta(x_m)}{1 - \delta} + \frac{\delta\beta}{2(1 - \delta)} - \frac{u_\theta(x(\theta))}{1 - \delta} \right] + \frac{\delta\beta}{2(1 - \delta)},$$

where we use the fact that the continuation value of a challenger must be less than the politician's ideal payoff. Note that we can choose $\delta\beta > 2(u_\theta(x(\theta)) - u_\theta(x_m))$ for all types θ to make the first term in brackets on the right-hand side positive and the second term arbitrarily large. In particular, the default policy choice of every politician type is winning when office incentives are sufficiently high.

Now consider the problem of the active lobby group $G = G(\theta)$ when office incentives are high, and suppose toward a contradiction that the lobby group's policy offer $\lambda_G^\pi(\theta)$ is not winning. Then the participation constraint of the politician requires that the group compensate the politician for losing. Letting ξ_θ^π denote the default policy of the politician, the payment $\mu_G^\pi(\theta)$ to the politician is at least equal to

$$\frac{1}{\gamma} \left[\frac{u_\theta(\xi_\theta^\pi) + \beta}{1 - \delta} - [u_\theta(x(\theta)) + \beta + \delta V_\theta^{C,\pi}(\Psi)] \right]$$
$$\geq \frac{1}{\gamma} \left[\frac{u_\theta(x_m)}{1 - \delta} + \frac{\delta\beta}{2(1 - \delta)} - \frac{u_\theta(x(\theta))}{1 - \delta} \right] + \frac{\delta\beta}{2\gamma(1 - \delta)},$$

where we use the fact that the default policy is winning and $u_\theta(\xi_\theta^\pi) \geq u_\theta(x_m)$. Of course, the policy offer $\lambda_G^\pi(\theta)$ cannot be better for the lobby group than its ideal point, x_G, and the continuation value of a challenger must be less than the group's ideal payoff. Thus, the equilibrium payoff to

the lobby group from offering $(\lambda_G^\pi(\theta), \mu_G^\pi(\theta))$ is less than or equal to

$$\text{lobby}_{equil} = \frac{u_G(x_G)}{1-\delta} - \frac{\delta\beta}{2\gamma(1-\delta)} - \frac{1}{\gamma}\left[\frac{u_\theta(x_m)}{1-\delta} + \frac{\delta\beta}{2(1-\delta)} - \frac{u_\theta(x(\theta))}{1-\delta}\right].$$

But the lobby group could instead offer the default policy ξ_θ^π, along with a payment of zero, effectively choosing not to lobby. Since the default policy is closer to x_G than the median policy, the lobby group's payoff from this deviation is no less than

$$\text{lobby}_{dev} = \frac{u_G(x_m)}{1-\delta}.$$

Comparing these payoffs, we find that

$$\text{lobby}_{equil} - \text{lobby}_{dev} = \left[\frac{u_G(x_G)}{1-\delta} - \frac{\delta\beta}{2\gamma(1-\delta)} - \frac{u_G(x_m)}{1-\delta}\right] + \frac{1}{\gamma}\left[\frac{u_\theta(x(\theta))}{1-\delta} - \frac{u_\theta(x_m)}{1-\delta} - \frac{\delta\beta}{2(1-\delta)}\right].$$

Choosing $\delta\beta$ to also satisfy $\delta\beta > 2\gamma(u_G(x_G) - u_G(x_m))$, we see that $\text{lobby}_{equil} - \text{lobby}_{dev} < 0$, but this implies that the lobby group can profitably deviate by refraining from lobbying, avoiding the necessity of compensating the politician for lost office benefit. This is impossible in equilibrium, and we conclude that when office incentives are sufficiently high, lobby groups always offer winning policies to incumbents from party π. Importantly for the proof of Proposition 4, when $\delta\beta$ is chosen in the above manner for a given value $\gamma > 0$, these conclusions carry over to all smaller $\gamma' \in (0, 1)$. This completes the first step.

Step 2: Next, we argue that the challenger continuation value for the median voter is independent of the incumbent's party. Recall that the win set consists of the policies x that give the median voter a payoff at least equal to her continuation value of a challenger, i.e., $\frac{u_m(x)}{1-\delta} \geq V_m^{C,\pi}(\Psi)$. We have shown that the challenger is always offered a winning policy and remains in office thereafter, regardless of her type, so that the median voter's continuation value of a challenger takes the simple form

$$V_m^{C,\pi}(\Psi) = \frac{1}{1-\delta}\int_N u_m(\lambda_{G(\theta)}^{1-\pi}(\theta))h^{1-\pi}(\theta)d\theta.$$

By Step 1, for each G and each $\theta \in \Theta_G^{1-\pi}$, we have $\lambda_G^{1-\pi}(\theta) \in W^{1-\pi}(\Psi)$. Then for all θ in the support of $h^{1-\pi}$, we have $u_m(\lambda_{G(\theta)}^{1-\pi}(\theta)) \geq (1-\delta)V_m^{C,1-\pi}(\Psi)$, and we conclude that $V_m^{C,\pi}(\Psi) \geq V_m^{C,1-\pi}(\Psi)$. A symmetric argument for the opposite inequality then yields $V_m^{C,\pi}(\Psi) = V_m^{C,1-\pi}(\Psi)$, and we henceforth drop the incumbent's party from the superscript, writing simply $V_m^C(\Psi)$ for the median citizen's continuation value of a challenger.

Step 3: Next, we show that only the endpoints of the win set are offered in equilibrium, that median indifference holds, and that the win set is independent of the incumbent's party. Note that $u_m(\lambda_{G(\theta)}^{1-\pi}(\theta)) \geq V_m^C(\Psi)$ holds for all θ in the support of $h^{1-\pi}$ and

$$V_m^C(\Psi) = \frac{1}{1-\delta}\int_N u_m(\lambda_{G(\theta)}^{1-\pi}(\theta))h^{1-\pi}(\theta)d\theta,$$

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so continuity of $\lambda_{G(\theta)}^{1-\pi}(\theta)$ at all $\theta \neq \theta_m$ implies that the equality

$$V_m^C(\Psi) \;=\; \frac{u_m(\lambda_{G(\theta)}^{1-\pi}(\theta))}{1-\delta}$$

holds for all $\theta \neq \theta_m$ in the support of $h^{1-\pi}$. In particular, using (4), we have

$$\frac{u_m(\lambda_L^{1-\pi}(\theta))}{1-\delta} \;=\; V_m^C(\Psi) \;=\; \frac{u_m(\underline{w}^{1-\pi})}{1-\delta} \qquad (19)$$

for all $\theta \in \Theta_L^{1-\pi}$, and thus $\lambda_L^{1-\pi}(\theta) = \underline{w}^{1-\pi}$ for all such θ. For all $\theta \in \Theta_R^{1-\pi}$, we have

$$\frac{u_m(\lambda_R^{1-\pi}(\theta))}{1-\delta} \;=\; V_m^C(\Psi) \;\leq\; \frac{u_m(\overline{w}^{1-\pi})}{1-\delta}.$$

By (4), the latter inequality can hold strictly only if $\overline{w}^\pi = 1$, but $x_m \leq \lambda_R^{1-\pi}(\theta) \leq 1$, so we cannot have $u_m(\lambda_R^{1-\pi}(\theta)) < u_m(1)$. We conclude that

$$\frac{u_m(\lambda_R^{1-\pi}(\theta))}{1-\delta} \;=\; V_m^C(\Psi) \;=\; \frac{u_m(\overline{w}^{1-\pi})}{1-\delta}, \qquad (20)$$

and thus for all $\theta \in \Theta_R^{1-\pi}$, we have $\lambda_R^{1-\pi}(\theta) = \overline{w}^{1-\pi}$.

An analogous argument for party π shows that for all $\theta \in \Theta_L^\pi$, we have

$$\frac{u_m(\lambda_L^\pi(\theta))}{1-\delta} \;=\; V_m^C(\Psi) \;=\; \frac{u_m(\underline{w}^\pi)}{1-\delta}, \qquad (21)$$

so that $\lambda_L^\pi(\theta) = \underline{w}^\pi$, and for all $\theta \in \Theta_R^\pi$, we have

$$\frac{u_m(\lambda_R^\pi(\theta))}{1-\delta} \;=\; V_m^C(\Psi) \;=\; \frac{u_m(\overline{w}^\pi)}{1-\delta}, \qquad (22)$$

so that $\lambda_R^\pi(\theta) = \overline{w}^\pi$ for all such θ. Thus, for all θ in the support of h^π, $\pi \in \{0,1\}$, lobby groups offer only the endpoints of the win set. Finally, the assumption that the support of h^π contains an open set around θ_m implies that $\Theta_G^\pi \neq \emptyset$ for each group and party, so that (19)–(22) do not hold vacuously. We conclude that $u_m(\underline{w}^\pi) = u_m(\overline{w}^\pi)$ and $u_m(\underline{w}^{1-\pi}) = u_m(\overline{w}^{1-\pi})$, so that median indifference holds, and that in fact $\underline{w}^\pi = \underline{w}^{1-\pi}$ and $\overline{w}^\pi = \overline{w}^{1-\pi}$, so that the win set is independent of π. This establishes that when $\delta\beta$ is sufficiently large, every simple lobbying equilibrium is strongly partisan.

Step 4: To complete the proof of Proposition 3, assume without loss of generality that the inequality $u_m(x_L(\theta_m)) \geq u_m(x_R(\theta_m))$ holds, i.e., the policy that maximizes joint surplus with group R is at least as good for the median voter as the policy that maximizes joint surplus with L.[28] Let $\underline{w}$ and $\overline{w}$ satisfy $\underline{w} < x_m < \overline{w}$, $x_L(\theta_m) \leq \underline{w}$, $\overline{w} \leq x_R(\theta_m)$, and $u_m(\underline{w}) = u_m(\overline{w})$. Choosing $\delta\beta$

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[28]Since we have established median indifference, $u_m(\underline{w}) = u_m(\overline{w})$, our assumption that $u_m(1) \geq u_m(0)$ does not affect the analysis, so this hypothesis is indeed without loss of generality for this argument.

as above, we specify σ so that the default policy of each type θ of politician is the closest policy in the win set to the ideal point $x(\theta)$. Specify lobby offers so that for all $\theta < \theta_m$, group L offers $\underline{w}$ and compensates the politician for choosing $\underline{w}$ rather than ξ_θ; and for all $\theta \geq \theta_m$, group R offers $\overline{w}$ and compensates the politician. And we specify the win set as $W = [\underline{w}, \overline{w}]$. Beliefs μ are specified as in the proof of existence. This specification satisfies the conditions for simple lobbying equilibrium, and in particular, the optimal offer for each lobby group is to pull every politician type to the endpoint of the win set closest to the ideal point x_G of the group, and the median voter is indifferent between re-electing an incumbent who chooses $\underline{w}$ or $\overline{w}$ and replacing the incumbent with a challenger. The most polarized equilibrium is given by the choice of $\underline{w}$ and $\overline{w}$ such that $\underline{w} < x_m < \overline{w}$ and $u_m(\underline{w}) = u_m(\overline{w})$ and such that one (or both) of the inequalities $x_L(\theta_m) \leq \underline{w}$ and $\overline{w} \leq x_R(\theta_m)$ bind. This completes the proof of Proposition 3.

Step 5: Proposition 4 follows by the fact that the equilibrium conditions are maintained as $\gamma \to 0$, and because the surplus-maximizing policies $x_L(\theta_m)$ and $x_R(\theta_m)$ converge to the median policy as $\gamma \to 0$. Thus, the inequalities in (5) are satisfied only at policies approaching the median, which implies $\underline{w} \to x_m$ and $\overline{w} \to x_m$, as required.

A.4 Non-monotonic lobby transfers: Proof of Proposition 5

Given any $\delta\beta$, it is clear that lobbying expenditures go to zero as $\gamma \to \infty$, for the equilibrium transfer $\mu_G^\pi(\theta) = m$ satisfies

$$\gamma m = U_\theta^\pi(\xi_\theta^\pi(P)|\Psi) + B^\pi(\xi_\theta^\pi(P)|\Psi) - U_\theta^\pi(y|\Psi) - B^\pi(y|\Psi),$$

and thus it is bounded above by

$$\frac{1}{\gamma}\left[\max_{\theta,x,x'} \frac{u_\theta(x) - u_\theta(x') + \beta}{1 - \delta}\right],$$

which goes to zero as $\gamma \to \infty$. Now, let $\gamma \to 0$. By Proposition 3, we can set $\delta\beta$ large enough that every simple lobbying equilibrium is strongly partisan, with win set $[\underline{w}, \overline{w}]$ such that $u_m(\underline{w}) = u_m(\overline{w})$. Moreover, we can set $\delta\beta$ large enough that for all θ, the default policy is winning, i.e, $\xi_\theta \in [\underline{w}, \overline{w}]$. Indeed, this will hold regardless of the win set if for all θ, we have

$$\frac{u_\theta(x_m) + \beta}{1 - \delta} > \frac{u_\theta(x(\theta))}{1 - \delta} + \beta,$$

which holds if

$$\delta\beta > \max_\theta u_\theta(x(\theta)) - u_\theta(x_m).$$

Consider any π and any $\theta \neq \theta_m$ in the support of h^π. Proposition 4 implies that for small enough γ, we have $x(\theta) \notin [\underline{w}, \overline{w}]$. Then, since the equilibrium is strongly partisan, the type θ politician is lobbied to the endpoint of the win set closest to their ideal policy, i.e., $\lambda_{G(\theta)}^\pi(\theta) = \xi_\theta$, in which case

$\mu_{G(\theta)}^{\pi} = 0$. Finally, consider the median politician type. Since equilibria are strongly partisan, we have $\lambda_R^{\pi}(\theta_m) = \overline{w}$. Because the lobby group can offer the median politician their default policy x_m at zero cost, the group's payoff from the offer $(\lambda_R^{\pi}(\theta_m), \mu_R^{\pi}(\theta_m))$ must satisfy

$$u_{\overline{\theta}}(\overline{w}) - u_{\overline{\theta}}(x_m) \;\; \geq \;\; \mu_R^{\pi}(\theta_m).$$

By Proposition 4, we have $u_{\overline{\theta}}(\overline{w}) \to u_{\overline{\theta}}(x_m)$, and we conclude that $\mu_R^{\pi}(\theta_m) \to 0$.

A.5 Extreme Strongly Partisan Equilibria: Proofs of Propositions 6 and 7

Let γ become large, let δ and β vary arbitrarily with γ subject to $\delta \geq \epsilon > 0$, and consider any selection of simple lobbying equilibria. We go to a subsequence such that the win set given an incumbent from each party converges: for $\pi \in \{0, 1\}$, we have $\underline{w}^{\pi} \to \underline{\tilde{w}}^{\pi}$ and $\overline{w}^{\pi} \to \tilde{w}^{\pi}$. To show that equilibria become strongly partisan, we consider a particular party π and begin by observing that by (4), there are three cases: (i) $\underline{w}^{\pi} = 0$ and $\tilde{w}^{\pi} = 1$, or (ii) $\underline{w}^{\pi} > 0$ and $\tilde{w}^{\pi} < 1$, or (iii) $\underline{w}^{\pi} > 0$ and $\tilde{w}^{\pi} = 1$. Note that for every politician type $\theta \geq \theta_m$, the maximizer of joint surplus with lobby group R goes to one as γ becomes large, and likewise for types $\theta < \theta_m$ and group L, i.e., $x_R(\theta) \to 1$ and $x_L(\theta) \to 0$. Furthermore, in case (iii), we have

$$u_m(0) \;\; < \;\; u_m(\underline{w}^{\pi}) \;\; \leq \;\; u_m(\tilde{w}^{\pi}) \;\; = \;\; u_m(1),$$

so that the median voter is not indifferent between the lobby groups. Thus, the proof of Proposition 6 considers only cases (i) and (ii), while Proposition 7 is proved following case (iii).

Case (i): $\underline{w}^{\pi} = 0$ and $\tilde{w}^{\pi} = 1$. Lobby offers then converge to the extremes of the policy space: $\lambda_L^{\pi}(\theta_m) \to 0$ and $\lambda_R^{\pi}(\theta_m) \to 1$. From (2), we have

$$
\begin{aligned}
u_m(\underline{w}^{\pi}) \;\; = \;\; (1 - \delta)V_m^{C,\pi}(\Psi) \;\; \geq \;\; & \int_{\theta' \in \mathcal{E}^{1-\pi}(\Psi)} u_m(\underline{w}^{1-\pi})h^{1-\pi}(\theta')d\theta' \\
& + \int_{\theta' \notin \mathcal{E}^{1-\pi}(\Psi)} \left[(1-\delta)u_m(0) + \delta(1-\delta)V_m^{C,1-\pi}(\Psi)\right]h^{1-\pi}(\theta')d\theta' \\
= \;\; & \int_{\theta' \in \mathcal{E}^{1-\pi}(\Psi)} (1-\delta)V_m^{C,1-\pi}(\Psi)h^{1-\pi}(\theta')d\theta' \\
& + \int_{\theta' \notin \mathcal{E}^{1-\pi}(\Psi)} \left[(1-\delta)u_m(0) + \delta(1-\delta)V_m^{C,1-\pi}(\Psi)\right]h^{1-\pi}(\theta')d\theta',
\end{aligned}
$$

where we use $V_m^{C,1-\pi}(\Psi) = \frac{u_m(\underline{w}^{1-\pi})}{1-\delta}$ in the equality. Since $u_m(\underline{w}^{\pi}) \to u_m(0)$, the above inequality yields

$$u_m(0) \;\; = \;\; \lim_{\gamma \to \infty} u_m(\underline{w}^{1-\pi}) \;\; = \;\; \lim_{\gamma \to \infty} (1-\delta)V_m^{C,1-\pi}(\Psi),$$

which implies that $\underline{w}^{1-\pi} = 0$ and $\tilde{w}^{1-\pi} = 1$. Thus, the equalities defining case (i) hold also for

party $1 - \pi$, and by an analogous argument, we find that

$$u_m(0) = \lim_{\gamma \to \infty}(1 - \delta)V_m^{C,\pi}(\Psi).$$

Furthermore, this implies that lobby offers to incumbents from party $1 - \pi$ converge to the extreme policies: $\lambda_L^{1-\pi}(\theta) \to 0$ for all $\theta < \theta_m$, and $\lambda_R^{1-\pi}(\theta) \to 1$ for all $\theta \geq \theta_m$.

Now, consider the choice of default policy by an extreme politician type $\theta \in \{\underline{\theta}, \overline{\theta}\}$. By the above arguments, and the assumption that the support of $h^{1-\pi}$ contains an open set around θ_m, we have shown that

$$\lim_{\gamma \to \infty}(1 - \delta)V_\theta^{C,\pi}(\Psi) = H^{1-\pi}(\theta_m)u_\theta(0) + (1 - H^{1-\pi}(\theta_m))u_\theta(1) < u_\theta(x(\theta)), \qquad (23)$$

reflecting the fact that if an incumbent from party π is replaced by a challenger from party $1 - \pi$, then her replacement will be lobbied to policies close to zero by lobby group L or to policies close to one by lobby group R. Then we have for type $\overline{\theta}$,

$$\liminf_{\gamma \to \infty} u_{\overline{\theta}}(\overline{w}^\pi) + \beta \geq \liminf_{\gamma \to \infty} u_{\overline{\theta}}(x(\overline{\theta})) + \beta$$
$$> \lim_{\gamma \to \infty}\left[(1 - \delta)(u_{\overline{\theta}}(x(\overline{\theta})) + \beta) + \delta(1 - \delta)V_{\overline{\theta}}^{C,\pi}(\Psi)\right],$$

where the second inequality uses (23) and, for strictness, $\delta \geq \epsilon$. Then inequality (9) holds strictly for γ sufficiently large (substituting the equilibrium challenger distributions for $\mathbf{P}$). Using an analogous argument for type $\underline{\theta}$, we conclude that when γ is sufficiently large, all politician types are either winners or compromisers, i.e., $\underline{c}^\pi = 0$ and $\overline{c}^\pi = 1$, and by a symmetric argument, $\underline{c}^{1-\pi} = 0$ and $\overline{c}^{1-\pi} = 1$.

Referring to the analysis of group R's optimal lobby offer in the proof of Proposition 1 in Subsection A.1, recall from (15) that given $\theta \geq \theta_m$, $\Phi_R^\pi(\theta|\mathbf{P}, \mathbf{M})$ is the difference between lobby group R's payoff from the optimal winning policy and the group's payoff from the optimal losing policy. Substituting in the equilibrium values of $(\mathbf{P}, \mathbf{M})$, and normalizing by $(1 - \delta)$, this difference is at least

$$u_{\overline{\theta}}(y_w^\pi(\theta)) + \frac{1}{\gamma}u_\theta(y_w^\pi(\theta)) - (1 - \delta)\left[u_{\overline{\theta}}(y_\ell^\pi(\theta)) + \frac{1}{\gamma}u_\theta(y_\ell^\pi(\theta))\right] \qquad (24)$$
$$-\delta(1 - \delta)\left[V_{\overline{\theta}}^{C,\pi}(\Psi) + \frac{1}{\gamma}V_\theta^{C,\pi}(\Psi)\right].$$

Since $\lim y_w^\pi(\theta) = \lim y_\ell^\pi(\theta) = 1 = x(\overline{\theta})$ as $\gamma \to \infty$, and using (23) and $\delta \geq \epsilon$, the limit infimum of the above quantity is greater than or equal to

$$\epsilon\left[u_{\overline{\theta}}(1) - (H^{1-\pi}(\theta_m)u_{\overline{\theta}}(0) + (1 - H^{1-\pi}(\theta_m))u_{\overline{\theta}}(1))\right],$$

which is strictly positive. Therefore, for γ sufficiently high, lobby group R offers winning policies

to all types $\theta \geq \theta_m$ in the support of h^π, and similarly, lobby group L also offers winning policies to all types $\theta < \theta_m$ in the support of h^π. Moreover, the same analysis shows that the lobby groups offer only winning policies to incumbents from party $1 - \pi$.

Now we can invoke the argument in the proof of Proposition 3 in Subsection A.3. The first step of that argument establishes that the lobby groups offer only winning policies, and the remaining steps leverage that insight to show that simple lobbying equilibria are strongly partisan. The arguments from Steps 2 and 3 apply here, and we conclude that every simple lobbying equilibrium is strongly partisan.

Case (ii): $\underline{w}^\pi > 0$ and $\tilde{w}^\pi < 1$. For γ large, if group R offers a winning policy, then the win set constraint binds, and we have $\lambda_R^\pi(\theta) = \overline{w}^\pi$, and losing offers coincide with $x_R(\theta)$ and converge uniformly to one. Likewise, if L offers a winning policy, then it is $\underline{w}^\pi$, and all losing offers, $x_L(\theta)$, converge to zero. Moreover, since $\overline{w}^\pi < 1$ for γ large, (4) implies $u_m(\underline{w}^\pi) = u_m(\overline{w}^\pi)$ in equilibrium. Then for γ large, the median voter's normalized continuation value of a challenger given an incumbent from party $1 - \pi$ satisfies

$$(1-\delta)V_m^{C,1-\pi}(\Psi) = \int_{\theta' \in \mathcal{E}^\pi(\Psi)} u_m(\overline{w}^\pi) h^\pi(\theta') d\theta' \tag{25}$$
$$+ \int_{\theta' \notin \mathcal{E}^\pi(\Psi)} [(1-\delta)u_m(x_{G(\theta')}(\theta')) + \delta(1-\delta)V_m^{C,\pi}(\Psi)] h^\pi(\theta') d\theta'$$
$$\geq \int_{\theta' \in \mathcal{E}^\pi(\Psi)} u_m(\overline{w}^\pi) h^\pi(\theta') d\theta'$$
$$+ \int_{\theta' \notin \mathcal{E}^\pi(\Psi)} \left[(1-\delta)u_m(1) + \delta u_m(\overline{w}^\pi) \right] h^\pi(\theta') d\theta',$$

where we use (2), translated to party $1 - \pi$, and $V_m^{C,\pi}(\Psi) = \frac{u_m(\overline{w}^\pi)}{1-\delta}$. Going to a subsequence if necessary, we can assume that $\int_{\theta' \in \mathcal{E}^\pi(\Psi)} h^\pi(\theta') d\theta'$ converges to some $\alpha \in [0,1]$. Then taking limits as $\gamma \to \infty$, we have

$$\lim_{\gamma \to \infty} (1-\delta)V_m^{C,1-\pi}(\Psi) \geq \alpha\big(u_m(\tilde{w}^\pi)\big) + (1-\alpha)\big((1-\epsilon)u_m(1) + \epsilon u_m(\tilde{w}^\pi)\big) > u_m(1),$$

where we use $\delta \geq \epsilon > 0$. At the same time, we have

$$\lim_{\gamma \to \infty} (1-\delta)V_m^{C,1-\pi}(\Psi) = \lim_{\gamma \to \infty} u_m(\overline{w}^{1-\pi}) = u_m(\tilde{w}^{1-\pi}).$$

We conclude that $u_m(\tilde{w}^{1-\pi}) > u_m(1)$, and thus $\tilde{w}^{1-\pi} < 1$. By (4), we then have $u_m(\underline{w}^{1-\pi}) = u_m(\overline{w}^{1-\pi})$ for large enough γ, and taking limits, this implies

$$u_m(\underline{w}^{1-\pi}) = u_m(\tilde{w}^{1-\pi}) > u_m(1) = u_m(0).$$

Finally, this yields $\underline{w}^{1-\pi} > 0$, and it follows that the inequalities defining case (ii) also hold for party $1 - \pi$.

Modifying the inequalities in (25), observe that when γ is sufficiently large, we have

$$
\begin{aligned}
V_m^{C,1-\pi}(\Psi) &= \int_{\theta' \in \mathcal{E}^\pi(\Psi)} \frac{u_m(\overline{w}^\pi)}{1-\delta} h^\pi(\theta') d\theta' \\
&\quad + \int_{\theta' \notin \mathcal{E}^\pi(\Psi)} [u_m(x_{G(\theta')}(\theta')) + \delta V_m^{C,\pi}(\Psi)] h^\pi(\theta') d\theta' \\
&\leq \int_{\theta' \in \mathcal{E}^\pi(\Psi)} \frac{u_m(\overline{w}^\pi)}{1-\delta} h^\pi(\theta') d\theta' \\
&\quad + \int_{\theta' \notin \mathcal{E}^\pi(\Psi)} [u_m(\overline{w}^\pi) + \delta V_m^{C,\pi}(\Psi)] h^\pi(\theta') d\theta' \\
&= V_m^{C,\pi}(\Psi),
\end{aligned}
$$

and therefore $V_m^{C,1-\pi}(\Psi) \leq V_m^{C,\pi}(\Psi)$. Since we have shown the inequalities for case (ii) hold for party $1 - \pi$, a symmetric argument delivers the opposite inequality, $V_m^{C,\pi}(\Psi) \geq V_m^{C,1-\pi}(\Psi)$, and we conclude that, in fact, the continuation value of a challenger is independent of party, i.e., $V_m^{C,\pi}(\Psi) = V_m^{C,1-\pi}(\Psi)$. Thus, we have

$$
u_m(\overline{w}^\pi) = u_m(\underline{w}^\pi) = u_m(\underline{w}^{1-\pi}) = u_m(\overline{w}^{1-\pi})
$$

for γ large, and it follows that the win set is independent of the incumbent's party; for the remainder of this case, we write it as $[\underline{w}, \overline{w}]$. Using the first equality in (25), we then have

$$
\begin{aligned}
\frac{u_m(\overline{w})}{1-\delta} &= \int_{\theta' \in \mathcal{E}^\pi(\Psi)} \frac{u_m(\overline{w})}{1-\delta} h^\pi(\theta') d\theta' \\
&\quad + \int_{\theta' \notin \mathcal{E}^\pi(\Psi)} \left[u_m(x_{G(\theta')}(\theta')) + \delta \frac{u_m(\overline{w})}{1-\delta} \right] h^\pi(\theta') d\theta'.
\end{aligned}
$$

Note that for all $\theta' \notin \mathcal{E}^\pi(\Psi)$, we have $u_m(\overline{w}^\pi) > u_m(x_{G(\theta')}(\theta'))$, and so the above equation implies that for all θ in the support of h^π, the active group lobbies the type θ politician to $\overline{w}$ or $\underline{w}$. A symmetric argument holds for party $1 - \pi$, and therefore the equilibrium is strongly partisan.

We have established that in cases (i) and (ii), if γ is sufficiently high, then all equilibria are strongly partisan. To complete the proof of Proposition 6, we must show that the most polarized equilibria become extremist. Again, we let γ become large, and we let δ and β vary arbitrarily with γ subject to $\delta \geq \epsilon > 0$. For arbitrary $\eta > 0$, we will demonstrate that for sufficiently high γ, for all $\delta \geq \epsilon > 0$, and for all β, there is a strongly partisan equilibrium with win set $[\underline{w}, \overline{w}]$ satisfying $\underline{w} \in (0, \eta)$ and $\overline{w} \in (1 - \eta, 1)$. We will specify Ψ so that: (i) the default policy of each type θ of politician is the policy in the win set closest to the ideal point $x(\theta)$, (ii) for all $\theta \leq \theta_m$, group L offers $\underline{w}$ and compensates the politician for choosing $\underline{w}$ rather than ξ_θ; and for all $\theta \geq \theta_m$, group R offers $\overline{w}$ and compensates the politician, (iii) the median voter is indifferent between the endpoints of the win set, so that $u_m(\underline{w}) = u_m(\overline{w})$, and (iv) citizen beliefs and votes are specified as in the proof of existence. By construction, the median voter's continuation value of a challenger is $\frac{u_m(\underline{w})}{1-\delta} = \frac{u_m(\overline{w})}{1-\delta}$, so it remains to be shown that $\underline{w}$ and $\overline{w}$ can be chosen sufficiently close to zero

and one, respectively, to satisfy the optimality conditions for default policies and lobby offers.

First, we must show that if the endpoints of the win set are sufficiently extreme, then the default policy choice of every politician type θ is compromising, i.e., $\overline{c}^{\pi} = 1$ and $\underline{c}^{\pi} = 0$ for each party. It suffices to show that the extreme politician types, $\overline{\theta}$ and $\underline{\theta}$, prefer to compromise. By (9), this holds for type $\overline{\theta}$ if

$$u_{\overline{\theta}}(\overline{w}) \;\; > \;\; (1-\delta)u_{\overline{\theta}}(1) + \delta(1-\delta)V_{\overline{\theta}}^{C,\pi}(\Psi),$$

and this in turn holds if

$$u_{\overline{\theta}}(\overline{w}) \;\; > \;\; (1-\epsilon)u_{\overline{\theta}}(1) + \epsilon(1-\delta)V_{\overline{\theta}}^{C,\pi}(\Psi), \tag{26}$$

where we use $\delta \geq \epsilon$. By the above construction, the normalized continuation value of a challenger is the expected payoff from a lottery with support $\underline{w}$ and $\overline{w}$, namely,

$$(1-\delta)V_{\overline{\theta}}^{C,\pi}(\Psi) \;\; = \;\; H^{1-\pi}(\theta_m)u_{\overline{\theta}}(\underline{w}) + (1 - H^{1-\pi}(\theta_m))u_{\overline{\theta}}(\overline{w}).$$

It follows that $u_{\overline{\theta}}(\overline{w}) > (1-\delta)V_{\overline{\theta}}^{C,\pi}(\Psi)$, so that the inequality (26) holds as long as $\overline{w}$ is close enough to one. Similarly, the type $\underline{\theta}$ politician prefers to compromise if $\underline{w}$ is close enough to zero, as required. Thus, we can choose $\underline{w}$ and $\overline{w}$ with $\underline{w} \in (0, \eta)$, $\overline{w} \in (1 - \eta, 1)$, and $u_m(\underline{w}) = u_m(\overline{w})$ to satisfy (26) along with the corresponding inequality for $\underline{\theta}$. Note that this incentive to compromise holds regardless of the magnitude of γ, and that the normalized continuation value $(1-\delta)V_{\overline{\theta}}^{C,\pi}(\Psi)$ is determined by the win set only; in particular, it is independent of γ, δ, and β.

Next, we must show that for the choice of win set above, if γ is sufficiently large, then the optimal lobby offers are indeed the endpoints of the win set. Of course, we have $x_L(\theta_m) \to 0$ and $x_R(\theta_m) \to 1$ as $\gamma \to \infty$. Thus, we can choose γ such that for all $\theta \leq \theta_m$, we have $x_L(\theta) \in [0, \underline{w})$; and such that for all $\theta \geq \theta_m$, we have $x_R(\theta) \in (\overline{w}, 1]$. In turn, this implies that if optimal lobby offers are winning, then the lobby groups offer the endpoints of the win set. Recall that lobby group R offers a winning policy if (24) is positive. Taking limits as in case (i), and using the fact that the normalized continuation value $(1-\delta)V_{\overline{\theta}}^{C,\pi}(\Psi)$ is independent of γ, δ, and β, it follows that the optimal offer for the group is indeed winning, and a symmetric argument holds for lobby group L. We conclude that given an interval with endpoints close to the extreme policies (and such that the median voter is indifferent between the endpoints), if the effectiveness of money is sufficiently large, then there is a strongly partisan simple lobbying equilibrium with win set equal to that interval. Since the most polarized equilibrium is at least as extreme, this completes the proof of Proposition 6.

Case (iii): $\underline{w}^{\pi} > 0$ and $\tilde{w}^{\pi} = 1$. To prove Proposition 7, we must argue that in this case, every sequence of non-strongly partisan simple lobbying equilibria become weakly extremist. To this end, note that

$$\lim_{\gamma \to \infty} (1-\delta)V_m^{C,\pi}(\Psi) \;\; = \;\; \lim_{\gamma \to \infty} u_m(\underline{w}^{\pi}) \;\; \leq \;\; \lim_{\gamma \to \infty} u_m(\overline{w}^{\pi}) \;\; = \;\; u_m(1).$$

Furthermore, we have $\underline{w}^{1-\pi} > 0$ and $\tilde{w}^{1-\pi} = 1$, for otherwise the arguments for cases (i) and

(ii) (reversing the roles of π and $1 - \pi$) would imply that the equilibria are strongly partisan for sufficiently large γ. And then the inequalities above, stated with respect to party $1 - \pi$, deliver the inequality $\lim_{\gamma \to \infty} (1 - \delta) V_m^{C, 1 - \pi}(\Psi) \leq u_m(1)$, as required.

A.6 Limits of Strongly Partisan Equilibria: Proof of Proposition 8

Let γ become large, and consider any selection of simple lobbying equilibria with convergent win sets, i.e., for each party π, we have $\underline{w}^\pi \to \underline{w}$ and $\overline{w}^\pi \to \tilde{w}$. By Proposition 6, equilibria are strongly partisan for γ sufficiently high. We claim that in the dichotomous model, it is a simple lobbying equilibrium for the type $\underline{\theta}$ politician to choose $\underline{w}$ and the type $\overline{\theta}$ politician to choose $\tilde{w}$, with win set $[\underline{w}, \tilde{w}]$, and with citizen beliefs and votes following off-path policies specified as in the proof of existence. The challenger continuation values determined by this specification are given by

$$\tilde{V}_\theta^C = \frac{H^{1-\pi}(\theta_m) u_\theta(\underline{w}) + (1 - H^{1-\pi}(\theta_m)) u_\theta(\tilde{w})}{1 - \delta}.$$

Thus, we have

$$V_\theta^{C,\pi}(\Psi) = \frac{H^{1-\pi}(\theta_m) u_\theta(\underline{w}) + (1 - H^{1-\pi}(\theta_m)) u_\theta(\overline{w})}{1 - \delta} \quad \to \quad \tilde{V}_\theta^C$$

for each party π. In particular, since $u_m(\underline{w}^\pi) = (1 - \delta) V_m^{C,\pi}(\Psi) = u_m(\overline{w}^\pi)$ in equilibrium, it follows that

$$u_m(\underline{w}) = (1 - \delta) \tilde{V}_m^C = u_m(\tilde{w}),$$

so that the median voter is indifferent between the endpoints $\underline{w}$ and $\tilde{w}$ and a challenger in the dichotomous model. In particular, the equilibrium condition on the win set $[\underline{w}, \tilde{w}]$ holds. We must verify that policy choices in the dichotomous model are optimal. It suffices to show that the type $\overline{\theta}$ politician cannot gain by deviating to $x(\overline{\theta}) = 1$, as an analogous argument applies to the type $\underline{\theta}$ politician. Thus, we must show

$$\frac{u_{\overline{\theta}}(\tilde{w})}{1 - \delta} \geq u_{\overline{\theta}}(1) + \delta \tilde{V}_{\overline{\theta}}^C, \tag{27}$$

where the left-hand side of the inequality is the discounted payoff from compromising to $\overline{w}$, and the right-hand side is the payoff from shirking. As in the proof of Proposition 1, let $\Phi_R^\pi(\theta_m)$ be lobby group R's payoff from the optimal winning policy minus the payoff from the optimal losing policy, given politician type θ_m. Since equilibria are strongly partisan for γ sufficiently large, $\Phi_R^\pi(\theta_m)$ takes the simple form

$$\frac{1}{1 - \delta} \left[u_{\overline{\theta}}(\overline{w}) + \frac{1}{\gamma} u_m(\overline{w}) - \frac{1}{\gamma} u_m(x_m) \right],$$

minus

$$u_{\overline{\theta}}(x_R(\theta_m)) + \delta V_{\overline{\theta}}^{C,\pi}(P) + \frac{1}{\gamma} \left(u_\theta(x_R(\theta_m)) + \delta V_m^{C,\pi}(\Psi) \right) - \frac{1}{\gamma} \left(\frac{u_m(x_m)}{1 - \delta} + \frac{\delta \beta}{1 - \delta} \right),$$

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and moreover $\Phi_R^\pi(\theta_m) \geq 0$. Taking the limit as $\gamma \to \infty$ and using $x_R(\theta_m) \to 0$, we conclude that

$$\frac{u_{\overline{\theta}}(\tilde{w})}{1-\delta} - u_{\overline{\theta}}(1) - \delta \tilde{V}_{\overline{\theta}}^C \;=\; \lim_{\gamma \to \infty} \Phi_R^\pi(\theta_m) \;\geq\; 0,$$

which delivers (27), as required.

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